

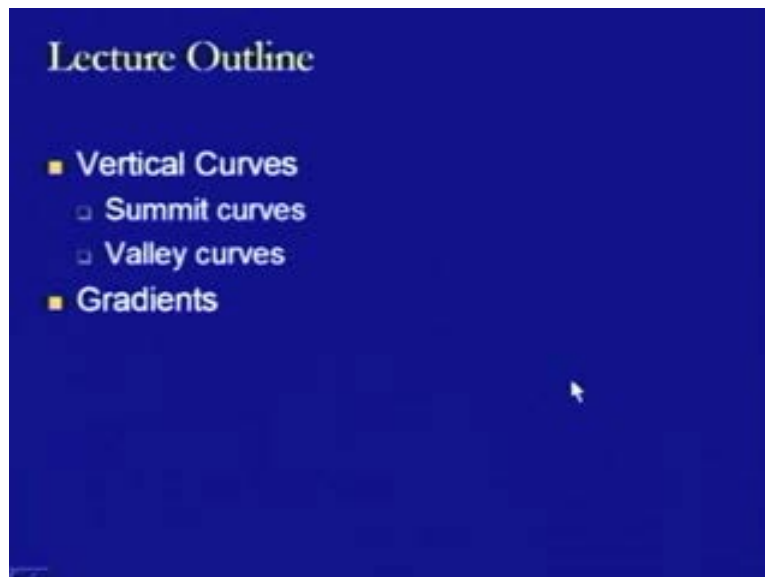
Transportation Engineering - II
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Lecture – 18
Vertical Curves and Gradients

Dear students, I welcome you back to the lectures in the lecture series on course material of Transportation Engineering – II. In today's lecture we will be discussing about the vertical curves and the gradients. We have already discussed about the different other features which are associated with the geometric design of railway tracks. We have already discussed about horizontal curves, we have discussed about the alignment, we have discussed about the superelevation, we have also discussed about the transition curves and then we have discussed about the various types of clearances which needs to be provided while designing any of the track or the tracks laid simultaneously parallel to each other on the straight or the curved sections.

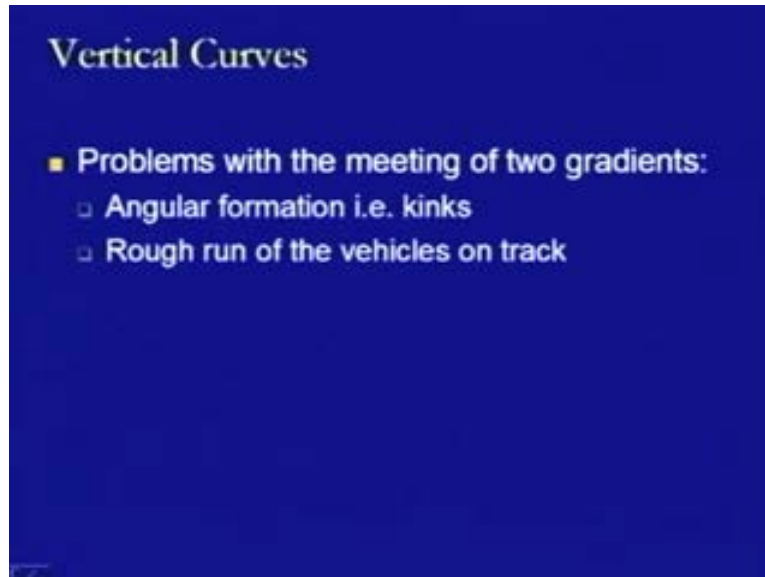
In today's lecture, we will be giving more emphasis to the remaining two aspects of any design of the railway track that is vertical curves and gradients.

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In the light of this, today's lecture has been arranged as, we will be talking about the vertical curves, we will be talking, within the vertical curves, about the summit curves, the valley curves and we will be talking about the gradients or gradients and their types.

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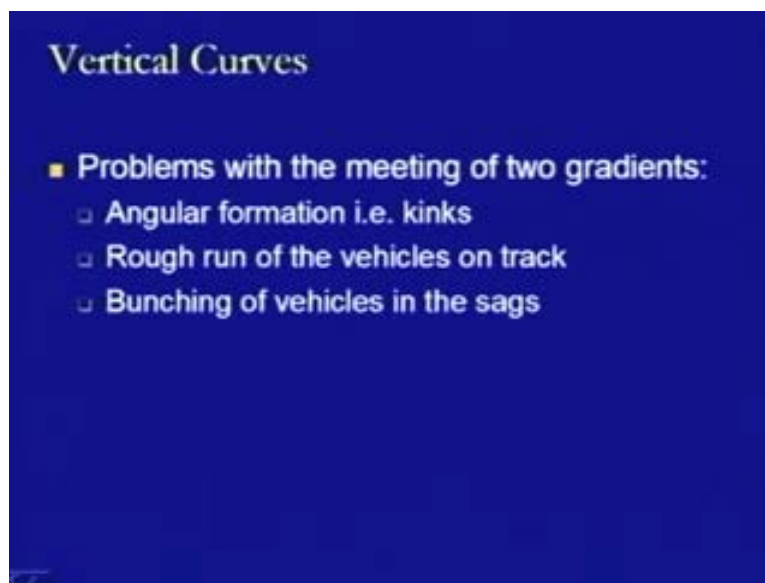
So, now we will be starting with the vertical curves. The vertical curves basically they are provided, because of the problem with the meeting of two gradients. In the case where there are two gradients, gradient is the rise or the fall between the two points. If we join those two points by a straight line, then whatever is the rise by which the next point is rising above the first point or is falling down with respect to the first point, then that is defining the gradient. So, therefore when we are talking about this gradient and the gradient or this straight line trajectory has been provided, then wherever the two, these straight line trajectories are joining, may be in falling, may be in rising or may be in rising or falling, then there are certain problems.

So, we have to look at first of all those problems and then, so as to solve those problems the vertical curves are provided. So, in this, first of all we will be looking at the problems. The very first problem is that wherever the two gradients are meeting each other, that is a point where the kink will get formed. Now, here we are talking everything in terms of the

vertical profile that is everything is being done in vertical direction. So, instead of horizontal direction like this, now we are talking about vertical direction in this form.

So, we have say, a rising gradient or a falling gradient and then, wherever these two are meeting there will be a kink. Kink is a point where there is an abrupt change in the direction and when there is an abrupt change in the direction that becomes a, in the form of gradients, an angular formation and this angular formation has their safety implications. It is a difficult condition in the case of the gradients or in the case of the vertical profile of alignment. Another point here is the rough run of the vehicle on the track. Now, obviously when there are kinks and the vehicle has to traverse that kink and the vehicle is the jerk when to traverse that kink, then that is what is the rough run. So, wherever such type of conditions has been provided, then the vehicle will be having a rough run, no smooth movement of the vehicle will be possible at all.

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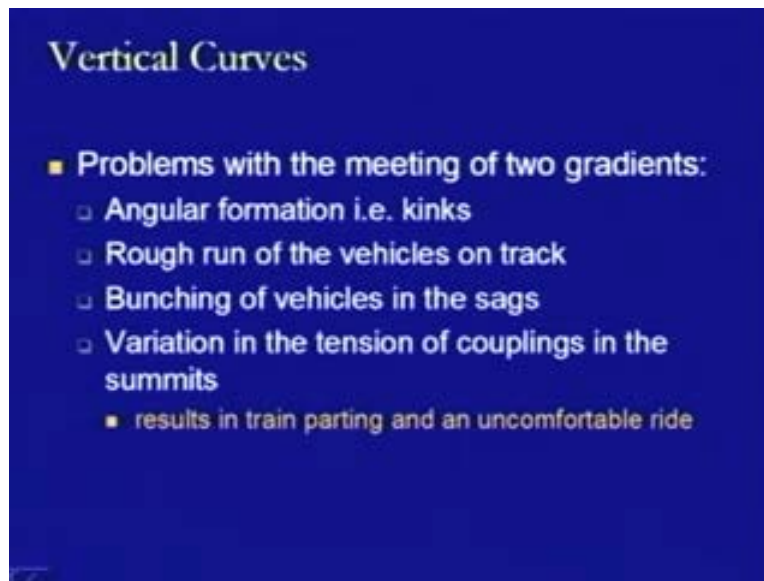
Another point, another problem which will be happening because of the gradients is bunching of vehicles in the sags. Sags are defined as the depressed portions of the gradients. What we can assume here is that if there is a falling line which is meeting the upward line, then the point at which these two lines are meeting that is there is a falling

line like this and there is an upward line like this, so these two are meeting at this point and this is the lowest point in this trajectory. So, this is what is sag. So, at this point of sag there will be bunching of vehicles.

What will happen is that all the vehicles which are coming towards this location, whether they are coming from this downwards gradient or whether they are going on this upward gradient, all the vehicles have the tendency to go towards the lowest point of that trajectory and this is just because of the properties or just because of the gravity due to which all these vehicles will try to align with respect to the sag. It means there will be accumulation and accumulation of a number of vehicles within the sag area and this is what is known as the bunching of the vehicles. The vehicles will try to come as near as possible to each other in the sag area as compared to any other area and there is a sort of flexibility which is being provided, where the two vehicles are connected with each other.

Generally, we have a loose shunted condition. So, in that loose condition, the bogies remain at a certain distance from each other. But then, there is a suspension effect or springing effect by which the connectivity through which they have been joined together they can be pushed inside and when there is bunching of vehicles, then what will be happening is that those connectivities will remain inside of the wagon itself and there will be very little distance which will be left between the two bogies. So, this is another effect of the sag or of the gradients which have been provided and this may also have its safety implication in the sense that there is, as we are moving towards the sag all the vehicles or wagons will be trying to hit it.

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Then, there is a variation in the tension of couplings. This variation in the tension of the couplings will be there, of course in two conditions. One is that, we were talking about the summit. In the case of the summit that is this is the topmost location, where the two gradients are meeting. So, if we have upward gradient which is going like this and it is followed by another gradient which is coming like this in the downward direction, so in that sense wherever these two gradients are meeting that is the upmost point and this upmost point is termed as the summit.

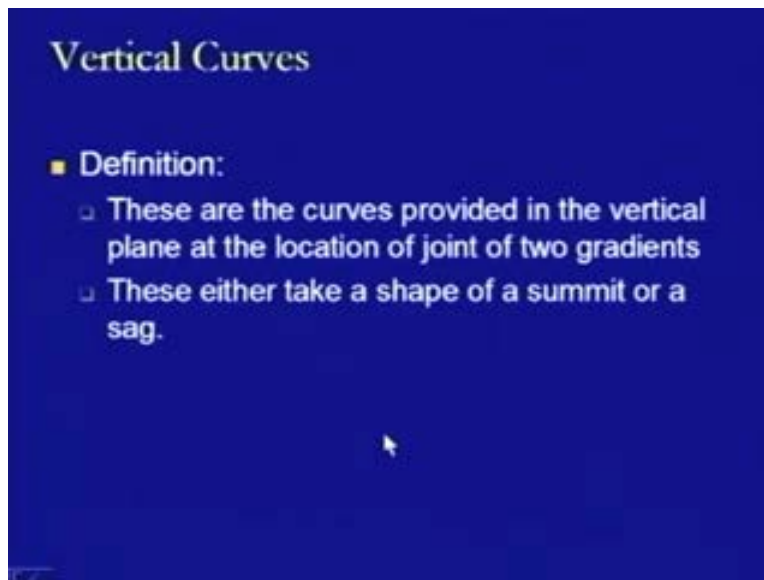
So, if a vehicle is negotiating on this upward gradient, comes to the summit and then it tries to go to the other direction, in both the conditions there will be tension on the couplings by which the wagons have been joined with each other. There will be a pushing effect which will be there at the point of contact of two wagons, because of this gradient. Because when we are going to that upward of this gradient, their gravity will be acting in the downward direction, so it will try to pull all these wagons in the downward direction, because of the pull of the Earth's gravitational law.

So, when this is being pulled in the downward direction, whereas we are trying to move towards the other direction, so there will be a tension at the coupling. This is what is this

tension and this variation may be there depending on how many wagons are there on one side of this summit as compared to the other side of the summit and it may result in the train parting or it may result in the uncomfortable ride. So, these are the two things which may happen. At times, it may be found that the couplings have broken down because of heavy tension and therefore, the wagons have got detached from the rest of the train.

Another condition which can be there is that as you are negotiating this summit curve and because the tension is acting in the opposite directions, therefore the ride will not be as comfortable as it can be in any of the smooth leveled track conditions. So, these are some of the problems which are there with the meeting of the different gradients and so as to meet out all these things, certain measures have to be taken, so that there is a smooth variation from one gradient to the another gradient and that is where the smooth movement can take place with respect to the different type of the forces which may be acting and this is met out by the vertical curves.

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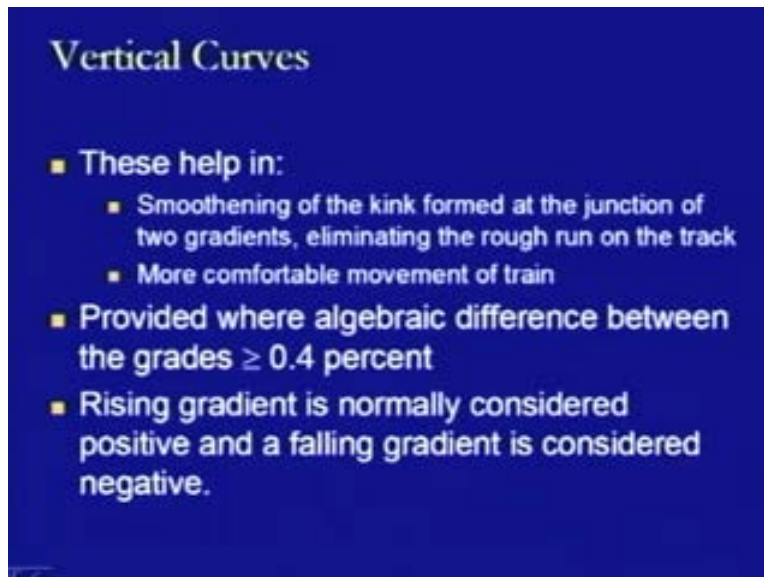


So, the vertical curves by definition, these are the curves provided in the vertical plane at the location of joint of two gradients, as we have discussed about the different problems and we have seen that whether it is a sag condition or it is a summit condition, in both the

conditions there are kinks being formed and there are certain problems which are associated with that one. So, at the point where the two gradients are being joined, may be at a sag or may be in a summit, at that location if a curve is being provided, then that curve is termed as vertical curve.

Now, these either take a shape of a summit or a sag depending on the location. As I have told that if we are going in the upward direction and then, it is followed by a downward direction movement, then the point at which they are meeting that is a summit condition, whereas if we are going in the downward direction and then we start going in the upward direction, then at the point at which this change is occurring that is a sag. So, we have the summit and we have the sag condition. So, these are the two types of the shape which can be there.

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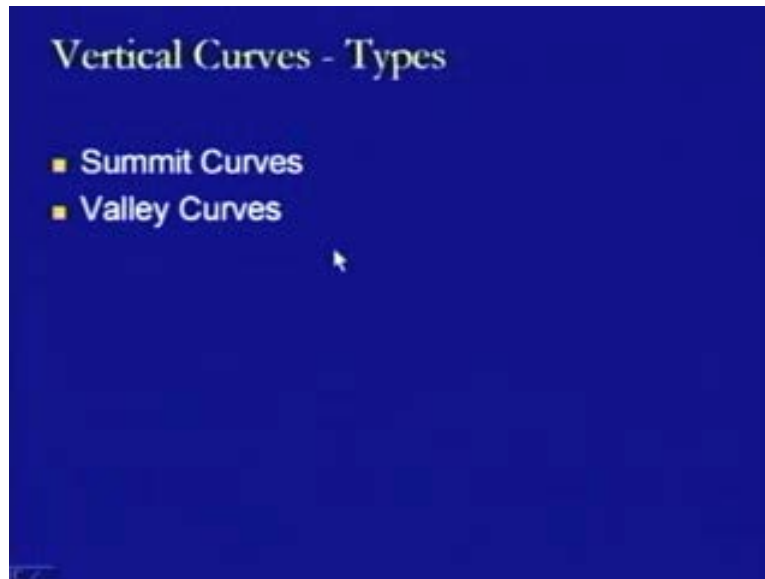
Then, these help in smoothing of the kink, which are formed at the junction of two gradients. This is one aspect that it helps in the smoothing of the kink. The kink is being eliminated and it is being smoothed out and as soon as the kink is being smoothed out, it helps in the smooth running of the track, therefore the rough run is being eliminated. Further, it becomes more comfortable movement or operation of the

trains, the passengers or the freight, who are moving in the wagons they will not be feeling any awkward or any jerk condition while traversing the change over from one direction to another direction in the vertical profile.

These vertical curves needs to be provided where the algebraic difference between the grades is greater than or at the minimum is equals to 0.4%. This is the restriction with respect to the vertical curves. If the difference is more than this one, then probably there is no requirement of providing the vertical curves and it will be as smooth as possible or the jerks which will be provided by the movement over that kink will not be that heavy that it may create the discomfortable or uncomfortable condition for the passengers. So, we have to look at the algebraic difference between the grades. That is when we are talking about an upgrade and we are talking about a downgrade or we are talking about a downgrade followed by an upgrade, then in both the conditions whatever is the difference between this gradient value and this gradient value it should be greater than or minimum should be equal to 0.4%.

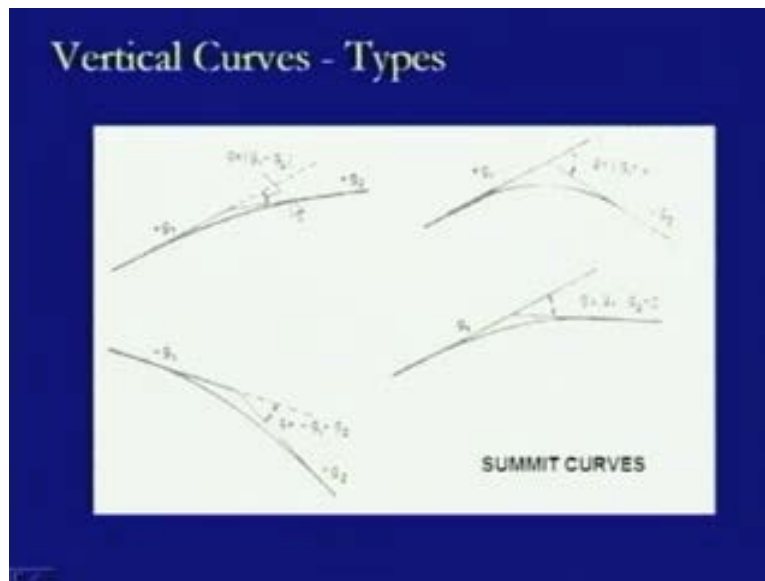
Then, rising gradient is normally considered positive and a falling gradient is considered negative. This is the normal nomenclature which is used while defining any of the gradients used in the vertical curves. So, we have to look at the gradients which we will be discussing a little later. So, whatever guidance has been provided, if we are moving from one point to the other point and while doing so the level is increasing, then this is termed as positive. Similarly, when we are moving from one point to the other and the level is decreasing, then it is termed as negative.

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Now, we come to different types of vertical curves. As we have seen in the lecture outline also, we have two types of curves - the summit curves and the sag curves and the valley curves or the sag curves are one and the same thing, because they are provided at the lowest point, whereas summit curves are those which are provided at the highest point. So, we will be discussing about all these types of curves one by one and we will also look at the profiles which are there by which these can be defined; so, the summit curves and the valley curves.

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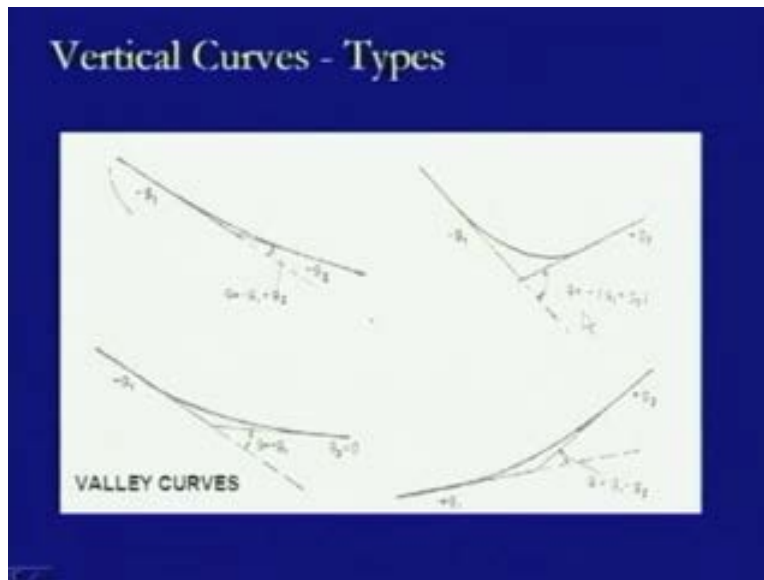


Now, here in this case, this diagram tries to show different conditions in which the summit curves can be formed. We have this condition, where there is a gradient g_1 which is going in the upward direction and that is why it is being shown as false to g_1 and this is followed by another gradient which is also going in the upward direction, but at a lower level or it is less steeper as compared to the previous one. So, in this case, this is being designated as plus g_2 . So, both these gradients, they are meeting at this point. So, this is the point of contact and this is the kink which will be there in the natural design of the vertical curves if being provided in this form. But, there is smoothing out this kink and for doing that we are providing a curve like this. This is the curve and this is what is the vertical curve.

Now, in this case, there is a change of direction from plus g_1 to plus g_2 and this change of direction is depicted here by this angle that is g and which is nothing but it is the algebraic difference between the two gradients. So, there is a gradient plus g_1 minus the next gradient plus g_2 will be, that is g_1 minus g_2 will be the value of g that is this deflection. Similarly, there is another case where there is an upgrade which is going like this as plus g_1 and there is a downgrade which is going like this as minus g_2 and again in this case they have been joined together by a curve so as to omit this kink.

Now, when we are omitting this kink we are changing the direction by this one to this one by an angle this one. This is g and this angle g is nothing again, but it is just the algebraic difference between these two values and therefore, it will be plus g_1 minus g_2 ; minus of minus g_2 , so therefore it will be g_1 plus g_2 . Then, there is another condition where both the gradients are downgrades. So, this is also minus g_1 and this is minus g_2 and again a curve is there, so as to omit the kink and the deflection angle will be minus g_1 minus minus g_2 means plus g_2 and here, we have one gradient which is upgrade, but the other gradient is horizontal. So, when the horizontal is being provided with this one, the gradient here will be zero and in this case, the deflection will be nothing but it will be equal to the gradient being provided in the previous case. That is g is equals to g_1 . So, these are the different conditions of summit curves.

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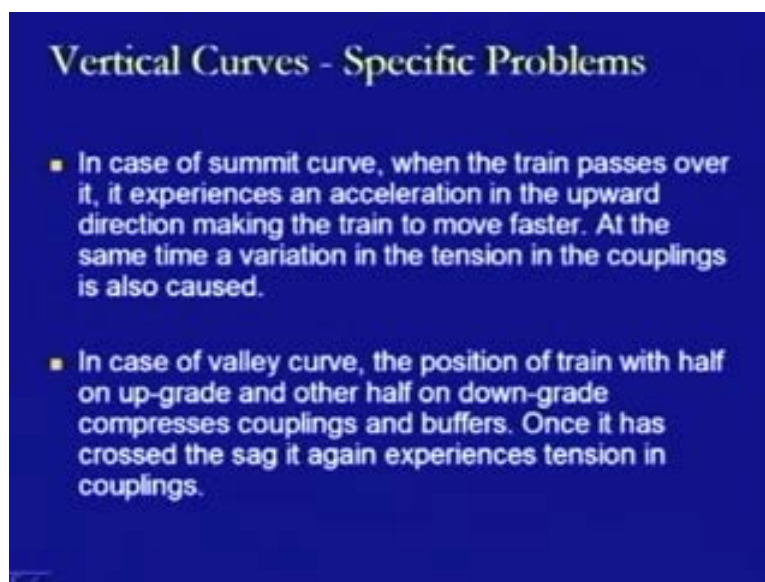
Now, let us look at similar different conditions for the valley curves. In the case of the valley curve, what we are looking at is that they are mostly associated with the down gradients. Here, we see that this down gradient which is also again followed by a down gradient, so this is minus g_1 and this is minus g_2 and they are meeting at this point. So, therefore we are trying to remove or eliminate this kink by the provision by smoothening here that is by providing a curve. So, this is the curve and this is what is the valley curve

or the sag curve and here again, we are having a deflection and this is to be taken again as the algebraic difference between the two gradients. So, what we do is this is minus g_1 minus minus g_2 . Therefore, it will be minus g_1 plus g_2 .

Similarly, here we have a downgrade which is followed by an upgrade. So, this is minus g_1 this is plus g_2 and being smoothed down by this curve like this. So, the kink is removed and we have this angle. So, this is minus g_1 minus plus g_2 . So, therefore this will be minus g_1 minus g_2 or minus, in bracket g_1 plus g_2 . So, we will be looking at another orientation here. In this orientation, we are having a downgrade and this is being followed by a horizontal. So, therefore this is minus g_1 and this is g_2 , it is zero. So, the deflection angle will be equal to only this value and this will be minus g_1 .

Another case is where an upgrade is being followed by another upgrade and therefore, it is smoothed again by a valley curve and in this case the deflection angle will be given by plus g_1 minus plus g_2 , therefore it will be g_1 minus g_2 . So, these are different cases in which the summit curves or the valley curves can be formed. There may be different other orientations too which can be seen, where some orientations already have been taken here, mostly which are in use at different sections.

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Vertical Curves - Specific Problems

- In case of summit curve, when the train passes over it, it experiences an acceleration in the upward direction making the train to move faster. At the same time a variation in the tension in the couplings is also caused.
- In case of valley curve, the position of train with half on up-grade and other half on down-grade compresses couplings and buffers. Once it has crossed the sag it again experiences tension in couplings.

Now, we come to the specific problems in the case of the vertical curves. The problems are, in case of the summit curve when the train passes over it, it experiences an acceleration in the upward direction. What happens is that when you are going on a summit curve like this, then there will be a centrifugal force which will be acting in the upward direction because of its curvature like this and due to this reason there it is experiencing an acceleration in the upward direction. This makes the train to move faster. At the same time, a variation in the tension in the couplings is also caused, because the train is moving like this and there is an upward motion acceleration in this direction. Therefore, the couplings get stressed in this direction as well as in this direction. So, there is variation in the tension in the couplings. The variation will be coming, because there will be different number of wagons on this side as compared to the other side and that will be bringing the variation.

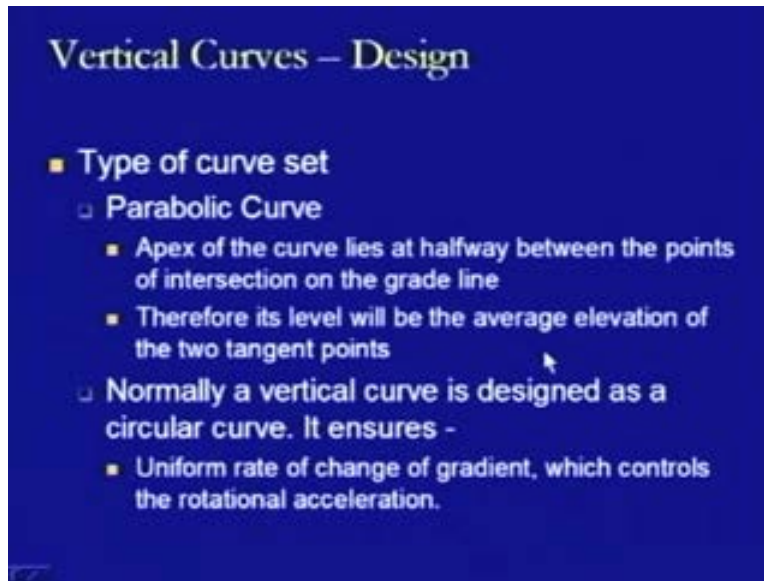
Similarly when we talk about the valley curve, then in the case of the valley curve, the position of train with half on upgrade and other half on the down grade compresses couplings and buffers. Now, in this case, we are talking about a curve like this. So, this is a valley curve. So, in this valley curve if the train, half of the train is on this portion and half of the train is on this portion, then what is happening is that, as we have seen there is bunching effect. Everything is coming very near to each other and in that condition there will be compression of the couplings as well as the buffers.

The buffers are those things which are provided between the two wagons, so that the two wagons can be kept away from each other. You must have seen where the two wagons are attached or where the two bogies of the compartment are joined together, you must have seen the part which moves inside or outside with the jerks. So, that is what is a buffer and then, there are some couplings which are attached, so as to join one bogie with another bogie. So, there is a compression of those couplings and buffers in this condition. Once it has crossed the sag, it again experiences tension in couplings. That is if we are moving on this one and instead of half of this and half of this, now we have crossed this sag and we are moving in upward direction here or upward direction here, then everything is going in the downward direction, because of the gravity. So, there is a

tension now being experienced by the couplings. So, this is another problem which is associated in the case of the valley curves. So, in the case of summit curve, it is totally a problem of tension in the couplings, because acceleration in the upward direction only helps in making the movement more smoother, whereas in the case of the sag curves both the things are there. There is bunching and there that is causing a compression on the buffers and there is a tension in couplings.

Now, we come to the design of the vertical curves.

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Vertical Curves – Design

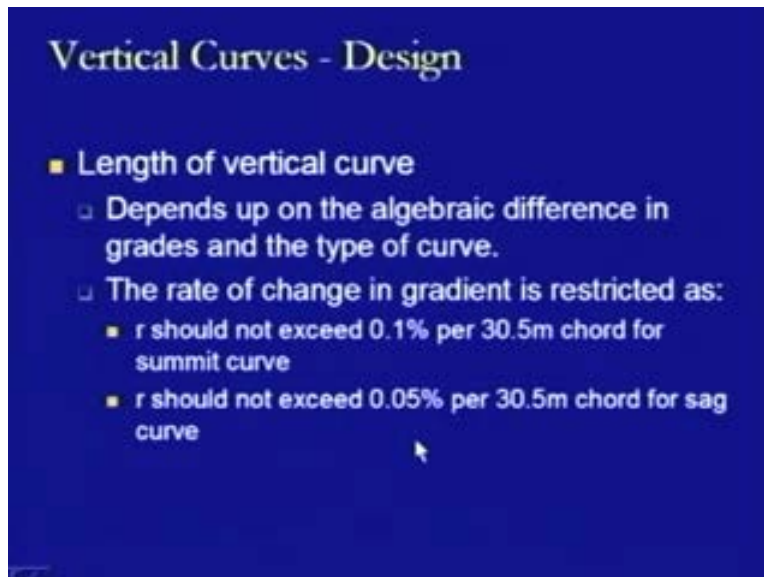
- **Type of curve set**
 - **Parabolic Curve**
 - Apex of the curve lies at halfway between the points of intersection on the grade line
 - Therefore its level will be the average elevation of the two tangent points
 - **Normally a vertical curve is designed as a circular curve. It ensures -**
 - Uniform rate of change of gradient, which controls the rotational acceleration.

It depends on what type of curve we are choosing. There can be a parabolic curve. In the case of parabolic curve, what happens is the apex of the curve lies at halfway between the points of intersection on the grade line. So, if we have this as a grade line and then, if we join these two points by a straight line, then the apex of the curve will lie halfway between these two points. This is one of the properties of the parabolic curve by which we can design it. Therefore, its level will be the average level of the two tangent points. So, this is an obvious condition, because if we have this grade like this and this is another grade and a curve has been provided in this form, so this is the apex and if we join them like this, then this is the central point. If we join this apex and central point, then the

value of this one will be only the average of gradient RL of this one as well as the elevation of the first point and the elevation of the second tangent point.

There is another thing related to the provision of curves is normally a vertical curve is designed as a circular curve. The reason behind is that it ensures a uniform rate of change of gradient. That already we have seen when we have discussed about the circular curves and this helps in controlling the rotational acceleration and this rotational acceleration is related with centrifugal force which is acting on any of the circular curve. So, this is one of the properties of the circular curve which is used in the design of the vertical curves.

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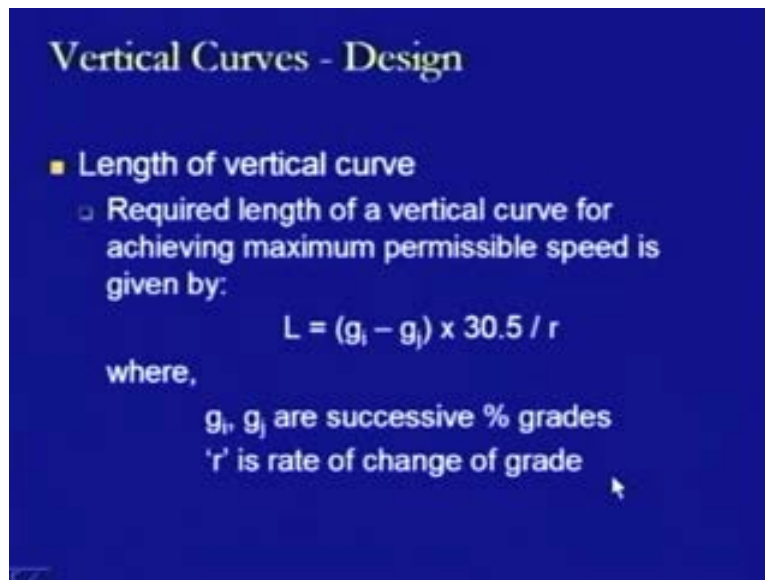
Vertical Curves - Design

- Length of vertical curve
 - Depends up on the algebraic difference in grades and the type of curve.
 - The rate of change in gradient is restricted as:
 - r should not exceed 0.1% per 30.5m chord for summit curve
 - r should not exceed 0.05% per 30.5m chord for sag curve

Now, here we are looking at the length of the vertical curve. This length of the vertical curve depends on the algebraic difference in the grades as well as the type of curve. These are the two factors which are going to govern the length of the vertical curve and another thing which will be creating an effect is the rate of change in gradient. In this, rate of change in gradient is defined as a small r , as written here and this is restricted as this. This r should not exceed 0.1% per 30.5 meters chord for the summit curve.

This is one of the restrictions in the case of the summit curve, whereas in the case of the sag curve, the restriction is that this r or the rate of change in gradient should not exceed 0.05% per 30.5 meter of the length of the chord. So, these are the two values which are creating a restriction in the case of the summit curve and the sag curve respectively and using this value of r , the value of the difference, algebraic difference in the gradients and the type of the curve to be used, we can calculate the length of the vertical curve.

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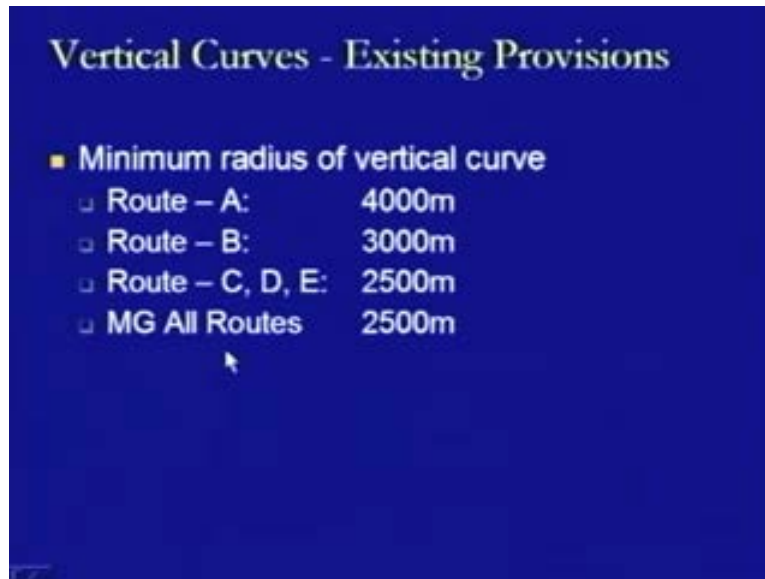


Vertical Curves - Design

- Length of vertical curve
 - Required length of a vertical curve for achieving maximum permissible speed is given by:
$$L = (g_i - g_j) \times 30.5 / r$$
where,
 - g_i, g_j are successive % grades
 - ' r ' is rate of change of grade

So, this length of the vertical curve required on any curve for achieving the maximum permissible speed can be computed as L is equal to g_i minus g_j that is the algebraic difference of the two gradients which have been provided and in between which the curve has to be provided and this is multiplied with 30.5 divided by r . So, this is the length of the chord and this is the rate of change of that chord. So, we are taking this value and by using this value we can compute the length of the vertical curve, where g_i and g_j are the successive percent grades. They are not in radians; they are in radians when we take it in the form of a percent fractional form. r is the rate of change of grade.

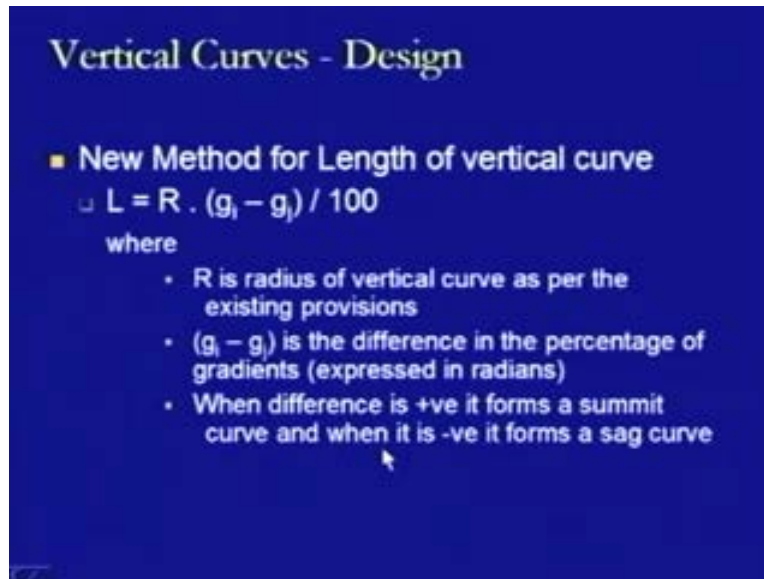
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Vertical Curves - Existing Provisions	
■ Minimum radius of vertical curve	
□ Route – A:	4000m
□ Route – B:	3000m
□ Route – C, D, E:	2500m
□ MG All Routes	2500m

Now, this is one of the provision by which we can compute the length of the vertical curve. There are some other provisions which have come up and the existing provision in the case of the vertical curves we will be looking at now. The minimum radius of the vertical curve has been defined already and this is defined on the basis of the route; so, the gauge as well on that gauge what is the route. So, in the case of route A of broad gauge, this minimum radius of vertical curve has been defined as 4000 meters, whereas in the case of route B this is being defined as 3000 meters. Then, for route C, D and E, it is defined as 2500 hundred meters. So, these are the values of the minimum radius of the vertical curve for different routes of the broad gauge. In the case of the meter gauge for all routes, for all type of the routes that is Q, R or S routes, this is being defined as 2500 meters.

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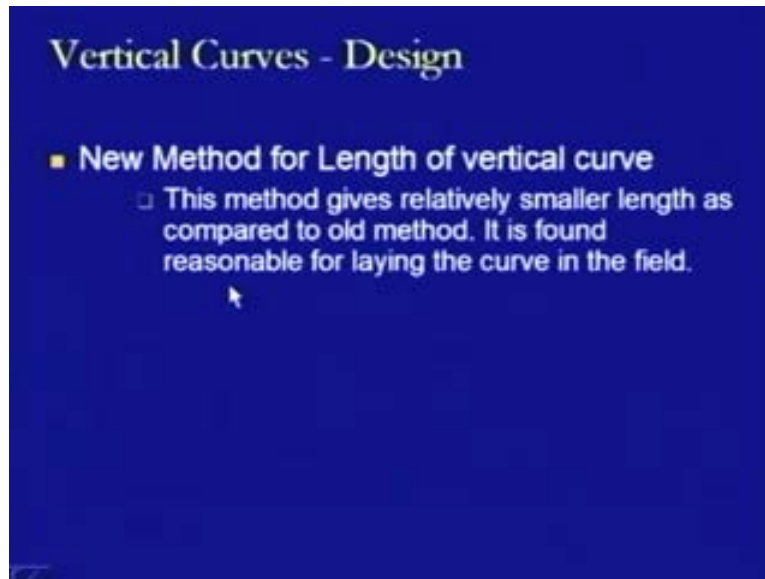


Vertical Curves - Design

- New Method for Length of vertical curve
 - $L = R \cdot (g_i - g_j) / 100$
where
 - R is radius of vertical curve as per the existing provisions
 - $(g_i - g_j)$ is the difference in the percentage of gradients (expressed in radians)
 - When difference is +ve it forms a summit curve and when it is -ve it forms a sag curve

Coming back to the design and the length of the vertical curve, another method which is being given now and is in use is L is equal to radius R multiplied with g_i minus g_j and divided by 100. So, in this case R is the radius of the vertical curve as per the existing provisions, g_i minus g_j this is the difference of the percentage of gradients which is expressed in radians and 100 is just trying to take it converting this value into the radians here. When we take the difference, if it is positive then it forms a summit curve and when this difference is coming as negative, then it forms a sag curve. This is how we can identify whether this is a summit curve or a sag curve. So, this is a new method by which we can compute the value.

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This new method for the length of the vertical curve gives the length which is relatively smaller or in some of the conditions we will find that there is large variation between the length which we have computed using the previous formula or which we are now computing this using this new formula. This is what is a comparison between the two methods by which we can compute the values. But now-a-days, we are using the latter formula compared to the former one and it is being observed that while laying these type of the vertical curves in the field, they have been found reasonable.

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Vertical Curves - Setting

- Various methods of curve setting in field are:
 - The Tangent correction method
 - The chord deflection method
- The tangent correction method is considered simpler than the other methods and is more convenient for the field staff.

Now, we come to the next aspect of the vertical curve, the setting of the curve in the field. There are different methods by which the curves can be set in the field. They are the tangent correction method and the chord deflection method. Now, the tangent correction method is considered simpler than the other method and it is also more convenient for the field staff to lay the curve using this method.

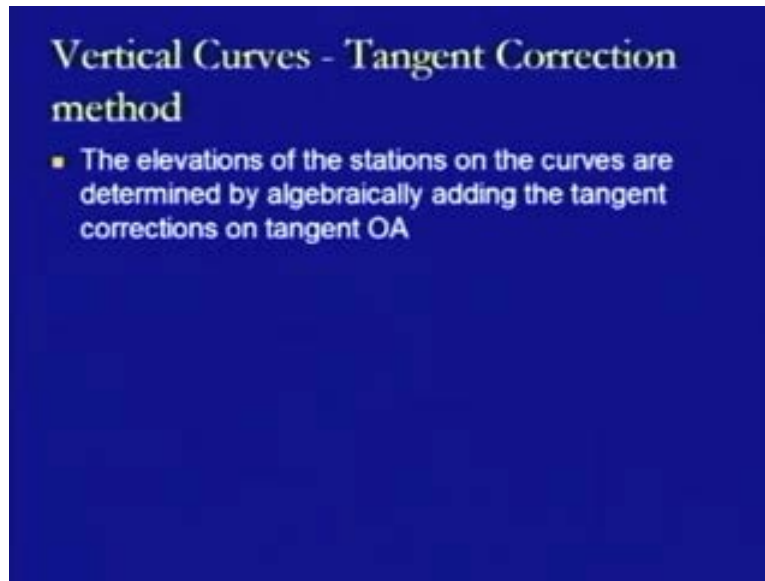
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Vertical Curves - Tangent Correction method

- Compute the length of vertical curve
- Work out chainage and reduced levels of tangent points and apex
- Apply tangent corrections as
$$Y = c x^2$$
where
$$c = (g_1 - g_2) / 4n$$
 - y is the vertical ordinate,
 - x is the horizontal distance from the springing point
 - g_1 is gradient -1 (positive for rising gradient)
 - g_2 is gradient -2 (negative for falling gradient)
 - n is number of chords up to half the length of the curve

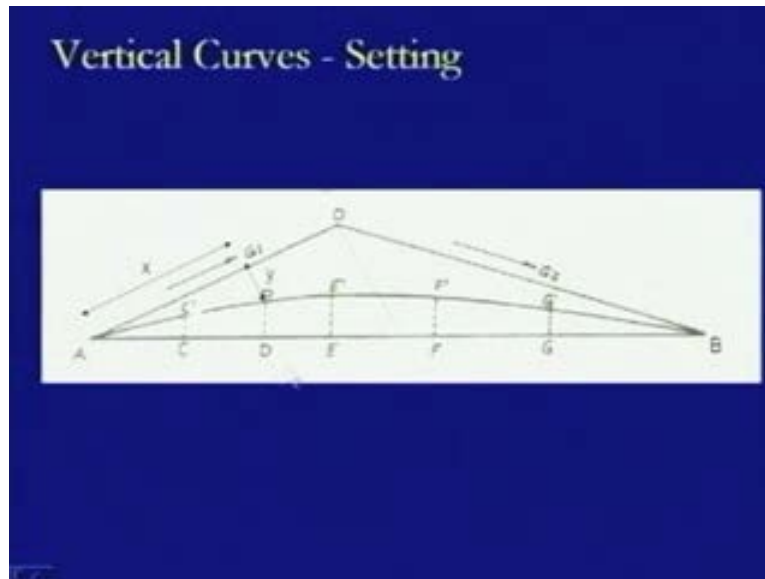
So, in the case of tangent correction method, we can compute the length of the vertical curve by the following process. We have to work out the chainage and the reduced levels of the tangent points and the apex and apply the tangent correction and this can be applied as Y is equals to $c \times x$ square where c is computed as g_1 minus g_2 divided by $4n$. Y is the vertical ordinate, x is the horizontal distance from the springing point, g_1 is the gradient 1 which is positive for rising gradient, g_2 is gradient 2 which is negative for falling gradient and n is the number of chords up to half the length of the curve. This is how all the parameters being used in the above equations have been defined.

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The elevations of the stations on the curves are determined by algebraically adding the tangent corrections on tangent OA. So, this is how the elevations of the stations can be computed and once we have got this elevation, then we can draw the curve in the field.

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This is the diagram which tries to define the same thing. Here this curve is to be drawn. This is the tangent 1, this is another tangent on this side. This is up gradient plus g_1 , this is down gradient minus g_2 . This is the tangent point 1 and this is the tangent point 2 and they have been connected by a chord like this. So, when we are moving, we have the RL of this point that is the point of intersection of the two grades. We have the RL of the tangent point 1 and then tangent point 2 that is t_a and t_b and once when we have these RL's, then as we go in this forward direction by a distance x , then we can compute this ordinate Y by using the formula, as we have seen previously. Once we have this formula, then we can go perpendicular to this tangent line and we will get this point. Using this point, then similarly we can find out this point or we can find out the rest of the points and then if we join them we will be getting the vertical curve.

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Vertical Curves

- **Chainages of Points**
 - Chainage of point 'A' = chainage of intersection point (O) - L/2
 - Chainage of point 'B' = chainage of intersection point (O) + L/2
- **Reduced Levels**
 - RL of point 'A' = RL of intersection point (O) - L/2 . (g/100)
 - RL of point 'B' = RL of intersection point (O) - L/2 . (-g/100)

Now, in this case of vertical curve we have to find out the chainages and the RL's of the different points and this is how we do it. So, the chainages of the points will be the chainage of point A that is the first tangent point is the chainage of point of intersection that is O where the two gradients are cutting each other minus L by 2, where L is the length of the curve. So, if we do this, because the apex is falling at the half of the length of the curve this is what is the property we have already seen. So, using that property we are trying to find out the chainage of point A.

Then, the chainage of point B, it will be in the forward direction and that is why it will be the chainage of the intersection point O plus L by 2. So, this is the chainage. Chainage means the distance, so we are coming back in this direction to the point A from point O. But in the case of point B, we are going forward. That is why, here it is taken as negative and this is taken as positive.

Then, another thing which is to be done is to find out the reduced levels or the elevations of different points along this curve. In this case, the RL of point A is RL of intersection of point O minus the distance multiplied by the grade that is L by 2 multiplied with g 1 divided by 100. Here, it is taken g i means the first gradient divided by 100. So, it is being

transformed into radians instead of percent value. So, this multiplied with the distance will give us the value by which it has to be reduced. So, this is how we can find out the RL of point A that is the initial starting of the curve.

Similarly, in the case of RL of point B, it will be the RL of intersection of point O minus L by 2 multiplied with the gradient in that direction and this is minus g divided by 100. So, this is how we can find out the RL of another point B and this is the end of the curve for second tangent point.

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Vertical Curves

- Increase in RL for length of curve 'L'
= RL of 'B' - RL of 'A'
- Increase in RL for distance 'c' (fraction of L)
= increase in RL for 'L' x c / L
- 1st offset on vertical curve
= $c(L - c) / 2R$
where $c = 0.2L, 0.4L, \dots, L$

Then, the increase in the RL for any length of the curve L is to be computed. Here, this will be nothing but RL of B minus RL of A. So, if we have got the RL of B and RL of A as computed by the previous two formula, then we can find out the increase in the length of curve L and this increase in the RL for any distance c which is some fraction of the curve length L , then that can be also computed and this will be increase in RL for L multiplied with the fraction c divided by L means we are trying to convert the overall increase in the RL, which was related to the length L in terms of the fraction. So, increase in RL divided by L multiplied by c is what we are doing here.

Now, for the first offset on the vertical curve this will be given by c multiplied by L minus c divided by $2R$, where c is some value of some fraction of L , may be like $0.2L$, $0.4L$, $0.6L$, $0.8L$, likewise we can take the values and if you want further more accuracy we can have 0.1 , 0.2 , 0.3 times of the L and we place those values in this equation and we will get the offsets for the different points on the vertical curve.

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Vertical Curves

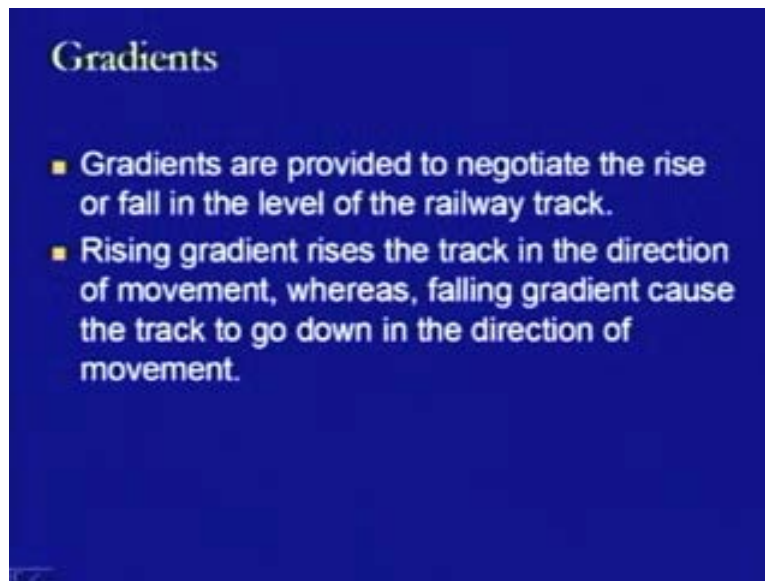
- RL of point on curve
= RL of point on grade line + offset

where

$$\text{RL of point} = \text{RL of previous point (say A)} + \text{increase in RL for 'c'}$$

Then, the RL of point on curve will be RL of point on the grade line plus offset, where the RL of point on the grade line will be the RL of the previous point say A plus increase in RL for the distance c which is a fraction of the overall length L . So, this is how we compute it. First of all, already we have computed this value using the RL of point O and then, it is to be increased by some value which is the value for distance c . So, this was all about the vertical curves and the provision of the vertical curves. Now once we have looked at the vertical curves what we have found is that they are dependent on the values of the gradients. Now, we have to look at what can be the value of the gradient which can be provided and this is the one aspect which we will be trying to look in the successive discussion over the gradient.

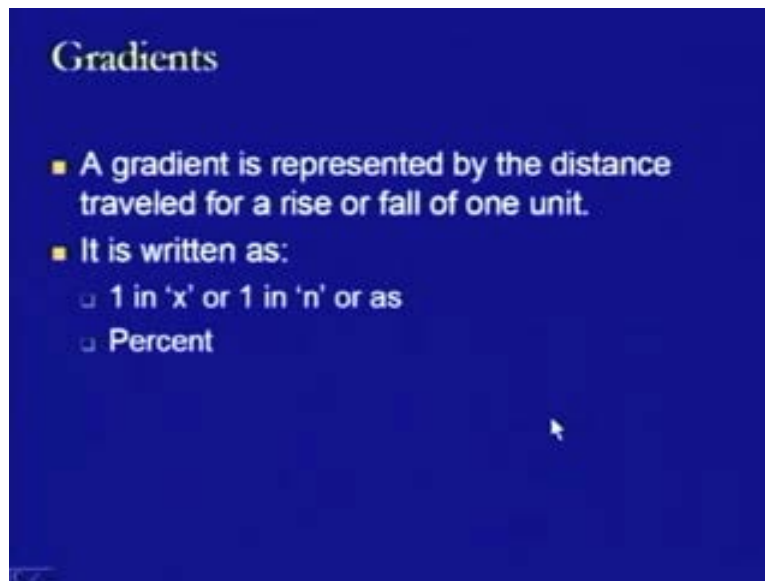
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So, starting with these gradients, gradients can be defined as that they are provided to negotiate the rise or fall in the level of the railway track. Now, as the railway track joins two points, any two points, say A and B and if there is a difference in the elevation of these two points A and B, then the line which is connecting them is what is gradient. So, this is what is trying to provide to negotiate. Now, this may be as a rise or this may be as a fall, as we have seen in the case of the vertical curves. So, if the second point is at a higher level as compared to the first point, then it is going to be a rise, whereas the second point is at a lower level as compared to the first point, then it is fall. So, it means there can be a rising gradient or there can be a falling gradient, which is connecting the two points on any railway track.

The rising gradient rises the track in the direction of movement, whereas the falling gradient causes the track to go down in the direction of the movement. So, this is how any rising or falling gradient is defined.

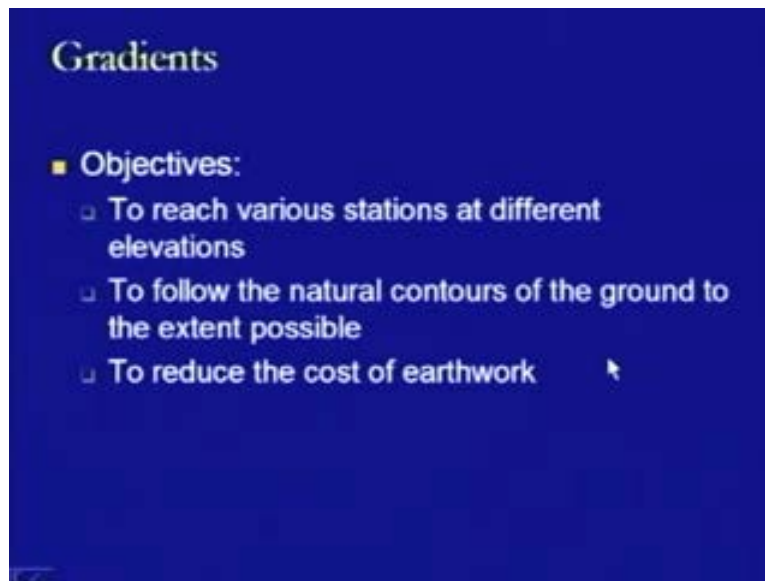
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Further, a gradient is represented by a distance traveled for a rise or fall of 1 unit. This is the way how the gradients are defined. If we are moving a certain distance in any of the direction and after moving that distance we find that there is a rise or there is a fall by 1 unit, by 1 unit means whatever unit of distance we are considering whether we are considering meter or we are considering centimeters or we are considering millimeters, on the basis of that say if we move some of the distance and after that we find that there is a change by 1 meter, then it means that is what is going to be the gradient in that form. So, this is how we can represent it. So, this is 1 meter with respect to so much horizontal distance.

So, it will be written as 1 in x, where x will be the horizontal distance or it is at times also written as 1 in n, where n also is a number which is trying to specify that if you move this much **value** distance, then you are going to have a rise or a fall of 1 unit or sometimes it is also designated as a percent value. So, these are the two different ways in which it is designated.

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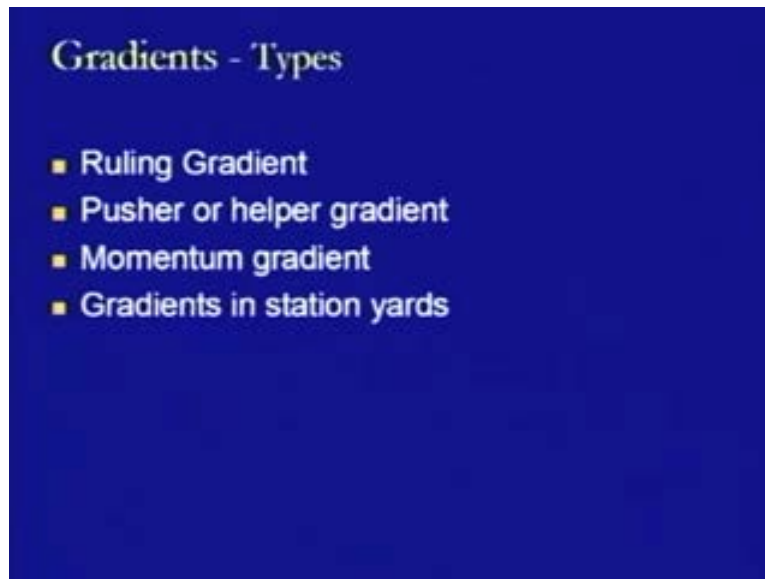


Now, why we are providing the gradients? Just look at some of the objectives of provisions of gradients. We are interested in reaching the different locations which are at different elevations. This is one of the primary objectives. Specifically in the case of hilly areas or in the case of the mountainous region or where the gradients are or where the cross slope of the or where the topographical conditions are such that the two points at vast difference of elevation.

Another one is that to follow the natural contours of the ground to the extent possible, so we are providing the gradients as we have seen in the case of the fixing of the alignment, where the mountainous gradients were discussed and we have the zigzag positions or the switch back conditions. In both of those conditions, as far as possible we tried to follow the contours and if we go along the natural contours, then they provide the flexibility of traversing the contours so as to attain the height and this has already been shown and discussed with you. So, this is what is being tried to mention here that they try to follow the natural contours of the ground to the extent possible and in this form they try to negotiate the elevations.

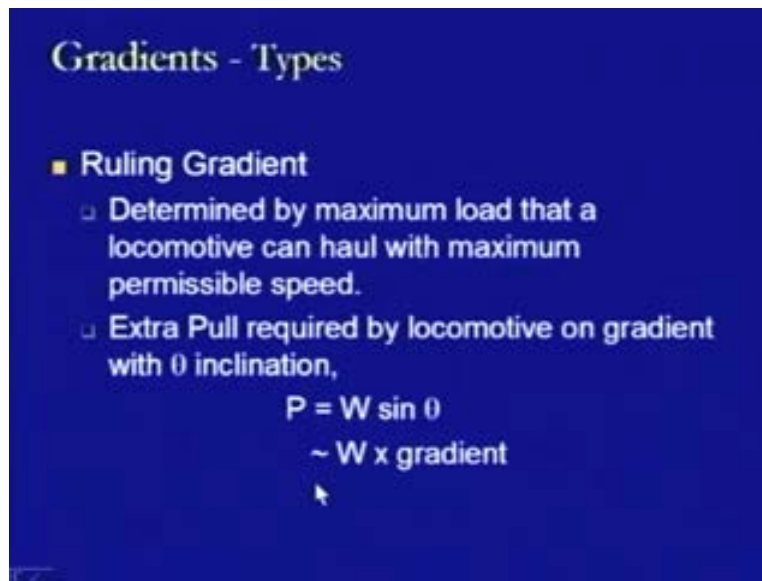
Further, it tries to reduce the cost of the earthwork. Now, if we have to go from one point to another point and there is very, very large difference between the elevation of these two points, then finally what we have to do is we have to do a lot of filling between these two points. So, instead of doing that one, if we follow the contours and go slowly in that point, then we can reduce the cost of the earthwork in that sense.

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So, looking at these objectives, now we come to the different type of the gradients which are generally provided. We have the ruling gradient, we have the pusher or the helper gradient and we have the momentum gradient and there are some gradients which are provided in the station yards. So, they are the specific type of the gradients being provided in the station areas, basically what we can see. So, the previous three conditions are generally being provided when the connectivities between the two places are provided.

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Gradients - Types

- **Ruling Gradient**
 - Determined by maximum load that a locomotive can haul with maximum permissible speed.
 - Extra Pull required by locomotive on gradient with θ inclination,
$$P = W \sin \theta$$
$$\sim W \times \text{gradient}$$

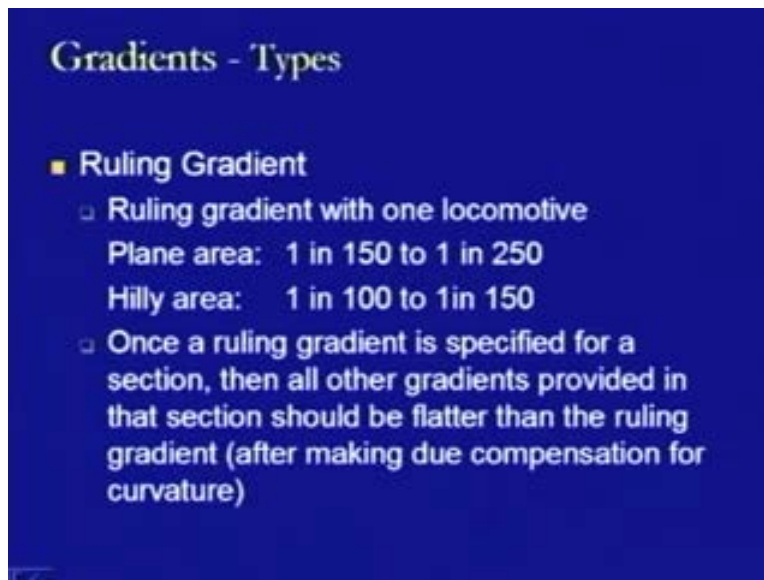
So, we will be starting with the different type of the gradients now and the first gradient which we are discussing here is ruling gradient. This is the design gradient basically, because it is determined on the basis of the performance of the locomotive and at the same time, it tries to look at the total amount of load which that locomotive can take up along with it, while negotiating any gradient without any loss or major loss in the speed of the movement. So, that is what is basically a ruling gradient. So, it is determined by the maximum load that a locomotive can haul with maximum permissible speed.

So, we are not interested in the reduction of the speed, we are not interested in the reduction of the total load which is to be carried out, but at the same time the locomotive should be able to traverse that rise or fall along with both the constraints. So, in this case, whatever is the value which is coming of that gradient in terms of so much rise or fall with respect to the horizontal distance being moved, that is going to be the ruling gradient and looking at these performance features of the geometric as well as of the locomotive characteristics, this is defined as a design gradient for any railway track. So, most of the railway tracks, which needs to be designed, should try to keep the gradients within the ruling gradient condition.

Extra pull required by the locomotive of the gradient with any inclination, theta inclination we have to look, we have to consider this value. That is what is the total amount of the pull which is required as we have seen and this is obviously equal to the resistance being offered by the gradient and this resistance being offered by the gradient, we have seen previously, it equals to W multiplied by g divided by 100.

So, in this case, this extra pull will be defined as P is equal to W into $\sin \theta$ or this $\sin \theta$ may be, because the θ is very small, it can also be transformed into $\tan \theta$ and when we transform into $\tan \theta$ then it is nothing but a gradient only, because the gradient is generally defined as say, 1 in x and 1 in x is nothing but $\tan \theta$. So, that is why what happens is that this extra pull is nothing but, this is W multiplied with the gradient. So, this is how these two things are getting correlated. W is the load which is to be carried out, P is the power which is required and g which is the gradient, which is to be traversed.

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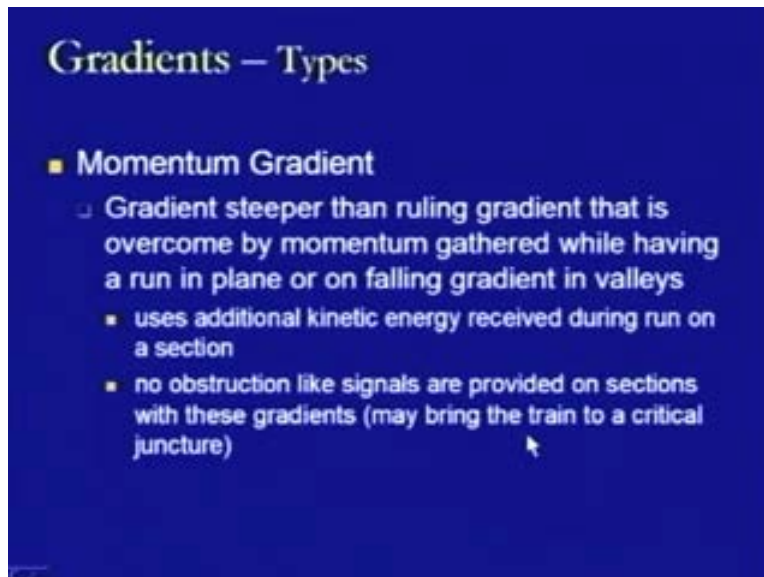


Then, in the ruling gradient, ruling gradient with one locomotive in the plane area is generally defined or restricted within the range of 1 in 150 to 1 in 250, whereas in the case of hilly area, it is 1 in 100 to 1 in 150. Now, once the ruling gradient is specified for

a section, then all other gradients which are provided in that section should be flatter than the ruling gradient. So, this is one aspect which as I have discussed already, because this is the design gradient, therefore any gradient which is to be provided should be flatter than this gradient. Then only we will be within the performance limits of any locomotive.

Now, one restriction which is being restricted here is that if there is any curvature along with the gradient, then some compensation has to be made in terms of the two types of the resistances being offered here now. One is the resistance offered by the gradient and another, the additional resistance which is offered by the curvature. So, to look at this one, the gradient has to be reduced by some value, so that the locomotive can traverse this gradient, where the gradient as well as the curvature is being provided without any loss of power.

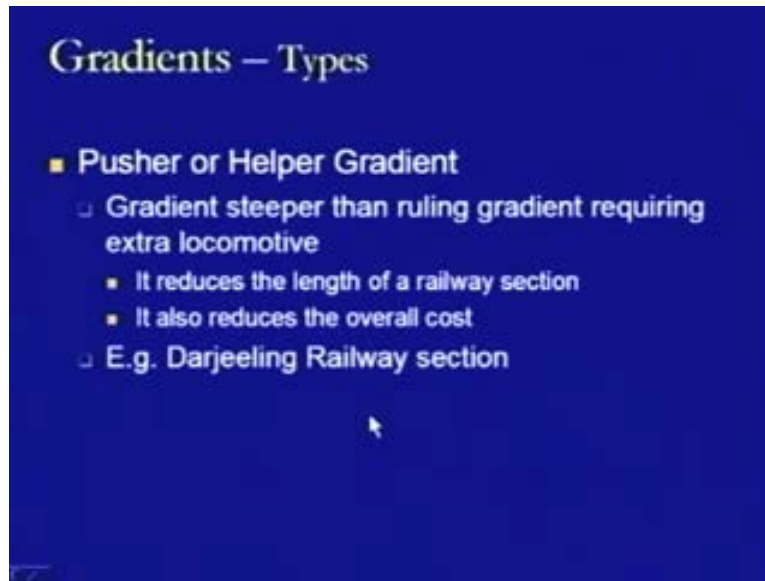
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So, another type of gradient is momentum gradient. In the case of momentum gradient, the gradients are steeper than the ruling gradient and this is overcome by the momentum which is gathered while having a run in plane or on falling gradient in valleys. What happens in this case is that there are certain locations where the ruling gradients will not be helping. Therefore, if a gradient is steeper, then the ruling gradient is to be provided.

But because this will be creating a loss in the power of the locomotive just before a momentum gradient, what we do is we provide a plane gradient so that the locomotive can recoup with the power or the falling gradient is provided which provides the additional kinetic energy, so that with the help of this additional kinetic energy the steeper gradient can be traversed and in such cases where these momentum gradients are provided, no obstructions like signals are provided on sections where these gradients are. Otherwise, it will bring the train to a critical juncture that is it will not be in a position to move forward.

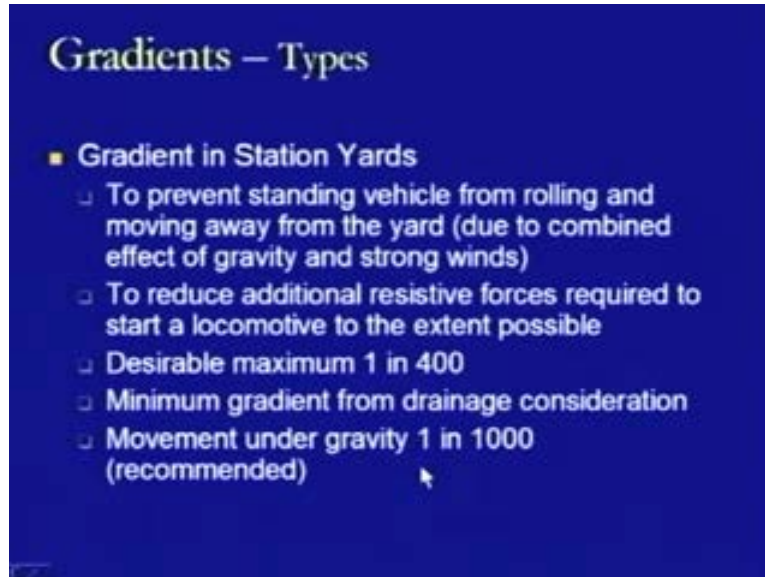
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Then, the next type of gradient is pusher or helper gradient. In the case of pusher or helper gradient, this is another steeper gradient than the ruling gradient and in this case, we require an additional extra locomotive. So, it is further higher condition or steeper condition of the gradients and it reduces the length of the railway section and it also reduces the overall cost. What happens is that, otherwise if we have to go traversing a very large section, instead of doing that, here we just provide an additional locomotive and this helps in taking the train to a further higher level and at the same time, we have to see whether it is reducing the length of the railway section or not and it is also creating an effect in terms of the overall cost and there are certain sections where this extra

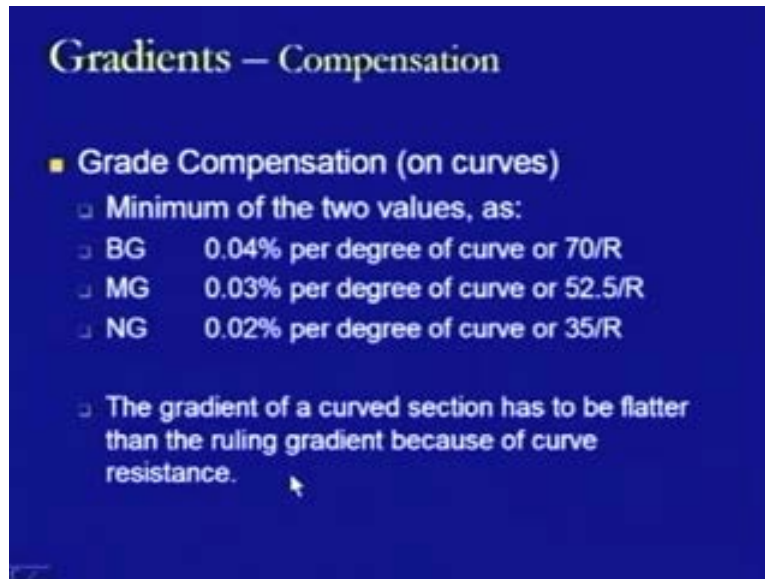
locomotive is provided, where the pusher or helper gradient is being provided. One of that example is the Darjeeling railway section.

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Now, in the case of the station yards, the gradients are provided to prevent the standing vehicle from rolling and moving away from the yard, because there is strong wind or gravity effect. Because of that, if we are providing the gradients, then the vehicles should not move away. We have to look at the gradient which is sufficient enough, so as to keep the things in position. So, to reduce the additional resistive forces which are required to start a locomotive, to the extent possible we have to provide these gradients and this one, the desirable maximum value is 1 in 400. So, another value is to be provided which is the minimum value and this minimum gradient is defined on the basis of the drainage considerations. That is what is the gradient which is required, so as to drain off the water and this value is 1 in 1000. So, in between these two values the gradient is to be provided in a station yard.

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Gradients – Compensation

- **Grade Compensation (on curves)**
 - Minimum of the two values, as:
 - BG 0.04% per degree of curve or $70/R$
 - MG 0.03% per degree of curve or $52.5/R$
 - NG 0.02% per degree of curve or $35/R$
 - The gradient of a curved section has to be flatter than the ruling gradient because of curve resistance.

Finally, the last thing which is to be discussed is the grade compensation, where the curves are being provided. As we have discussed that due to the curves there is an additional resistance which is coming and so as to remove this additional resistance, the gradients are eased out and the value by which the gradients are eased out is the minimum of 0.04% per degree of curve or 70 divided by R in the case of broad gauge or 0.03% per degree of curve or 52.5 divided by R in case of meter gauge and 0.023% per degree of curve or 35 divide by R in case of narrow gauge. So, this is what is about the grade compensation and whatever we have discussed in today's lecture is related to the left over components of the geometrics that is the vertical curves and the gradients. With this we complete the geometric features of the railway track and we will be starting with the other features in the coming lectures and I say good bye to you. Thank you.