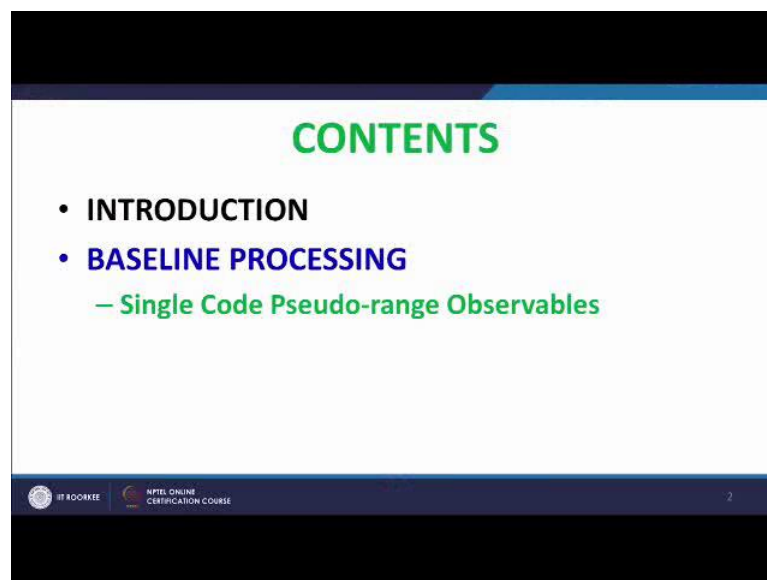


**GPS Surveying**  
**Dr. Jayanta Kumar Ghosh**  
**Department of Civil Engineering**  
**Indian Institute of Technology, Roorkee**

**Lecture – 14**  
**GPS Data Processing- II (Baseline Processing)**

Welcome friends. Today I am going to discuss the 14th class on GPS surveying. In today's class I would like to take up further on GPS data processing and under GPS data processing we will be talking on baseline processing.

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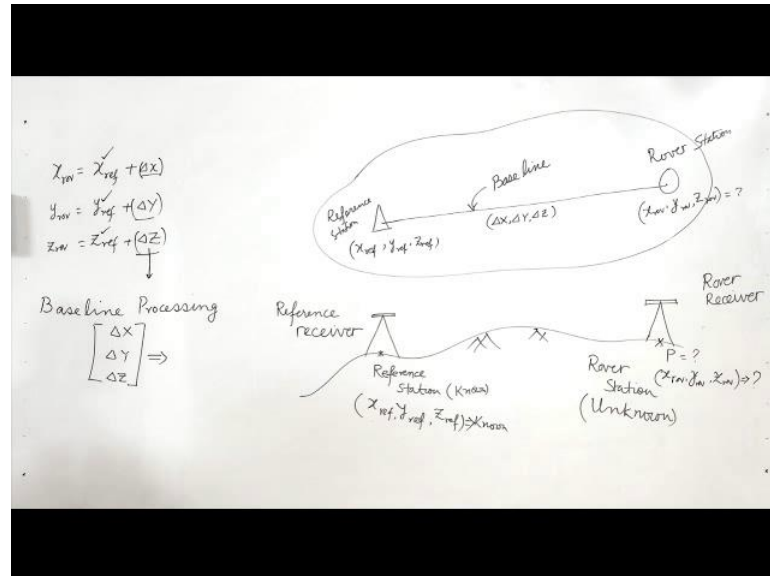


Now, this class we will like to first discuss on the introduction; that means, I would like to say about baseline that is processing and finally, I will like to let you understand how the baseline processing is being carried out by taking a single coat pseudo range GPS observable from which we do end up with some parameters which is required to find out the baseline.

Now, in case of GPS surveying baseline is the most fundamental unit or the primary unit that is considered in GPS surveying. So, it is a very important thing to know and understand about this processing how you arrive at the baseline parameters. Now whenever we go for relative mood of GPS surveying we do end up with some baseline.

Now, as you remember in now in my previous or some other classes I have told you that in case of relative positioning we take a location which is known as reference station.

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That means, reference station means the location of these points is known. So, at this reference station we set up an instrument GPS instrument and so it is called reference receiver. And corresponding, another receiver we take suppose this is the point which position we need to find out, so this position is considered as rover station and a receiver is placed on this rover station and in this case we call this receiver as rover receiver and this is a station whose position is unknown. So, known if we say X or Y ref reference and Z ref are the location of these station; that means, these parameters are known and if we say X rover, Y rover and Z rover are the co-ordinates of the point p which is the rover station these are not known. So, we have to find out.

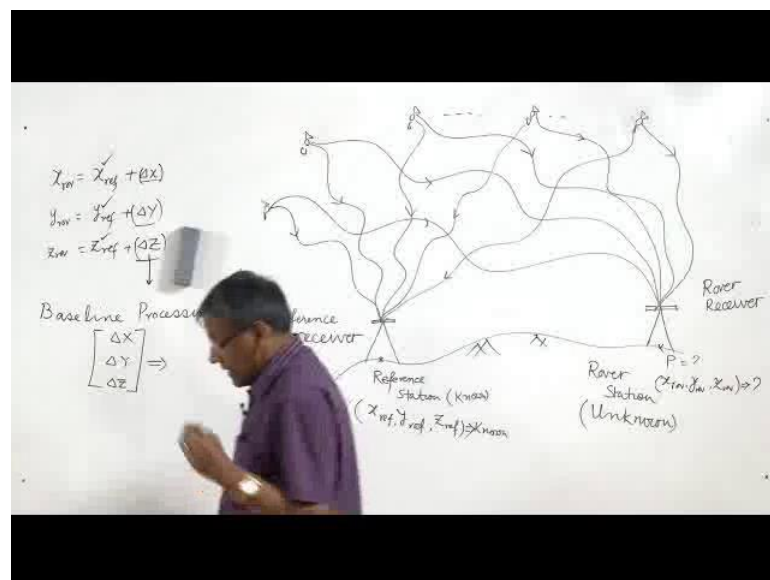
Now, in through baseline processing actually we do. So, if we see this is the elevation if we say it in plane metric view. So, corresponding to this point this is the station and corresponding to this one, this is the station. So, this is the reference station and this is the rover station and a line joining these two stations this is called baseline. So, we have to find out the parameter of this baseline. So, if we say that the location is ref like this r Y ref Z ref then X Y Z. So, we want to find out this.

Now, if the difference is given by del X, del Y, del Z, then this X rover will be X reference plus del X Y rover equal to Y reference plus del Y Z rover equal to Z reference

plus del Z. Now you can see here that these parameters are known, we want to find out these X Y Z. Now if we find know what is the del X del Y and del Z then you will be able to get X Y Z. So, these del X del Y del Z are the parameters which define the baseline. So, it is called baseline processing; that means, by finding out the parameters of the baseline we find out the location of the rover station. So, it is called baseline processing.

So, by baseline processing in baseline processing actually we do not find out del X del Y del Z. Now after finding out this we will be able to get the location of the rover station. Now what happens in the (Refer Time: 07:27) positioning.

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The signals from numbers of satellites like suppose these are the satellites there may be n numbers of satellites, n numbers of satellites. Suppose all these satellites are visible from this reference as well as this rover station; that means, signals from all these satellites are arriving to both reference and rover stations. So, it will be coming like this. So, signals are coming. So, there may be n numbers of satellites, similarly signals are coming to the like this from both n set of satellites from all over this n set of satellites signals are coming to both reference and rover receiver .

Now, as you know in these signals we have three parts of carrier phase observable and five types of pseudo phase observable. Now all these individual components we made be used to find out this baseline parameters. Either it can be used individually or it can be

used in multiple ways; that means, we may use a single coat or we may use multiple coat observables or we may use a single coat coupled with, carrier phase of observables may be one carrier phase observables or may be one carrier phase observables. How, for all processing as I told you in point positioning it is that we go for code single coat c a observables processing as the most fundamental and that is required.

So, depending upon the components of the signals that we will consider or depending upon the method of surveying like in this case suppose both are static condition. So, we call it as static surveying, it may be kinematic surveying, it may be stop and go as you have learned in earlier classes. So, whatever is the mode of surveying alternatively, fundamentally that we have two sets of observation. One set will come from the reference station and another set will be coming from rover station, and that set we will be considering only the C A code observables of the reference station signal and the C A code observables of the rover receiver signals and those observables will be used to find out the baseline parameters.

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The whiteboard contains the following handwritten equations:

$$\begin{aligned} \text{Known} \\ \boxed{PR_{ref}^i(t)} &= \rho_{ref}^i(t) + c \cdot \delta t_{ref} - c \cdot \delta t^i + I_{ref}^i(t) + T_{ref}^i(t) + dt_{ref}^i + dt^i + dt_{ion}^i + e_{ref}^i(t) \leftarrow \\ \boxed{PR_{rover}^i(t)} &= \rho_{rover}^i(t) + c \cdot \delta t_{rover} - c \cdot \delta t^i + I_{rover}^i(t) + T_{rover}^i(t) + dt_{rover}^i + dt^i + dt_{ion}^i + e_{rover}^i(t) \\ \boxed{PR_{ref}^i(t) - PR_{rover}^i(t)} &= \text{Known} \\ PR_{ref, rover}^i(t) &= \left( \rho_{ref}^i(t) - \rho_{rover}^i(t) \right) + c \left( \delta t_{ref} - \delta t_{rover} \right) + \left( I_{ref}^i(t) - I_{rover}^i(t) \right) \\ &\quad + \left( T_{ref}^i(t) - T_{rover}^i(t) \right) + \left( dt_{ref} - dt_{rover} \right) + \left( dt_{ion}^i - dt_{ion}^i \right) + e_{ref}^i(t) - e_{rover}^i(t) \end{aligned}$$

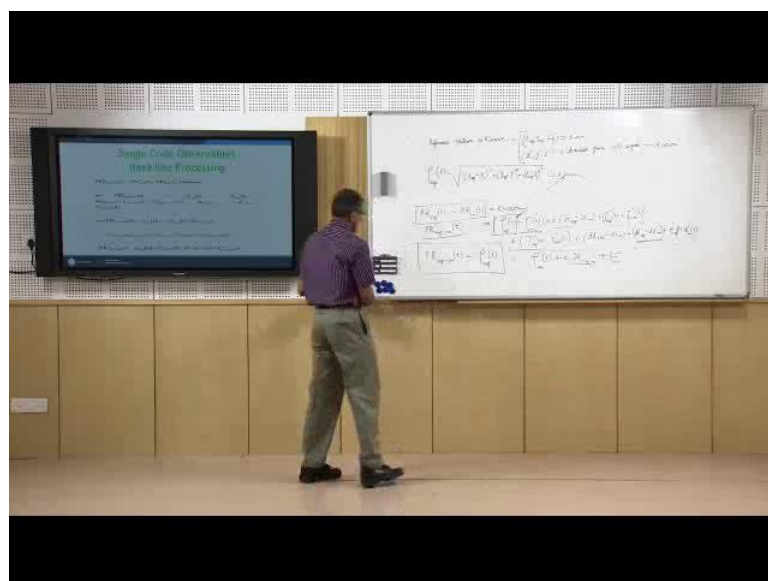
Now, as you know that the pseudo range observable from satellite i, any satellite suppose i to our reference receiver r e f at any epoch t can be given by the geometric range of the satellite from the reference receiver at the epoch t plus, the receiver clock error we have used the symbol del t reference then the satellite clock error. So, error; then the ionospheric error from satellite to reference receiver then, tropospheric error then

receiver hardware error then satellite hardware error then multipath error then the random error. So, all these constitute, so this is how we do defining the GPS observable this is the parameter which we get in the GPS signal, but this signal GPS observable actually consists of all these constituents.

Similarly, if we take the pseudo range of the rover receiver from the same satellite, but only to the rover receiver at the same popup time we will get similar expression. So, ionospheric error, tropospheric error, rover receiver hardware error, satellite error, then multipath error - now this is also known, these two are known, these are known. Now if we take the difference.

So,  $P$  that is what is our relative positioning, which is known because both unknowns, so it is known. So, will be equal to our difference in geometric range then you say into receiver and reference clock error then these two will cancel out. And the ionospheric error difference then tropospheric error difference between reference and rover then our difference of receiver reference and rover receiver hardware these two will cancel which are the satellite hardware error, then multipath error is also will be there and of course, that random error will be there. So, these errors will be minimized, but we do not know about this anyway. So, this is the known part. So, this can be written as pseudo range reference comma rover from the same satellites I had an epoch  $t$ . So, this is known.

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Now, as we know that the reference station is known means we know the value of X reference Y reference Z reference this is known, and since the satellite positions of the X i, Y i, Z i, this is also available from GPS signal. So, it is known. So, the both of these are known and as we know that the geometric range of the reference receiver from the satellite is nothing but square root of X reference minus X i whole square Y reference minus Y i whole square. So, this is known.

So, in this expression this part is known, this is also known and this is the thing which really we want to find out also this part is not known this is also to be determined. So, the expression may be now changed to PR reference rover i t minus reference i t, this part is known again is equal to minus of rover t plus c into clock error reference rover this is also known. And for others, for rest of these let us consider it an error term E. So, whole of our GPS baseline observations can be reduced to this expression, this is the expression we get.

Now, as we already told the X rover is equal to X reference plus del X Y rover is equal to Y reference plus del Y Z rover is equal to Z reference plus del Z where X del X del Y del Z are the baseline parameters.

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The image shows a whiteboard with handwritten mathematical equations. At the top, the coordinates of the rover are expressed in terms of reference station coordinates and baseline parameters:

$$\begin{aligned} X_{rov} &= X_{ref} + \Delta X \\ Y_{rov} &= Y_{ref} + \Delta Y \\ Z_{rov} &= Z_{ref} + \Delta Z \end{aligned}$$

Below this, the observation equation is derived. The left side is the difference in pseudorange between the rover and the reference station:

$$PR_{ref,rov}^i(t) - \rho_{ref}^i(t)$$

The right side is the geometric range minus the reference station clock error, plus an error term E:

$$= \underbrace{f_{rov}^i(\Delta X, \Delta Y, \Delta Z, \Delta t)}_{\rho_{rov}^i(t)} - c \cdot \Delta t_{ref,rov} + E^i$$

So, this can be expressed as this is known, this can be expressed as a function of rover with respect to del X del Y del Z. Both of these together can be expressed like del d t

suppose. So, our whole relation now can be considered as some known parameter this and a function like this and an error term this.

Now, if we remember in the last class we have seen in a single code GPS processing, data processing we also arrived at same expression pseudo range, this is known, this is known and this is the expression, a function based on the receiver position and the clock error receiver and an error.

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**Single Code Positioning.....**

$$PR_r^i(t) = \rho_r^i(t) + c\delta t_r + e_r^i(t)$$

$$PR_r^i(t) = f_r^i(x, y, z, \delta) + e_r^i(t)$$

$$f_r^i(x, y, z, \delta) = f_{computed}^i + \frac{\partial f_r^i}{\partial x} \Delta x + \frac{\partial f_r^i}{\partial y} \Delta y + \frac{\partial f_r^i}{\partial z} \Delta z + \frac{\partial f_r^i}{\partial \delta} \Delta \delta$$

$$f_{computed}^i = \sqrt{(x^i - x_0)^2 + (y^i - y_0)^2 + (z^i - z_0)^2} + c\delta_0 = \rho_0^i + c\delta_0$$

$$\frac{\partial f_r^i}{\partial x} = -\frac{(x^i - x_0)}{\rho_0^i} = -\frac{(x_0 - x^i)}{\rho_0^i}; \quad \frac{\partial f_r^i}{\partial y} = -\frac{(y^i - y_0)}{\rho_0^i} = -\frac{(y_0 - y^i)}{\rho_0^i};$$

$$\frac{\partial f_r^i}{\partial z} = -\frac{(z^i - z_0)}{\rho_0^i} = -\frac{(z_0 - z^i)}{\rho_0^i}; \quad \frac{\partial f_r^i}{\partial \delta} = c.$$

$$PR_r^i(t) = f_{computed}^i + \frac{\partial f_r^i}{\partial x} \Delta x + \frac{\partial f_r^i}{\partial y} \Delta y + \frac{\partial f_r^i}{\partial z} \Delta z + \frac{\partial f_r^i}{\partial \delta} \Delta \delta + e_r^i$$

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Now here is also, this is the relation. Similarly we can get it now here we can now this is the equivalent expression, but as it is non-linear, so also this is non-linear because our function is square root of this, this, and this. So, now, we have to convert this to a linear function, first we have to linearise it and we have to model developed by linear model then we solve it by least square method.

Now, if we want to solve this by least square analysis this non-linear model or this non-linear equation first has to be linearised then has to develop a linear model then we should go for least square analysis method, as we did in single code positioning. And we can do in the same way only in this case the parameters are del X del Y del Z which is the error between the assumed value of the receiver and the correct value of the receiver, but in this case this is the baseline parameter, and in this case del X del Y del Z we have to made it to zero, so iterative process, but in this case we have to find out the value of del X del Y del Z as the unknown parameters. So, this is the fundamental difference,

otherwise the mode of linearization and the development of the linear model is identical with that what we find in single code positioning.

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### Single Code Positioning.....



The difference between  $PR_r^i(t)$  and  $f_{computed}^i$ , called as residual observation ( $O^i$ ) can be computed as

$$O^i = PR_r^i(t) - f_{computed}^i$$

$$O^i = \frac{\partial f^i}{\partial x} \Delta x + \frac{\partial f^i}{\partial y} \Delta y + \frac{\partial f^i}{\partial z} \Delta z + \frac{\partial f^i}{\partial \delta_r} \Delta \delta_r + \epsilon_r^i$$

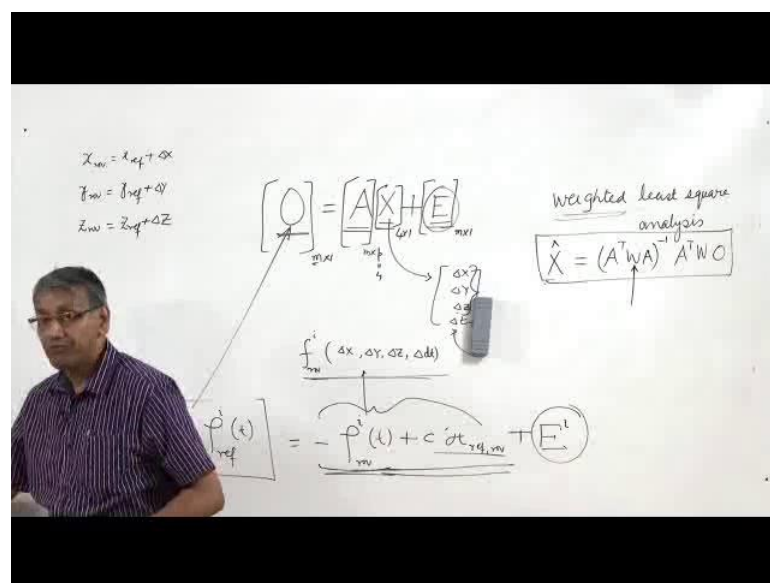
$$O^i = \begin{bmatrix} \frac{\partial f^i}{\partial x} & \frac{\partial f^i}{\partial y} & \frac{\partial f^i}{\partial z} & \frac{\partial f^i}{\partial \delta_r} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta \delta_r \end{bmatrix} + \epsilon_r^i$$

One such relation can be obtained for observation from each satellite.



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So, you can see here in the similar way we have to proceed in this case also. So, here also we will end up with ultimately an observable equation is equal to  $A X$  plus  $E$  where  $O$  is the residual observation,  $A$  is the design matrix,  $X$  is the unknown parameter matrix, and  $E$  is the error matrix.

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Weighted least square analysis

$$\hat{X} = (A^T W A)^{-1} A^T W O$$

$$\hat{p}(t) = -\hat{f}(t) + \hat{c}(t) + \hat{E}(t)$$



Now in this case the residual observation is this one and your design parameters will be that unknown parameters will be our  $\Delta X$ ,  $\Delta Y$ ,  $\Delta Z$ ,  $\Delta t$  where this is the baseline parameter, these three are the baseline parameters and it is the time parameter and this is the error. So, this is the thing what we did, we knew. In the last class I have explained all these thing.

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**Single Code Positioning.....**

At any epoch of observation, if a receiver receives signals from  $m$  SVs, then a system of  $m$  equations will be obtained and can be represented as

$$\begin{bmatrix} O^1 \\ O^2 \\ O^3 \\ \vdots \\ O^m \end{bmatrix} = \begin{bmatrix} \frac{\partial f^1}{\partial x} & \frac{\partial f^1}{\partial y} & \frac{\partial f^1}{\partial z} & \frac{\partial f^1}{\partial \delta_r} \\ \frac{\partial f^2}{\partial x} & \frac{\partial f^2}{\partial y} & \frac{\partial f^2}{\partial z} & \frac{\partial f^2}{\partial \delta_r} \\ \frac{\partial f^3}{\partial x} & \frac{\partial f^3}{\partial y} & \frac{\partial f^3}{\partial z} & \frac{\partial f^3}{\partial \delta_r} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f^m}{\partial x} & \frac{\partial f^m}{\partial y} & \frac{\partial f^m}{\partial z} & \frac{\partial f^m}{\partial \delta_r} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta \delta_r \end{bmatrix} + \begin{bmatrix} \epsilon_r^1 \\ \epsilon_r^2 \\ \epsilon_r^3 \\ \vdots \\ \epsilon_r^m \end{bmatrix}$$

The equation can be written in matrix symbol as  $O = AX + \epsilon$ . This is the linear observation model based on pseudo-range.

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So, now you can see here that this is the observable equation that we get or the linear model we have developed out of the observation equation. So, here the observable equation for a single satellite, now the signal coming from a single satellite - this is the observable equation linear model and for  $m$  satellites this is the relations we have derived in the last class. So, which can be converted to this - this is the in matrix notation and this is the linear model.

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**Single Code Positioning.....**

The linear observation model based on pseudo-range can be solved to determine the unknown parameters by applying normal equation method of least square solution. Let the solution of the unknown parameters be denoted by  $\hat{X}$ , which can be obtained as

$$\hat{X} = (A^T A)^{-1} A^T O$$

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And this linear model has been used to find out the unknown parameter X, the position of the receiver by using least square analysis normally equation method and we have end up with this.

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**Single Code Observables  
Base line Processing**

$$[PR_{ref,rov}^i(t) - \rho_{ref}^i(t)] = f_{rov}^i(\Delta X, \Delta Y, \Delta Z, \Delta \delta_i) + E_{ref,rov}^i(t)$$

$$O = AX + E$$

a linear relation where known parameters are included in the O matrix having dimension  $d \times 1$ , where  $d$  is the number of linearly independent data. The design matrix A has dimension  $d \times p$  where  $p$  is the number of parameters. The parameter corrections  $(\Delta X, \Delta Y, \Delta Z, \Delta \delta_i)$  are represented by X matrix of dimension  $p \times 1$ . The errors associated with observables and pre-processing are represented by the E matrix, which has the same dimension as O.

In case of baseline processing, using "weighted least squares" approach to account for the correlation in data, estimated parameters can be obtained using

$$\hat{X} = (A^T W A)^{-1} A^T W O$$

where, W is the data weight matrix, to be derived later on, and O is a vector containing the residual observations.

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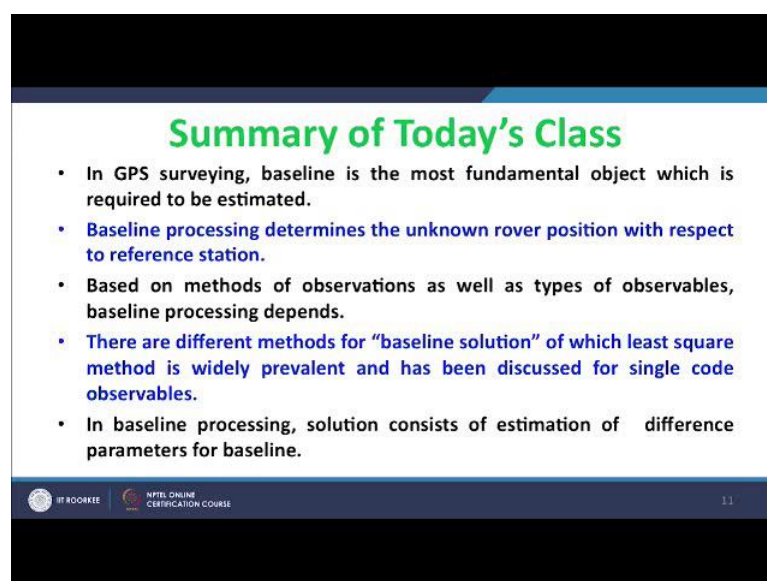
Similar to this, in case of baseline processing also we had ended up with this relation. So, this is the non-linear equation that is based on single code GPS observable for baseline processing. Now since this is a non-linear method. So, using the identical way what we did in the single code point positioning this non-linear model first has to be linearised

and then end up with a linear model like this. Only difference is that this  $X$  is the baseline parameters.

Now, this observation matrix that observable is that what the dimension  $m$  into  $1$  where  $m$  is the number of satellites from which observables are coming and  $A$  is the aim into  $p$  suppose,  $p$  is the number of unknown, which is  $4$  in this case. So, I can write it equal to  $4$  and  $X$  is again  $4$  into  $1$  and the  $E$  matrix is having the  $m$  into  $1$ ; that means, one error associated with each satellite observable.

In case of the baseline using another important difference from point position is that, in case of baseline processing we go for weighted least square analysis. Instead of simple we do give some weighted least square analysis; that means, we do provide some weight to the observables coming from satellites. So, observables from each satellite are not having the same quality, they will be having the different amount of errors. So, their weights will be different, the satellites having less error, observable will be more weighted and vice versa. So, by using the weighted least square analysis we do get the unknown parameter matrix is equal to  $A^T W A^{-1} A^T W O$ . So, this is the final output that we will get in case of baseline processing. So, where  $W$  is the weight matrix and it depends upon the quality of the observable that is being received from the GPS satellite.

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The slide features a white background with a blue header and footer. The title 'Summary of Today's Class' is in green. The main content is a bulleted list of five points in blue text. The footer contains logos for IIT Roorkee and NPTEL Online Certification Course, along with the page number 11.

### Summary of Today's Class

- In GPS surveying, baseline is the most fundamental object which is required to be estimated.
- Baseline processing determines the unknown rover position with respect to reference station.
- Based on methods of observations as well as types of observables, baseline processing depends.
- There are different methods for "baseline solution" of which least square method is widely prevalent and has been discussed for single code observables.
- In baseline processing, solution consists of estimation of difference parameters for baseline.

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With this I want to conclude today's class, but before that I would like to summarize today's class. Baseline is the most fundamental or most primary parameter in GPS processing. So, it needs to be taken into account with importance, with high importance. Now whenever we go for relative positioning, we do end up with some baselines and depending upon the purpose for which the relative positioning is being done. The quality of baseline has to be looked into. We do have baselines of different types, depending upon the methods of surveying or depending upon the type of observables that we will take for processing the baseline. There are many methods which may be used to process the baseline of which the least square analysis method is the most widely used.

So, however in this class we had discussed about how to process the GPS data to end up with the solution for baseline parameter using single code GPS observables, and at the end of this baseline processing we end up with a parameters which defines the different components of a baseline which further may be added to the original reference coordinate to find out the location of the rover receiver.

Thank you, hopefully we will meet you next class. Again, in the next class I am going to discuss on Quality Assessment.

Thank you.