Global Navigation Satellite Systems and Applications

Prof. Arun K. Saraf
Department of Earth Sciences
Indian Institute of Technology, Roorkee

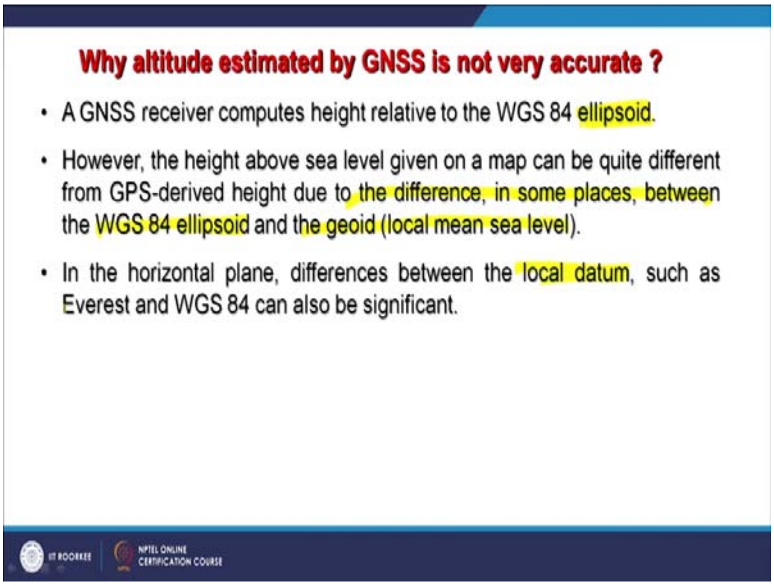
Lecture – 16

Why altitude estimated by GNSS receivers is not very accurate

Hello everyone and welcome to 16th lecture of Global Navigation Satellite Systems and Application course. And in this one, we are going to discuss a very important topic, very pertinent to the estimations of position and elevation. Basically it is related with the elevation the z value and that is why I have kept the topic that why altitude estimation by these GNSS receivers is not very accurate.

So, x, y position we have been discussing errors and everything but what about the z position that is elevation above mean sea level. This is what generally we think in that way.

(Refer Slide Time: 01:18)



Why altitude estimated by GNSS is not very accurate ?

- A GNSS receiver computes height relative to the WGS 84 ellipsoid.
- However, the height above sea level given on a map can be quite different from GPS-derived height due to the difference, in some places, between the WGS 84 ellipsoid and the geoid (local mean sea level).
- In the horizontal plane, differences between the local datum, such as Everest and WGS 84 can also be significant.

So, as you know that most of the GNSS receivers, they computes the height relative to WGS 84 ellipsoid or we can say above the ellipsoid. The key point here is that the measure the height or computes the height above the ellipsoid. An ellipsoid is not a real surface, it’s basically a mathematical surface and when you compare with the topographic surface or a toposheets and then there are differences between heights which has been estimated by GNSS receiver and the real height which is mentioned in the toposheets or topographic maps.

So, however the height above sea level given on a map can be quite different from GPS-derived height due to the difference in some places between the WGS 84 ellipsoid and the geoid; that is a local mean sea level and geoid is a gravity service that we will be also seeing in detail. So, now we are talking about three surfaces; one is ellipsoid or spheroid, another one is the geoid and third one is topographic surface. And, all three are different. One is based on completely theoretical model, another one is based on the gravity surface though currently it is relatively course resolution and third one is a topographic surface.

So in horizontal plane, the difference between the local datum such as Everest and WGS 84 can also be significant. The point here is that like if we take the older series of survey of India toposheets which were made using Everest spheroid, not WGS 84 spheroid and when we get these position estimations from these GNSS receivers for example, from GPS then they estimates the height using this WGS 84.

So, if I compare the height which I am getting through my GNSS receiver which is using GPS or some other signals and using that is WGS 84 so, when that height I compare with the height which for the same location given in this topographic map then there can be a difference. And, the difference is because the different spheroids in toposheets have been used that is the Everest spheroid instead of WGS 84.

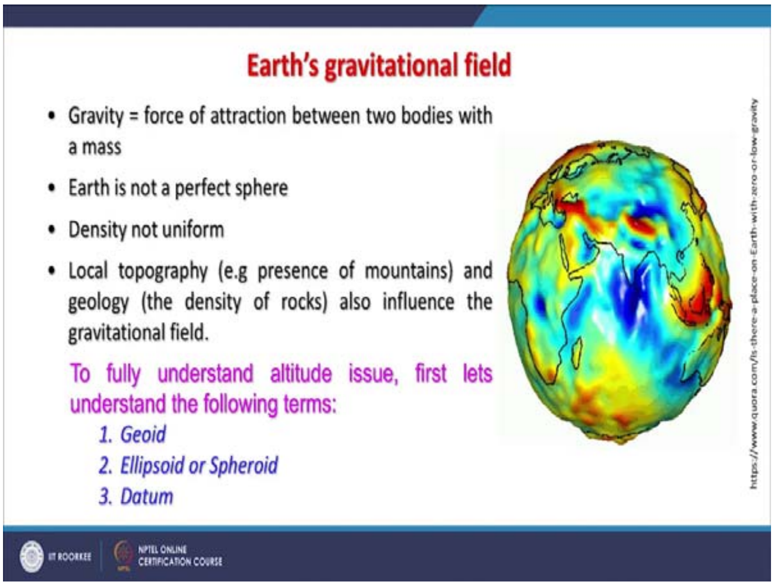
So, in order to make things compatible now like in India, survey of India has developed a another series of topographical maps or toposheets which instead of Everest spheroid, they have started using WGS 84 spheroid because these two spheroids are different. So, if you are getting data through one spheroid which is WGS 84 and compare with the data which has been produced through a different spheroid which is Everest then, obviously there will be differences. So, in order to avoid these differences, survey of India has decided to use WGS 84.

One more issue will come about the map projection because from 3D; the earth body is a 3D body and map is a 2-dimensional. So, when you project a 3D body into a 2-dimensional map, you use some certain projections. Each country is having their own projection in order to display or map their country in a true shape and size because each country is located uniquely on the globe and there cannot be one universal projection which is applicable to all countries.

So, therefore each country had developed their projection systems. So, India; in older toposheets, the India was using this marketer polyconic projection whereas in new series of toposheets now UTM which is Universal Transverse Mercator projection is being used which is an equal area projection in the equator and the middle latitudes but if we go for higher latitude though it is universal and by name it is universal but in say for polar regions, it distorts the continent like Antarctica very badly but for India and subcontinent region, this UTM can also works.

So, new series of toposheets from survey of India now are having WGS 84 spheroid and map projection is UTM. So that the advantage with this that these GNSS receivers which by default setting, they are using WGS 84 spheroid and also they are using UTM projection. And therefore, the position estimates based on these receivers can be now compared with the new series of toposheets and there should be then less differences between these two.

(Refer Slide Time: 06:50)



Earth's gravitational field

- Gravity = force of attraction between two bodies with a mass
- Earth is not a perfect sphere
- Density not uniform
- Local topography (e.g presence of mountains) and geology (the density of rocks) also influence the gravitational field.

To fully understand altitude issue, first lets understand the following terms:

1. *Geoid*
2. *Ellipsoid or Spheroid*
3. *Datum*

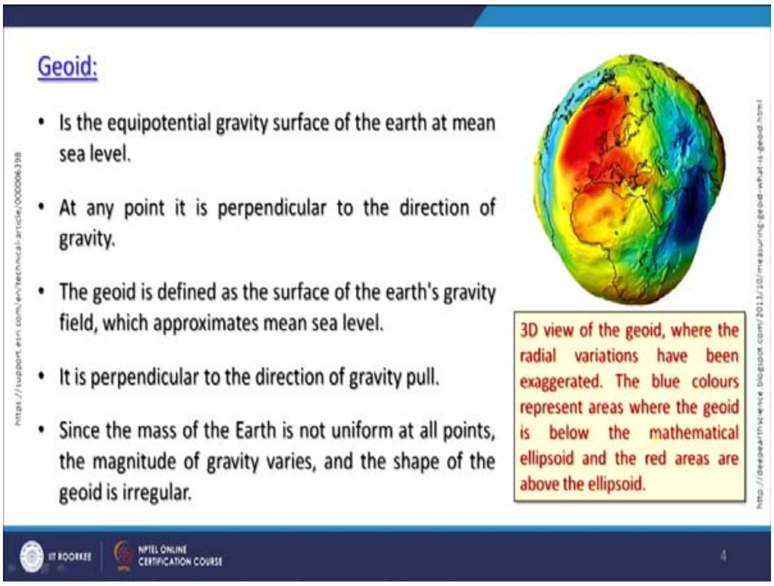
https://www.quora.com/Is-there-a-place-on-Earth-with-zero-or-low-gravity

Now, when we go for say geoid; geoid is gravities surface basically and gravity is a force of attraction between two bodies with a mass and the earth is not a perfect sphere as we all know about this. And, the density point of view which has been understood through gravity surface that the density is also not uniform because depending on the rocks basically and topography. And topography is also influenced by the rocks.

So, the local topography that is presence of mountains and geology that is lithology that is density of rocks also influences the gravitational field; like in Himalayan conditions, you are seeing altogether different gravity than compared to the coastal part. And, similarly there are desert areas and other things where you are seeing completely different topography and of course, different gravity fields. So, to fully understand this altitude or elevation values issues, first let us try to understand this 3-4 terms which we are going to use.

First is geoid, another one is ellipsoid or also called spheroid. So, both terms are used and then datum.

(Refer Slide Time: 08:07)



Geoid:

- Is the equipotential gravity surface of the earth at mean sea level.
- At any point it is perpendicular to the direction of gravity.
- The geoid is defined as the surface of the earth's gravity field, which approximates mean sea level.
- It is perpendicular to the direction of gravity pull.
- Since the mass of the Earth is not uniform at all points, the magnitude of gravity varies, and the shape of the geoid is irregular.

3D view of the geoid, where the radial variations have been exaggerated. The blue colours represent areas where the geoid is below the mathematical ellipsoid and the red areas are above the ellipsoid.

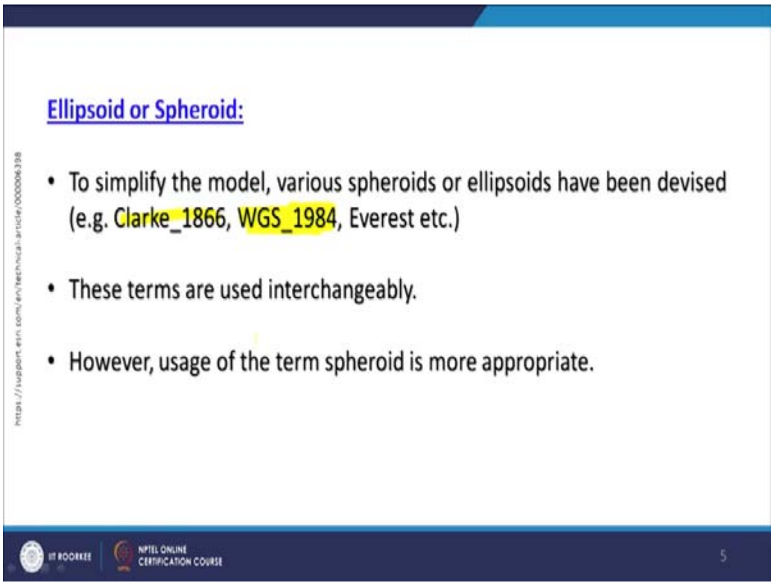
https://support.esri.com/en/technical-article/000006398

http://deepearthscience.blogspot.com/2013/10/measuring-geoid-what-is-geoid.html

So, first is this geoid. A geoid as mentioned earlier very briefly that it is an equipotential gravity surface of the earth at mean sea level and at any point, it is perpendicular to the direction of gravity. So, in that way it is measured. And, the geoid is defined as the surface of earth’s gravity field which approximate mean sea level. Now, again is perpendicular to the direction of gravity pull. And, since the mass of the earth is not uniform at all locations, at all points throughout the earth, the magnitude of the gravity also varies. And, thus therefore this shape of the geoid is very irregular and that is one reason.

Another one is that this gravity information, we are not having currently at very high resolution. So, slowly-2 things are getting improved and once we get a reasonably high resolution geoid body like this is shown here, then we may get a better position estimation especially related with the height; particularly about the height. So, this is basically 3D view of the geoid where the radial variations have been exaggerated just to understand better. The blue colour represents areas where geoid is below the mathematical ellipsoid. So, this ellipsoid has also been used and red areas are showing that is above the ellipsoid.

(Refer Slide Time: 09:43)



Ellipsoid or Spheroid:

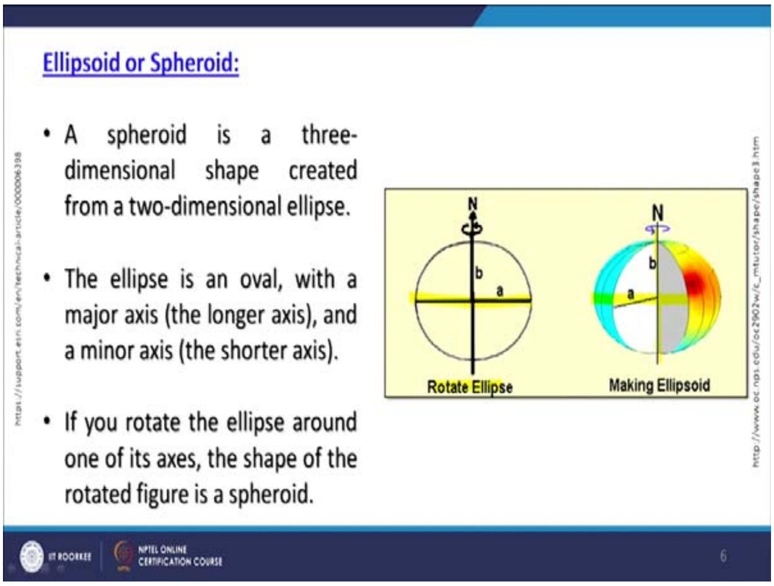
- To simplify the model, various spheroids or ellipsoids have been devised (e.g. Clarke_1866, WGS_1984, Everest etc.)
- These terms are used interchangeably.
- However, usage of the term spheroid is more appropriate.

https://support.esri.com/en/technical-article/000006398

Now, the second surface which we use to represent this 3D body of the earth is ellipsoid or spheroid which is say simply a mathematical model and various spheroids or ellipsoids have been devised. We have already discussed two spheroids like WGS 84 and Everest. There are many, for example also the Clarke_1866. The year that is mentioned here that there were a conference in 1984 and there they have decided the parameters which are being used in this WGS and therefore, that year has been attached with the naming of that spheroid.

So, this is world geodetic spheroid 1984. Similarly, for Clarke is 1866. And since very long times, things have not changed in Clarke but most common spheroid which is used in the all GNSS systems and many topographical maps is definitely WGS 84. And these terms, ellipsoid or spheroids are interchangeably used. However, usage of term spheroid is more appropriate in current discussion.

(Refer Slide Time: 11:07)



Ellipsoid or Spheroid:

- A spheroid is a three-dimensional shape created from a two-dimensional ellipse.
- The ellipse is an oval, with a major axis (the longer axis), and a minor axis (the shorter axis).
- If you rotate the ellipse around one of its axes, the shape of the rotated figure is a spheroid.

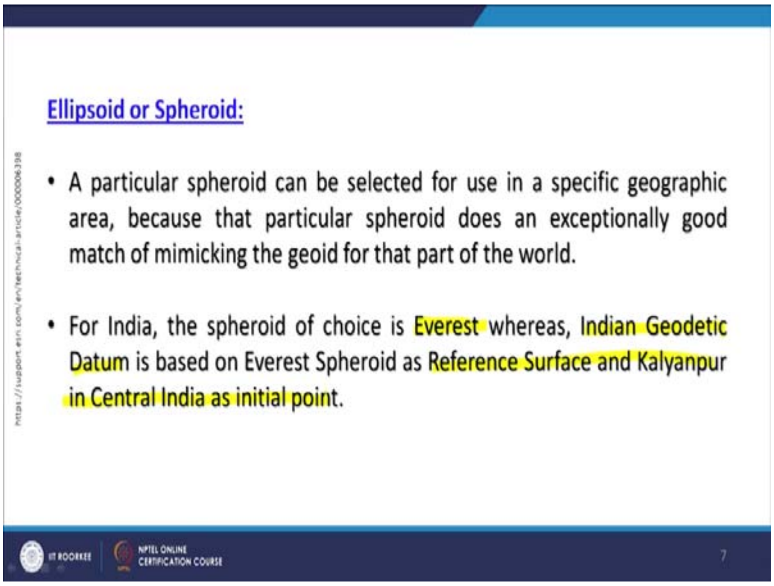
https://support.esri.com/en/technical-article/000006398

http://www.oc.nps.edu/oc2902w/c_mtutor/shape/shape3.htm

So, spheroid basically a 3-dimensional shape created from a 2-dimensional ellipse and this is what that when you rotate a ellipse along this vertical axis then ellipsoid can be made. And, this ellipsoid an oval with a major axis which is the longer axis here or here and a minor axis which is shorter which is basically representing roughly north and south.

And, if you rotate the ellipse around one of its axis, the shape of the rotated figure become spheroid and this is what spheroid which we are seeing.

(Refer Slide Time: 11:51)



Ellipsoid or Spheroid:

- A particular spheroid can be selected for use in a specific geographic area, because that particular spheroid does an exceptionally good match of mimicking the geoid for that part of the world.
- For India, the spheroid of choice is Everest whereas, Indian Geodetic Datum is based on Everest Spheroid as Reference Surface and Kalyanpur in Central India as initial point.

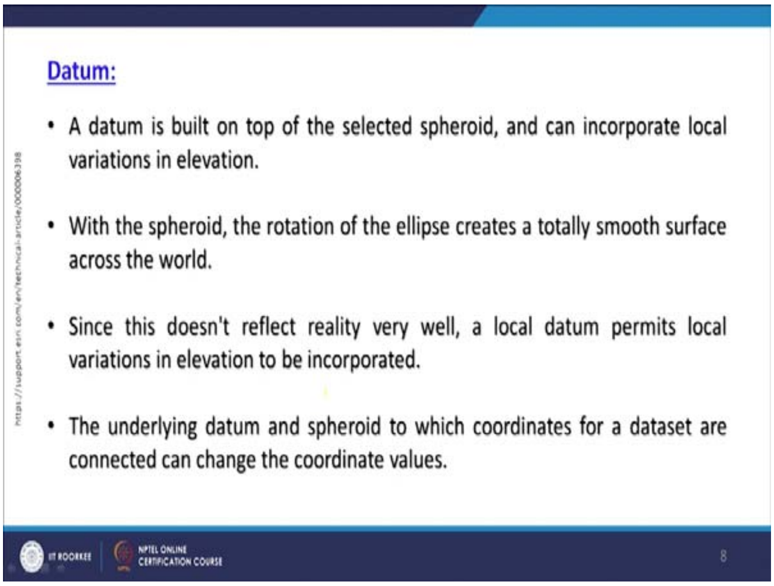
https://support.esri.com/en/technical-article/000006398

So, a particular spheroid can be selected for use in specific geographic area and that is why there are various spheroids also exist because that particular spheroid does not exceptionally good match for mimicking the geoid for that part of the world. Because as we have seen in case of geoid that different parts of the world are having different gravity and therefore the surface is very uneven.

Similarly, when we try to use one single spheroid for all geographic area then again there is a problem. So, that is why people evolve their own because ultimate aim in the map scenario case was to represent the political boundary of their country in its true shape and size because if shape changes then size will also changed and lot of disputes will then be there. Anyway, that is a completely separate discussion.

In India specifically, the spheroid earlier which was Everest whereas a Indian geodetic system was being used and the Reference Surface and Kalyanpur in Central India as initial point for this spheroid to make for rotation purpose.

(Refer Slide Time: 13:14)



Datum:

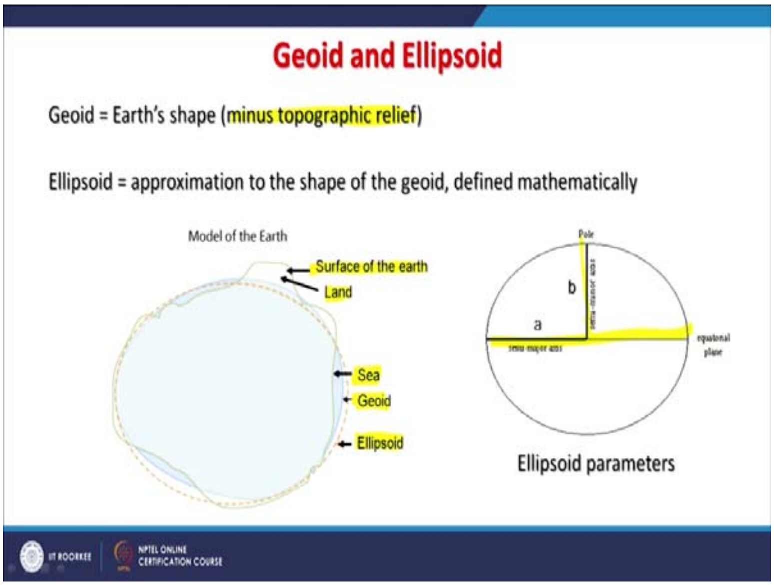
- A datum is built on top of the selected spheroid, and can incorporate local variations in elevation.
- With the spheroid, the rotation of the ellipse creates a totally smooth surface across the world.
- Since this doesn't reflect reality very well, a local datum permits local variations in elevation to be incorporated.
- The underlying datum and spheroid to which coordinates for a dataset are connected can change the coordinate values.

https://support.esri.com/en/technical-article/000006398

Now datum is built on top of the selected spheroid and can incorporate local variations in elevation and with the spheroid, the rotation of the ellipse which creates a totally smooth surface across the world. Since this does not reflect the reality very well that is how the earth surface looks in real term, a local datum permits local variations in elevation to be incorporated. And, therefore the datum is also used. So, underlying datum and spheroid to which coordinates of a datasets are connected can change the coordinate values.

So, if we compare using a different datum, again there will be a problem.

(Refer Slide Time: 14:05)



Geoid and Ellipsoid

Geoid = Earth's shape (minus topographic relief)

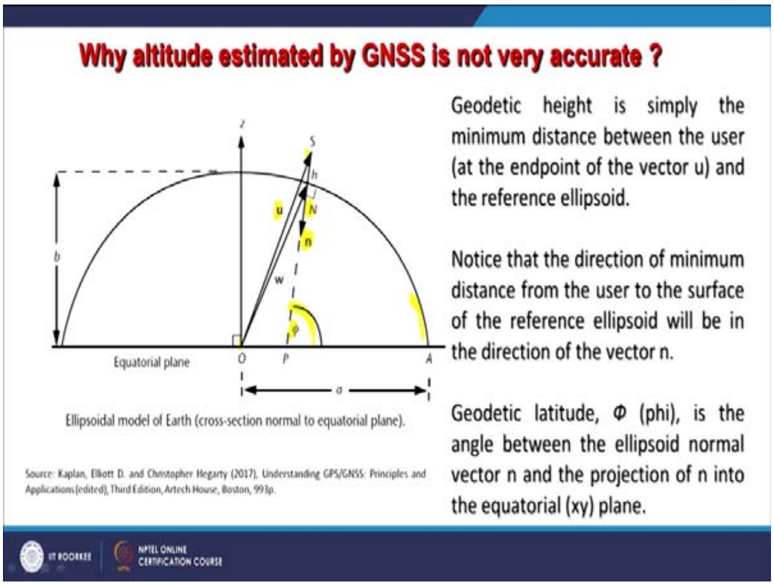
Ellipsoid = approximation to the shape of the geoid, defined mathematically

Ellipsoid parameters

So, this geoid is an earth shape minus topographic relief. So, you say because this topographic relief which is available in land part that is minus. Now, these three surfaces which we are seeing here; the geoid is here which is a gravity surface, ellipsoid is here which is a mathematical surface, this is the sea surface, this is of course, the schematic is there and surface of the earth which is above mean sea level and this part is land.

So, this is a sort of model of the earth which is having all these geoid, ellipsoid, sea, land and ultimately topographic surface of the earth. So, ellipsoid basically in a brief approximation to the shape of geoid defined mathematically. And we have already seen that a semi major axis is there, semi minor axis is there when you rotate along this minor axis or major axis you end up with the spheroid.

(Refer Slide Time: 15:10)



Why altitude estimated by GNSS is not very accurate ?

Ellipsoidal model of Earth (cross-section normal to equatorial plane).

Source: Kaplan, Elliott D. and Christopher Hegarty (2017), Understanding GPS/GNSS: Principles and Applications (edited), Third Edition, Artech House, Boston, 993p.

Geodetic height is simply the minimum distance between the user (at the endpoint of the vector u) and the reference ellipsoid.

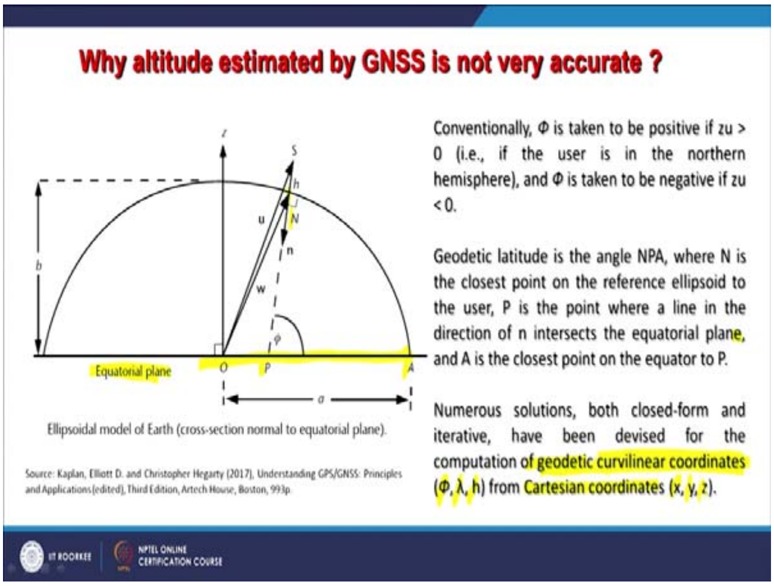
Notice that the direction of minimum distance from the user to the surface of the reference ellipsoid will be in the direction of the vector n.

Geodetic latitude, Φ (phi), is the angle between the ellipsoid normal vector n and the projection of n into the equatorial (xy) plane.

So, we continue on this one that the geodetic height is simply the minimum distance between the users at the end point of vector u which is this one. So, this is vector u starting from here and ending from here. And reference ellipsoid which is here shown that this is reference ellipsoid.

Now, the direction of minimum distance from the user to the surface of the reference ellipsoid will be in the direction of vector n which is this smaller one. And, the geodetic latitude that is phi which is given here, the angle between A, P and N or h that is a latitude, the phi; the angle between ellipsoid normal vector that normal vector n which is this one and the projection of n into a equatorial xy plane. So, when it is projected that angle becomes phi here.

(Refer Slide Time: 16:24)



Why altitude estimated by GNSS is not very accurate ?

Ellipsoidal model of Earth (cross-section normal to equatorial plane).

Source: Kaplan, Elliott D. and Christopher Hegarty (2017), Understanding GPS/GNSS: Principles and Applications (edited), Third Edition, Artech House, Boston, 993p.

Conventionally, Φ is taken to be positive if zu > 0 (i.e., if the user is in the northern hemisphere), and Φ is taken to be negative if zu < 0.

Geodetic latitude is the angle NPA, where N is the closest point on the reference ellipsoid to the user, P is the point where a line in the direction of n intersects the equatorial plane, and A is the closest point on the equator to P.

Numerous solutions, both closed-form and iterative, have been devised for the computation of geodetic curvilinear coordinates (Φ, λ, h) from Cartesian coordinates (x, y, z).

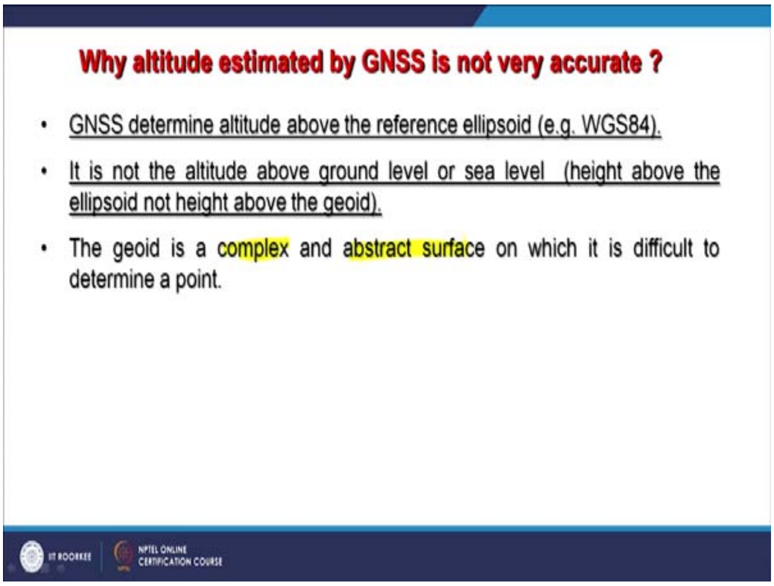
Now, conventionally phi is taken to be positive if a zu is greater than 0. So that means that when it is greater than 0, we take it positive and phi is taken to be negative if zu is less than 0. So, basically when this angle makes the difference between in different scenarios. And one has to remember whether one is in the northern hemisphere or southern hemisphere. So, for example, in case of northern hemisphere, this is the scenario, in southern hemisphere the scenario can be different.

Now, geodetic latitude is the angle which is NPA. So, P is here, N is here and that is the phi where N is the closest point on the reference ellipsoid here and P is the point where a line in the direction of N intersects with the equatorial plane which is this one and A is the closest point on the equator of P. So, this is what it is equatorial plane and this is the angle phi which is there.

So, there are various solutions, both from closed form and iterative are available which have been devised for the computation of geodetic curvilinear coordinates. The issue is here that earth is not flat; it is a 3D body and not a perfect spheroid. So because of these, the complications arises and therefore, curvilinear coordinates we get, which are geodetic curvilinear coordinate that is phi, lambda and height that is z value which we try to convert into a Cartesian coordinate system which is a 2-dimensional system using x, y and z that is latitude, longitude and altitude or elevation above mean sea level.

So, from 3D to 2D, this is how things can be achieved.

(Refer Slide Time: 18:59)



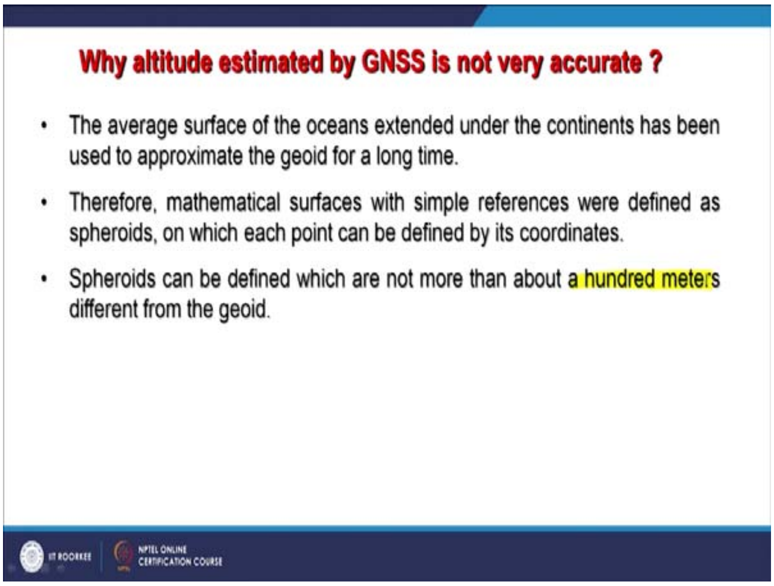
Why altitude estimated by GNSS is not very accurate ?

- GNSS determine altitude above the reference ellipsoid (e.g. WGS84).
- It is not the altitude above ground level or sea level (height above the ellipsoid not height above the geoid).
- The geoid is a complex and abstract surface on which it is difficult to determine a point.

Now further, to determine the altitude above the reference ellipsoid for example using WGS 84, it is not the basically altitude above ground level or sea level which is measured. But this is the height above the ellipsoid not the height of the geoid is measured. And that is why there is issue limitation of getting elevation value from GNSS. Limitation in that sense that the elevation values or z values which are coming through GNSS are not as accurate as supposed to be and they are also not as accurate as x, y values.

There is lot of improvement as we have seen in earlier discussions in case of x, y positioning but the z positioning, still lot of work need to be done. So, the geoid as we have understood is a very complex and basically abstract surface because it is model surface on which is difficult to determine a point.

(Refer Slide Time: 19:57)



Why altitude estimated by GNSS is not very accurate ?

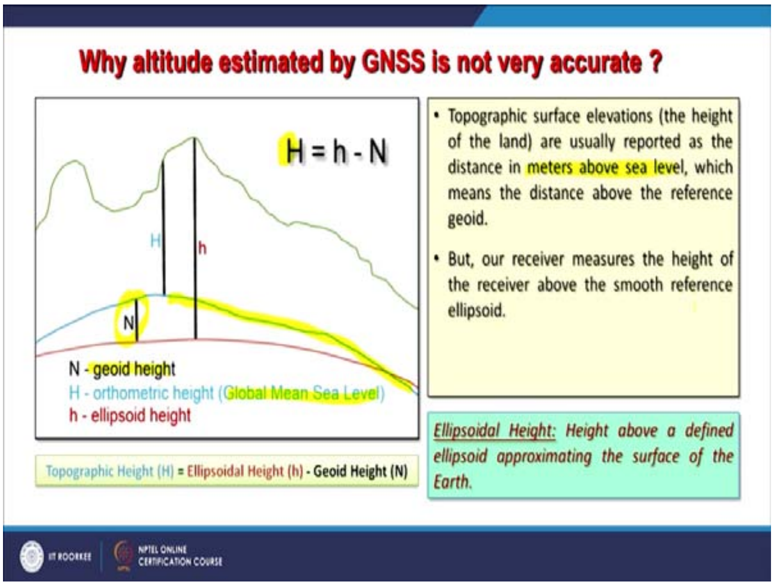
- The average surface of the oceans extended under the continents has been used to approximate the geoid for a long time.
- Therefore, mathematical surfaces with simple references were defined as spheroids, on which each point can be defined by its coordinates.
- Spheroids can be defined which are not more than about a hundred meters different from the geoid.

IIT ROORKEE | NPTEL ONLINE CERTIFICATION COURSE

And the average surface of oceans extend under the continents has been used to approximate the geoid for very long time. Now, therefore, mathematical surfaces with simple references were defined as spheroid on which each point can be defined by it is coordinates. So, spheroid can be defined which are not more than about a hundred meters different from the geoid. But, hundred meters is a big difference in terms of position estimation or height estimations through a GNSS receiver.

Generally, we do not get that that big error but still, the height or elevation values or z values which we get from GNSS is still not reliable. Why it is not reliable, we will try to understand further.

(Refer Slide Time: 20:52)



Why altitude estimated by GNSS is not very accurate ?

N - geoid height
H - orthometric height (Global Mean Sea Level)
h - ellipsoid height

Topographic Height (H) = Ellipsoidal Height (h) - Geoid Height (N)

- Topographic surface elevations (the height of the land) are usually reported as the distance in meters above sea level, which means the distance above the reference geoid.
- But, our receiver measures the height of the receiver above the smooth reference ellipsoid.

Ellipsoidal Height: Height above a defined ellipsoid approximating the surface of the Earth.

IIT ROORKEE | NPTEL ONLINE CERTIFICATION COURSE

There are three surfaces here shown is; the red one is the ellipsoid height and this is a section and then you are having height H that is in blue is the orthometric height that is vertically upward from the global mean sea level. So, that is there in the blue and then you are having green that is topographic surface on which we are having receiver or position ourselves and N is the geoid height.

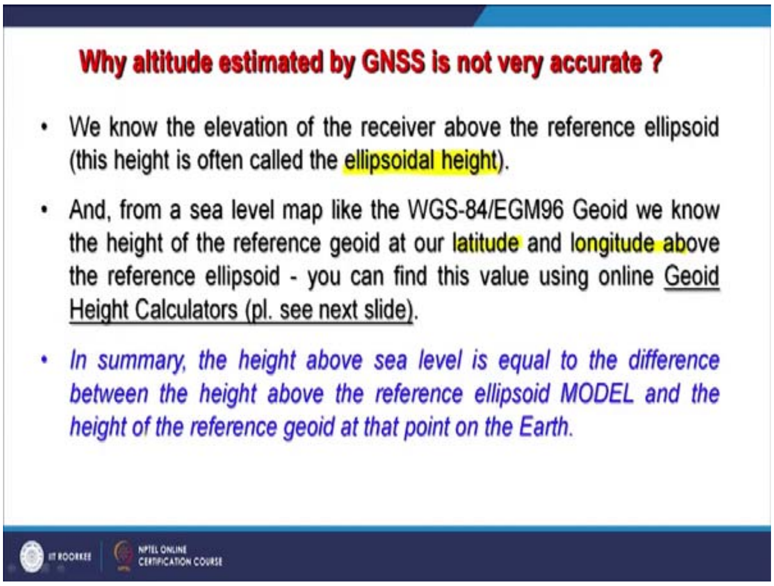
So, what we basically would like to have the H; that is H equal to this total height from the ellipsoid height which is a mathematical surface to a topographic height minus this height which is the geoid height. So, this is what we estimate. So, topographic height if we want to calculate then we have to have an ellipsoidal height that is small h and then geoid height that is N here. So, if we subtract from small h to minus N that is ellipsoid height minus geoid height then we get the topographic height.

So, ellipsoid height which is basically height above defined ellipsoid approximating the surface of the Earth. Now, this is again as mentioned that this is an ellipsoidal height; ellipsoid itself or spheroid itself is a mathematical surface. So, the local variations cannot be incorporated and therefore, that ellipsoidal height which we were here cannot vary as per the topography as you can also see here that the red curve is not varying as topography is varying or topographic surface.

So, that what we basically are looking the height over this topographic surface. So, topographic surface elevation that is height of the land above mean sea level are usually reported as distance in metres above mean sea level; basically the above mean sea level, this is the term which we use which means the distance above the reference geoid and this blue one is the geoid which is reference geoid is there and but our receiver measures the height of the receiver above the smooth reference surface.

So, what our system should have been measuring; the height above the geoid but they are measuring height above the spheroid because this is what we have discussed in the beginning that the height which is GNSS receivers are giving above say world WGS 84, not this gravity surfaces. But slowly-2, these are being modeled and definitely there are lots of improvements when we started in 90s these GPS systems. And now things have definitely improved but more improvements are required towards the elevation values.

(Refer Slide Time: 24:21)



Why altitude estimated by GNSS is not very accurate ?

- We know the elevation of the receiver above the reference ellipsoid (this height is often called the ellipsoidal height).
- And, from a sea level map like the WGS-84/EGM96 Geoid we know the height of the reference geoid at our latitude and longitude above the reference ellipsoid - you can find this value using online Geoid Height Calculators (pl. see next slide).
- *In summary, the height above sea level is equal to the difference between the height above the reference ellipsoid MODEL and the height of the reference geoid at that point on the Earth.*

IIT ROORKEE | NPTEL ONLINE CERTIFICATION COURSE

Now we know that the elevation of the receiver above the reference ellipsoid; this height is often called the ellipsoidal height rather than height above mean sea level. And, from a sea level map like WGS or EGM96 geoid, we know the height of the reference ellipsoid at our latitude and longitude above the reference ellipsoid. So, latitude, longitude values nowadays, we are getting very accurate but z value is a problem. So, you can find this value by using online geoid height calculations.

So, if you are getting a height value or elevation value from a receiver, one can employee a calculator which calculates height instead of spheroid, it will calculate the height above geoid and what are the inputs are required you basically provide the latitude, longitude.

So, latitude here, you provide the longitude here and then GPS elevations whatever you get from your GNSS receiver and when you submit, you get the height above geoid rather than height above spheroid.

But, this is again because a formula is fixed. So, here the local variations of geoid surface or geoid body which we have seen earlier are not incorporated. And if they are incorporated but they are not in that accuracy form also so, this is a problem here.

So in summary, the height above sea level is equal to the difference between the height above the reference ellipsoid model and the height of the reference geoid at that point on the earth surface. So, this is what the scenario. So, this brings to the end of this discussion about that why elevation values generally are not as accurate as x, y values are. We have discussed the errors associated with x, y values in length. Very quickly in this particular discussion, we have also discussed the issues related with the z values and as usual I am leaving with a cartoon for you to smile.

Thank you very much.