

**INDIAN INSTITUTE OF TECHNOLOGY ROORKEE**

**NPTEL**

**NPTEL ONLINE CERTIFICATION COURSE**

**Digital Image Processing of  
Remote Sensing Data**

**Lecture – 13  
Frequency Domain Fourier Transformation**

**Dr. Arun K. Saraf  
Department of Earth Sciences  
Indian Institute of Technology Roorkee**

Hello everyone and welcome to the 13<sup>th</sup> lecture of digital image processing of remote sensing data course and in this particular discussion we are going to discuss frequency domain Fourier transformation. In the previous lecture we have discussed about the spatial domain how to use spatial domain concept and we can design different kinds of special filters, convolution filters and can improve our images as per our requirements, So similar things can also be done but instead of exploiting the spatial domain.

We can exploit the frequency domain during Fourier transformation and we can also achieve quite good results. So and this discussion we are going to focus on this, we know that the wave which is shown here.

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## Fourier Series and Transform

Jean-Baptiste Fourier (1768) mentioned that *"any periodic function can be expressed as a weighted sum of sines and/or cosines (Fourier series)"*.

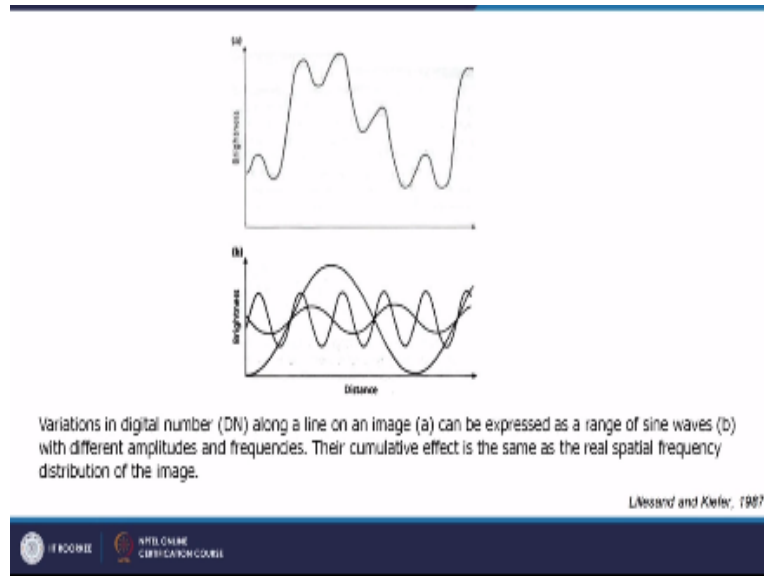


The function at the bottom is the sum of the four Functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

And can be segmented in two different ways, so this wave the lower one is the sum of all these waves and this is what basically it was in 1768 by Jean Baptiste Fourier, he mentioned that any periodic function can be expressed as a weighted sum and this is what the weighted sum of sines and/or cosines and that we call as furiously.

So this is the fundamental was laid down in 1768 and they at the nest function, at the bottom and which I have just mentioned is basically, is the sum of four functions about, which you can see here of different wavelength. So Fourier idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism did not appreciate at that time.

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But later on now in digital image processing, it has been adopted though we are here, we are mainly thinking in a linear fashion but there also for remote sensing data which is a two-dimensional still we can imply this thing, which we will see and very soon. So the same example is given here that, this is the combined sum of all these wave lengths and waves which are shown here.

So the variations and digital numbers along the line on this image a can be express in range sign wave which is shown here in the lower figure of different amplitudes and frequencies. So the different amplitude and frequencies when you sum you get this thing so that cumulative effect is the same as the real special frequency distribution of the image.

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- Other functions can be expressed as an integral of sines and/or cosines multiplied by a weighing function (Fourier Transform)
  - Functions can be recovered by the inverse operation with no loss of information
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Now other functions can also be expressed as an integral of sines or cosines multiplied by a weighing function that is Fourier transforms and function can be recovered by inverse operations there is no loss of information it. This means basically that what we do through from a special domain, we transform into frequency domain with the filtering there and once the filtering is done, we reverse the transformation or inverse transformation Fourier transformation and we restore the image with better results or with our filtered image.

So this is a two steps process, first is they or rather three steps process, first is that is from a special transformation from a special domain to frequency domain, do the filtering within the frequency domain and then come back into the special domain and they, of course the main aim which of entire this course is to enhance the quality of images, so that we can do the better interpretations. So any image or natural scene and can be regarded as and reconstructed.

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- Any image or natural scene can be regarded as, and reconstructed from a 'spectrum' of sine waves with different directions, wavelengths and amplitudes.
  - Such a reconstruction is known as a Fourier synthesis, and the disassembly of an image into a family of sine waves is a Fourier analysis.
  - The spots, lines and bars that form the targets for tests of acuity, and form images too, are themselves made up of sine waves.
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From a spectrum of sine waves with different directions wavelength and amplitude, few examples we have seen and such a reconstruction is known as Fourier sines and disassembling of an image into family of sine waves for Fourier analysis, because the filtering once you transform from spatial to frequency domain, then filtering becomes much easier and you can select where things have to be filtered out and later on you transfer back or inverse transformation.

So it sports lines and bars that forms the targets for the test of acuity and form images too are themselves made up of sine waves and the small dots and clearly spaced lines are dominated by high amplitude at high frequency and this is why they are difficult to recognize.

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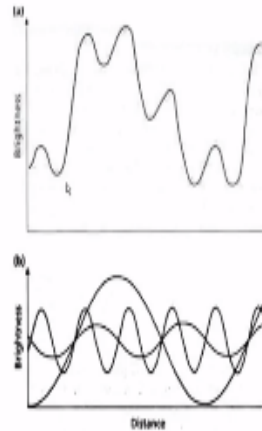
- Small dots and closely spaced lines are dominated by high amplitudes at high frequency. This is why they are difficult to recognize.
- As size increases, high frequency components become lower in amplitude and those in the frequency range to which the eye is attuned increase.
- Perception and distinction become easier.
- Beyond this, however, very low frequency components increase in amplitude; effectively degrading perception and distinction.

So as size increases of these dots high frequency components become lower in amplitude and those frequency range to which the eye is attuned increase and perception and distinction become easier of these digital images and beyond this however very low frequency components increase in amplitude effectively liberated perception and distinctions.

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- A mathematical technique for separating an image into its various frequency components is Fourier analysis.

- The basic principle is illustrated in left figure for a single dimension.



Variations in digital number (DN) along a line on an image (a) can be expressed as a range of sine waves (b) with different amplitudes and frequencies. Their cumulative effect is the same as the real spatial frequency distribution of the image.

So mathematically the technique for separating an image into various frequency component is basically the Fourier analysis and there are the basic principle is illustrated in this figure, as also we have seen this figure, that the we are and you know this a cumulative image which is sum of all these images and so that we can take out different frequency components and later on once, we remove that suppose noise is coming and from only one frequency or one amplitude a particular frequency wave.

So we can remove that one and then were combine and get a new results filtered results, so the Fourier transformation of an image is a two dimensional, so things have to be little different the result of such as separation expresses the spatial attributes of an image in terms of their frequencies amplitude and their orientation.

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- The Fourier transform of an image, the result of such a separation, expresses the spatial attributes of an image in terms of their frequencies, amplitudes and their orientation.
  - It is a transformation that enables certain groups of frequencies and directions to be emphasized or suppressed by algorithms known as filters.
  - Those that emphasize high frequencies and suppress low frequencies are high-pass filters. Similarly there are medium- and low-pass filters.
  - Moreover, selected ranges of spatial frequencies can be removed or retained in the resulting image, using band-stop and band-pass filters.
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And it is a transformation that enables certain groups of frequencies and direction to be emphasized, same as in and the ultimate aim is to filter the image made certain enhancement in the means, so same as in special filtering technique we want we try to emphasize some features may be regional features or local features and we want to be emphasized certain, so similar things we will do but different way instead of a special domain we will work in frequency domain.

So both that emphasize high frequencies and suppresses low frequencies are high-pass filter similarly in the spatial domain also you have seen high-pass filters and low-pass filter, similarly there are medium and low-pass filters in frequency domain to move over and selected any ranges of special frequencies, can be removed or retained in the resulting image using band stop and band pass filters.

Basically the concept of this filtering techniques have come from electronics there in like an audio or video some noise is coming you want to remove. So you identify and that frequency and then remove that frequency wave from the signal. So the process is analogous to electronic filtering as I have just mentioned in amplifiers to reduce his and Rumble they enhance the bass or treble and so on in a sound recording and filtering.

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- The process is analogous to the electronic filtering in amplifiers to reduce hiss and rumble, enhance the bass or treble and so on in a sound recording.
- Filtering can be implemented through the Fourier transform, when it is said to operate in the frequency domain, or in the spatial domain of the image itself by a process known as convolution.
- Frequency-domain filtering is more powerful, but is also the more expensive of computer time, involves highly complex mathematics and the result of a transform is not easily visualized in terms of the image itself. Most image processing systems routinely use convolution filters, with the option of frequency-domain filtering for special purposes.



So the process is analogous to electronic filtering as I have just mentioned in amplifiers to reduce hiss and rumble they enhance the bass or treble and so on in a sound recording and filtering can be implemented through the Fourier transform when it is said to operate in frequency domain or any spatial domain of image cell processors convolution, convolution filtering we have already discussed and in frequency domain filtering is more powerful.

Because you are thus you know that which are the features are causing noise in your scene which are the features which are causing distortions in the scene so the identification of those things is much easier in frequency domain than in spatial domain that is why it is more powerful compared to spatial domain but the processing is more required better understanding of images and further processes is required therefore people generally prefer spatial and domain filtering the other than frequency domain.

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- Instead of using spatial domain, i.e. the  $(x, y)$  coordinate space of images, an alternative coordinate space that can be used for image analysis is the frequency domain.
- In this approach, an image is separated into its various spatial frequency components through application of a mathematical operation known as the **Fourier Transform**.

But it is more powerful than even so instead of using a spatial domain that is the X&Y coordinate space of an image and alternate coordinate system that can be used for image analysis that is the frequency domain and in this approach an image as mentioned earlier also is separated into various spatial frequency components through application of mathematical operation that is known as Fourier transform.

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- Conceptually, this operation amounts to fitting a continuous function through the discrete DN values if they were plotted along each row and column in the image.
- The “peaks and valleys” along any given row or column can be described mathematically by a combination of sine and cosine waves with various amplitudes, frequencies and phases.

And conceptually this operation amounts to fitting a continuous function through the discrete digital number values if they are plotted along each row and column in the image and the peaks and valleys along any given row or column can be described mathematically by a combination of sine and cosine waves with various amplitudes frequencies and faces.

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- The **frequency domain** refers to the plane of the two dimensional discrete Fourier transform of an image.
- The purpose of the Fourier transform is to represent a signal as a linear combination of sinusoidal signals of various frequencies.

And the frequency domain field refers to the plane of two dimensional discrete Fourier transform of an image and the purpose of the Fourier transform is to represent a signal as a linear combination of sinusoidal signals of various frequencies.

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## Introduction to the Fourier Transform and the Frequency Domain

- The one-dimensional Fourier transform and its inverse

- Fourier transform (continuous case)

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx \quad \text{where } j = \sqrt{-1}$$

- Inverse Fourier transform:

$$e^{j\theta} = \cos \theta + j \sin \theta$$

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} du$$

- The two-dimensional Fourier transform and its inverse

- Fourier transform (continuous case)

$$F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j2\pi(ux+vy)} dx dy$$

- Inverse Fourier transform:

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux+vy)} du dv$$



So this is how mathematically that there are different options are available like one dimensional Fourier transform and it is inverse equation both are given here similarly two dimensional Fourier transform and it is inverse here it is also continuous case it is also continuous case even later we will see the speed case so this is one dimensional Fourier transform this is two dimensional Fourier transform equations are given.

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## Introduction to the Fourier Transform and the Frequency Domain

- The one-dimensional Fourier transform and its inverse

- Fourier transform (discrete case) DTC

$$F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M} \quad \text{for } u = 0, 1, 2, \dots, M-1$$

- Inverse Fourier transform:

$$f(x) = \sum_{u=0}^{M-1} F(u) e^{j2\pi ux/M} \quad \text{for } x = 0, 1, 2, \dots, M-1$$



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Here this is one-dimensional and Fourier transform but discrete case not continuous case and both equations Fourier transform equation and inverse depletion so there are various options are available here depending on requirement of your filtering for the image so after an image is separated into a component spatial frequencies.

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- After an image is separated into its component spatial frequencies, it is possible to display these values in a two-dimensional scatter plot known as a Fourier spectrum.
- The lower frequencies in the scene are plotted at the centre of the spectrum and progressively higher frequencies are plotted outward.
- Features trending horizontally in the original image result in vertical components in the Fourier spectrum; features aligned vertically in the original image result in horizontal components in the Fourier spectrum.

It is possible to display these values in a two-dimensional scatter plot known as a Fourier spectrum. This Fourier spectrum basically will allow us to design a filter and then they move the mice or whatever features which are which you want to de-emphasize so lower frequencies in scene are plotted at the center of a spectrum. This is important and to remember that the lower frequencies in the scene are plotted at the center of spectrum and progressively higher frequencies are plotted outward.

The in which this is in a two-dimensional scatter plot or a in a career spectrum and this features trending or gently in horizontally in the original image result in vertical components say and rotation of the components here or X is here that the originally when they are trending in the oriental they will appear in this Fourier spectrum along with the vertical component and features vice versa is also true that features a line vertically no real an image result in original old oriental components in the Fourier spectrum.

So there is a rotation by 90 degree this one has to remember while interpreting the Fourier spectrum so if the Fourier spectrum of an image is known it is possible to regenerate the original image through the application of an inverse Fourier transform so for one-dimensional and Fourier transforms both inverse and forward we have all seen for two dimensional continuous and discrete.

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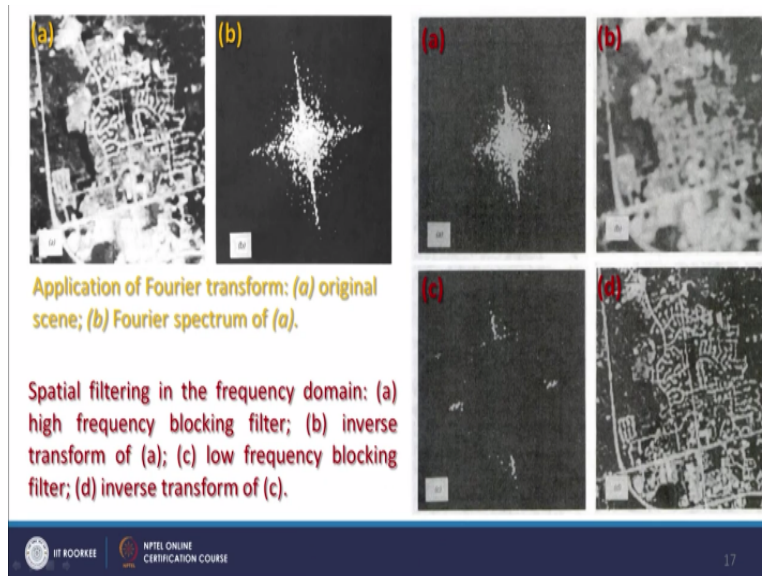
- If the Fourier spectrum of an image is known, it is possible to regenerate the original image through the application of an inverse Fourier transform.
- This operation is simply the mathematical reversal of the Fourier transform.
- Hence, the Fourier spectrum of an image can be used to assist in a number of image processing operations.
- For example, spatial filtering can be accomplished by applying a filter directly on the Fourier spectrum and then performing an inverse transform.



And this operation is simply the mathematical reversal of Fourier transform and the Fourier transform of an image can be used to assist in a number of image processing operations and example like a spatial filtering can we increase by applying the filter directly on the Fourier spectrum and then performing a inverse transform inverse transform has to be performed that is the last step in flow.

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Now example this is an image and this is the Fourier spectrum and the edge mentioned that the command the components or features which are horizontal here or along the XY they will appear here and the features which are in vertical directions they are appearing here and you also mentioned earlier that the low-frequency features as mentioned here that the new frequencies in scene are plotted at the center of the spectrum.

So the low-frequency are plotted at the center here and high frequencies are at the periphery or outside away from the center so this is how the Fourier spectrum is interpreted now filtering if you want to filter now we know which, which direction so it becomes very easy now so if we want to filter say the features which are roughly say north south not north south and south, south, south, south east direction then we will remove DZ from the Fourier spectrum.

And once we go for reverse inverse transformation then these features a directional features will be an ad emphasized so the features which are here if I remove from here in the Fourier spectrum then the resultant image will not show them what they will be do emphasize the examples are here given again that this is the original image this is the Fourier spectrum.


After filtering and that then inverse transformation this is the result so the high frequency and low frequency features which are in the centre has been removed and only high frequency features here have been kept which are away from the center and therefore in the resultant image you are seeing this kind of very high frequency features a spatial filtering in the frequency domain which is the a high frequency blocking filter which is given here and in the end B inverse

transform image and low frequency blocking filter an inverse transformation so this frequency filtering operation is carried out.

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The frequency filtering operation is carried out as per the following steps:

1. The original image is transformed to its frequency representation using Fourier transform.
2. Image processing – selecting an appropriate filter transfer function and multiplying it to the elements of the Fourier spectrum.
3. The inverse Fourier transform is performed to return to the spatial domain for display purposes.



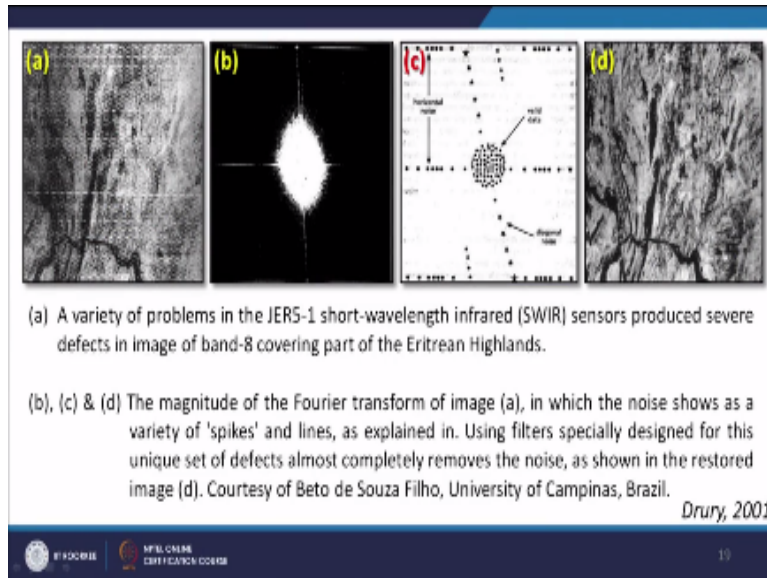
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As per the following steps the original image is transformed to its frequency domain representation using Fourier transform then image processing selecting an appropriate filter transform functions and multiplying it to the elements of Fourier spectrum the inverse spectrum and that in inverse Fourier transform is performed to return to the spatial domain for display purposes.

So you restore to the image after filtering and this is what the insert it this is what the inverse Fourier transform few more examples are given and like here this is the image there is the Fourier spectrum you want to remove certain frequencies you want to emphasize certain frequencies you can do it as in this example is done.

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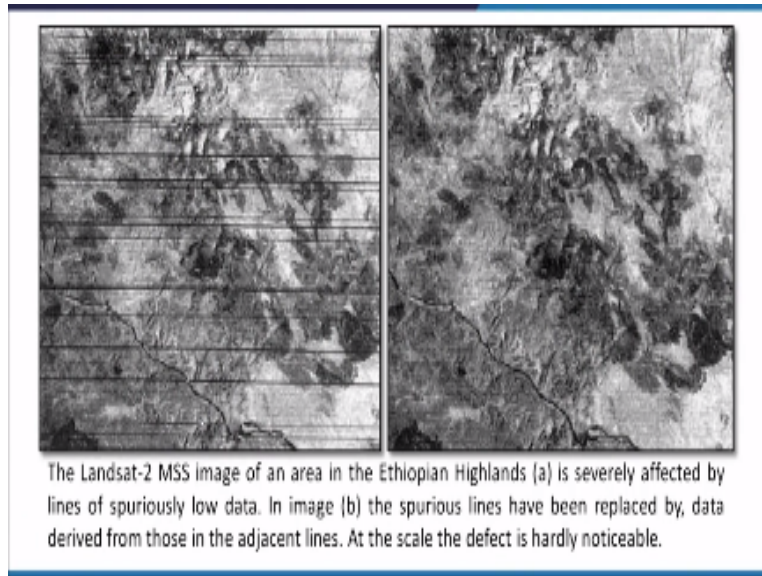


And then you know this high frequency features have been removed from the perimeter side and like under than only low frequency features, so reasonable features have been emphasized whereas high frequency features have been or local speckle kind of thing have been removed and the high frequency features have been de-emphasized and you see a very smooth image.

So this image for interpretation is noted that good because it is suffering from lot of distortion lot of errors, like these almost where missing lines or horizontal lines are there. They will appear here in the Fourier transform as vertical lines, so if I remove this then I am removing basically these lines and similarly if I remove the outside in this is a Fourier scatter plot and away from the center then I have removed the high frequency features and this is what you see here.

So on a different this is the example of JER 5 and 51 shortwave infrared images and on which is the band it covering the tree and highland so this is one example of how beautifully an image with Fourier transformation and filtering can be improved for image. Another example from Land sat MMS in Ethiopian Highlands this is severally which is similarly affected by spurious blow data.

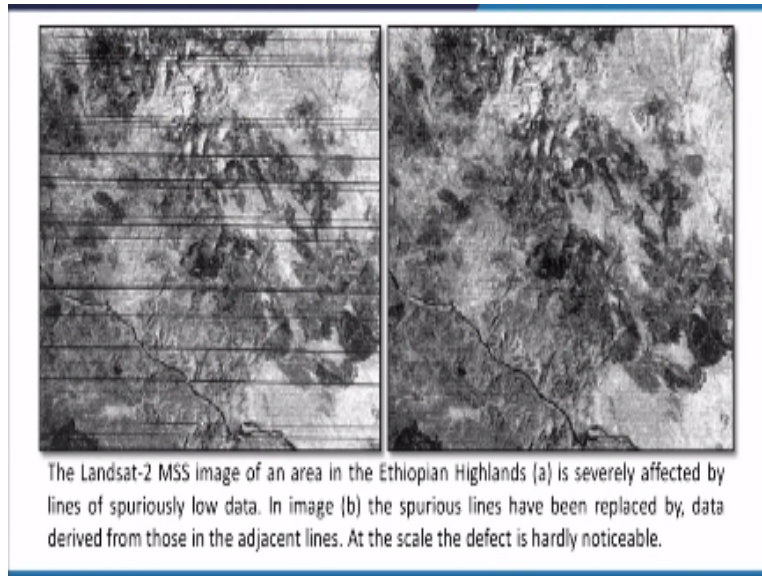
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And this and the Fourier transform if you spectrum if you look you would find vertical lines kind of thing you can remove enduring filtering process go for inverse Fourier transform and this is what you see. So all lines have been removed here you get very good results, whereas in case of especial filtering what we do we take out the average of adjacent pixels?

Which are not missing and then we take an average and substitute value for missing pixels, then there is basically sort of cosmetic and not a pro scientific method in that way and if there are many lines are missing then special techniques will not work. But here in Fourier transformation it is very easy to remove.

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Such distortions or noises which are present in the data and this is a very common feature of remote sensing data, many times because of some malfunctioning of sensors or CCDs some transmission problems you may get certain lines, which need to be removed in order to restore the original quality of an image, so that may be the image of particular date or particular area maybe very important and therefore and the technique is available the best suitable technique for here is the Fourier transformation. So this is methods involves an identifying frequency it is just a recap of the thing.

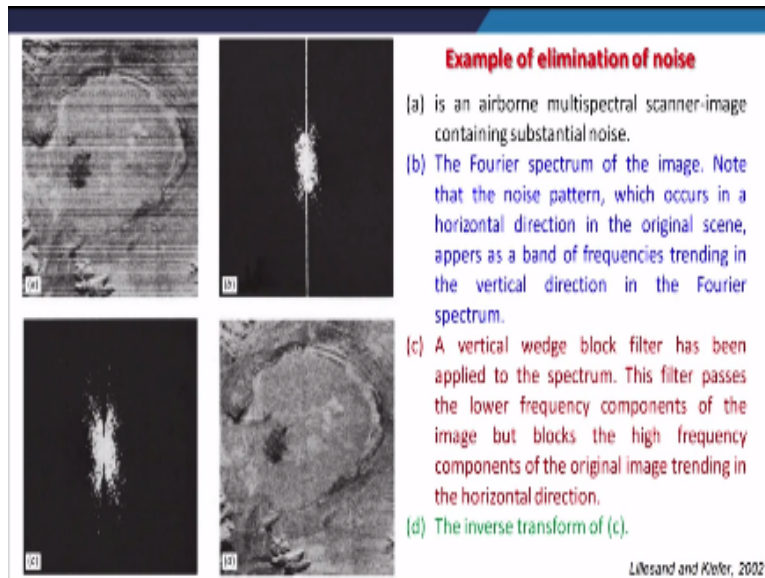
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- Methods involves on identifying the frequency and amplitude of striping and other all pervasive, systematic defects, which are superimposed on the real variation, by analysing a Fourier transform of the image.
- The periodic noise can then be identified as a signal and suppressed by filtering in the frequency domain.
- The inverse transform to the spatial domain then restores the cleaned up image.
- In the frequency domain the noise is isolated as distinct signals, and almost wholly removed by specially designed filters that operate in the frequency domain.

Identifying the frequency and amplitude of striping, and other person a person pervasive systematic effects, systematic effects distortion defects can be removed very easily, super impose on the real variation by analyzing a Fourier transform of the image and the periodic noise can then be identified as a signal and suppose by filtering in frequency domain. Again go back for inverse transform to special domain then these stores a cleaned up image and in frequency domain that noises isolated as a distinct signals and almost wholly remove by is basically designed filters that operate the frequency domain.

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If you more like one example here horizontal lines are there in as mentioned earlier that in Fourier spectrum they will appear very strongly in form of vertical lines, so once these vertical lines are removed inverse transformation is performed, this is what you see. So this it is very hard to interpret this image and there if you make interpretations you may make some wrong interpretations, the wrong decisions about the features which are present but if you filter it through transformation function.

Then you may restore in very good quality, so this is a) is airborne multispectral scanner image, containing substantial noise as you can see, b) the Fourier spectrum of the image that the noise pattern which occurs in a horizontal direction in the image will appear vertical in a will appear as a vertical direction in the Fourier spectrum. Now this c) is as I mentioned that the vertical wedge block and filters that has been applied to that this block filter has been applied and this filter passes the lower frequency components of the image but blows the high frequency because the lines are appearing as high frequency components of the original image and this is inverse transform of C.

So the image receive installation is the very good quality of this analysis, so Fourier analysis is useful in a host of image processing operations in addition to spatial filtering and image restoration applications.

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- Fourier analysis is useful in a host of image processing operations in addition to the spatial filtering and image restoration applications.
- However, most image processing is currently implemented in the spatial domain because of the number and complexity of computations required in the frequency domain.

However most image processing it is currently implemented in the spatial domain also earlier I mentioned that many people prefer and because, the fine is easier to understand the spatial domain but if they adopt and go for a Fourier transformation functions and reach and remove them wise is high frequency features another then it is equally good or much better more powerful technique is definitely Fourier transform, because it is most number of complexity or computation required in frequency domain.

Mainly digital image processing software which is not having advanced features may not have even frequency domain filtering techniques or this Fourier transformation. So this situation is likely to change as we improve in the computer hardware and software and more research may be required and to make these things easier. So this brings to the end of this filtering techniques in pervious lecture we have discussed especial filtering and in this one we have discussed the frequency filtering techniques both are very useful, very powerful but frequency domain filtering for certain line, missing line higher frequency features it is much more powerful.

But it requires more computation time and better understanding and may be all digital images processing software may not support, whereas very standard normal lowed digital image processing software will definitely support special domain filtering techniques. So and if one is having both in their systems then both should be tried only image and see that and do the comparison and see that which one is getting better results. I am sure you would get definitely better results that they will apply for Fourier transformation concepts so thank you very much.



**For Further Details Contact**  
**Coordinator, Educational Technology Cell**  
**Indian Institute of Technology Roorkee**  
**Roorkee – 247 667**  
**E Mail: [etcell,iitrke@gmail](mailto:etcell,iitrke@gmail.com), [etcell@iitr](mailto:etcell@iitr.ac.in), [ernet,in](mailto:ernet@iitr.ac.in)**  
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**Online Editing**

Jithin. K

**Graphics**

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