Foundation Engineering
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Module - 03
Lecture - 11
Stability of Slopes

Hello viewers, welcome back to the lectures on Stability of Slopes, today it is our sixth lecture, in our previous lectures, we have already discussed the infinite slopes. The these slopes, in these slopes we assume the failure surface to be parallel to the ground surface. And we had considered many cases, of the drainage conditions, then we started with the finite slopes, finite slopes are smaller in aerial extent. Generally, they are manmade structure, there we discussed the Culmann's method

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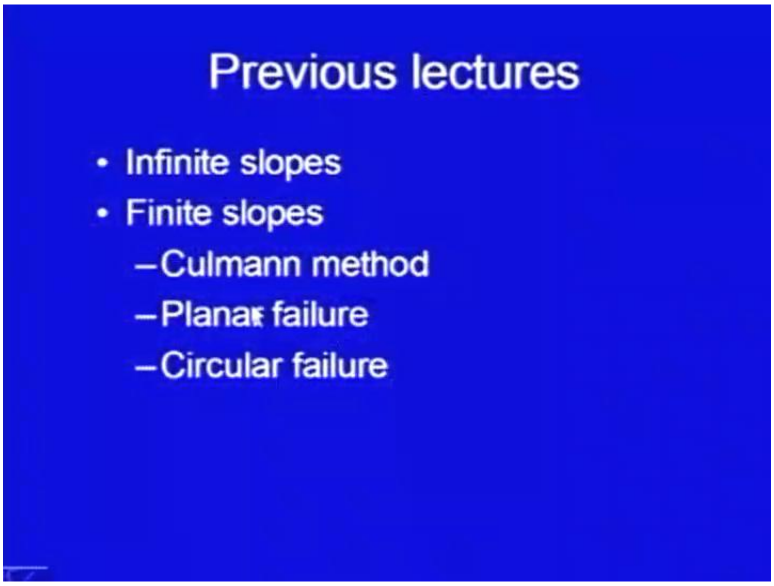

Then, we have already discussed planar failure and then we started discussing about the circular failure, where the failure surface is taken circular.

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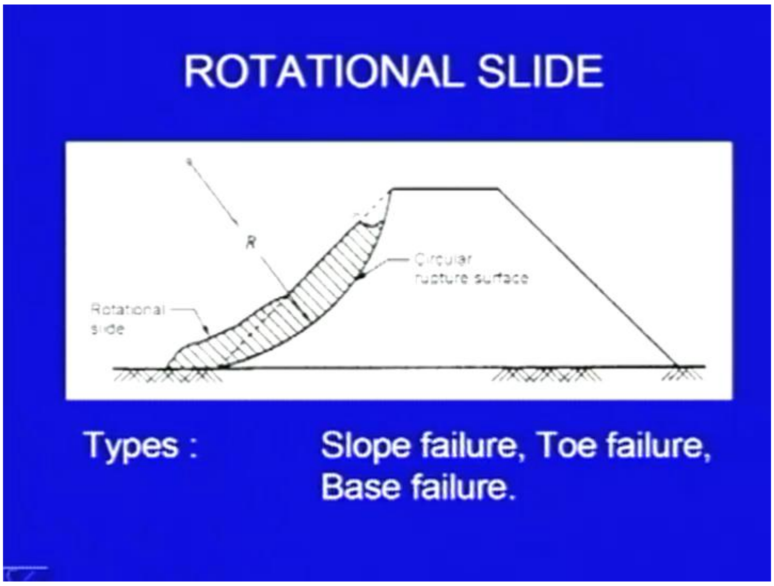

This was the case of a rotational slide, here it is an embankment, and this is the failure surface, which we have discussed last time. And in 2 D this surface is circular, in 3 D it is cylindrical surface, perpendicular to the plane of paper we are taking 1 unit dimension. The general term is rotational failure or rotational slide, and we also discussed different types sub categories, these were slope failure toe failure and base failure.

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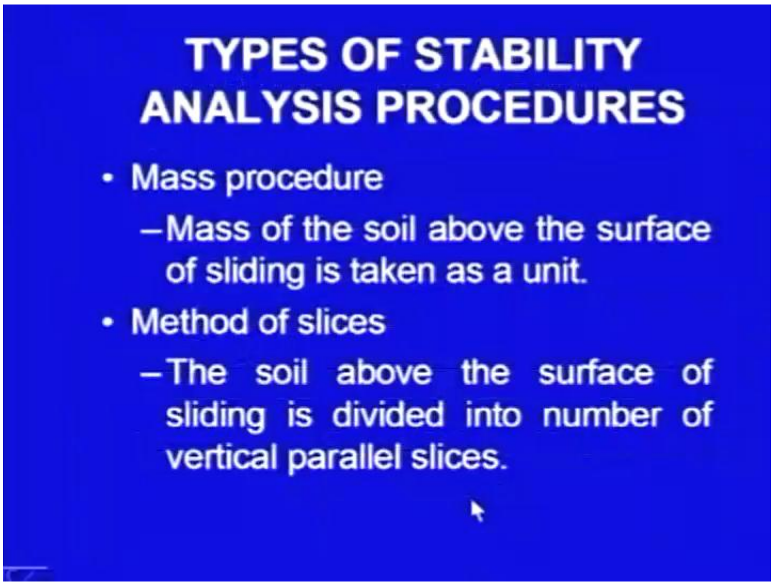

Then, we discussed about the types of the stability analysis procedures, the first category is of those methods, where the mass of the soil above the surface of sliding is taken as a whole, as a unit the entire body is considered as a free body, and it is equilibrium is considered for analysis. In the second method, which is called as method of slices, the mass is divided into number of vertical parallel slices, so we had started discussions on the first method mass procedure.

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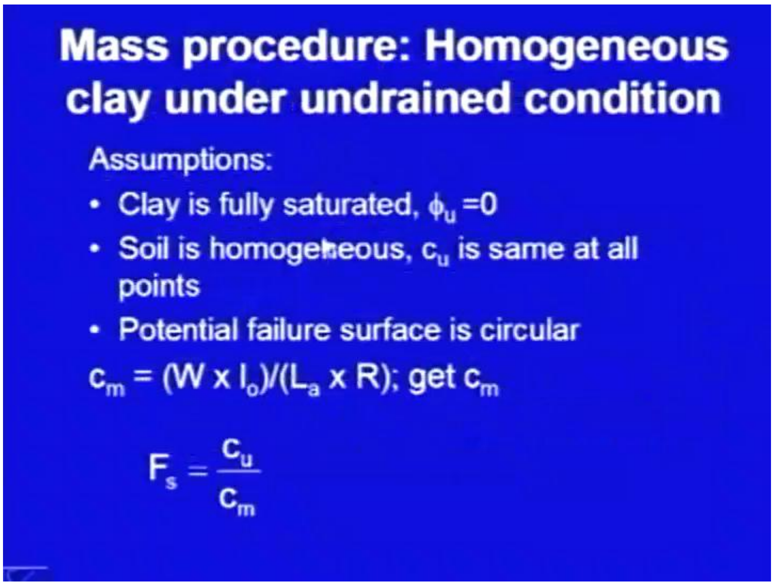

In mass procedure, we had taken the first case, it was homogeneous clay under un drained condition the slope was having clay, which is assumed to be fully saturated. That means, phi u was 0, soil was assumed to be homogeneous, C u was same at all the points on the failure surface and the potential failure surface, was assumed to be circular.

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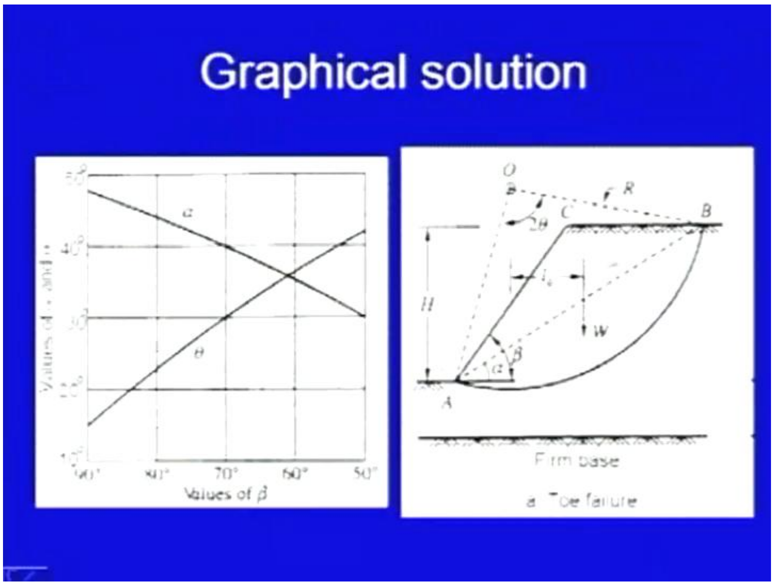

The method of analysis included, finding out the moment, which are trying to rotate this slide, this is the example here. So, we took this circular surface and then, here it is the weight, it is lever arm is L 0, this is the center of the circle, this circle we have taken arbitrarily as a first trial circle we have taken, this is the radius. So, we find out the weight of this wedge, and the moment, which this weight is trying to generate and this moment is being registered by the cohesive forces, which are acting all along the periphery of this arc.

So, if C m is the cohesion ((Refer Time: 04:12)), if C m is the cohesion, then you can find out, the cohesive forces and their moments. So, C m into, into the length of arc into radius, that gives you the resisting moment, and for equilibrium case, when the limiting limit equilibrium condition is satisfied, these two moments are taken equal, means the resisting moment and driving moments, they are taken equal. And, from there, you can find out, what is the value, which is required for the cohesion.

So, this much value of the cohesion is required to keep it in equilibrium, so this is mobilized cohesion. So, once we get, that C m, factor of safety can, easily be found out, by dividing C u by C m, C u is the available strength, available un drained shear strength, So, C u upon C m this is the mobilized value, so F s factor of safety is C u upon C m. So this is what, we had discussed last time, and I was discussing about the graphical solutions.

Because, now what we have to do is, after taking one circle and doing lot of calculation, we have to take another trial circle, another center, and the same procedure has to be repeated, number of times, and then we have to find out, that circle, which is having minimum factor of safety. So, the graphical solutions are very easy in these cases, so here, it has been produced, the graphical charts are available, so on the x axis, here you can see it is, slope angle beta.

And, on y axis, these are two parameters alpha and theta, please refer this figure, theta is this angle 2 theta, 2 theta is the angle subtended by the arc here, and alpha is this angle, which the base this particular line, it meets with the horizontal. So, the method becomes very simple, this is the case of the toe failure, what we have to do is corresponding to the given beta value, we have to read alpha and theta. Say, for example, if I take, alpha is equal to 55 degree somewhere here.

Then, corresponding to that, I will read, here alpha, alpha will be somewhere more than 30, and then also, I will read, here theta, this is almost very near to 40, somewhat less than 40. Once you get, this theta and alpha, now you can very easily draw this triangle, draw a line at an angle alpha, get point B, and then this angle is 2 theta, you can very easily get this angle, and then, you can draw this triangle, and get the center, and then rest of the analysis, can be done very easily.

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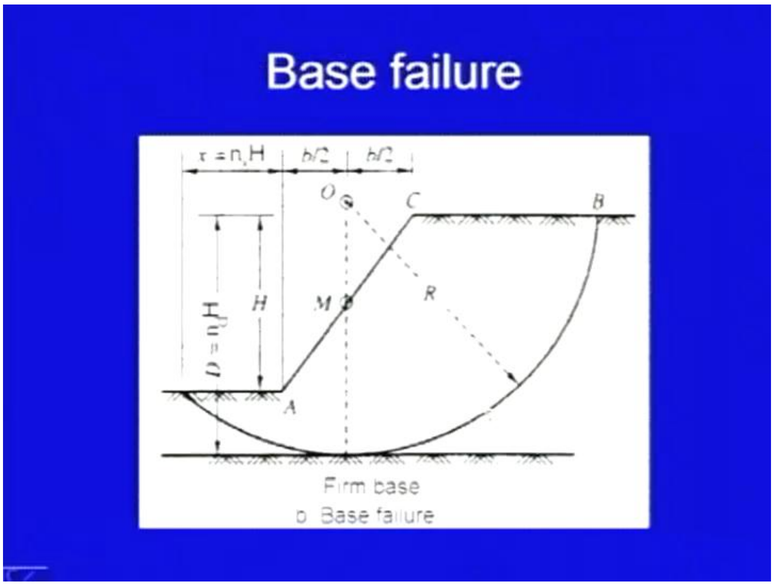

If it is a, case of the base failure, when the circle is passing through the base, then what we do is, the center has to, the center of the circle has to lie on a line, which is drawn from midpoint M of the slope, and this line is vertical. It is, because the circle has to pass in this manner, this is the firm base, it is a base failure, so circle is tangential to the base, and base we are assuming to be horizontal, so the circle has sorry, the center has to lie here, this angle, this line and the, horizontal line they should be perpendicular to each other.

So, here, the O is the center of the circle, and these are the parameters, H is the height of the slope and the base is situated at n d times, H depth below this level, so n d is a parameter. Another parameter, here in this analysis in this graphical solution is n x here, so x is n x into H, this gives you the point, where this, circular surface intersects the, the ground, that distance of this point, from toe is n x into H, so these are the parameters n x, n d.

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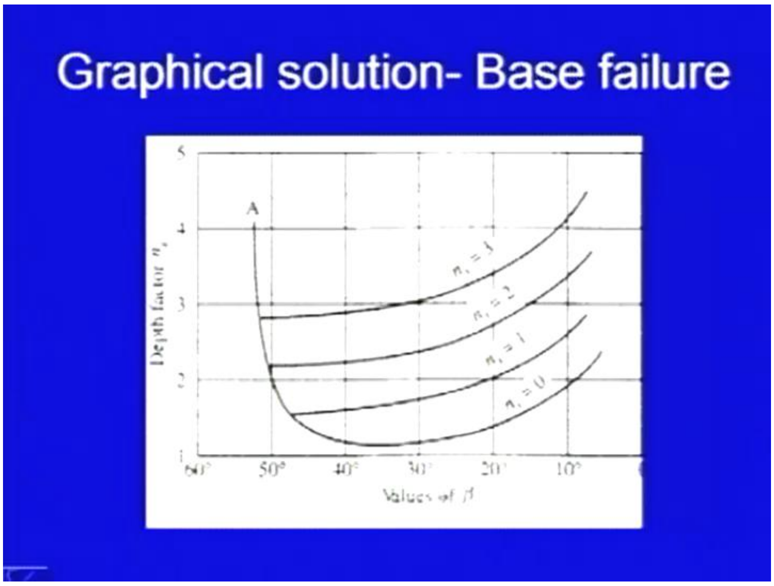

And then, one can the graphical solutions are available here, for any value of beta, for these are beta values here on x axis, on y axis these are n d values, and these are the different curves for, different n x values. So, for any beta value, say for example, 30 degree, and for any n d value, say for example, 2, so n d is equal to 2, beta is equal to 30, here it is the point, so this is roughly, this curve is n x is equal to 1, this curve is n x equal to 2, so you can interpolate in between them, what is n x.

And then, you can find out, once n x is known, you can find out, the point, where the circle is touching, the, this, this point, how far away ((Refer Time: 09:46)), from the toe it is touching, the ground and then, you can complete this circle.

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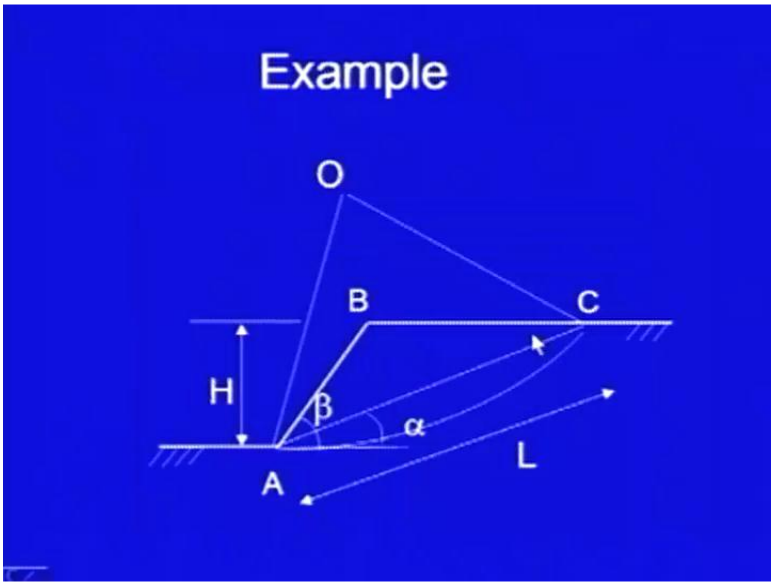

Let us, take an example, this is an example of toe failure, the, this is the general diagram, H is the height, beta is the slope angle.

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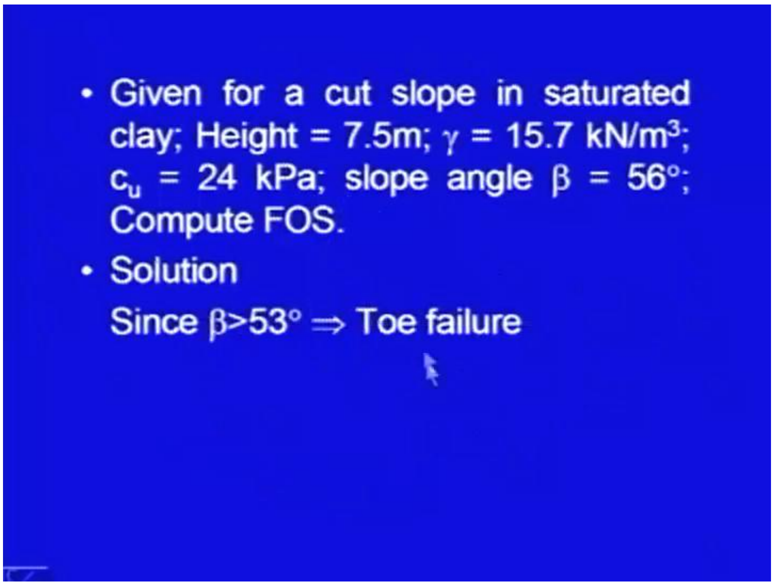

And, these are the data, given for a cut slope, in saturated clay, the height of the slope is 7.5 meter, and the saturated unit weight is 15.7 kilo Newton per meter cube, C u is 24 k

Pa, slope angle beta is 56 degree, and we are asked to find out the factor of safety. Now, I told you, in the last class, that for beta more than 53 degree, the slope in this case of, in this clays condition, will always be a toe failure. So, here, beta is more than 53 beta is 56, so it is more than 53, so it is a toe failure, so let us, use the graphical solution for toe failure.

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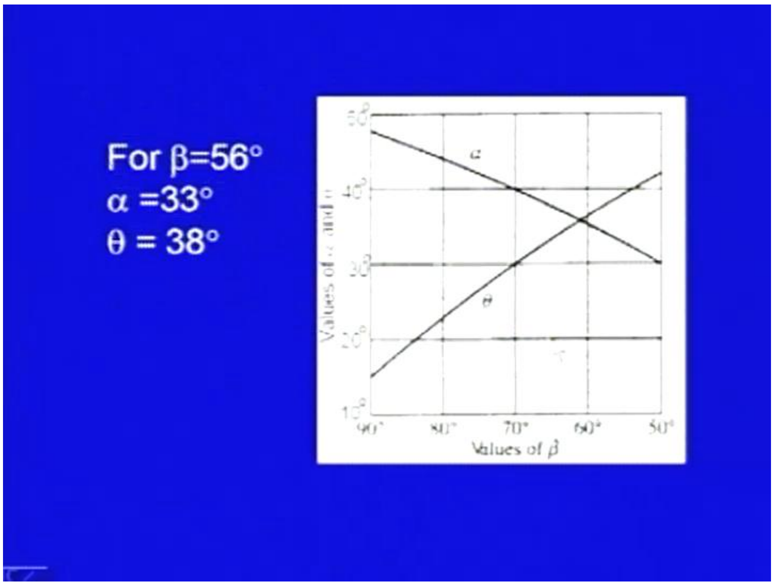

So, for 56, so ((Refer Time: 11:04)) this is 50, here it is 60, so somewhere here, the 56 will be there, and corresponding to that, the alpha value roughly comes out to be 33, and here, this is theta, it comes out, roughly about 38.

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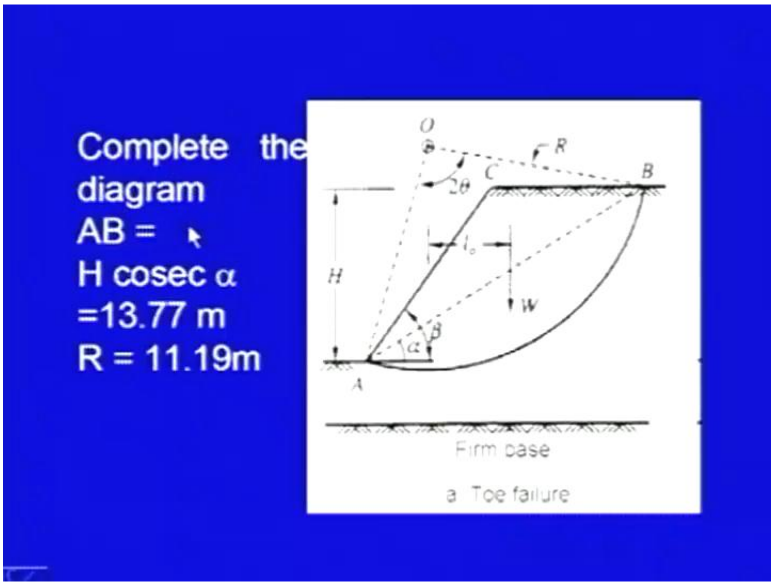

So, now you know these, two angles, you can complete the diagram, you can make a trial, you can solve this problem graphically also, or you can solve it analytically. So, this angle is known to you, alpha is known to you, 2 theta is known to you, once 2 theta is known to you, you can find out this angle, so and then, rest of the things have to be done. So, here, A B this comes out to be H, into cosec of alpha, this comes out to be 13.77 meter, and the radius in this particular case comes out to be 11.19 meter.

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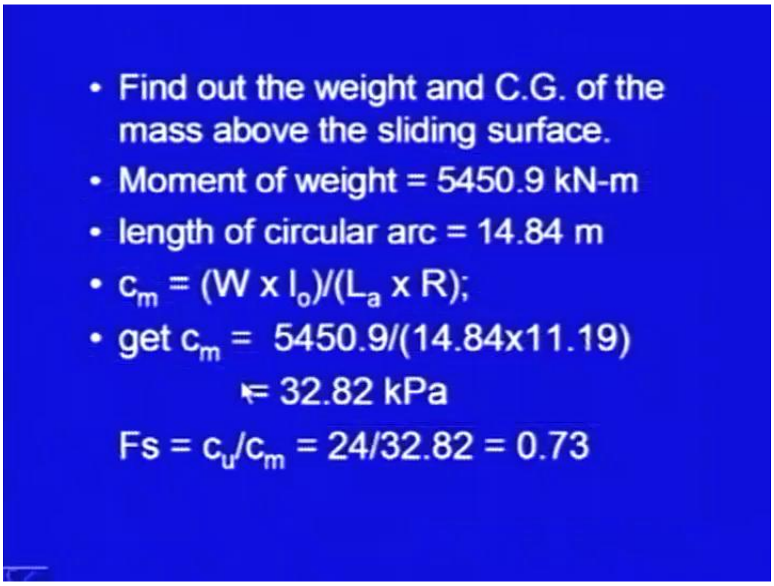

I am not showing the rest of the calculations, they are simple, you can do the geometrical ((Refer Time: 12:12)), you can use the geometry. And then, you can find out the weight and C.G. of the mass above the sliding surface, then you can find out the, moment of the weight, as I discussed, this moment will be weight into it is C.G. from the, center of the circle. In this case, I got this value, then length of circular arc also, you can get very easily, and then, you can get the mobilized cohesion, the cohesion, which is required to keep, this slope in equilibrium.

So, C m comes out to be 32.82 k Pa, so this is the value of C m, which is required, and the available value, in given in the problem was 24 k Pa. So, you can see here, available value is 24, we required around 32.82, factor of safety is 0.73, so it is not safe, we need more, the for the stability of the slope, higher value is required, and the available strength is 24, so factor of safety is 0.73.

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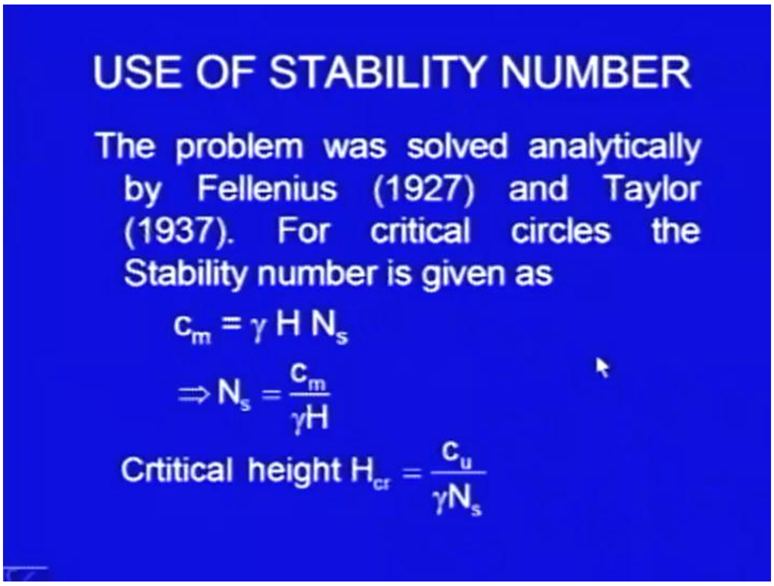

Now, there is another way of solving these problems, this problem was solved by, a solved analytically, by Fellenius and Taylor, and for critical circles, they have, they have given the solution, in terms of the stability number. The stability number is defined as, N s is equal to C m upon gamma H, this is we have already defined it, so for case of the mobilized cohesion N s is will be equal to C m upon gamma H. And, for the case of the critical height, so when critical height means, maximum height if you want to calculate, what is the maximum permissible height, so factor of safety will be equal to 1.

C u and C m will become equal, so in that case, if you take it on this side, so critical height, will be equal to C u upon gamma into N s.

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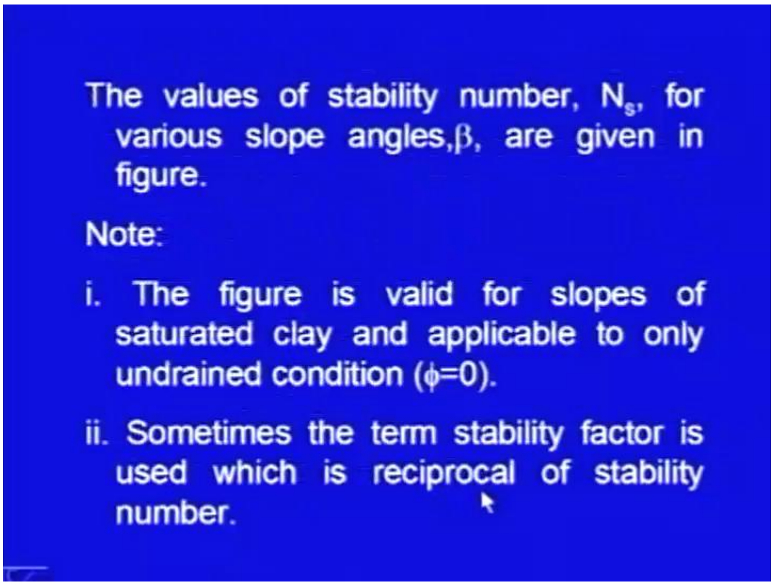

So, this N s has been given, the values of stability number N s, for various slope angles beta are given in the figure, I will show that figure to you. This figure is valid, this chart is valid, for slopes of saturated clay and applicable to only un drained condition, this is important point, this particular chart is for this specific condition. One more thing here, I would like to mention is that, sometimes in the literature, you will find another term, stability factor, so this stability factor is nothing but, it is the reciprocal of the stability number, so we will use the stability factor also, in future.

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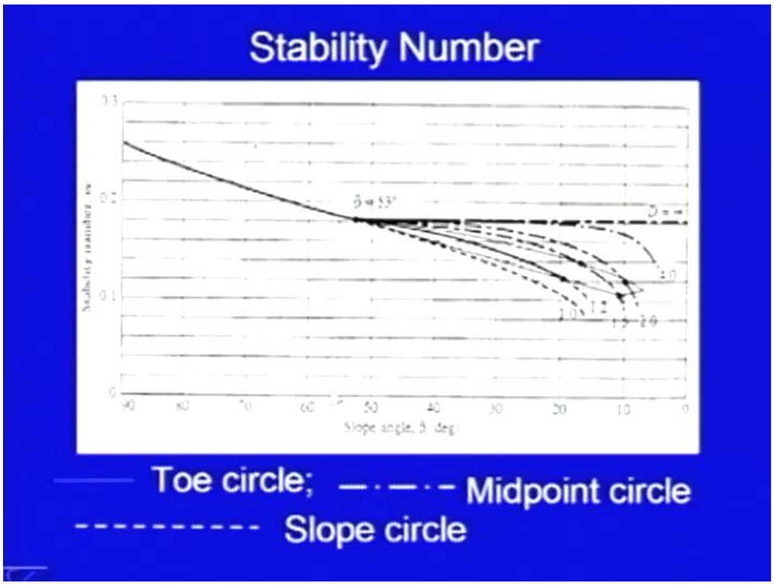

Here, it is the chart of the stability number, on the x axis, the slope angle is given in degrees, and on y axis it is stability number, so it is 0.1, 0.2, 0.3, value. And, you can see, from around 53, beta equal to 53, and more than 53, you have only one line, this means this is for the toe failure. So, this, this solid line is for toe failure, for steeper slopes when, if beta is more than 53 only the toe failure occurs, if it is less than 53, then this can be base failure or slope failure.

So, the charts are available, for both the cases, if it is a midpoint circle, midpoint circle means, say it is a base failure, in case of the base failure, the circle, the critical circle is called as midpoint circle. And, slope failure, if it is the circle is passing above the toe that is slope failure, so for that, different lines are available, so we have what we have to do is for corresponding value of beta, say for example, beta is 60 degree. So, if you take the intersection here, straight way, you get, the value of stability number and that, stability number is related with C gamma and H.

So, you can have, a relationship between C gamma H, you can have a parametric analysis, if gamma H, something is given, you can find out C, if C is given, you can find out H, and any, any sort of analysis, can be done.

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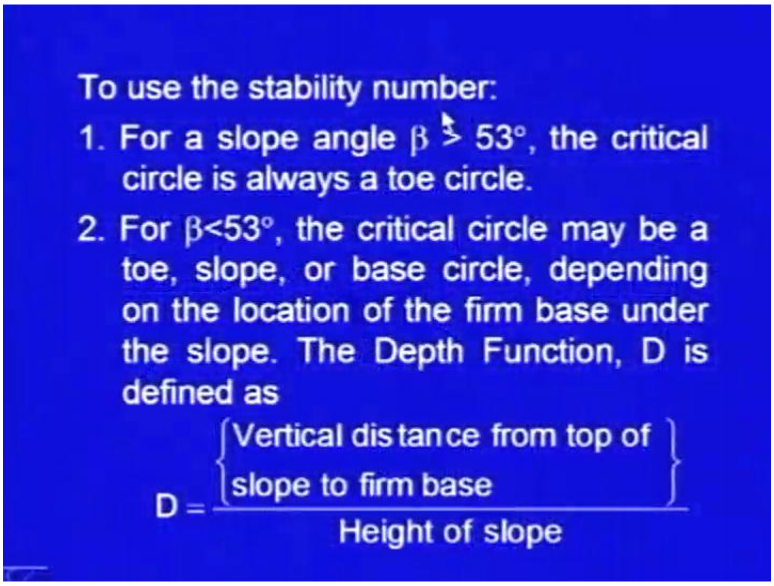

To use this stability number, for a slope, beta more than 53 degree, the critical circle is always a toe circle, that is, what I told you just now. And, if beta is less than 53, then the critical circle, may be a toe circle, it may be a slope failure, or it may be a base circle, depending on the location of the firm base, under the slope. The depth function D is defined as, so here, you can see the, there is a little bit difference, in the parameters of this graphical solution, please note down here what is D, you have to be cautious, when you use these results.

So, D here is defined as, vertical distance from top of the slope to the firm base, and it is divided by height of the slope, so it is the ratio, of the depth, of the firm base and the height of slope, that is D.

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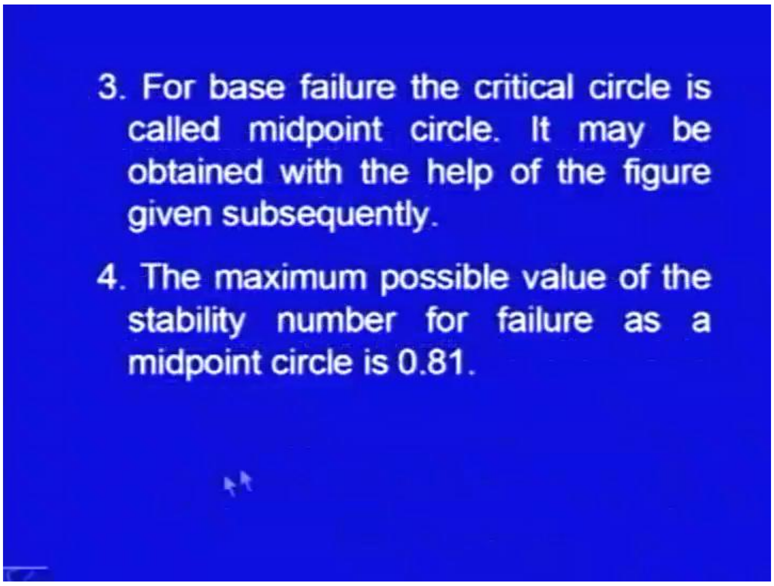

For base failure, the critical circle is called midpoint circle, this I told you, for when it is passing through the base, the critical circle will be called as midpoint. It may be obtained, with the help of the figure, which the figure will be given subsequently, i will just, show you it to you. Last point is that maximum possible value of the stability number, for failure, as a midpoint circle is 0.81.

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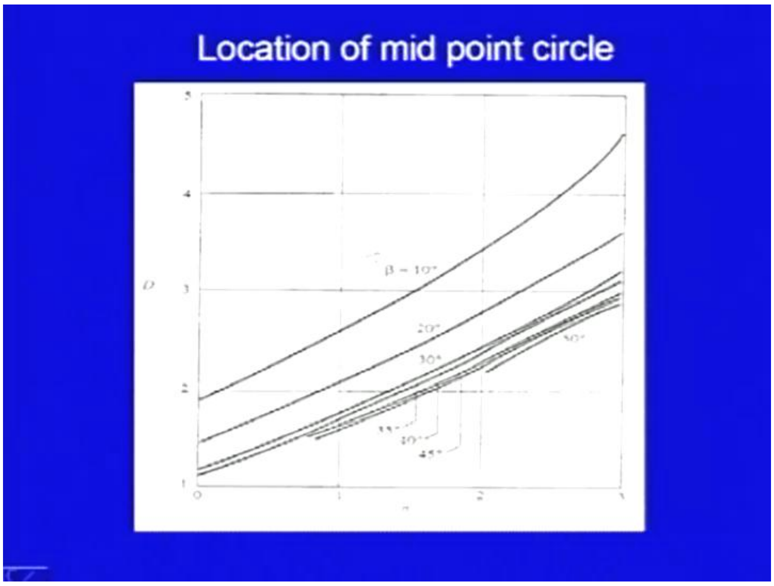

Here, this is the chart, which is, which can be used to find out the, location of the midpoint circle, I defined the parameter D there, D was the ratio, of the depth of the firm base to the height of the slope. So, D is here, and another parameter N is here, and for different beta values, these are the curves, so for example, if with, if beta is equal to 10, and D is equal to let us say 3, then for 10, you can find out, what is N.

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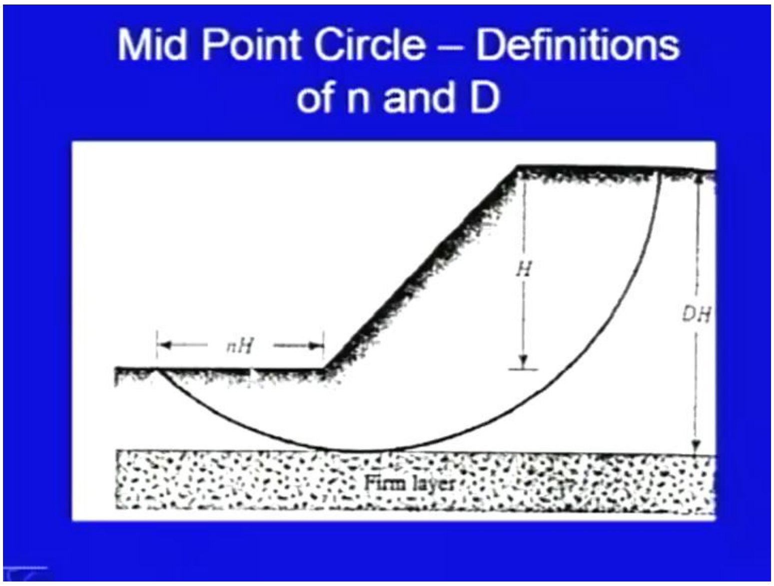

This is the, this figure gives you, again the definitions, N is, N which I defined there, N is the, the coefficient, which you have to multiply with H, to get this distance, the distance, where the circle, this circle touches the ground. So, relationship ((Refer Time: 19:37)) between N D, and beta is available, so if you know, the depth of the base, and if you know, the H value, you can very easily find out. So, to, to find out this, you have to use this graph, so for the given, N value for given beta value, you can have D or vice versa.

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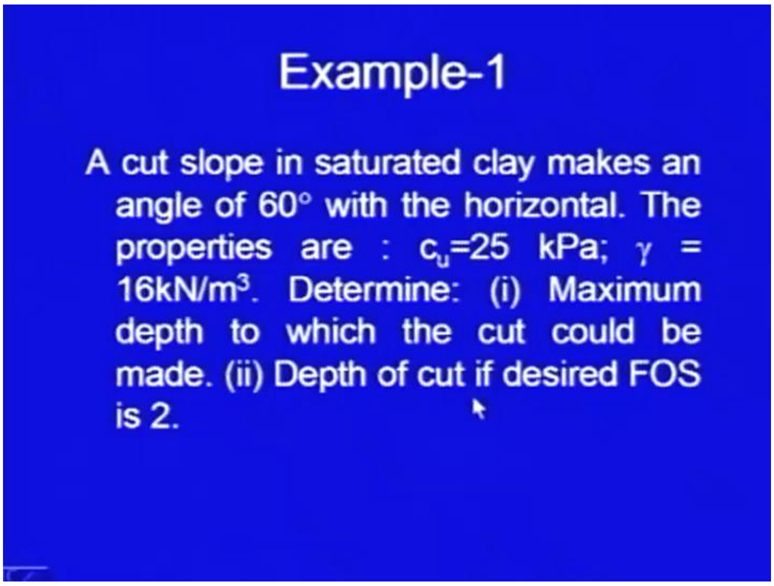

Let us, solve an example here, it is given, that a cut slope, in saturated clay, makes an angle of 60 degree with the horizontal, the properties of the clay are, C u is given as 25 k Pa, k Pa means kilo Newton per meter square. Gamma is 16 kilo Newton per meter cube we have to determine the maximum depth to, which the cut could be made, the critical depth, and secondly, the depth of the cut, if we need to have a, factor of safety of 2.

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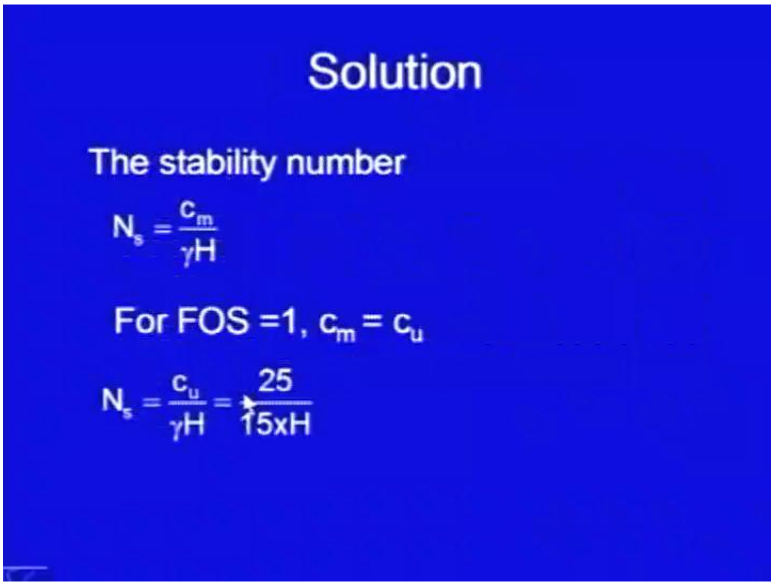

We will solve this problem by, using the stability number, the stability number is defined as, N s is equal to C m upon gamma into H, and when the factor of safety is 1, C m will be equal to C u, so N s is equal to C u upon gamma into H here, will be critical, H critical, so put these values, C u is 25, gamma is 15, H is unknown, so this is N s.

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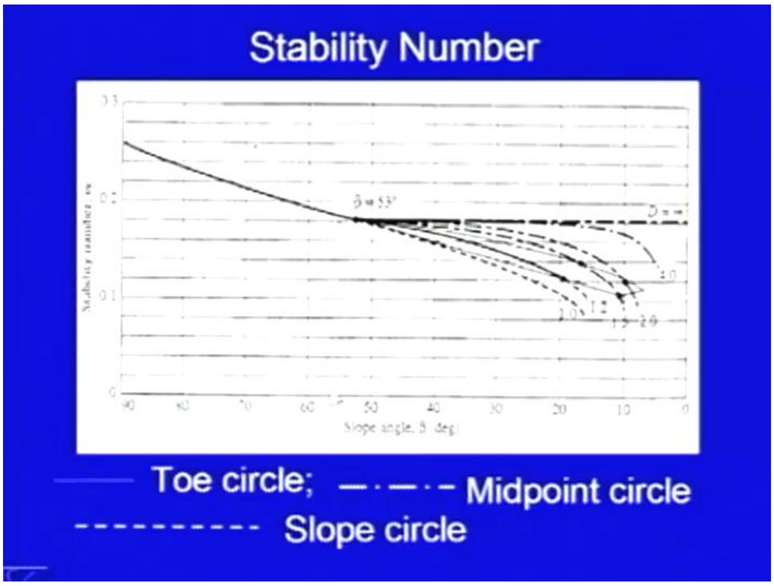

Now, from this chart, you can see the value of the stability number, corresponding to the given beta values.

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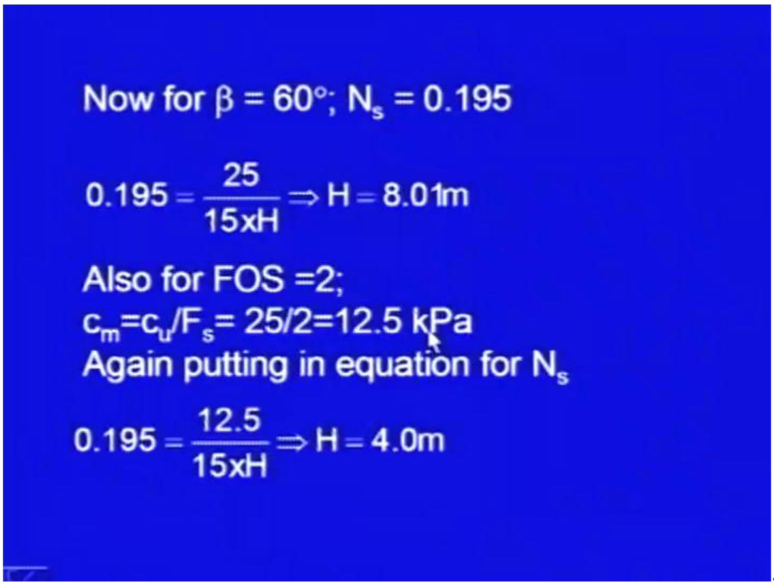

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Beta is given as 60 degree, so for 60 degree, you can see from here, this is 50, 60, and as I go up, this is the point of intersection, and from here, this is less than 0.2, little less than 0.2. So, here you can interpolate it, and I got this value, as 0.195, put this 0.195 in the previous equation, so point 0.195 will be equal to, C gamma, C upon gamma H, and that gives you, H is equal to 8.01 meter, so it is very convenient, to use these charts. Now, for the second part of the problem, it was, told that the factor of safety needed is 2, so that means, now, you can find out C m, C m is equal to ,C u divided by factor of safety.

Factor of safety is nothing but, the ratio of, ratio of available strength, divided by the mobilized strength, so from here, I get C m value, and C m value comes out to be, 12.5 k Pa. So, mobilized now is 12.5, the rest of the analysis is same, the stability number depends on beta, it is same here 0.195, and C is 12.5, gamma is 15, H is the new height, which is corresponding to 2 factor of safety, and by solving this equation, you will be getting the height, so this is the safe height, for safe factor of safety.

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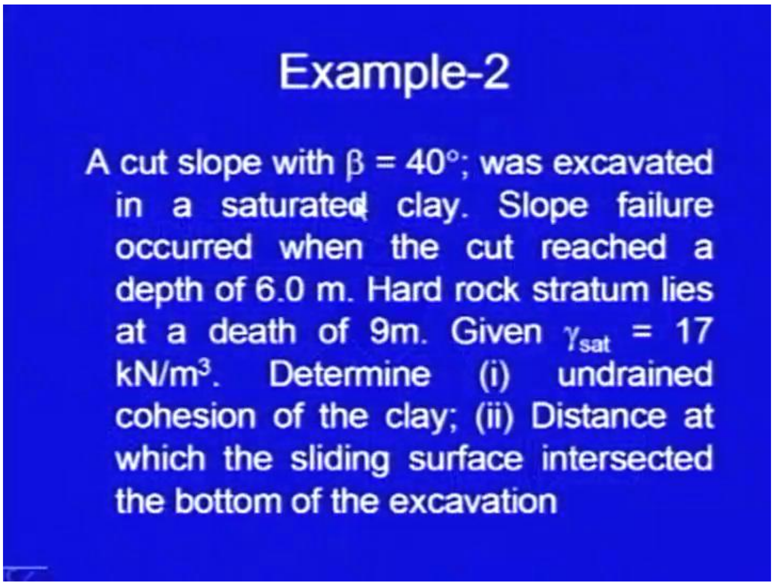

Let us, have another example, this is a cut slope, now the angle is small, angle is beta, beta is 40 degree angle, this was excavated in a saturated clay, slope failure occurred, when the cut reached a depth of 6.0 meter, Now, this is given, that when it was, 6.0 meter of the depth was reached, the failure took place, means the limiting condition was reached.

Hard rock stratum, lies at a depth of 9 meter, now this is given here, the firm base is, available at a depth of 9 meter, given gamma saturated is 17, determine, un drained cohesion of the clay, now here, it is the property, which we are trying to determine using the back analysis. Secondly, the distance at, which the sliding surface intersected, the bottom of the excavation.

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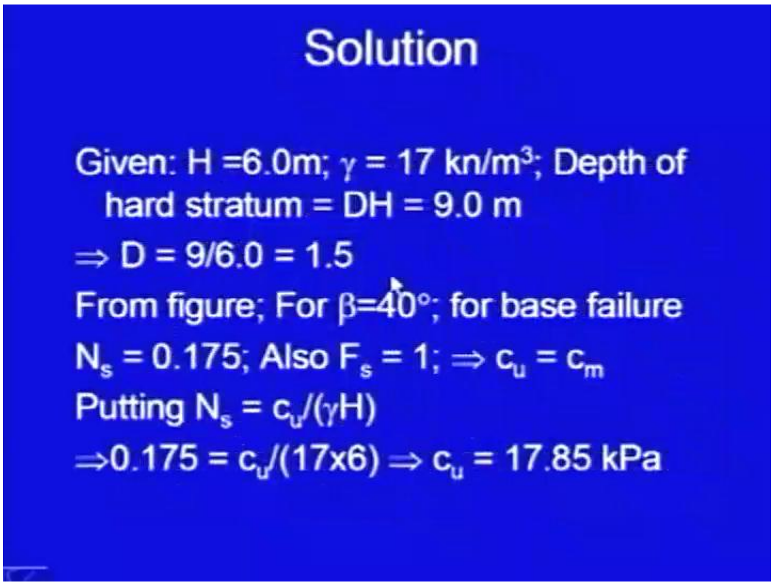

Now, it is given, that limiting height now is 6.0 meter, gamma is 17, and depth of the hard stratum is 9, now if you remember, the nomenclature, which we had used, was that, D into H is the, depth of the hard stratum. So, D into H is equal to 9, so this parameter D, of the chart, comes out to be 9 upon 6, that is 1.5, now from figure, from the figure, which I was discussing, where D and N, and the stability numbers were available. So, for if beta is equal to 40 degree and now I use the base failure, the, these, the curve the chart for the base failure is used, and you will be getting N s is equal to 0.175.

Also, factor of safety is 1 here, because it was, the limiting case, so C u is equal to C m, so the expression of the stability number will be, N s is equal to C u upon gamma H. So, 0.175 is equal to C u upon 17 into 6, and that gives you the, value of C u, as 17.85 k Pa.

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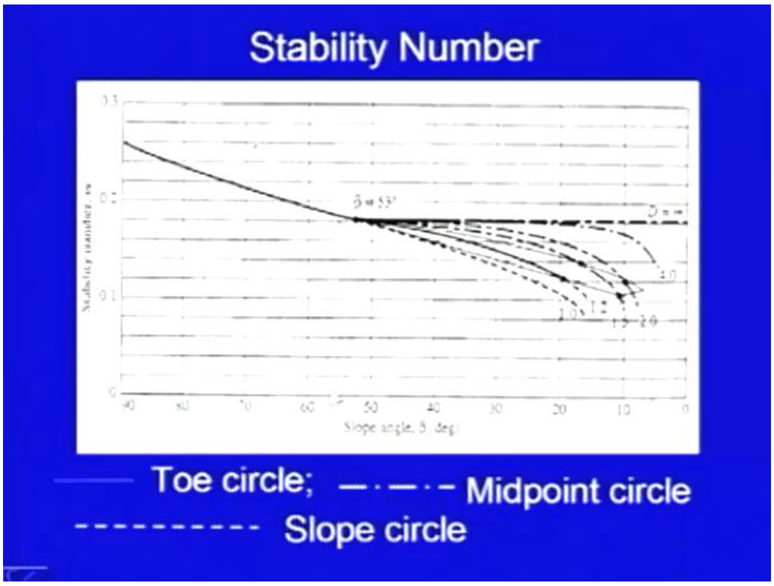

This is the chart, which I had used, here ((Refer Time: 25:38)) for beta equal to 40 degree, you can check, for base failure, so beta equal to 40 degree. So, here it is 0, 10, 20, 30, 40 I have to go, along with this direction, and here, you have to use the base circle, base circle means, midpoint circle here. So, this is the, these are the charts for different D values, and D here, it is given as ((Refer Time: 26:06)), D we have calculated as 1.5, so this is D is equal to 4, and 1.5 is here this one, 4, 2, 1.5, 1.2.

So, corresponding to 1.5, we get somewhere here, so somewhere here, and that value one can easily interpolate, that value was 1.175, so once, N s is available, now this expression, is simple to use, and you get value of C u.

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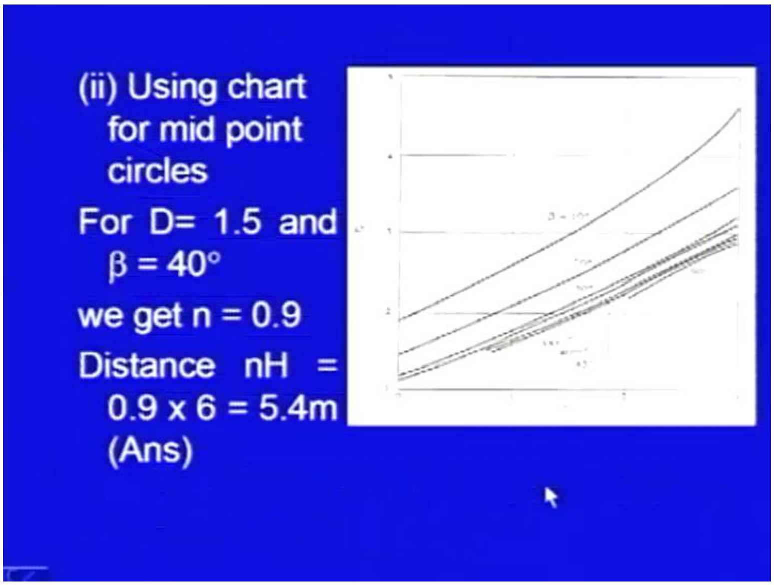

Now, the in the second part of the question, we have been asked to find out the point, where the failure circle is meeting the ground, so this is the chart, I have already shown it, on the x axis, you have N, on y axis, you have D. So, now, in this particular case, D is 1.5 and beta equal to 40, so D equal to 1.5 is somewhere here, so corresponding to this, and beta equal to 40, so 40 is this circle, so somewhere here, you get the point, and we get, n roughly, about 0.9.

So, distance n H, as per that, as per this graphical solution, comes out to be, 0.6, sorry 0.9, that is n into H, H is 6, that is 5.4 meter, so you get the final answer, where the circle touches the ground.

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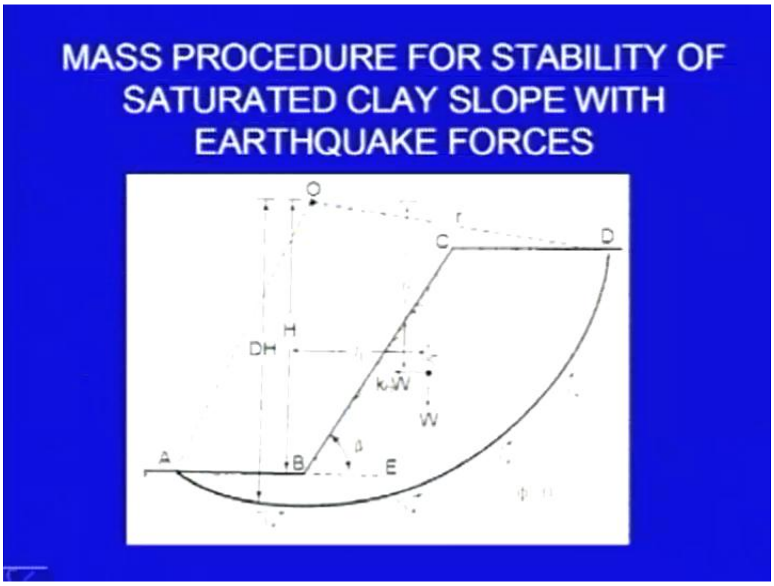

So, that was the, case of the, of the saturated clay, let me, now go to the next case, it is again a saturated clay, but we are now taking into account the earthquake forces also. So, here, it is the mass procedure, once again it is mass procedure, we are not dividing the mass into the slices, so it is mass procedure, for stability of saturated clay slope, with earthquake forces. So, here, it is the slope, the B point is the toe of the slope, and here this is a, trial circle we have taken, centre is O, and the CG is, CG of the mass above this failure surface, entire mass is here, weight W is acting in downward direction, at the CG.

The additional component, which we are going to get, in this case, is the force due to earthquake, and that force, is taken as, the weight of this, wedge multiplied by a coefficient KH. KH is the ratio of the acceleration due to earthquake, divided by, by the acceleration due to gravity, so this KH multiplied by W, that gives you, the force, which acts here, and the force is taken to be acting, in this direction, so it is destabilizing it, so you can find out now, the factor of safety, by considering all these forces.

We have to consider the moment about point O, about point O this cylinder is trying to rotate, resisting forces, will be available from the cohesion, which is acting along the periphery of the cylinder. This weight, will be acting in the downward direction, so the lever arm, will be this distance, the perpendicular distance between these two lines, one line is passing through the center, another through the CG. And, the earthquake force KH into W, it is acting in this direction, in the horizontal direction, so for this, this is the lever arm.

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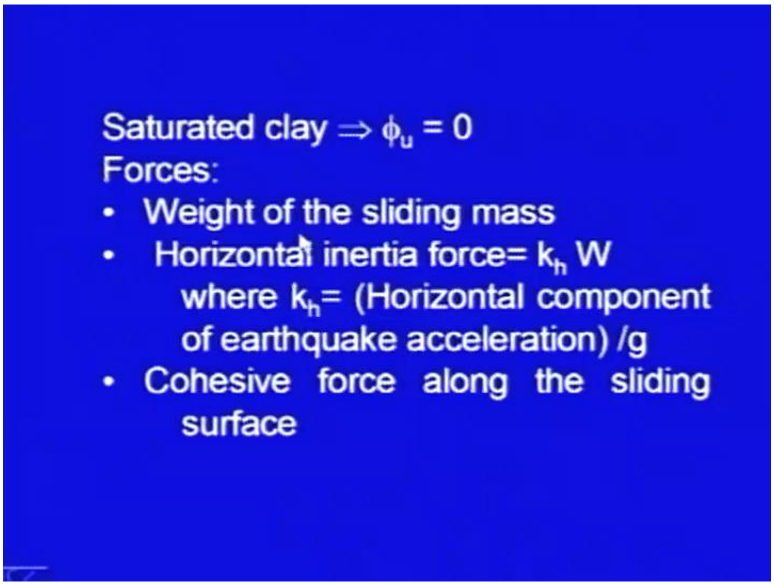

So, again we are assuming phi u is equal to 0, and the forces, as I discussed, are the weight of the sliding mass. Number 2, horizontal inertia force KH into W, where KH is the coefficient, it is the ratio of horizontal component of earthquake acceleration and g, and third force, is again the same cohesive force, along the sliding surface.

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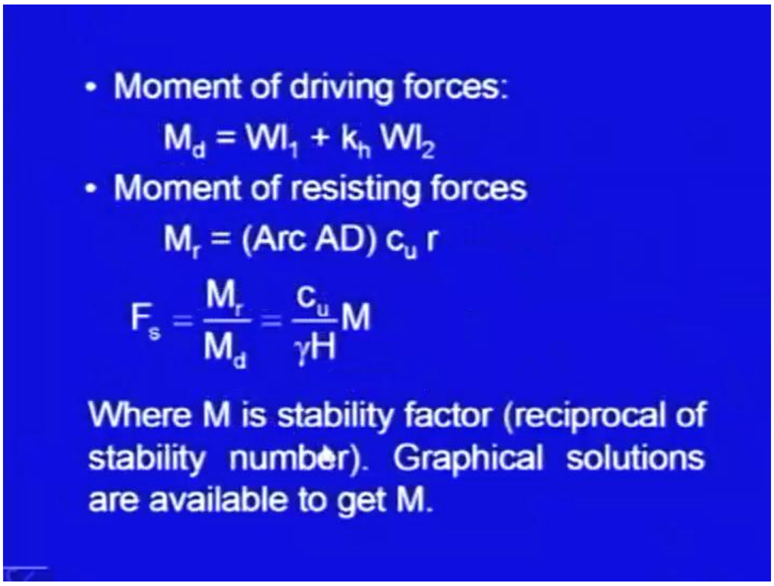

We will be taking the moments, so moment of driving forces M d will be equal to W into l 1, l 1 is the lever arm for weight, and l 2 is the lever arm for the force, due to earthquake, so W into l 1, plus KH into W into l 2. The moment of the resisting forces, will be the length of the arc, along which these forces are acting, and then, you have to multiply, them by C u, we are assuming, it is limiting case, otherwise C m can be taken, when it is a safe case, and into radius, so that gives, you the resisting moment.

And, the factor of safety expression is very simple F s is equal to, the dry the resisting moments, upon the driving moments. And, again, we have to use the same procedure, as we did last time, we have to find out the weight, we have to find out this CG, we have to do this analysis analytically or graphically, some standard graphical solutions are available, and in those solutions, it is, this factor of safety has been defined as, C u upon gamma H into M.

This is the way, in this, in this form, the graphical solutions are available, here, where M is stability factor, it is nothing but, the reciprocal of stability number. So, graphical solutions are available, and from those graphical charts, from those solutions, you can find out M, once M is available. Then, you can have a relationship between, factor of safety, C u gamma and H, and you can do the any analysis, any parametric analysis, some of them may be known to you, some of them may be unknown, you can find out the unknowns, graphical solutions are available, to get this stability factor M.

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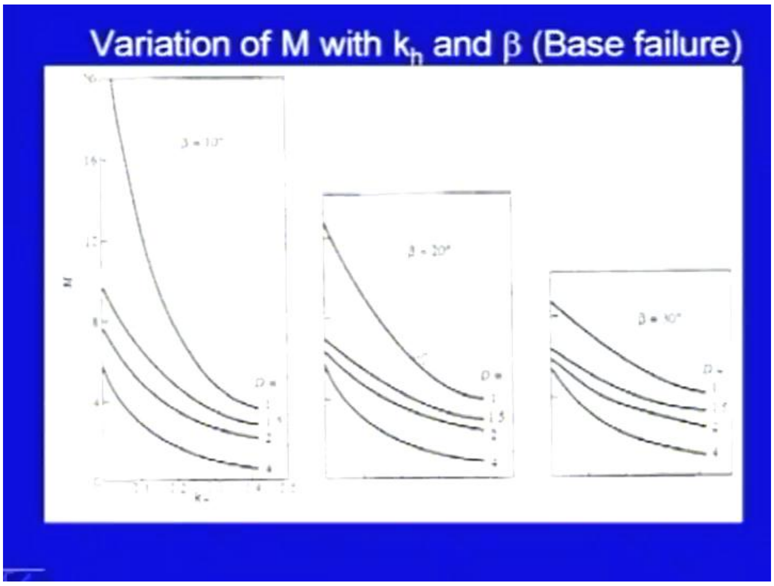

Here, are the solutions, on the x axis, you have the coefficient KH, on y axis, it is the stability factor M, and these curves, there are different curves, you can see one, two, three, four, four curves are there. There are for, they are for different values of D, D is the parameter, which we recently defined, the first curve, you can see, is for D equal to 1, then, second one, is for 1.5, then 2 and 4 here, and this first chart, is for beta equal to 10 degree.

So, similarly, here, now the values are not written here, on y axis and x axis they are the same, KH here, M here, and this particular value is 4, this particular value is 8, and this is 12. Similarly, the third chart, so the second chart is for beta equal to 20 degree, beta equal to 30 degree.

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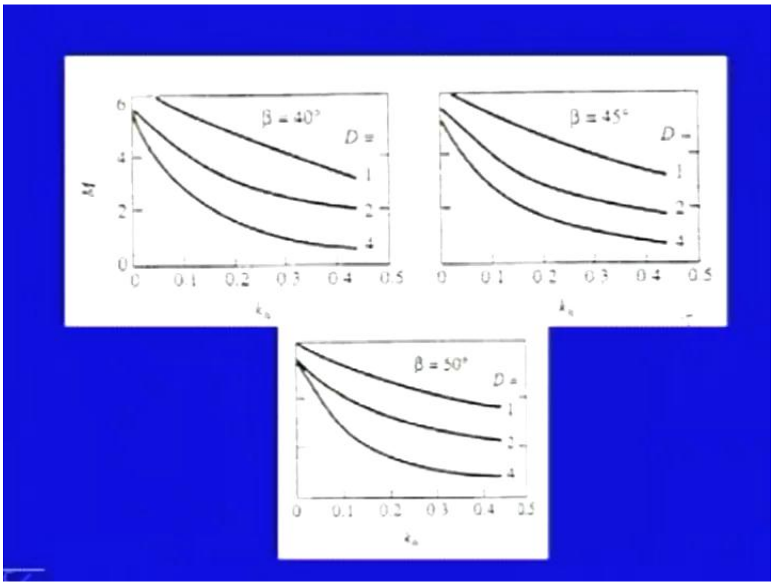

And then, you have beta equal to 40 degree, 45 degree, and 50 degree, on x axis, you always have KH, y axis, M and there are different curves, for different D values, so far intermediate values, you can do the interpolation.

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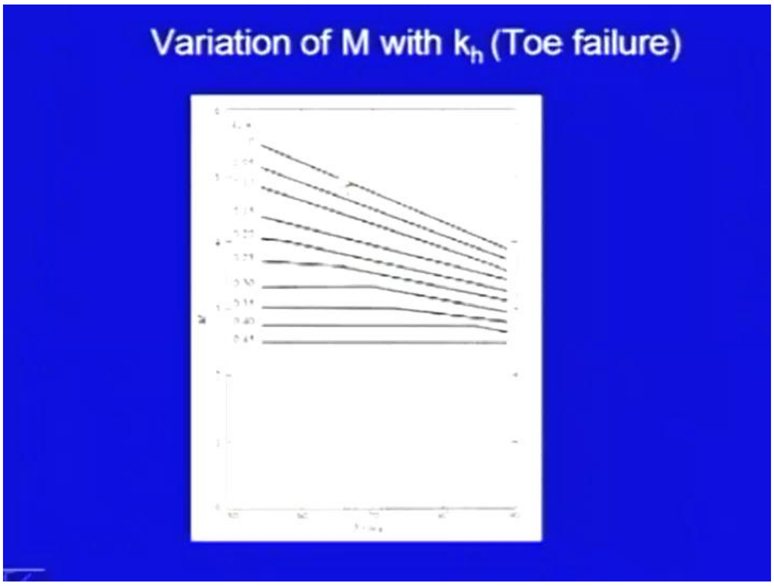

Here, the variation of the stability factor M is given with KH, for the toe failure, the earlier cases, were up to, you can see up to beta equal to 50 degree, if the beta is more than 53, it is the toe failure, which is going to take place. So, here on the x axis, it is beta, and on y axis it is M, and these are the curves, for different KH, so obviously, there is no parameter D, for toe failure. So, for different KH values, any KH value, for corresponding beta value, you can find out the M, and then, you can do the analysis.

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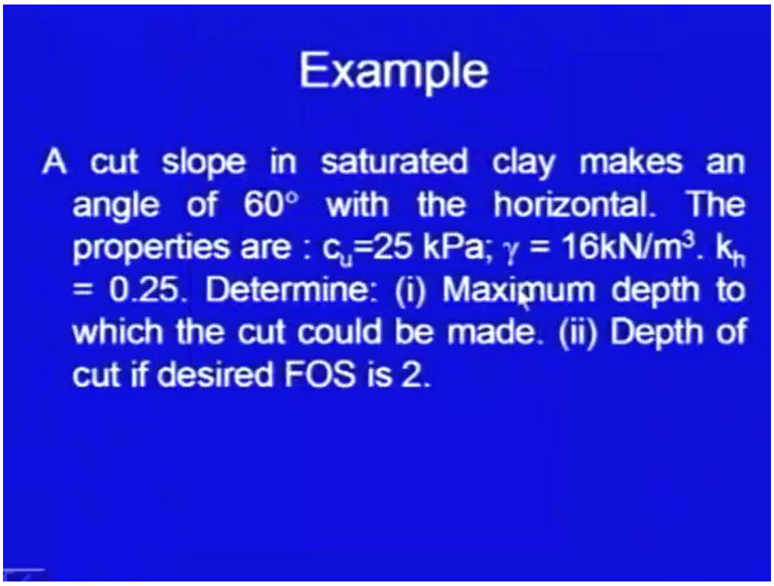

I will demonstrate, the applicability of this graphical methods, through this example, it is given here, a cut slope in saturated clay, makes an angle of 60 degree, with the horizontal. The properties are C u equal to 25 k Pa, gamma equal to 16 kilo Newton per meter cube, KH is given now, 0.25, determine maximum depth, to which the cut could be made, and depth of cut, if desired factor of safety is 2.

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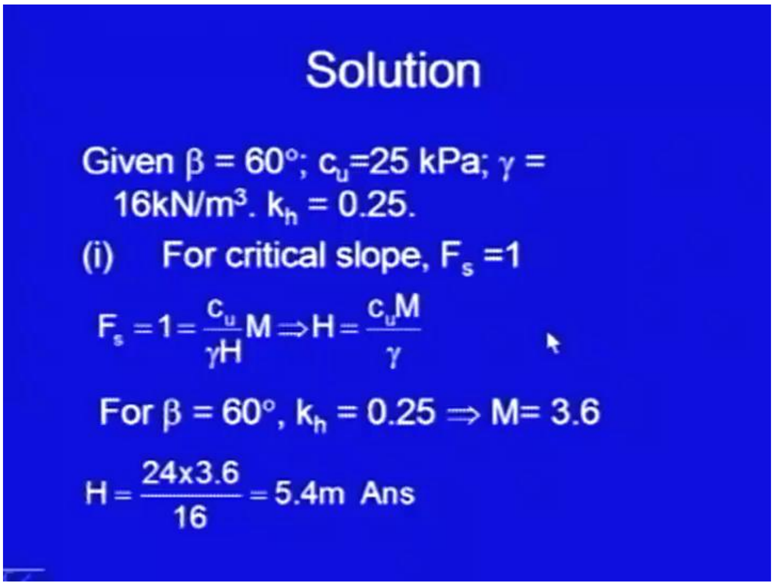

So, now, given beta is equal to 60, C u is this much, these are the parameters given, for critical slope, C u will be equal to C m, factor of safety is 1. So, F s is equal to, the equation is cu upon gamma H into M, so H is equal to H becomes, C u into M, divided by gamma. So, for beta equal to 60 degree, and KH is equal to 0.25, i have recently, previously shown you the curves, and for beta equal to 60, you can directly get, value of M.

And, beta is more than 50 here, you can see, so it is the toe failure case, so M is equal to 3.6, so when you put it, in this equation, you will be getting, straight way, 5.4 meter this is the answer.

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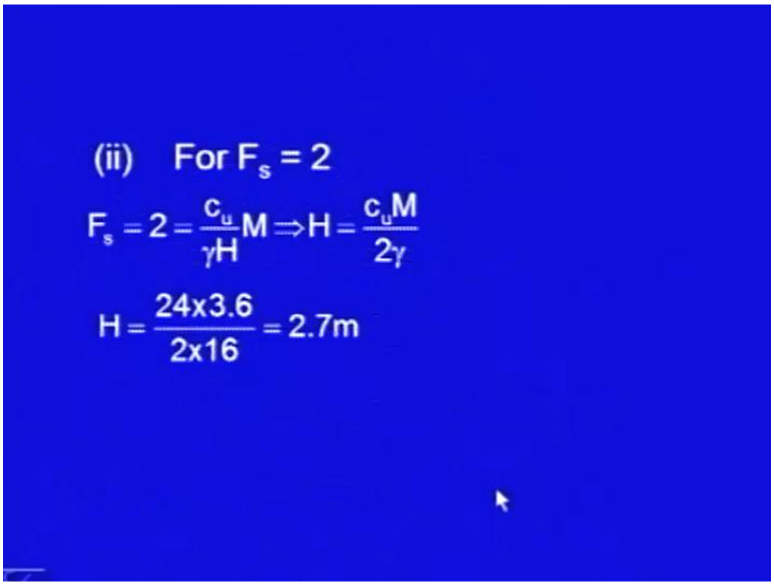

So, this is very convenient, method very convenient tool, these graphical methods are very convenient, for doing the analysis. Similarly, you can do the analysis, when the factor of safety is 2, so F s is equal to 2, and in terms of the stability factor, it is C u upon gamma H into M. So, H becomes equal to here, from this equation, you can get H is equal to C u into M divided by 2 gamma, M, you already know, for those values, and then, H you can calculate, and finally, H is equal to 2.7 meter.

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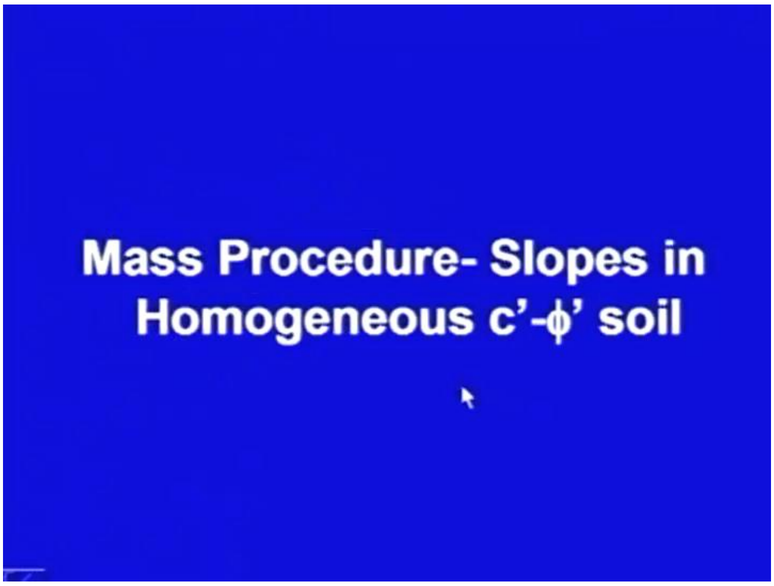

So, these were the cases, where we had, clay soil only, and it was completely saturated, and phi value was taken as 0, so only C u was applicable. Now, we are proceeding, to the next case, again we are taking the mass procedure, means we will be, treating the soil mass as a whole, but now, the soil is homogeneous, c dash, phi dash soil.

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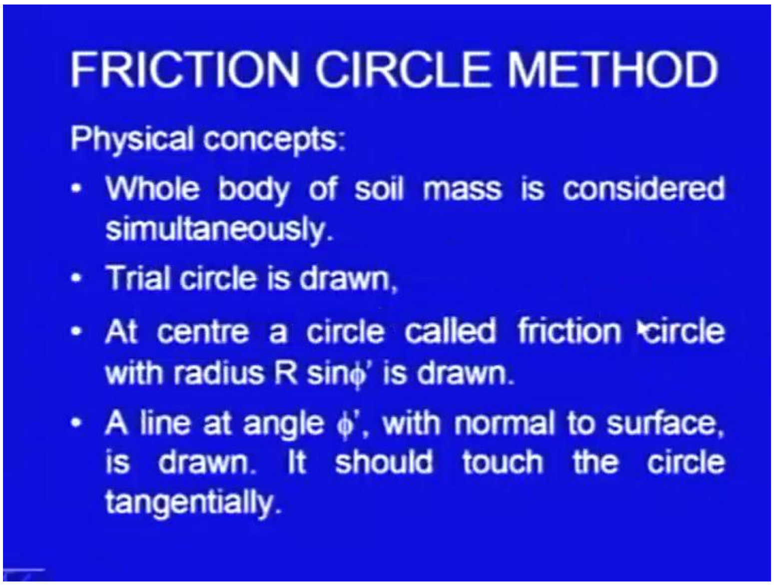

So, it has both the properties now, c dash as well as phi dash, the method, first method, which we are going to discuss is, friction circle method. Let me, first discuss the physical concepts of this method, whole body of the soil mass is considered simultaneously, as i told you, it is a mass procedure, so the entire body of the soil, which is trying to rotate, is considered as a free body then trial circle is drawn as usual, now at the centre, a circle called friction circle, with radius R sin phi dash is drawn.

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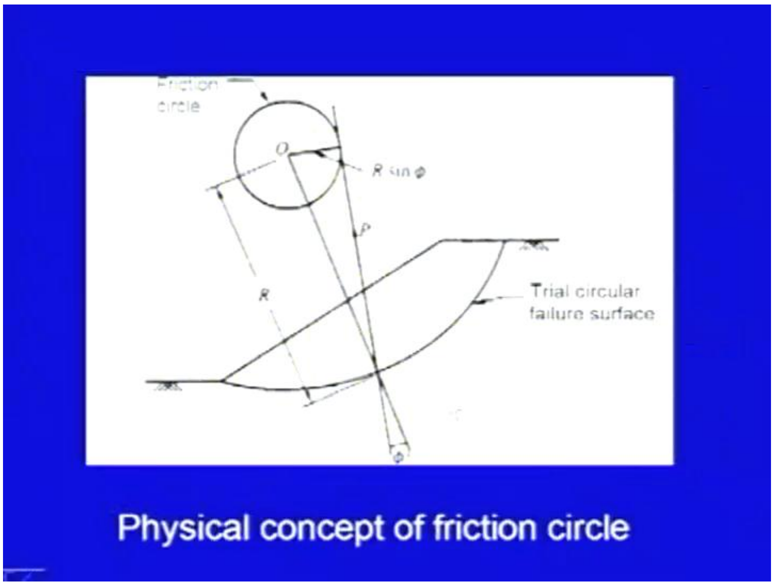

Let me, show this figure, here it is, the trial circle, we have taken the first trial, and this is the center of the circle. Now, here, if I am, if I take, the frictional force will be acting in this direction, the reaction will be taking in this direction, the resultant of these two, will be inclined at an angle phi, this is very know well known fact. What I am doing here is, let me, draw a circle here, and the radius of this circle, I have purposely kept as R sin phi dash.

So, what happens is that, if you draw a line here, if you draw a line here, at an obliquity of phi, this angle will be phi, and this line, should touch this circle, the reason is very simple. We have taken the radius of the circle R sin phi dash, and this distance is R, this angle is phi dash, and this triangle is a right angle triangle, so automatically, this becomes R sin phi dash, so here, if a line, a line, at an angle phi dash, with normal to the surface is drawn, it should touch the circle tangentially, mathematically, we can very easily prove it, this circle is called as friction circle.

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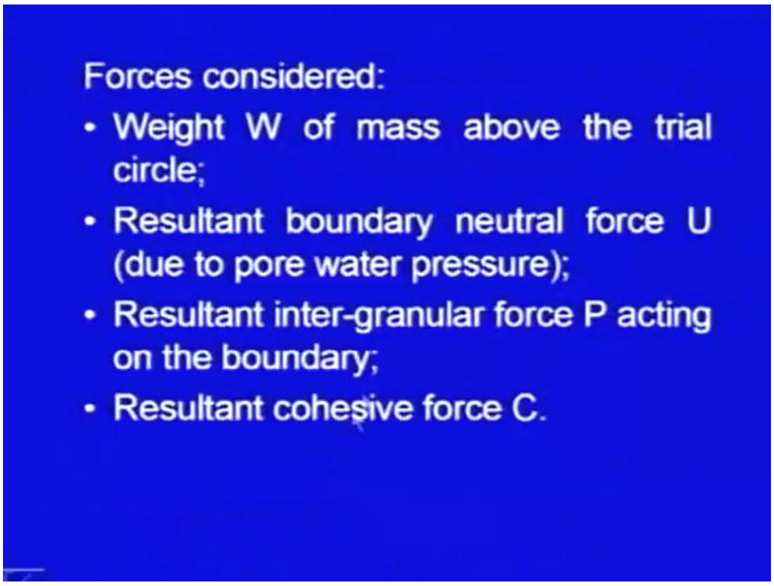

Now, the force is, which we are considering, weight W of the mass, above the trial circle, that is first force. Secondly, the resultant boundary neutral forces, there will be the forces, due to force, due to pore water pressure, so the resultant of all those forces, this is the resultant U, resultant boundary neutral force U. Thirdly, resultant inter granular force P acting on the boundary, at the frictional, at the boundary of the, of the circle, and resultant cohesive force.

So, we are treating these forces separately, cohesion we are taking separately, and inter granular force, here friction, and the reaction, we will be taking, combinedly.

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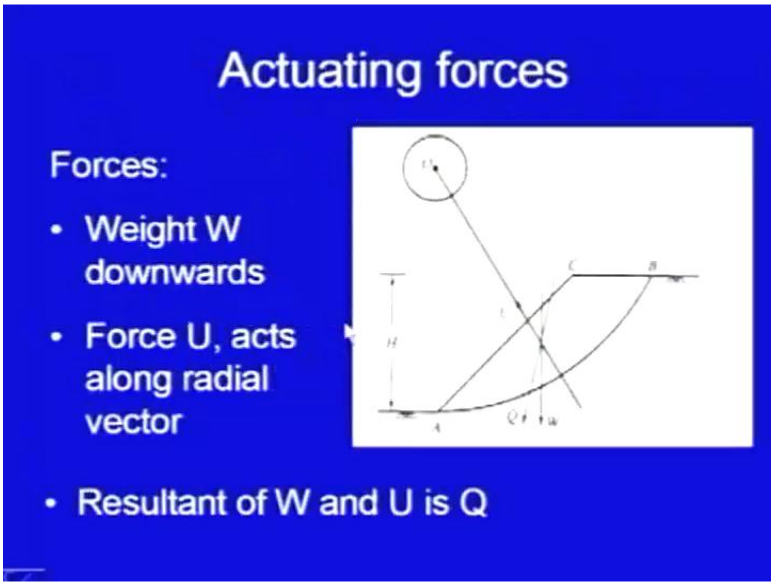

Let us, discuss these forces, one by one, actuating forces, so the actuating forces, the forces, which are trying to destabilize, is weight W, so weight W, let us say, this is the CG of this wedge, cylindrical wedge, and weight W is acting in the downward direction. Second is, force U, and U is the water pressure, it has to act normal to the surface, so necessarily, this vector U, will be passing through the centre of the circle, trial circle, so force U, acts along the radial vector.

So, we have to find out, these two forces, from the geometry, as well as from the flow conditions, and then, let us say, the resultant of W and U is Q. So, W is acting, here in this direction, downward direction, U is acting in, radial direction, and here it is the, resultant Q.

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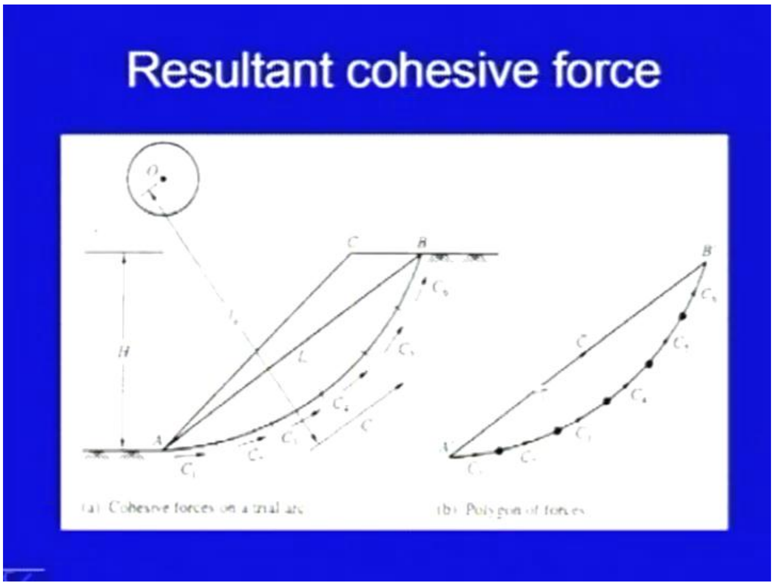

Now, come to the cohesive forces, this is the trial circle, and let us divide, if I take very small segments, all along this trial circle, trial all along this arc. So, this is small segment, another small segment, third small segment, and so on, so let us, say the cohesion, along this along these small segments are C 1, C 2, C 3 and so on. If, I plot these cohesions, separately here, and then plot this polygon, so C 1 will be here, C 2 is here, C 3 is here, and so on.

And, then, you can find out, their resultant, and also, the magnitude of this resultant, direction as well as magnitude. So, if I, if you compare, this figure, and this figure, the resultant should be, equal to in magnitude, it should be equal to this length, and it is direction will be, in this direction. So, the resultant of these cohesive forces will be equal in magnitude, and it will be, parallel to this line AB, AB is the chord here.

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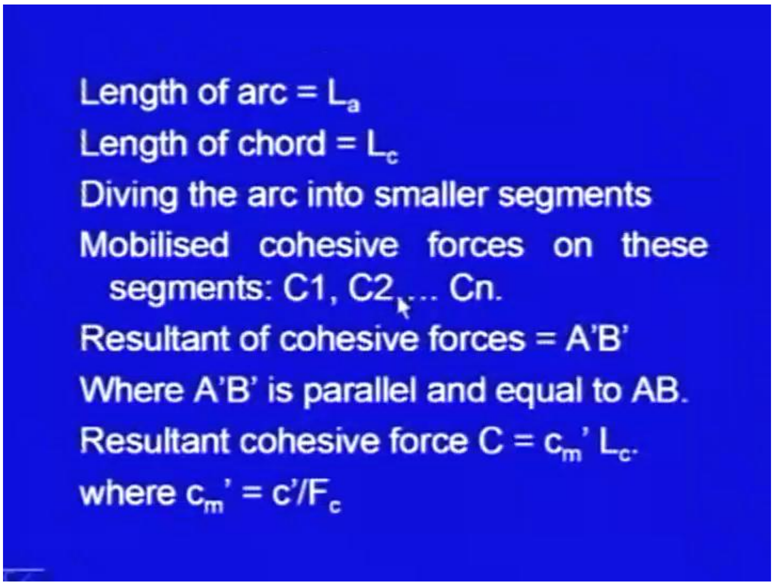

So, let us, say the length of the arc is L a, this is the length of the arc, and this is the chord, and you know that, length of arc is always more than length of chord, length of chord is let us, say L c. Dividing the arc into smaller segments, and then taking the mobilized cohesive forces, on these segments as C 1, C 2 and C n as I discussed, Resultant of all these, small cohesive forces is A dash B dash, ((Refer Time: 45:26)) it is shown here, A dash is this point, B dash is this point, so this is the resultant, where A dash B dash is parallel and equal to AB, this is necessarily it will be, parallel and equal to this chord AB.

The resultant cohesive forces, therefore, the resultant of all these cohesive forces, C will be equal to C m dash, C m dash is the mobilized cohesion, in effective terms, into length of chord, because it is magnitude is available now, available with us. So, the resultant cohesive force will be, equal to C m dash into L c, where C m dash is nothing, but, dash upon factor of safety, against cohesion, c dash upon F c

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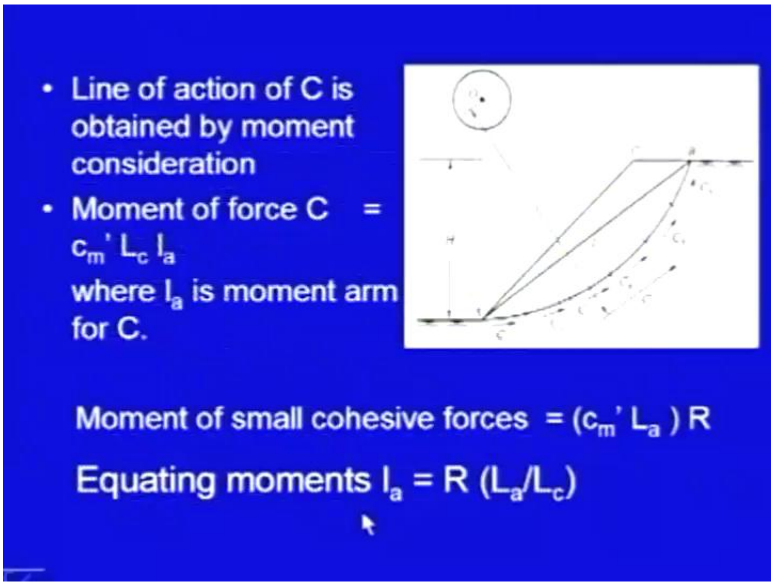

Now, we know the magnitude of the cohesive force, we know its direction, but still we do not know, where it is acting, the line of action of C is obtained by moment consideration. The moment of this force will be equal to C m dash we are taking moment about the point O, because this cylindrical surface, cylindrical volume is trying to rotate. So, moment of this cohesive force, the resultant force will be equal to C m dash into L c, that is the magnitude, and let us say, L a, is the lever arm, or the moment arm, from the centre of the circle, so from a, L a, is the distance, where this C is acting.

Now, this moment, should be equal to, the moment of small cohesive components, which we took last time. So, if I take these components, C 1, C 2, C 3 and O and so on, up to C m, then all these forces are acting, perpendicular to the radial vector, and their lever arm will be equal to, the radius of this particular trial circle. So, the moments of small cohesive forces will be equal to, C m dash into L a, what i have done here is, total arc length, if you, if you take them separately, so it will be C m dash into L 1, let us say L 1 is this small length.

Then C m 2, c m dash into L 2 and so on, so this complete length is L a, so when we, when we sum up, so it will be the total moment, the resisting moment, of the cohesive forces will be, C m dash into this length of arc into radius. And, when it is in equilibrium, these two, these two values, they have to be equal, we have to find out C here. So, by equating, these two moments C m dash, into L c into small l a, into C m dash capital L a, into R.

When we equate it C m dash will cancel, and you will be getting the lever arm, of the, the, of the force C, will be equal to capital R, into L a upon L c, where L a, is the length of arc, this is the length of arc. L c is length of chord, and you know, length of arc is always more than length of chord, so La should be higher than R, so L a is higher than R, so somewhere, here it is acting. So, you can find out, the line of action, of the cohesive forces, it will be somewhere, beyond this radius.

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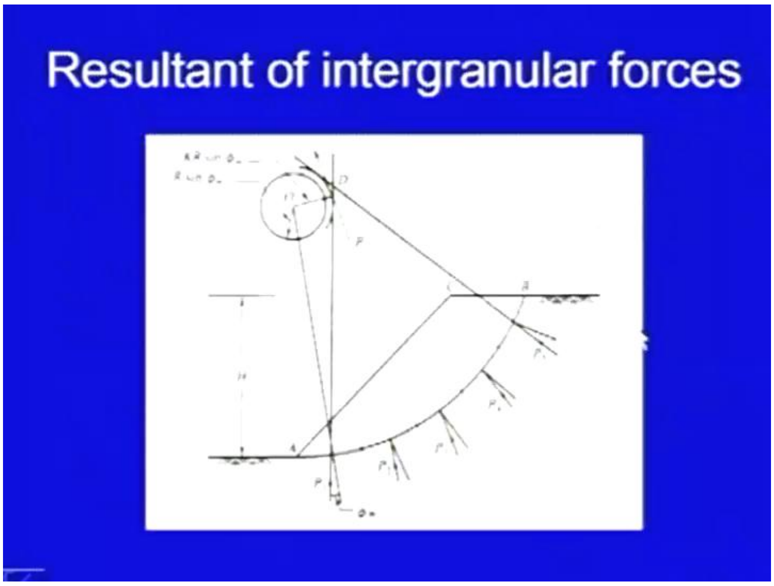

Now, let us, come to the resultant of inter granular forces.

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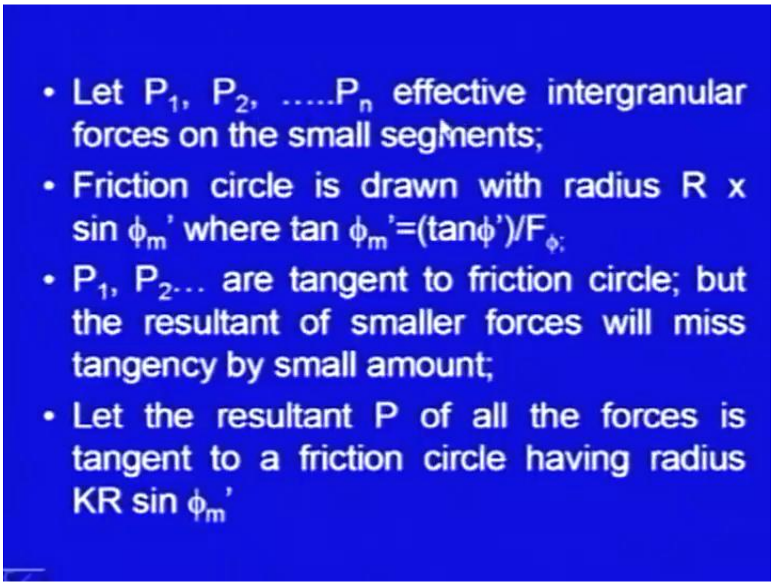

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Let P 1, P 2, P n these are the effective inter granular forces on small segments, so these are very small segments, we have taken, and effective inter granular forces, the frictional component, this is P 1 here, P 2 here, P 3 here and so on. Friction circle is drawn, with radius R into sin of phi m dash, here phi m dash is the mobilized angle. Let us say, this here the, the slope is safe, and the mobilized shear the angle, friction angle is phi m dash, so the radius will be R into sin of phi m dash, where tan of phi m dash is equal to tan of phi dash divided by F phi.

So, this is the factor of safety against friction, and factor of safety against friction is defined as, tan phi dash divided by tan phi m dash, so here, tan phi m dash, will be equal tan phi dash upon F of phi. So, now P 1, P 2 are tangent to friction circle, that I discussed, but the resultant of these smaller forces, will miss the tangent, by will miss tangency, by a small amount, so here, I have shown it. If, you draw the small component, then the, the resultant that, inter granular force, will be tangent to this small, smaller friction circle.

But, when you take, all the, these components, when you take the resultant, it has been found, that the resultant will miss the tangency, by a small amount here. Let the resultant P, of all the forces is tangent to a friction circle, having radius K into R into sin of phi m dash. So, let us say, there is a factor K, which we have to multiply, so that, we get a circle, modified I can call it, we can call it modified friction circle, this one the little bit larger one, so that this resultant passes through the, is tangentially, touches the circle.

So, the inner circle is the original friction circle, the radius is R sin phi m dash, and outer circle is little bit bigger, radius is K times, this radius of the previous circle, and this value of K, you can find out from the charts, it is available.

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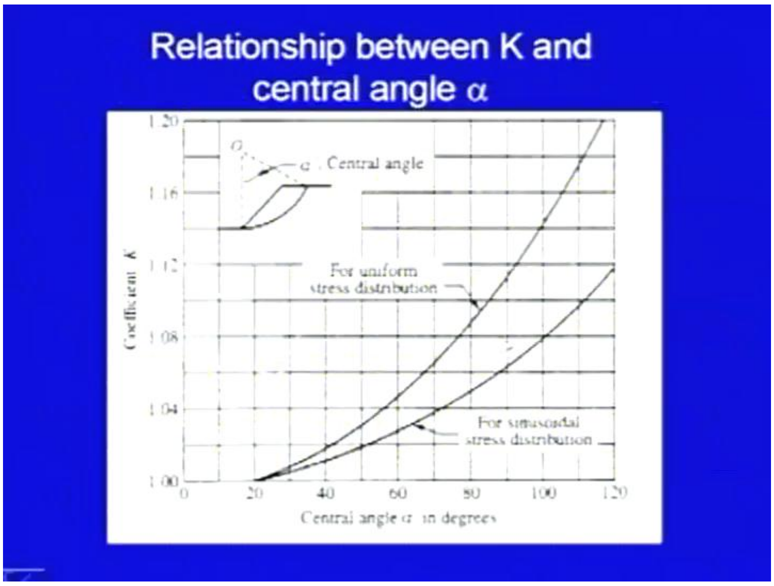

This is the relationship between the K, and central angle alpha, so on x axis, you have central angle alpha, for the particular case, which we have taken, for the particular trial circle, this alpha, you can find out. And, once that, alpha is known, then you can find out the value of K, so here, on x axis, you have angle alpha, y axis, you have coefficient K, and you can see there, are two charts, one two curves, one is for uniform stress distribution, and second is for, sinusoidal stress distribution.

So, it depends on, what kind of the stress distribution has been assumed, from this chart, one can find out the value of coefficient K. So, you can see, it is varying from 1.00 and up to 1.20, so the radius will be, 1 times to 1.2 times the radius of the original friction circle, so using this chart, you can find out the value of K.

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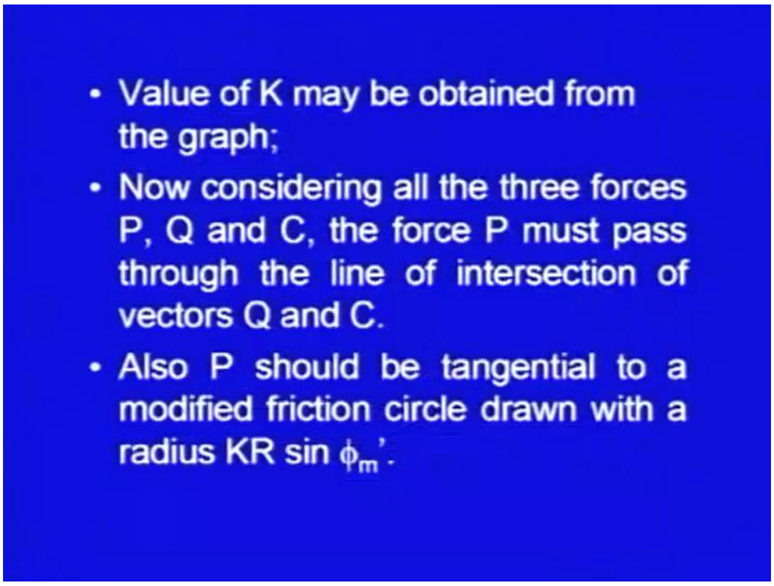

Now, considering all the three forces, P Q and C, the force P must pass through the, line of intersection of the vectors Q and C, this is the condition, which should be, satisfied.

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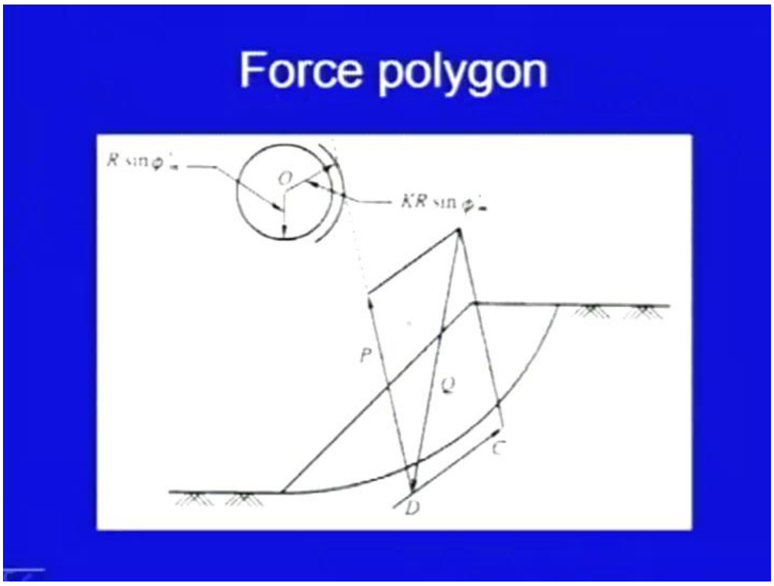

So, we know, now the P direction of P is available with us, this one can, we can draw this modified, friction circle, this direction is available with us, this line of action of C is also available with us. And, also P should be tangential, to the modified friction circle drawn with a radius KR sin phi m dash, so as per this diagram, this is the modified circle, friction circle, P should be tangential to this, and then, for the limit equilibrium case, this particular force polygon should be satisfied. So, you can, you can find out the, the unknowns, so this particular, force polygon, if this is Q, this is C and P, if this can be constructed.

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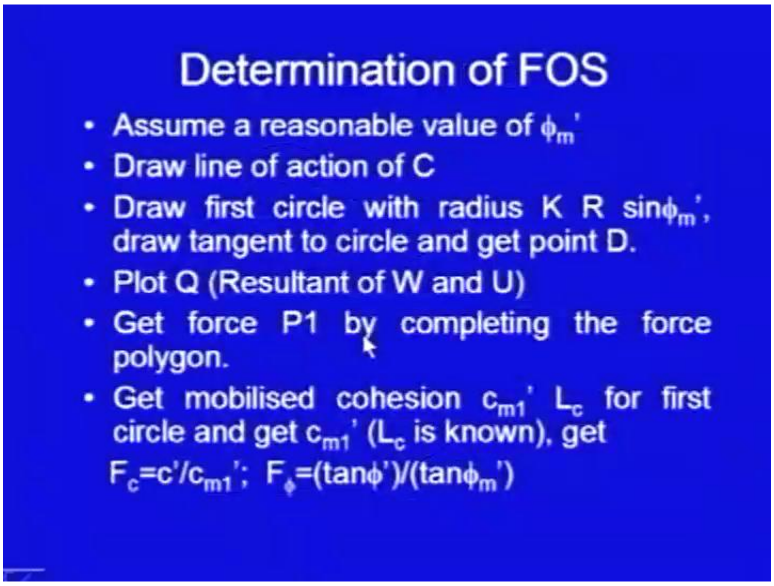

For, determining the factor of safety, so these are the basic concepts, for determining the factor of safety, what we have to do, is we start, with the reasonable value of phi m dash.

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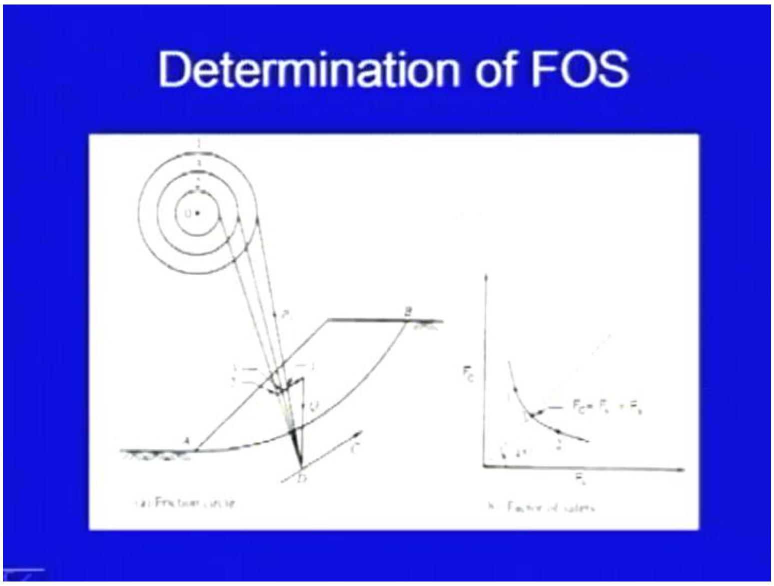

We use, any reasonable value, and then ((Refer Time: 55:14)) draw line of action of C, so this is the, line of action of C, which we have drawn, this is the slope, we have calculated, the how far it is from this radius, from this centre, and line of action has been drawn. Draw first circle, with radius K R sin phi m dash, and draw tangent to circle, and get point D, so this is the first circle, first trial circle, which we have drawn, this is the modified circle, friction circle, draw tangent to it, so that, gives you the direction of P, and get this point of intersection D.

Now, plot Q, the resultant of W and U, so here, now plot this is the force Q, which we have already calculated, then complete this polygon, draw a line parallel to C, so this force polygon, gives you, the force P, it gives you the force C, and you can find out, yes you can find out C, and P using this force polygon.

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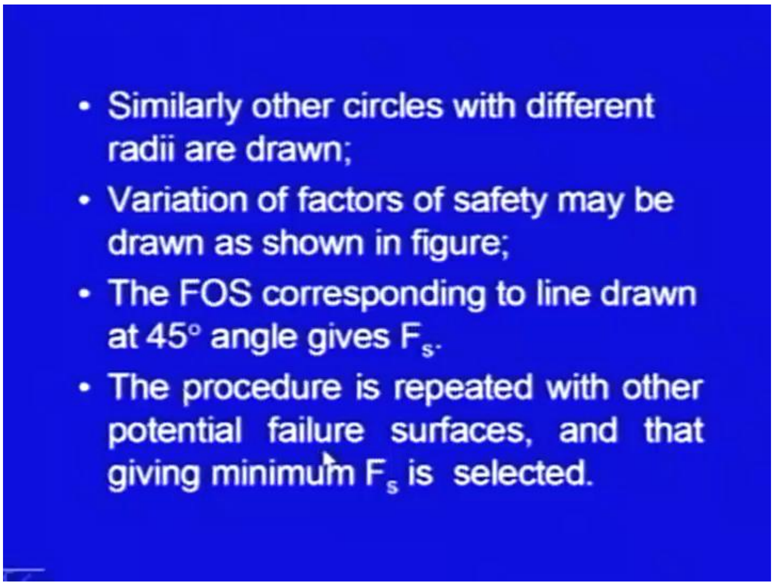

So, get force P 1, by completing the force polygon, and then get mobilized, cohesion C m 1 dash into L c, this is the capital C, the cohesion for this first trial, for the first circle. And, once this is available, then you can get, C m 1 dash, the mobilized cohesion, for the first circle, so get F c is equal to, once this C m 1 dash is available, then, you can get factor of safety, against cohesion, and also for the phi value, which we have assumed here, for the phi m value, which we assumed here, we can get F phi.

So, friends, today we have discussed, the finite slopes, and we have concluded the, slopes and saturated clays, we also considered the earthquake forces, and then we have started discussions on the C dash, phi dash and so else. And, we have started one method, friction circle method, and in next lecture, we shall continue with the same method.

Thank you.