

**Foundation Engineering**  
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**Module – 01**  
**Lecture - 03**  
**Shallow Foundation**

So, in the last lecture, we discussed the Terzaghi bearing capacity equation. And also some of the applications have been demonstrated by solving few problems. So, today I will continue with the same and discuss few more problems and then extend it as the theory as given by Meyerhof and for the eccentric load footings.

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Terzaghi's bearing capacity equation in  
case of general shear failure

$$q_{ult} = c N_c + q_0 N_q + 0.5\gamma B N_\gamma$$

For local shear failure

$$c' = (2 / 3) c \quad \text{and} \quad \tan\phi' = (2 / 3) \tan\phi$$

The bearing capacity factors  $N_c$ ,  $N_q$  and  
 $N_\gamma$  may be read from table

So, Terzaghi bearing, Capacity equation as we have seen, for the general shear failure case is given by  $q_{ultimate}$  equal to  $c N_c$  plus  $0.5 \gamma B N_\gamma$  plus  $q_0 N_q$ . Where  $N_c$ ,  $N_q$ ,  $N_\gamma$  factors are determined on the basis of the angle of shearing resistance tables have been provided and also the figures for the Terzaghi bearing capacity factors. In case of local shear failure  $c$  is taken as two-third of  $c$  and  $\phi$  is determined by this equation  $\tan \phi' = \text{two-third of } \tan \phi$ . So, these bearing capacity factors can be read from the table.

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The Terzaghi bearing capacity equation has been modified for other shapes of foundations by introducing the shape factors.

The Terzaghi bearing capacity equation, we have seen has been modified for the other shapes of foundations like, for the case of.

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### Square Foundations

$$q_{ult} = 1.3 c N_c + q N_q + 0.4 \gamma B N_\gamma$$

### Circular Foundations

$$q_{ult} = 1.3 c N_c + q N_q + 0.3 \gamma B N_\gamma$$

Square foundations for the case of circular foundations, in the case of square foundations, the equation comes out to be  $1.3 c N_c$  plus  $q N_q$  plus  $0.4 \gamma B N_\gamma$  whereas, for the case of circular foundations, it is  $1.3 c N_c$  plus  $q N_q$  plus  $0.3 \gamma B N_\gamma$ .

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## Rectangular Foundations

$$q_{ult} = c N_c (1 + 0.3 B/L) + q_0 N_q + 0.5 \gamma B N_\gamma (1 - 0.2 B/L)$$

where,

B= width or diameter and

L= Length of foundation

In the case of rectangular foundations, we use this bearing capacity equation in, which a it is given by  $c N_c (1 + 0.3 B \text{ upon } L) + q_0 N_q + 0.5 \gamma B N_\gamma (1 - 0.2 B \text{ upon } L)$ . Where B is the width of the footing or foundation or the diameter of the footing and L is the length of the foundation.

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## Skempton's Bearing Capacity Factor

For saturated clay soils, Skempton (1951) has proposed the following equation for strip foundation

$$q_{ult} = c N_c + \gamma D_f$$

We have also seen that, in the case of a cohesive soil Skempton has proposed the following equation for the case of strip footing,  $q_{ultimate} = c N_c + \gamma D_f$ .

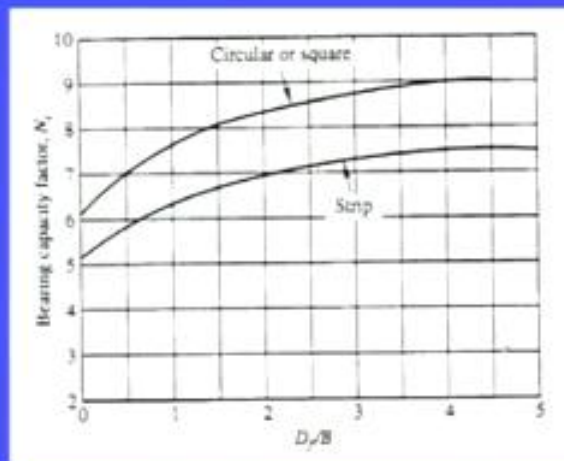
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The lower and upper limiting values of  $N_c$  for strip and square foundations may be written as follows:

Type of Foundation	Ratio of $D_f/B$	Value of $N_c$
Strip	0	5.14
	$\geq 4$	7.5
Square	0	6.2
	$\geq 4$	9.0

The lower and upper limiting values of  $N_c$  for strip and square footings, may be written in the follows, like for the case of different type of foundations depending upon  $D_f$  by  $B$  ratio. We can obtain or we can read the values of  $N_c$  from this particular table or.

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We can use this particular plot between bearing capacity factor  $N_c$  and  $D_f$  by  $B$  and  $D$  from using this, particular graph, we can obtain for different values of  $D$  by  $f$   $B$   $D$  by  $D_f$  by  $B$  ratio.

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The equation for rectangular foundation may be written as follows:

$$(N_c)_R = (0.84 + 0.16 \times B/L) (N_c)_S$$

where,

$(N_c)_R = N_c$  for rectangular foundation, and

$(N_c)_S = N_c$  for square foundation

The equation for rectangular foundation may be written, for the case of  $N_c$  it is given by  $0.84 + 0.16 B/L$  into  $N_{cS}$ , where  $N_{cS}$  is the  $N_c$  for square foundation,  $N_{cR}$  is the  $N_c$  for rectangular foundation. We have also discussed the effect of water table on bearing capacity of shallow foundation. We know that due to fluctuations in the water table during different seasons, the bearing capacity is affected. And this bearing capacity can be corrected for water table using following equation.

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For any position of water table within the depth  $(D_f + B)$ ,

$$q_{ult} = c N_c + q_o N_q R_{w1} + 0.5 \gamma B N_\gamma R_{w2}$$

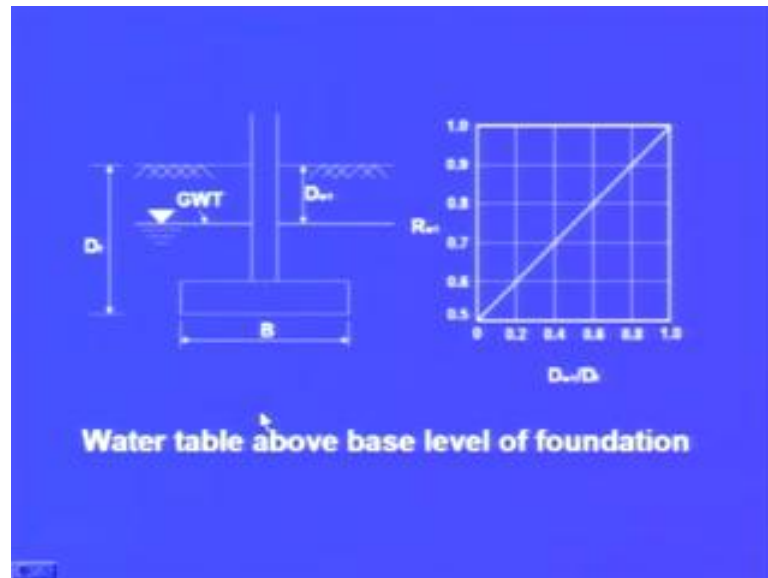
where,

$R_{w1}$  = reduction factor for WT above base level of foundation

$R_{w2}$  = reduction factor for WT below base level of foundation

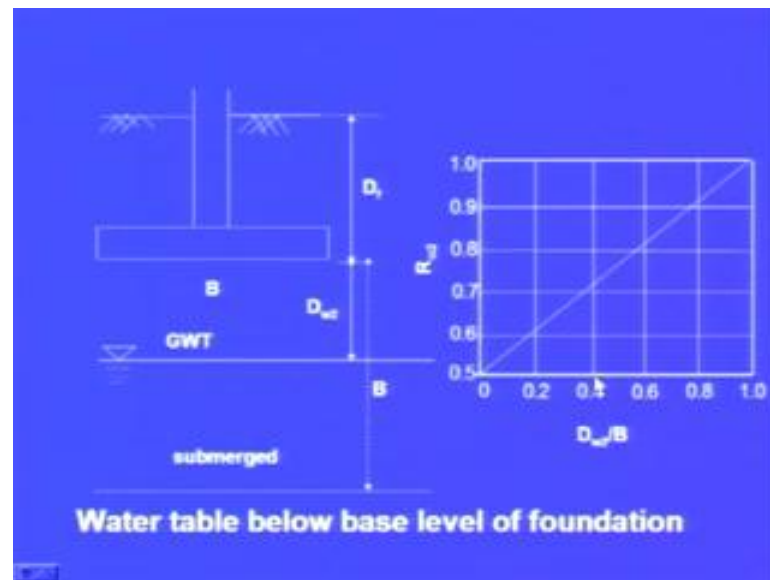
So, here if the water table is within the depth  $D_f$  plus  $B$ , we can use this particular occasion as  $q_{ultimate} = c N_c + q_0 N_q R_w 1 + 0.5 \gamma B N_\gamma R_w 2$ . Where  $R_w 1$  and  $R_w 2$  are the reduction factor for water table, when water table is above the level of foundation this is  $R_w 1$  and when water table is below, the base level of foundation  $R_w 2$ .

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Now, this  $R_w 1$  and  $R_w 2$  can be obtained using this chart; you can see from here this is the depth of foundation. This is width of foundation, this is the position of the ground water table, which is at a depth  $D_w$  below the ground surface and using this chart we can find this,  $R_w 1$  depending upon  $D_w$  by  $D_f$  ratio.

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Similar to this, if suppose water table is below the base level of the foundation. And it is at  $D_w/2$  below the base level. Then and it is within the width, within the depth equivalent to width of the foundation, then we can use this particular chart to find out  $R_w/2$ . So, here,  $R_w/2$  can be read or it can be calculated by the equation which we discussed in the last lecture.

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Equivalent effective unit weights may be used to determine ultimate bearing capacity

$$q_{ult} = c N_c + \gamma_{e1} D_f N_q + 0.5 \gamma_{e2} B N_\gamma$$

$\gamma_{e1}$  = weighted effective unit weight of the soil lying above the base level of foundation

$\gamma_{e2}$  = weighted effective unit weight of the soil lying within the depth  $B$  below the base level of foundation

$\gamma_{sat}$  = saturated unit weight of the soil below WT

$\gamma_{sub}$  = submerged unit weight of the soil

Another method, for considering water table is by making use of equivalent unit weight effective, unit weight  $\gamma_{e1}$  and  $\gamma_{e2}$ . And these equivalent effective unit

weights can be determined for the 2 cases. When the water table is above the foundation level and when the water table is below the foundation level. Now, various terms used are  $\gamma_{e1}$ ,  $\gamma_{e2}$ ,  $\gamma_{\text{saturated}}$  and  $\gamma_{\text{submerged}}$ .

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**CASE 1:** When the water table lies above base level of foundation or when  $D_{w1}/D_f \leq 1$ ,

$$\gamma_{e1} = \gamma_{\text{sub}} + (D_{w1}/D_f)(\gamma - \gamma_{\text{sub}})$$

$$\gamma_{e2} = \gamma_{\text{sub}}$$

For the case, when the water table lies above the base level of foundation or when  $D_{w1}/D_f$  is less than 1,  $\gamma_{e1}$  is taken as  $\gamma_{\text{submerged}}$  plus  $D_{w1}/D_f$  times  $\gamma_{\text{saturated}}$  minus  $\gamma_{\text{submerged}}$ . Whereas,  $\gamma_{e2}$  will simply be equal to  $\gamma_{\text{submerged}}$ .

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**CASE 2:** When the water table lies below base level of foundation or when  $D_{w2}/B \leq 1$ ,

$$\gamma_{e1} = \gamma$$

$$\gamma_{e2} = \gamma_{\text{sub}} + (D_{w2}/B)(\gamma - \gamma_{\text{sub}})$$

When the water table lies below base level of foundation or when  $D/B$  is less than or equal to 1 then  $\gamma_e$  is nothing but the unit weight of soil, above the foundation level. And  $\gamma_e$  is  $\gamma_{\text{submerged}} + D/B \gamma - \gamma_{\text{submerged}}$ .

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**EXAMPLE**

A square footing, founded at a depth of 1.5m below the ground surface in cohesionless soil, carries a column load of 1280 kN. The soil is submerged having an effective unit weight of 11.5 kN/m<sup>3</sup> and an angle of shearing resistance of 30°. Find the size of the footing for  $F_s = 3$ . Use Terzaghi's theory of general shear failure.

So, this is what we have discussed in the last class, last lecture. Now using the same concept we will solve some more few more problems and so, that the application of these particular occasions are very clear. One such problem is that of square footing founded at a depth of 1.5 meter below, the ground surface in cohesion less soil means,  $c$  equal to 0 it carries a column load of 1280 kilo Newton. So, the column load is given the soil is submerged having an effective unit weight of 11.5 kilonewton per meter cube and an angle of shearing resistance  $\phi$  equal to 30 degrees. We will have to find the size of the footing for a factor of safety equal to 3 and using the Terzaghi's theory of general shear failure.

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**SOLUTION :**

For square footing,

$$q_{ult} = \gamma D_f N_q + 0.4 \gamma B N_\gamma$$

Since the WT is close to the ground level,

$$\gamma = \gamma_{sub} = 11.5 \text{ kN/m}^3$$

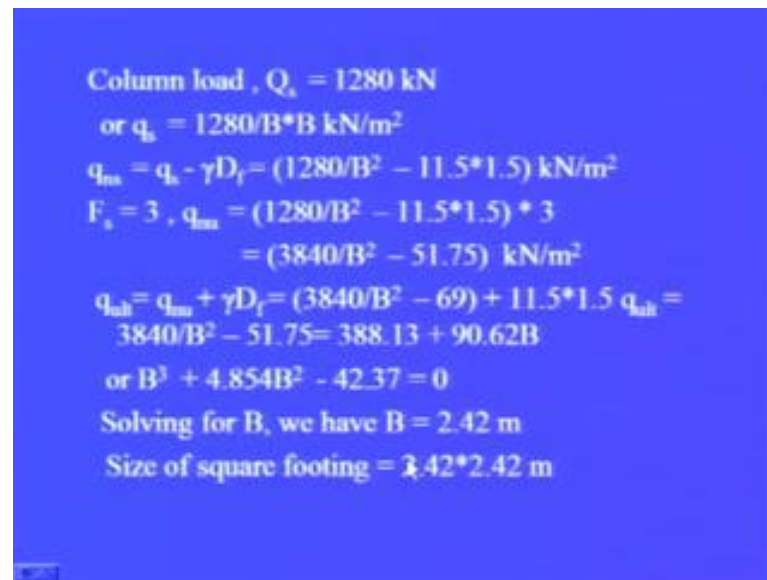
For  $\Phi = 30^\circ$ ,  $N_q = 22.5$ , and  $N_\gamma = 19.7$ .

Substituting the known values, we have

$$\begin{aligned} q_{ult} &= 11.5 * 1.5 * 22.57 + 0.4 * 11.5 * 19.7 * B \\ &= 388.13 + 90.62 B \quad \text{kN/m}^2 \end{aligned}$$

Now, in order to get solution, we know that the for the case of square footing in the cohesion less soil  $q$  ultimate will be given by,  $\gamma D_f N_q$  plus  $0.4 \gamma B N_\gamma$ . You can see from here, that this factor is 0.4 is not 0.5, which is in the case of strip footing. Since the water table is close to the ground level the unit weight will be the submerged unit weight and which is already given 11.5 kilonewton per meter cube. Now, for  $\phi$  equal to 30 degrees from the tables we discussed, we can find out  $N_q$  and  $N_\gamma$  these are 22.5 and 19.3 respectively. Now, when we substitute respective values in the ultimate bearing capacity equation we will find that the ultimate bearing capacity equation comes out to be 388.13 plus 90.62 B, where B is the width of the column footing.

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Column load,  $Q_u = 1280 \text{ kN}$   
or  $q_u = 1280/B^2 \text{ kN/m}^2$   
 $q_{nu} = q_u - \gamma D_f = (1280/B^2 - 11.5 \times 1.5) \text{ kN/m}^2$   
 $F_s = 3, q_{nu} = (1280/B^2 - 11.5 \times 1.5) \times 3$   
 $= (3840/B^2 - 51.75) \text{ kN/m}^2$   
 $q_{ult} = q_{nu} + \gamma D_f = (3840/B^2 - 69) + 11.5 \times 1.5$   
 $q_{ult} = 3840/B^2 - 51.75 = 388.13 + 90.62B$   
or  $B^3 + 4.854B^2 - 42.37 = 0$   
Solving for  $B$ , we have  $B = 2.42 \text{ m}$   
Size of square footing =  $2.42 \times 2.42 \text{ m}$

Now, from the data given as that of column load is 1280 kilo Newton. We can find out what should be the ultimate corresponding to this what should be the ultimate bearing capacity sufficient. So, that this can be taken up by the soil, so, in order to do that, first of all we will find out. What is the safe bearing load or safe bearing capacity? That is nothing, but this load divided by area of the footing, here we have assume width of the footing as  $B$ . So, the area of footing will be  $B$  into  $B$ , so, 1280 divided by  $B$  square. Now, once  $q$  shape is known we can find out  $q$  net shape that comes out to be  $q_s$  minus  $\gamma D_f$ . So, when we substitute values of  $\gamma D_f$ , we will get this as 1280 by  $B$  square minus 11.5 into 1.5 total in bracket kilonewton per meter square. Now, it is also given that, the factor of safety is equal to 3.

So, we can find out net ultimate using this factor of safety, we multiplied by factor of safety, and finally, we find out  $q$  ultimate that is  $q$  net ultimate plus  $\gamma D_f$ . So, when we substitute, we will end up with an equation, which will be 3840 by  $B$  square minus 51.75. Now, we have already, determined this ultimate bearing capacity from the Terzaghi equation. And when we substitute, we compare this Terzaghi equation with this value, we will get a cubic equation in this form  $B$  cube plus 4.854  $B$  square minus 52.37 equal to 0. This we solve by trial and error, by solving we will get width of the footing and that comes out to be 2.42. So, the size of the square footing, which will be sufficient for 1280 kilonewton load will be 2.42 meter by 2.42 meter. Another solved example which will make in which we make use of the Terzaghi bearing capacity equation is a

footing of 1.5 meter diameter it carries a safe load of 800 kilo Newton, that including its self weight.

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**SOLVED EXAMPLE**

A footing of 1.5 m diameter carries a safe load (including its self weight) of 800 kN in cohesionless soil. The soil has an angle of shearing resistance  $\Phi = 36^\circ$  and an effective unit weight of  $12 \text{ kN/m}^3$ . Determine the depth of foundation for  $F_s = 2.5$  by Terzaghi's general shear failure criteria.

So, it is the total in cohesion less, soil the soil has an angle of shearing resistance phi equal to 36 degrees. Now, as phi equal to 36 degrees, we will consider this case of the general shear failure and an effective unit weight of the 12 kilonewton per meter cube. Determine the depth of foundation for which  $F_s$  factor of safety is equal to 2.5 again using Terzaghi's criteria.

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**SOLUTION :**

Using Terzaghi's equation for  $c = 0$

$$q_{ult} = \gamma D_f N_q + 0.3 \gamma B N_\gamma$$

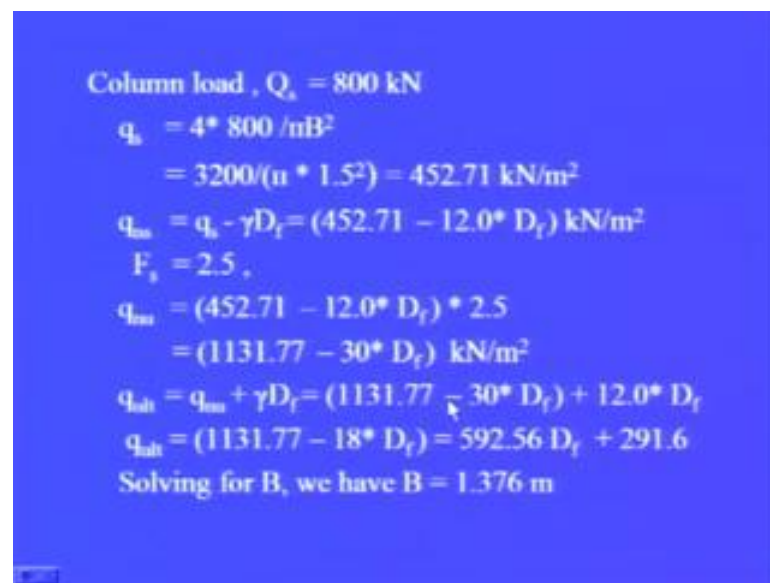
For  $\Phi = 36^\circ$ ,  $N_\gamma = 54$ , and  $N_q = 49.38$

By substituting the known values, we have

$$q_{ult} = 12 \times 49.38 D_f + 0.3 \times 12 \times 1.5 \times 54$$
$$= (592.56 D_f + 291.6) \text{ kN/m}^2$$

Now, using Terzaghi's equation for  $c$  equal to 0, we end up with equation as  $q$  ultimate equal to  $\gamma D_f N_q$  plus  $0.3 \gamma B N \gamma$ . Now, here you can see this is 0.3 which is for the case of a circular foundation. Now, similar to the other cases for different values of  $\phi$ , we can obtain  $N_q$  and  $N \gamma$  factor. So, here  $\phi$  for  $\phi$  equal to 36 degrees  $N \gamma$  and  $N_q$  can be directly read from the table. And when we substitute these values? We will get this ultimate bearing capacity, equation in which depth of foundation is unknown and that will be given by  $592.56 D_f$  plus  $291.6$  kilonewton per meter square.

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Column load,  $Q_s = 800 \text{ kN}$   
 $q_s = 4 \cdot 800 / \pi B^2$   
 $= 3200 / (\pi \cdot 1.5^2) = 452.71 \text{ kN/m}^2$   
 $q_{\text{net}} = q_s - \gamma D_f = (452.71 - 12.0 \cdot D_f) \text{ kN/m}^2$   
 $F_s = 2.5$   
 $q_{\text{net}} = (452.71 - 12.0 \cdot D_f) \cdot 2.5$   
 $= (1131.77 - 30 \cdot D_f) \text{ kN/m}^2$   
 $q_{\text{ult}} = q_{\text{net}} + \gamma D_f = (1131.77 - 30 \cdot D_f) + 12.0 \cdot D_f$   
 $q_{\text{ult}} = (1131.77 - 18 \cdot D_f) = 592.56 D_f + 291.6$   
 Solving for  $B$ , we have  $B = 1.376 \text{ m}$

Now, from the given data the column load is 800 kilo Newton. So, 800 kilonewton 4 into 800 kilonewton divided by pi by B square, that will be the  $q$  safe that comes out to be 3200 divided by pi into 1.5 square 452.71 kilonewton per meter square. Now,  $q$  safe is known, we can find out  $q$  net safe that comes out to be  $q$  safe minus  $\gamma D_f$ . If you know factor of safety we can find out  $q$  net ultimate, if you multiply this  $q$  net safe by factor of safety. So, we will get this as 1131.77 minus 30 into  $D_f$  in which  $D_f$  is the depth of foundation which is unknown.

Now, after getting this  $q$  net ultimate, we can find out  $q$  ultimate by adding  $\gamma D_f$  to the net ultimate. So, it comes out to be 1131.77 minus 30  $D_f$  plus 12 in into  $D_f$ . Now, we have already; obtain 1 expression for ultimate bearing capacity using Terzaghi criteria. When we compare this, with the that equation and we solve it by trial and error we will

get the width of the foundation, that is the diameter of the foundation as 1.376 meter it means, this much even if you provide 1.4 that will be for this particular case.

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### SOLVED EXAMPLE

If the ultimate bearing capacity of a 1 m wide strip footing resting on the surface of sand is  $250 \text{ kN/m}^2$ , what will be the net safe bearing pressure that a  $3 \times 3 \text{ m}$  square footing resting on the surface can carry with  $F_s = 3$ . Assume that the soil is cohesionless. Use Terzaghi's theory.

Now, if you see another example, again we use Terzaghi bearing capacity if the ultimate bearing capacity of 1 meter wide strip footing resting on the surface of sand is 250 kilonewton per meter square. What will be the net safe bearing pressure? That a 3 by 3 meter square, footing resting on the surface can carry. So, using the data of strip footing, we can use it for the square footing and obtain the solution.

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### SOLUTION :

We may write the following equations for cohesionless soil with  $D_f = 0$  :

$$q_{ult} \text{ (square)} = 0.4 \gamma B_1 N_\gamma$$

$$q_{ult} \text{ (strip)} = 0.5 \gamma B_2 N_\gamma$$

$$\begin{aligned} \text{Therefore, } q_{ult} \text{ (square)} / q_{ult} \text{ (strip)} \\ &= (0.4 \gamma B_1 N_\gamma) / (0.5 \gamma B_2 N_\gamma) \\ &= 0.8 B_1 / B_2 = 0.8 \times 3 / 1 = 2.4 \end{aligned}$$

We may write the following equations, for cohesion less soil with depth  $D_f$  equal to 0  $q_{ultimate}$  for square footing equal to  $0.4 \gamma B_1 N \gamma$ . Where  $B_1$  is the width of a square footing and  $q_{ultimate}$  for strip footing is  $0.5 \gamma B_2 N \gamma$ , where  $B_2$  is the width of strip footing. So, the ratio of  $q_{ultimate}$  of the square and  $q_{ultimate}$  of the strip, that will come out to be  $0.8 B_1$  upon  $B_2$  that is  $0.8$  into  $3$  upon,  $1$  that is equal to  $2.4$ .

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$$\begin{aligned}
 q_{ult} \text{ (square)} &= 2.4 q_{ult} \text{ (strip)} \\
 &= 2.4 * 250 = 600 \text{ kN/m}^2 \\
 q_{nu} &= q_{ult}, \text{ since } D_f = 0 \\
 q_{ns} &= q_{nu} \text{ (square)} / 3 \\
 &= 600 / 3 \\
 &= 200 \text{ kN/m}^2
 \end{aligned}$$

Now,  $q_{ultimate}$  of square will be equal to  $2.4$  times  $q_{ultimate}$  of the strip and that  $q_{ultimate}$  of the strip is given as  $250$ . So,  $q_{ultimate}$  of the square will be equal to  $2.4$  into  $250$  that is equal to  $600$  kilonewton per meter square. Now,  $q_{net ultimate}$  will be equal to  $q_{ultimate}$  since the footing is resting on the ground itself. And hence  $q_{net safe}$  can be equal to  $q_{net ultimate}$  of the square, divided by factor of safety that is  $3$ . That is equal to  $600$  by  $3$  comes out to be  $200$  kilonewton per meter square.

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### SOLVED EXAMPLE

A circular plate of diameter 1.05 m was placed on a sand surface of unit weight  $16.5 \text{ kN/m}^3$  and loaded to failure. The failure load was found to give a pressure of  $1500 \text{ kN/m}^2$ . Determine the value of the bearing capacity factor  $N_\gamma$ . The angle of shearing resistance of the sand measured in a triaxial test was found to be  $39^\circ$ . Compare this value with the theoretical value of corresponding to  $N_\gamma$ . Use Terzaghi's theory.

Now, in another solve problem, that is the case of a circular plate of diameter 1.05 meter was placed on a sand surface of unit weight 16.5 kilonewton per meter cube. And loaded to failure the failure load was found to give a pressure of 1500 kilonewton per meter square. We will have to determine the value of bearing capacity factor  $N_\gamma$ , the angle of shearing resistance of the sand measured in a triaxial test was found to be 39 degrees. We will have to compare this value with the theoretical value of corresponding to  $N_\gamma$  using Terzaghi theory.

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### SOLUTION :

Since the plate is placed on the surface  $D_f=0$ , the equation for the  $q_{ult}$  (circular) is

$$q_{ult} = 0.3 \gamma B N_\gamma = 0.3 * 16.5 * 1.05 * N_\gamma$$

since  $q_{ult} = 1500 \text{ kN/m}^2$ ,

$$\text{we have } N_\gamma = 1500 / 5.1975 = 289$$

For  $\Phi = 39^\circ$ ,  $N_\gamma = 88.8$ , which is very less than that obtained from the plate load test. This is partly due to the scale effect and partly due to sensitiveness of  $N_\gamma$  at higher values of  $\Phi$ .

Now, in order to get solution, we know that the plate is placed on the surface. So,  $D_f$  equal to 0 plate is circular. So,  $q_{ultimate}$  is  $0.3 \gamma B N \gamma$  and when we substitute the values. We will get  $0.316.5$  into  $16.5$  into  $1.05 N \gamma$ , since  $q_{ultimate}$  equal to  $1500$  kilonewton per meter square. So,  $N \gamma$  will come out to be equal to  $1500$  divided by  $5.1975$  that is equal to  $289$ . Now, for  $\phi$  equal to  $39$  degrees,  $N \gamma$  equal to  $88.8$ , which is very less than we obtained from the plate load test. This is partly due to the scale effect and partly due to the sensitiveness of  $N \gamma$  at higher values of  $\phi$ .

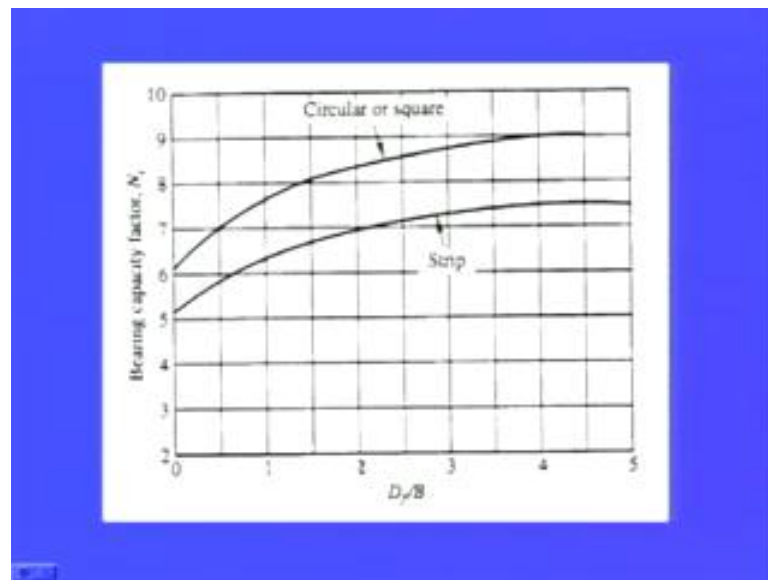
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**SOLVED EXAMPLE**

Find the net safe bearing load per meter length of a long wall footing 2 m wide founded on a stiff saturated clay at a depth of 1 m. the unit weight of the clay is  $17 \text{ kN/m}^3$ , and the shear strength is  $120 \text{ kN/m}^2$ . Assume the load to be applied rapidly such that undrained condition ( $\Phi = 0^\circ$ ) prevail. Use  $F_s = 3$  and Skempton's method.

The next solve problem is, about to find the net safe bearing load per meter length of a long wall footing 2 meter wide founded on a stiff saturated clay at a depth of 1 meter the unit weight of the clay is  $17$  kilonewton per meter cube. And the shear strength is  $1020$  kilonewton per meter square, assume the load to be applied rapidly such that undrained condition  $\phi$  equal to  $0$  prevail use factor of safety equal to  $3$  and use the Skempton's parameter.

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Now, Skempton parameter  $N_c$  can be obtained by this chart, which we have discussed in the last lecture.

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**SOLUTION :**

Using Skempton's equation we have

$$q_{ult} = cN_c + \gamma D_f \text{ or } q_{nu} = cN_c$$

From figure, for  $D_f/B = 0.5$ ,  
 $N_c = 6$  for strip footing.

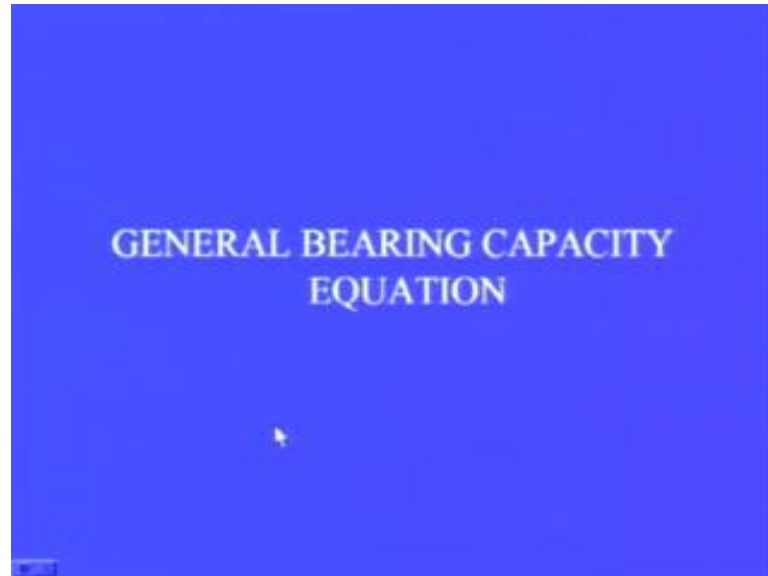
Therefore  $q_{ult} = 120 \times 6 = 720 \text{ kN/m}^2$

$$q_{ns} = 720/3 = 240 \text{ kN/m}^2$$

Now, using this chart and the equation given by Skempton we have find out this  $q$  ultimate equal to  $cN_c$  plus  $\gamma D_f$  where this  $N_c$  is obtained by the previous chart and as it is on the surface. So, this  $D_f$  equal to 0 we will end up with  $q$  net ultimate, that is equal to  $cN_c$ . Now, from this figure for  $D_f$  by  $B$  equal to 0.5  $N_c$  comes out to be 6 for square footing strip footing therefore,  $q$  ultimate equal to 120 into 6 that is 712

kilonewton per meter square  $q_{net}$  safe comes out to be 720 divided by three that is 240 kilonewton per meter square.

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So, for we have discussed about the Terzaghi, bearing capacity equation. And we have taken different problems, variety of problems by which we can either find out ultimate bearing capacity. We can find out net safe bearing capacity, we can find out safe bearing capacity and also in few of the problems, we have discussed such that depending upon the unknown. Let us say, whether it is width of the footing or the depth of the footing or the bearing capacity factor  $N_\gamma$ , we can obtain using Terzaghi bearing capacity equation. Similar to this we have also discussed about how to apply the water table correction? Whether, the water table is above the base of the foundation or below the base of the foundation.

Now, this general bearing this bearing capacity safe bearing capacity, equation has been extended by many researchers. And the, they have obtained general bearing capacity equation, the form of the equation has been found to be similar to that of Terzaghi bearing capacity equation. Only, difference is in the parameters like  $N_c$   $N_q$   $N_\gamma$  basically you will find that  $N_c$  and  $N_q$  are almost similar as that of Terzaghi. But there will be difference only in the value of  $N_\gamma$ , they have also extended it for the different safe factors. So, now, onwards I am going to discuss the modifications, which

are being done by various researchers in the Terzaghi bearing capacity equation. So, that is general bearing capacity equation.

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Meyerhof (1963) presented a general bearing capacity equation which takes into account the shape and the inclination of load. The general form of equation suggested by Meyerhof for bearing capacity is

$$q_{ult} = cN_c s_c d_c i_c + q'_o N_q s_q d_q i_q + 0.5 \gamma B N_\gamma s_\gamma d_\gamma i_\gamma$$

It was Meyerhof in 1963, who presented a general bearing capacity which takes into account the shape of shape and the inclination of the load the general form of equation. Suggested by Meyerhof for bearing capacity is given as  $cN_c s_c d_c i_c$  plus  $q'_o N_q s_q d_q i_q$  and plus  $0.5 \gamma B N_\gamma s_\gamma d_\gamma i_\gamma$  now here.

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where

$c$  = unit cohesion

$q'_o$  = effective overburden pressure at base level of the foundation

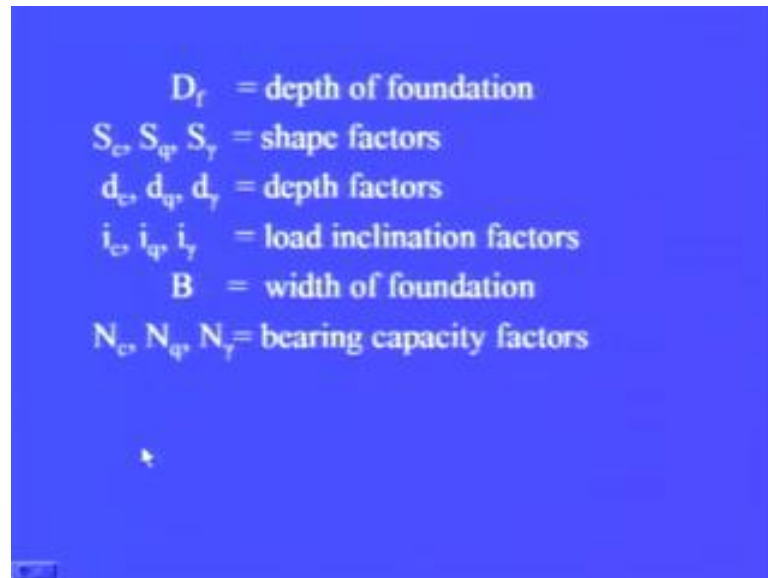
$$= \gamma D_f$$

$\gamma$  = effective unit weight above base level of foundation

$\gamma$  = effective unit weight of soil below foundation base

$C$  is the unit cohesion  $q_0$  is the effective overburden pressure, at the base level of foundation. Which can be determined, if we know effective unit weight  $\gamma_0$  and the depth of foundation  $D_f$ . Where  $\gamma_0$  is the effective unit weight above the base level of foundation and  $\gamma$  is the effective unit weight of soil below the base level of foundation.

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$D_f$  is the depth of foundation  $S_c, S_q, S_\gamma$  are the shape factors,  $d_c, d_q, d_\gamma$  are the depth factors,  $i_c, i_q, i_\gamma$  are the load inclination factors,  $B$  is the width of foundation and  $N_c, N_q, N_\gamma$  are the bearing capacity factors.

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- Hansen (1970) extended the work of Meyerhof by including two additional factors to take care of base tilt and foundations on slopes.
- Vesic (1973) used the same form of equation suggested by Hansen.
- All three investigators have used equations proposed by Prandtl (1921) for computing the values of  $N_c$  and  $N_q$  wherein the foundation base is assumed as smooth with the angle  $\alpha = 45^\circ + \Phi/2$ .

Hansen in 1970 extended the work of Meyerhof by including 2 additional factors to take care of the base tilt and the foundations on slopes. Another researchers Vesic in 1973 used the same form of equation, suggested by Hansen all 3 investigators have used equations proposed by Prandtl 1921 for computing the values of  $N_c$  and  $N_q$ . Where in the foundation base is assumed as smooth and it is incline at an the the sides of the wedge are incline at an angle of 45 plus phi by 2.

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However the equations used by them for computing the values of  $N_c$ ,  $N_q$  and  $N_\gamma$  are,

$$N_q = e^{(1.1 \tan \Phi)} N_\phi$$

$$N_\phi = \tan^2 (45^\circ + \Phi/2)$$

$$N_c = (N_q - 1) \cot \Phi$$

$$N_\gamma = (N_q - 1) \tan(1.4\Phi) \quad - \text{(Meyerhof)}$$

$$N_\gamma = 1.5(N_q - 1) \tan \Phi \quad - \text{(Hansen)}$$

$$N_\gamma = 2.0(N_q - 1) \tan \Phi \quad - \text{(Vesic)}$$



power half for  $\phi$  equal to 0. Whereas, Hansen has given  $1 - \tan \alpha \tan \phi$  upon  $A_f c_a$  into  $N_c$ . So, similar expressions have been suggested by Meyerhof Hansen and Vesic for  $I_\gamma$  expression is like this, for  $I_\phi$  expression is like this. So, using this table, we can find out the inclination factors the safe factors. And depth factors, which are to be used in the general bearing capacity equation the different terms which have been used in calculating the parameters are given as follows?

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The following terms are defined with regard to the inclination factors,  
 $Q_h$  = horizontal component of inclined load  
 $Q_v$  = vertical component of inclined load  
 $c_a$  = unit cohesion on the base of footing  
 $A_f$  = effective contact area of footing  
 $m = m_b = (2 + (B/L)) / (1 + (B/L))$   
                     with  $Q_h$  parallel to B  
 $m = m_b = (2 + (B/L)) / (1 + (B/L))$   
                     with  $Q_h$  parallel to L

Like  $Q_h$  is the horizontal component of the inclined load,  $Q_v$  is the vertical component of inclined load,  $c_a$  is the unit cohesion on the base of the footing.  $A_f$  is the effective contact area of the footing,  $m$  equal to  $m_b$  is given by this particular equation which says 2 plus B upon L divided by 1 plus B by L with  $Q_h$  parallel to B and if  $Q_h$  is parallel to L. Then it will be  $m$  equal to  $m_b$  2 plus B upon L plus 1 divided by 1 plus B upon L.

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Henson recommends the following equation for  $\Phi = 0$  case;

$$q_{ult} = cNc (1 + s_c + d_c - i_c) + q_0$$

$s$ ,  $d$  and  $i$  are the Henson's shape, depth and inclination parameters.

Henson recommends, the following equation for phi equal to 0 case, that is equal to  $q_{ult}$   $cNc$  in bracket 1 plus  $s_c$  plus  $d_c$  minus  $i_c$  plus  $q_0$ . Whereas,  $d$  and  $I$  are the Henson's shape depth and inclination parameters and these can be read from the table given in the book by V N S Moorthy.

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**Validity of the Bearing Capacity Equations**

- There has been little experimental verification of any of the methods except by using model footings.
- Terzaghi's equation, being the first proposed, has been quite popular with designers.
- Both Meyerhof and Hansen methods are widely used.
- The Vesic method has not been much used.

Validity of bearing capacity equations, there has been little experimental verification of any of the methods. Except by using model footings Terzaghi's equation being the first

proposed has been quite popular with the designers. Now, both Meyerhof and Hansen methods are widely used the Vesic method has not been much used.

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- It is good practice to use at least two methods and compare the computed values of  $q_{ult}$ .
- If the two values do not compare well, use a third method.

It is a good practice to use at least 2 methods and then compare the computed values of  $q_{ultimate}$ . Now, if the 2 method values do not compare well then we use a third method out of these, as I said the Terzaghi bearing capacity equation is widely used.

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**BEARING CAPACITY OF  
SHALLOW FOUNDATIONS AS  
PER IS:6403-1981**

Now, as per our Indian Standard 6 4 0 1 9 8 1, they have consider the equation which was given by Brinch and Hansen.

(Refer Slide Time: 28:35)

IS: 6403-1981 recommends following equation for the computation of net ultimate bearing capacity of a shallow foundation in general shear failure,

$$q_{nu} = cN_c s_c d_c i_c + q'_o (N_q - 1) s_q d_q i_q + 0.5 \gamma B N_\gamma s_\gamma d_\gamma i_\gamma R'_w$$

$N_c$ ,  $N_q$  and  $N_\gamma$  are bearing capacity factors suggested by Vesic (1973).  $R'_w$  is a factor which takes in to account the effect of water table.

And that equation is recommended for, used to determine net ultimate bearing capacity of a shallow foundation, in the case of a general shear failure case. And this equation is  $q_{nu}$  net ultimate equal to  $cN_c$  plus  $s_c d_c i_c$  plus  $q'_o (N_q - 1) s_q d_q i_q$  plus  $0.5 \gamma B N_\gamma s_\gamma d_\gamma i_\gamma R'_w$ . Where this is the effective overburden pressure at the level of foundation  $N_q$  minus 1  $s_q d_q i_q$  plus  $0.5 \gamma B N_\gamma s_\gamma d_\gamma i_\gamma R'_w$ . That is the reduction factor for water table or correction factor for the water table. Now, all these safe factors are given in the tabular form in IS 6403-1981. So, here again  $N_c$ ,  $N_q$  and  $N_\gamma$  are the bearing capacity factors, that are suggested by Vesic and  $R'_w$  is a factor which takes into account the effect of water table.

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$R'_w = 1$  if water table is likely to remain permanently at or below a depth of  $(D_f + B)$  below ground level or  $D'_w > B$  where  $D'_w$  is the depth of water table measured from base of foundation.

For  $D'_w = 0$   $R'_w = 0.5$

Linear interpolation between 0 and 1 for  $0 < D'_w < B$

$R_w$  will be equal to 1, if water table is likely to remain permanently at or below a depth of  $D_f$  plus  $B$  means, at a large depth below the ground level or  $D_w$  is greater than  $B$ . Where  $D_w$  is the depth of water table measured from the base level of foundation. For  $D_w$  equal to 0 means the water table is just at the level of foundation then this  $R_w$  is taken as equal to 0.5. Now, for the intermediate values we can use linear interpolation between 0 and 1, when the  $D_w$  is ranging between 0 and width of the foundations. So, this is the width and we will have to go up to the width of the depth equivalent to width of the foundation.

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For a cohesive soil, the net ultimate bearing capacity of a footing immediately upon construction ( $\Phi_u = 0^\circ$ ) is given by

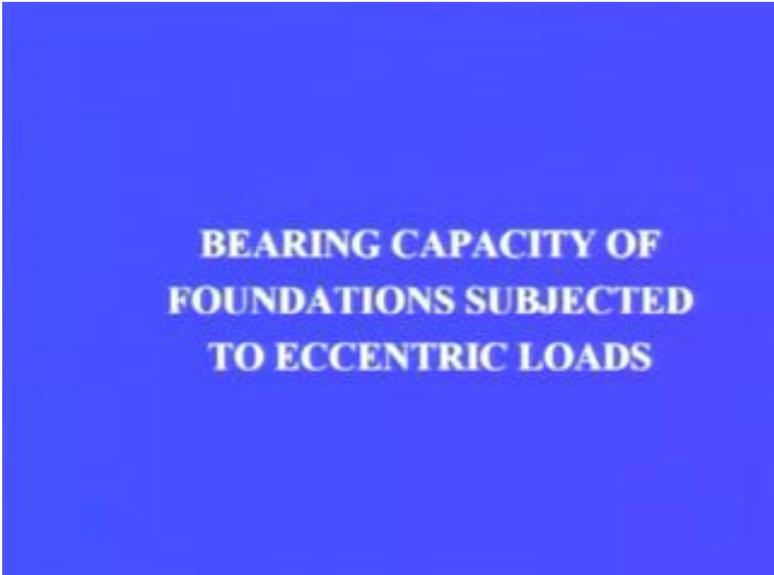
$$q_{nu} = c_u N_c s_c d_c i_c$$

where  $N_c = 5.14$  and  $c_u$  is obtained either from unconfined compression test or from correlations with static cone penetration resistance.

$c_u$  varies between 1/18 to 1/15 of static cone penetration resistance for normally consolidated clay and 1/26 to 1/22 of static cone penetration resistance for normally consolidated clay

For a cohesive soil, the net ultimate bearing capacity of a footing immediately, upon construction immediately, upon construction, we know that the undrained conditions prevail. So, we take  $\phi_u$  equal to 0 and for that case this net ultimate bearing capacity will be given by  $c_u N_c s_c d_c i_c$ . Where  $N_c$  is taken as 5.14 and  $c_u$  is obtained either from the unconfined compression test or from correlations with static cone penetration resistance, that is of field method to determine the resistance of the strata.  $c_u$  varies between 118 to 115 of static cone penetration resistance for normally consolidated clay whereas; it is 126 to 120 second of static cone penetration resistance value for over consolidated clays.

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**BEARING CAPACITY OF  
FOUNDATIONS SUBJECTED  
TO ECCENTRIC LOADS**

Now, so far we have considered the case of the foundations, in which the load is vertical, application of the load is vertical. However, inclination factor, inclination parameters given by Meyerhof Hansen and Vesic can be used for inclined loads. But when there are many situations, in which we get the load, which is applied on the foundation eccentrically.

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If a foundation is subjected to lateral loads and moments in addition to vertical loads, eccentricity in loading results.

The point of application of the resultant of all the loads would lie outside the geometric centre of the foundation.

So, in order to determine bearing capacity of foundations subjected to eccentric loads like due to such as if a foundation is subjected to lateral loads and moments in addition to vertical loads, eccentricity in loading results. It may be inclined load or it may be a moment, which is acting on the column. And then finally, foundation is subjected to that moment, the point of application of the resultant of all the loads, would lie outside the geometric centre of the foundation.

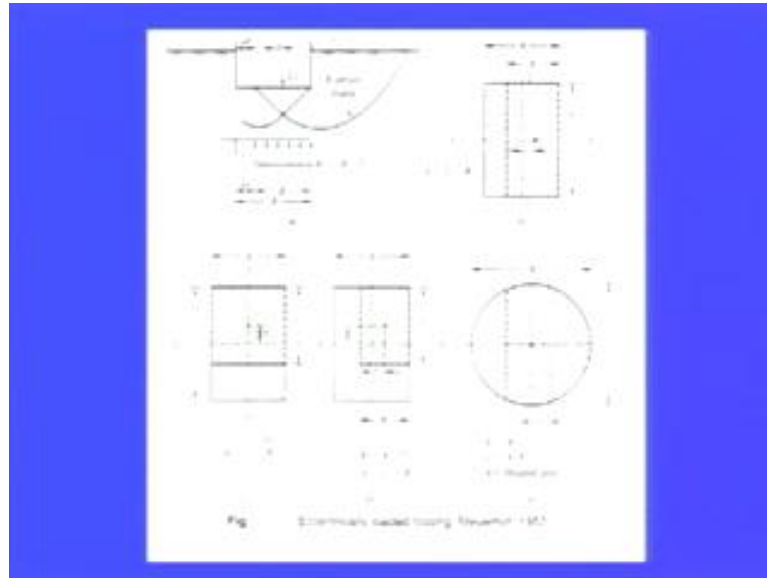
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The eccentricity  $e$  is measured from center of foundation to the point of application normal to the axis of the foundation.

The maximum eccentricity normally allowed is  $B/6$ , where  $B$  is the width of the foundation

The eccentricity  $e$  is measured from the center of foundation to the point of application. Normal to the axis of the foundation the maximum eccentricity normally, allowed is  $B/6$ , where  $B$  is the width of the foundation.

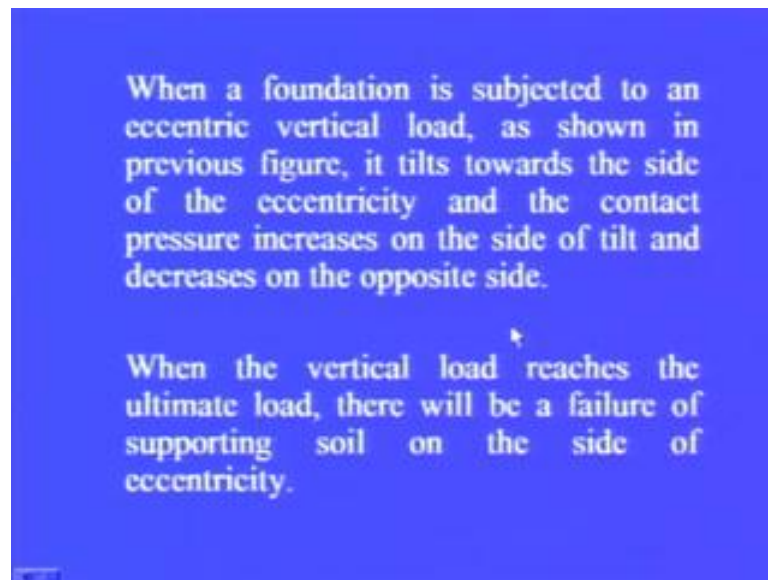
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Now, we can see from this particular figure, let us take the case of a foundation, which is resting at this particular depth. Now, this load  $Qd$ , which is applied not applied at the center, but it is centrally placed at this particular location. So, the eccentricity measured is  $e$ . Now, when this, when this eccentric load is applied then the pressure, which is exhausted on the side of the eccentricity is more than the pressure exhausted on the other side. And this becomes effective the width of the footing becomes, effective that comes out to be  $B - 2e$ . Now, the same case you can see for in the plan let us say we have a rectangular footing and eccentricity is in the direction of the width.

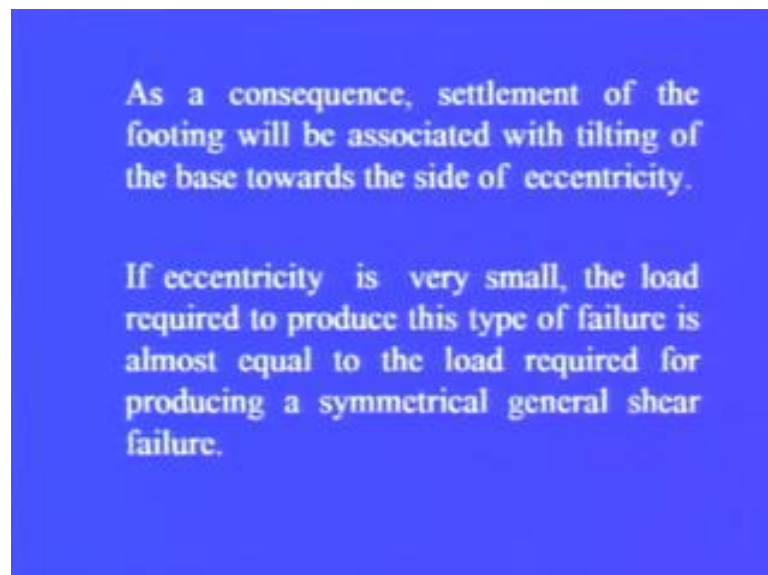
So, length will remain as it is, but width will be reduced by  $B - 2e$ , if this is  $e$  then it will come out to be  $2e$ . Now, in another case, let us say that the eccentricity is applied in the direction of the length. Then the effective length will be equal to  $L - 2e_y$ , where  $e_y$  is the eccentricity, which is measured from the center of this foundation. Now, it is quite possible, that the load, which is applied may be eccentric in both the directions. Like, if you have this point and this is the center of the footing, then this  $e_x$  in the  $x$  direction. That is the in the width direction and  $e_y$  is in the  $y$  direction, that is in the case of length direction similar is the case, when the circular foundation is considered.

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So, when a foundation is subjected to an eccentric vertical load as shown in previous figure, it tilts towards the side of the eccentricity. And the contact pressure increases on the side of tilt and decrease on the opposite side. Now, when the vertical load reaches the ultimate load, there will be a failure of supporting soil on the side of the eccentricity.

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As a consequence, settlement of the footing will be associated with tilting of the base towards, the side of eccentricity. If eccentricity is very small the load required produce,

this type of failure is almost equal to the load required, for producing a symmetrical general shear failure.

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Failure occurs due to intense radial shear on one side of the plane of symmetry, while the deformations in the zone of radial shear on the other side are still insignificant.

For this reason, the failure is always associated with a heave on that side towards which the footing tilts.

Failure occurs, due to the intense radial shear on 1 side of the plane of symmetry. While the deformations in the zone of radial shear on the other side are still insignificant for this reason, the failure is always associated with a heave on the on that side towards, which the footing is tilting.

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In order to take in to account the effect of eccentricity on the ultimate bearing capacity of the foundation, Meyerhof (1953) suggested to take effective footing dimensions as follows,

$$L' = L - 2e_y, \quad B' = B - 2e_x$$

The effective area is given by

$$A' = B' \cdot L'$$

where  $e_x$  and  $e_y$  are the eccentricities in the direction of axes.

In order to take into account, the effect of eccentricity, on the ultimate, bearing capacity of foundation, Meyerhof suggested to, take effective footing dimensions as follows. If the eccentricity is on the length direction, then and if, suppose the load is acting at a distance of  $e_y$  from the center of the footing  $L$  dash will be written as  $L$  minus  $2e_y$ , whereas, if it is in the width direction. And if the load is acting on at a distance of  $e_x$  from the center, then the effective width will be given by  $B$  minus  $2e_x$ . Now, if it is, eccentric in both the directions, so, we will be considering this effective length and then effective width. So, the effective area will simply be given by  $B$  dash into  $L$  dash, where  $e_x$  and  $e_y$  are the eccentricities in the direction of the axes.

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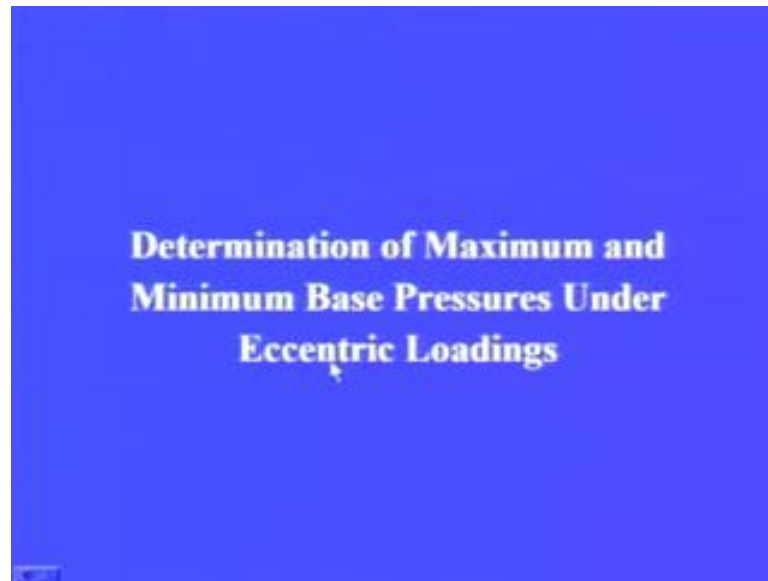
The Ultimate load bearing capacity of a footing subjected to eccentric loads may be expressed as

$$Q_{ult} = q_{ult} A'$$

where,  $q_{ult}$  = ultimate bearing capacity of the footing with the load acting at the center of the footing.

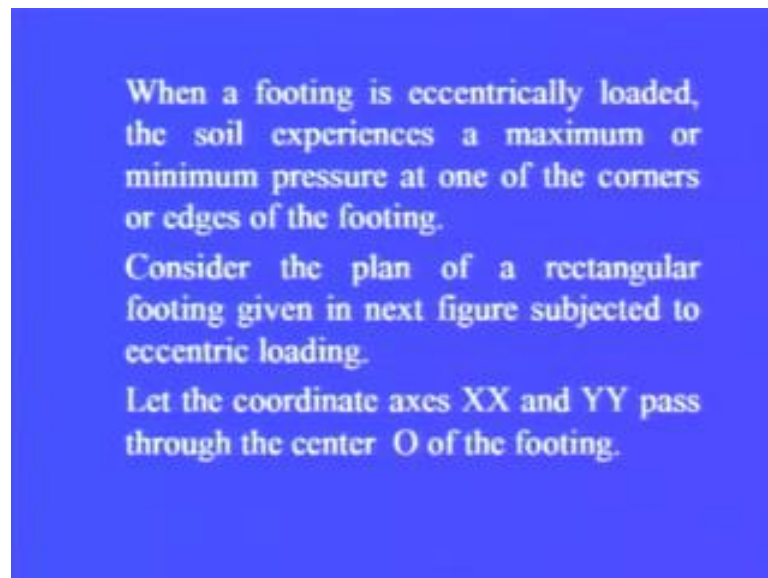
The ultimate load bearing capacity of a footing subjected to eccentric loads may be expressed as  $Q_{ultimate}$ . That will be equal to  $q_{ultimate}$  bearing capacity and this is the  $Q_{ultimate}$  load multiplied by the effective area.

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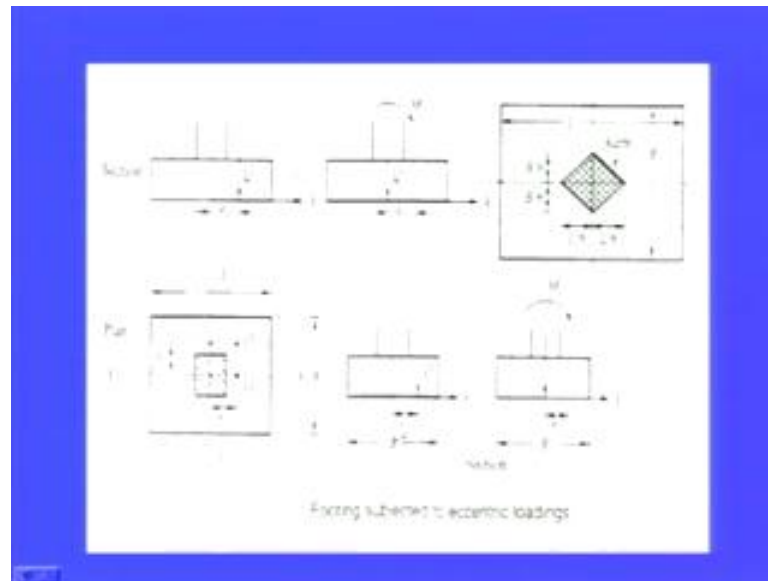
Now, in order to determine, a maximum and minimum base pressures, base pressures under eccentric loading.

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Now, when a footing is subjected to eccentrically loaded the soil experiences as we have seen a maximum or minimum pressure, at one of the corners or edges of the footing. Now, let us, consider the plan of a rectangular footing, given in the next figure subjected to eccentric loading. Let the coordinate axis are XX and YY pass through the center O of the footing.

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So, this is the case of a rectangular footing the dimensions are  $L$  and  $B$  and the coordinate axis are  $XX$  and  $YY$  passing through center of the footing. Now, corresponding elevation is shown here now it is the case when this load is acting at a distance  $e_x$  in the direction of  $x$ . Or it may be resulting due to load is applied from a at the center, but there is a moment  $M_x$  which is acting in this direction. So, there will be eccentricity  $e$ . similar to this is the case when the load there is 2 way eccentricity in the  $x$  direction.

As well as in the  $y$  direction when this particular foot, footing foundation is subjected to the moments in both the directions or the load is not applied at the center. And it is at a distance of  $e_x$  from the in the  $x$  direction and it at a distance of  $e_y$  from the  $y$  direction. Now, here you can see that there is a dashed area shown in which this allowed it eccentricity is  $B/6$  by  $x$   $B/6$  shear and  $B/6$  on the other the side. Similarly  $L/6$  shear and  $L/6$  on the other side this area is known as core as for as the bearing cap the load is applied within this area. Then you will find that no tension is developed anywhere in the foundation which will be more made more clear by the following discussion.

(Refer Slide Time: 39:40)

If a vertical load passes through O, the footing is symmetrically loaded. If a vertical load passes through  $O_x$  on the X-axis, the footing is loaded with one way eccentricity. The distance of  $O_x$  from O, designated as  $e_x$  is called eccentricity in X-direction. If the load passes through  $O_y$  on the Y-axis, the eccentricity is  $e_y$  in the Y-direction. If the load passes through  $O_{xy}$  the eccentricity is called with two way eccentricity or double eccentricity.

If a vertical load passes through O the footing is symmetrically loaded if a vertical loaded passes through  $O_x$  on the X axis the footing is loaded with 1 way eccentricity. The distance of  $O_x$  from O is designated as  $e_x$  is called the eccentricity in the X direction. Similarly, if the load passes through  $O_y$  on the Y axis the eccentricity is  $e_y$  in the Y direction if the load passes through  $O_{xy}$  the eccentricity is called with 2 way eccentricity or double eccentricity as we have seen in the previous figure.

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For the load passing through  $O_{xy}$  in the figure, the points C and D at the corners of the footing experience the maximum and minimum pressures respectively  
The general equation for pressure may be written as,  
$$q = (Q/A) \pm (Q \cdot e_x / I_y) \pm (Q \cdot e_y / I_x)$$
  
or  
$$q = (Q/A) \pm (M_x / I_y) \pm (M_y / I_x)$$

Now, for the load passing through Oxy in the figure the point C and D at the corners of the footing experience the maximum and minimum pressure respectively. And these maximum and minimum pressures can be determined by following equation where  $q$  is the pressure  $Q$  is the load  $A$  is the area and  $e_x$  and  $e_y$  are the eccentricities and  $I_x$  and  $I_y$  are the moment of inertia. So, this  $q$  will be given by  $Q$  upon  $A$  plus minus  $Q$  into  $e_x$  into  $x$  upon  $I_y$  plus minus  $Q$  into  $e_y$  into  $y$  upon  $I_x$  or  $Q$  upon  $A$  equal to  $q$  equal to  $Q$  upon  $A$  plus minus  $M_x$  into  $x$  where  $M_x$  is the moment. So, here this  $Q$  into  $e_x$  is nothing but the moment which is applied in the  $x$  direction similarly  $Q$  into  $e_y$  is nothing but  $M_y$  which is applied in the  $y$  direction.

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where,

- $q$  = contact pressure at a given point  $(x,y)$
- $Q$  = total vertical load
- $A$  = area of footing
- $M_x = Q \cdot e_x$  = moment about axis Y-Y
- $M_y = Q \cdot e_y$  = moment about axis X-X
- $I_x, I_y$  = moment of inertia of footing about XX and YY axes

So, these are self explanatory  $q$  equal to contact pressure at a given point whose coordinates are given by  $x$  and  $y$ .  $Q$  is the total vertical load  $A$  is the area of footing  $M_x$  is the  $Q$  into  $e_x$  moment about axis YY  $M_y$  is  $Q$  into  $e_y$ . That is the moment about axis XX and  $I_x$  and  $I_y$  are the moment of inertia footing about XX and YY axis.

(Refer Slide Time: 41:48)

$q_{\max}$  and  $q_{\min}$  at points C and D may be obtained by substituting in eqn., for  
 $I_x = (LB^3 / 12)$   
 $I_y = (BL^3 / 12)$   
 $x = (L / 2)$   
 $y = (B / 2)$

Q max and q minimum at point C and D may be obtained by substituting in equation for  $I_x$  that is equal to  $LB^3$  upon 12 and  $I_y$  that is equal to  $BL^3$  upon 12  $x$  equal to  $L$  by 2 and  $y$  equal to  $B$  by 2.

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We have,  
 $q_{\max} = (Q/A) * ( 1 + (6e_x / L) + (6e_y / B) )$   
 $q_{\min} = (Q/A) * ( 1 - (6e_x / L) - (6e_y / B) )$   
For one way eccentricity put either  $e_x = 0$ ,  
or  $e_y = 0$  in the equation.

When we substitute these values, we will get  $q_{\max}$  as  $Q$  upon  $A$  1 plus  $6e_x$  upon  $L$  plus  $6e_y$  upon  $B$   $q_{\min}$  will be equal to  $Q$  upon  $A$  1 minus  $6e_x$  upon  $L$  minus  $6e_y$  upon  $B$ . For one way eccentricity put either  $e_x$  equal to 0 or  $B e_y$  equal to 0 in the equation we will get  $q_{\max}$  and  $q_{\min}$ .

(Refer Slide Time: 42:35)

When  $e_x$  or  $e_y$  exceed a certain limit, the above equation gives negative value of  $q$  which indicates tension between the soil and the bottom of the footing.

The above equations are applicable only when the load is applied within a limited area known as the Kern as shown in figure so that the load may fall within the shaded area to avoid tension .

When  $e_x$  or  $e_y$  exceed a certain limit the above equation gives negative value of  $q$  which indicates tension between the soil. And the bottom of footing the above equations are applicable only when the load is applied within a limited area known as the Kern as shown in the figure. So that the load may fall within the shaded area to avoid any tension in the subsoil.

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### **Example: 1**

A water tank foundation has a footing of size  $6\text{m} \times 6\text{m}$  founded at a depth of  $3\text{m}$  below ground level in a medium dense sand stratum ( $\Phi = 33^\circ$ ) of great depth. The foundation is subjected to a vertical load at an eccentricity of  $B/10$  along one of the axes. The soil profile with remaining data is shown in figure on next slide. Estimate the ultimate load,  $Q_{ult}$ , by Meyerhof's method

Now, we can discuss this with an example. Let us consider this example in which a water tank foundation has a footing of size  $6\text{ meter by } 6\text{ meter}$  founded at a

depth of 3 meter below ground level in medium dense sand for which  $\phi$  equal to 33 degrees. That is extending up to a great depth the foundation is subjected to a vertical load at an eccentricity of  $b$  upon 10 along one of the axis the soil profile with remaining data is shown in figure on the next slide. Estimate ultimate load  $Q$  ultimate by Meyerhof method.

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**Solution:**

$$B' = B - 2e = 6 - 2(0.6) = 4.8\text{m}$$

$$L' = L = B = 6\text{m}$$

**Meyerhof equation**

$$q_{ult} = cN_c s_c d_c i_c + q_u N_q s_q d_q i_q + 0.5 \gamma B N_\gamma s_\gamma d_\gamma i_\gamma$$

Here  $c = 0$ ,  $i_c = i_q = i_\gamma = 1$

**From Meyerhof Tables**

For  $\Phi = 33^\circ$   $N_q = 26.3$  and  $N_\gamma = 26.55$

$$s_q = 1 + 0.1 N_\phi [B/L]$$

$$= 1 + 0.1 \tan^2 (45^\circ + 33^\circ/2)(1) = 1.34$$

So, in order to get the solution  $B'$  can be determined as  $B$  minus  $2e$  6 minus 2 into point 6 that is equal to 4.8 meter  $L'$  will remain as it is, because it is eccentric only in the width direction. So,  $L'$  equal to  $L$  that is equal to  $B$  because it is a square foundation equal to 6 meter. And the Meyerhof equation is given by  $cN_c s_c d_c i_c$  plus  $q_u N_q s_q d_q i_q$  plus  $0.5 \gamma B N_\gamma s_\gamma d_\gamma i_\gamma$  here  $c$  equal to 0. So,  $i_c$  equal to  $i_q$  equal to  $i_\gamma$  equal to 1 from Meyerhof tables for  $\phi$  equal to 33 degrees  $N_q$  comes out to be 26.3 and  $N_\gamma$  equal to 26.55  $s_q$  will be equal to  $1 + 0.1 N_\phi B$  upon  $L$  where  $\phi$  equal to 33 degrees. And  $\phi$  equal to  $\tan^2 45$  plus  $\phi$  by 2. So, substituting value of  $\phi$  we will get this  $s_q$  as 1.34.

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$$\begin{aligned} s_r &= s_q = 1.34 \text{ for } \Phi > 10^\circ \\ d_q &= 1 + 0.1 (N_\Phi)^{0.5} [D_f/B'] \\ &= 1 + 0.1 * 1.84 [3/4.8] = 1.115 \\ \text{Substituting } d_r &= d_q = 1.115 \text{ for } \Phi > 10^\circ \\ q_{ult} &= 18.5 * 3 * 26.3 * 1.34 * 1.115 \\ &\quad + 0.5 * 18.5 * 4.8 * 26.55 * 1.34 * 1.115 \\ &= 2181 + 1761 = 3942 \text{ kN/m}^2 \\ Q_{ult} &= B * B' * q_{ult} = 6 * 4.8 * 3942 = 114 \text{ MN} \end{aligned}$$

S gamma and sq will be equal to 1.34 for phi equal phi greater than 10 degrees and dq will be equal to 1 plus point 1 N phi to the power 0.5 Df upon B dash. So, here this is 1045 plus phi by 2 1 plus 0.1 when we substitute these values we will get dq as 1.115. Now, substituting d gamma equal to dq equal to 1.115 for phi greater than 10 degrees in the ultimate bearing capacity equation, we will get ultimate bearing capacity as 3942 kilonewton per meter square. And hence the ultimate load that will be equal to B into B dash here this is the effective area. And this effective area is reduced due to the eccentricity multiplied by ultimate bearing capacity. And that comes out to be 114 Mega Newton.

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### **Example:2**

A chimney, with a rigid base 2.5m square, is placed at a depth of 1m below the ground level. The soil is clay with an unconfined compressive strength of 60 kN/m<sup>2</sup> and the unit weight of 20 kN/m<sup>3</sup>. The weight of the chimney is 60 kN. The chimney has a resultant wind load of 19.5 kN acting parallel to one of sides of the chimney base at a height of 1.5 m above the ground surface. Determine the FOS with respect to bearing capacity. Use Meyerhof's recommendation.

There is another example of eccentricity a chimney with a rigid base 2.5 meter square is placed at a depth of 1 meter below the ground level. The soil is clay with an unconfined compressive strength of 60 kilonewton per meter square. And the unit weight of is 20 kilonewton per meter cube the weight of the chimney is 60 kilonewton. The chimney has a resultant wind load of 19.5 kilonewton acting parallel to 1 of the sides of the chimney base at a height of 1.5 meter above the ground surface. So, that we can find out what will be the moment? Determine the factor of safety with respect to bearing capacity again using Meyerhof recommendations.

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### **Solution:**

The wind load will have the effect of introducing both inclination and eccentricity of loading. The resultant of the wind load and weight force will be inclined at an angle  $\alpha$  to the vertical.

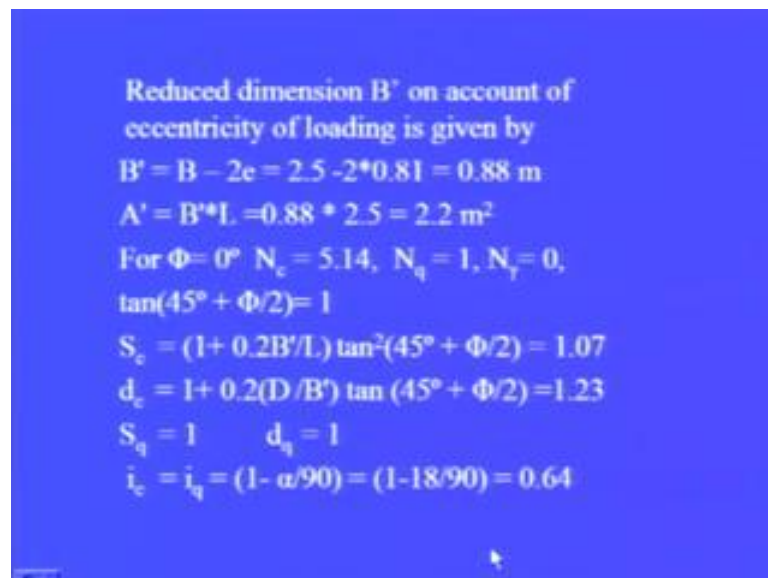
$$\tan \alpha = \text{horizontal wind force} / \text{vertical weight force} \\ = 19.5/60 = 0.325 \quad \text{and therefore } \alpha = 18^\circ$$

Height of the horizontal load above the base  
= 1.5 + 1 = 2.5 m. Eccentricity of the resultant load,  $e$  can be calculated from,

$$e/2.5 = \tan \alpha = 0.325 \quad e = 2.5 * 0.325 = 0.81 \text{ m}$$

Now, the wind load will have the effect of introducing both inclination and eccentricity of loading the resultant of the wind load. And the wind force will be inclined at an angle of alpha to the vertical where this alpha will be given by this particular equation  $\tan \alpha$  equal to horizontal wind force divided by vertical weight force that will be 19.5 divided by 60 equal to 0.325. And therefore, alpha equal to tan inverse of 0.325 comes out to be 18 degrees. Height of the horizontal load above the base of the footing is 1.5 plus 1. That is the depth equal to 2.5 meter eccentricity of the resultant load e can be calculated from e upon 2.5 equal to tan of alpha equal to 0.325 which we have obtained from here. So, e can be taken calculated as 0.81 meter.

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Reduced dimension  $B'$  on account of eccentricity of loading is given by

$$B' = B - 2e = 2.5 - 2 \times 0.81 = 0.88 \text{ m}$$

$$A' = B' \times L = 0.88 \times 2.5 = 2.2 \text{ m}^2$$

For  $\Phi = 0^\circ$   $N_c = 5.14$ ,  $N_q = 1$ ,  $N_\gamma = 0$ ,  
 $\tan(45^\circ + \Phi/2) = 1$

$$S_c = (1 + 0.2B'/L) \tan^2(45^\circ + \Phi/2) = 1.07$$

$$d_c = 1 + 0.2(D/B') \tan(45^\circ + \Phi/2) = 1.23$$

$$S_q = 1 \quad d_q = 1$$

$$i_c = i_q = (1 - \alpha/90) = (1 - 18/90) = 0.64$$

Once e is known we can find out reduced dimension B dash and that B dash will be equal to B minus 2 e. So, substitute value of eccentricity here that is 0.2 8 0.81, you will get B dash equal to 0.88 meter and effective area will be equal to B dash into L 0.88 into 2.5 that is equal to 2.2 meter square. Now, for phi equal to 0 Nc will be equal to 5.14 Nq equal to 1 and N gamma equal to 0 tan of 45 plus phi by 2 equal to 1 sc by using different equations suggested by Meyerhof. We can find out sc that comes to be 1.07 dc that comes out to be 1.23 and sq equal to 1 dq equal to 1. Similarly, we can also obtain inclination parameters ic and iq both are equal equal to 1 minus alpha by 90 and when we substitute value of alpha we will get this as 0.64.

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$$\begin{aligned} &\text{Meyerhof equation} \\ &q_{ult} = cN_c s_c d_c i_c + q_0 N_q s_q d_q i_q + 0.5 \gamma B N_\gamma s_\gamma d_\gamma i_\gamma \\ &c_u = 60/2 = 30 \text{ kN/m}^2 \\ &q_{ult} = 30 * 5.14 * 1.07 * 1.23 * 0.64 \\ &\quad + 20 * 1 * 1 * 1 * 0.64 \\ &\quad = 142.7 \text{ kN/m}^2 \\ &Q_u = q_{ult} A' = 142.7 * 2.2 = 314 \text{ kN} \\ &\text{Factor of safety} = 314/60 = 5.2 \end{aligned}$$

When we substitute all these parameters in the Meyerhof equation which is given by  $cN_c s_c d_c i_c$  plus  $q_0 N_q s_q d_q i_q$  plus  $0.5 \gamma B N_\gamma s_\gamma d_\gamma i_\gamma$ . And  $\gamma$   $c_u$  is equal to 60 by 230 kilonewton per meter square based on the unconfined compressive strength  $q_{ultimate}$ . When you substitute all these values here we will find that it comes out to be 142.7 kilonewton per meter square. And hence the ultimate load is equal to ultimate bearing capacity multiplied by effective area and that comes out to be 314 kilonewton. Now, for a factor of safety we will get the factor of safety we will get as 314 divided by 60 that is equal to 5.2. So, the factor of safety for this particular case will be 5.2.

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**Example:3**

Calculate net ultimate bearing capacity of a rectangular footing  $2\text{m} \times 4\text{m}$  in plan, founded at a depth of  $1.5\text{m}$  below the ground surface. The load on the footing acts at an angle of  $15^\circ$  to the vertical and eccentric in the direction of width by  $15\text{ cm}$ . The saturated unit weight of the soil is  $18\text{ kN/m}^3$ . The effective stress shear strength parameters  $c' = 15\text{ kN/m}^2$  and  $\phi' = 25^\circ$  can be used in analysis. Natural water table is at a depth of  $2\text{m}$  below the ground surface. Use IS: 6403-1981 recommendations.

Now, another example calculate net ultimate bearing capacity of a rectangular footing 2 meter by 4 meter in plan, founded at a depth of 1.5 meter below the ground surface. The load on the footing acts at an angle of 15 degrees to the vertical and eccentric in the direction of width by 15 centimeter. The saturated unit weight of the soil is 18 kilonewton per meter cube the effective stress shear strength parameters  $c'$  equal to 15 kilonewton per meter square. And  $\phi'$  equal to 25 degrees can be used in the analysis natural water table is at a depth of 2 meter below the ground surface. Now, we will have to determine the bearing capacity using the IS 6403 1981 recommendations which we have discussed previously.

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**Solution:**

$$q_{\text{lim}} = cN_c s_c d_c i_c + q_u (N_q - 1) s_q d_q i_q + 0.5 \gamma B N_\gamma s_\gamma d_\gamma i_\gamma R'_w$$

$$c = c' = 15 \text{ kN/m}^2 \quad \Phi = \Phi' = 25^\circ$$

$$D'_w = 2.0 - 1.5 = 0.5 \text{ m}$$

For  $\Phi = 25^\circ$ ,  $N_c = 20.7$ ,  $N_q = 10.7$  and  $N_\gamma = 10.9$

$$\gamma_{\text{sat}} = \gamma = 18 \text{ kN/m}^3 \quad q = 18 \times 1.5 = 27 \text{ kN/m}^2$$

For  $D'_w/B = 0.25$   $R'_w = 0.625$  (by interpolation)

$$e_x = 0.15 \text{ m}; \quad \text{effective width } B' = B - 2e_x = 1.70 \text{ m}$$

$$s_c = s_q = 1 + 0.2 B'/L = 1.025$$

$$s_\gamma = 1 - 0.4 B'/L = 0.83$$

Now, in order to get solution, what we will have to do? We will have to find? All the parameters which are there in the bearing capacity equation like  $s_c$   $d_c$   $i_c$  safe factors  $s_q$   $d_q$   $i_q$  and  $s_\gamma$   $d_\gamma$   $i_\gamma$  and the water table correction factor. Now,  $c$  is equal to 15 kilo Newton per meter square  $\Phi$  equal to  $\Phi'$ . That is given 25 degrees  $D_w$  dash is the depth of the water table below the base level of the foundation at that comes out to be 2 minus 1.5 that is equal to 0.5 meter. Now, for  $\Phi$  equal to 25 degrees, we can obtain from the table  $N_c$  equal to 20.7  $N_q$  equal to 10.7 and  $N_\gamma$  equal to 10.9  $\gamma$  saturated is given as 18 kilo Newton per meter cube.

So, we can find out the,  $q$  or  $q_0$  that is equal to 18 into 1.5 that is 27 kilo Newton per meter square. Now, from the graph by interpolation, we can find out the value  $R_w$  dash for  $D_w$  dash by  $B$  ratio equal to 0.25 that comes out to be 0.625 eccentricity is given in the  $x$  direction as 0.15 meter 15 centimeter. So, effective width will be equal to  $B$  dash minus  $2e_x$  that is equal to 1.7 meter. And safe factor will be given by  $1 + 0.2 B$  dash upon  $L$  here, effective width will be used in the expression and it comes out to be 1.025 similarly  $s_\gamma$  comes out to be 0.83 using this particular expression.

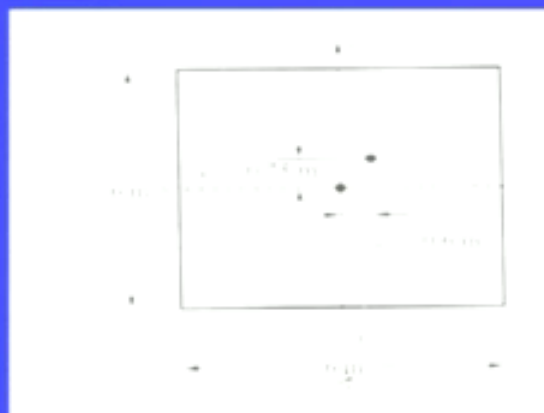
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$$\begin{aligned}
 d_c &= 1 + 0.2(D_f/B') \tan(45 + \Phi/2) = 1.28 \\
 d_q &= d_r = 1 + 0.1(D_f/B') \tan(45 + \Phi/2) = 1.14 \\
 i_c &= i_q = (1 - \alpha/90)^2 = (1 - 15/90)^2 = 0.69 \\
 i_r &= (1 - \alpha/\Phi)^2 = (1 - 15/25)^2 = 0.16 \\
 \text{Substituting these values,} \\
 q_{nu} &= 15 \times 20.7 \times 1.085 \times 1.28 \times 0.69 \\
 &\quad + 27 \times 9.7 \times 1.085 \times 1.14 \times 0.69 \\
 &\quad + 0.5 \times 18 \times 1.7 \times 10.9 \times 0.83 \times 1.14 \times 0.16 \times 0.625 \\
 &= 297.5 + 223.5 + 15.8 = 536.8 \text{ kN/m}^2
 \end{aligned}$$

Similarly, if we know phi and different parameters like, depth and width, we can obtain parameters  $d_c$ ,  $d_q$ ,  $i_c$  and  $i_r$ . Now, these values can be obtained by respective equations. So, here these comes out come out to be 1.28, 1.14, 0.69 and 0.16 is respectively, when we substitute these values in the net ultimate bearing capacity equation, when then after calculation. We will get this net ultimate bearing capacity as 536.8 kilo Newton per meter square.

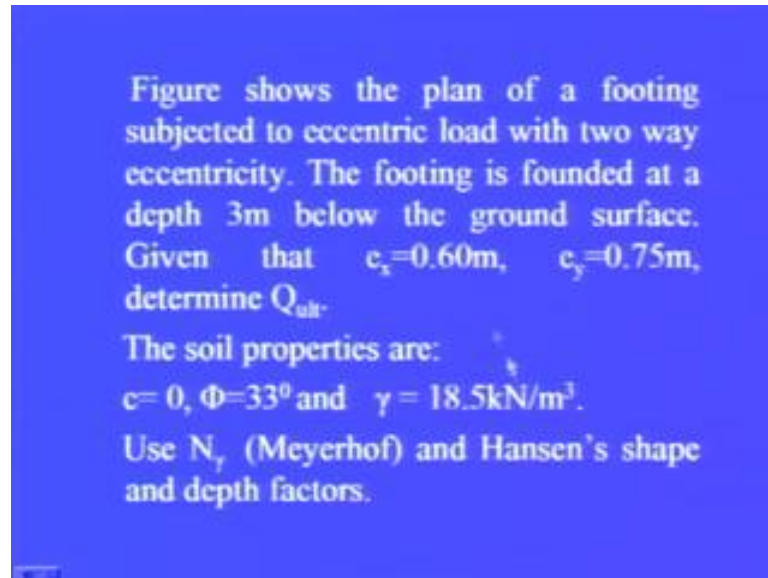
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#### Example: 4



Now, this is another example of 2 way eccentricity now, this is of footing of size 6 meter by 6 meter load is acting here, which is eccentric by 0.75 meter in this direction and by 0.6 meter in the other direction.

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Now, figure shows, the plan of a footing subjected to eccentric load with 2 way eccentricity the footing is founded at a depth of 3 meter, below the ground surface. Given that  $e_x$  equal to point 6 meter  $e_y$  equal to 0.75 meter, we will have to determine  $Q_{ultimate}$  load. The soil properties are given like this  $c$  equal to 0  $\phi$  equal to 33 degrees and  $\gamma$  equal to 18.5. We will have to use  $N_\gamma$  parameter given by Meyerhof and shape and depth factors given by Hansen.

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**Solution:**

For two way eccentricity, the effective length and breadth of the foundation are

$$B' = B - 2e_y = 6 - 2 \times 0.75 = 4.5 \text{ m.}$$

$$L' = L - 2e_x = 6 - 2 \times 0.60 = 4.8 \text{ m.}$$

Effective area,

$$A' = L' \times B' = 4.5 \times 4.8 = 21.6 \text{ m}^2$$

We have

$$q_{ult} = \gamma D_f N_q s_q d_q + 0.5 \gamma B' N_\gamma s_\gamma d_\gamma$$

Now, again in the case of 2 way eccentricity, we will have to find out effective width. And effective length which is given by B minus 2 ey L minus 2 ex when we substitute this we will get effective width as 4.5 meter effective length as 4.8 meter. So, effective area is the multiplication of these 221.6 meter and we have the ultimate bearing capacity equation as this.

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For  $\Phi = 33^\circ$ ,  $N_q = 26.3$  and  $N_\gamma = 26.55$

Using Hansen equations

$$s_q = 1 + (B'/L') \tan 33^\circ = 1.61$$

$$s_\gamma = 1 - 0.4(B'/L') = 0.63$$

$$d_q = 1 + 2 \tan 33^\circ (1 - \sin 33^\circ)^2 \times 3/4.5 = 1.183$$

$$d_\gamma = 1$$

$$\begin{aligned} q_{ult} &= 18.5 \times 3 \times 26.3 \times 1.61 \times 1.183 + \\ &\quad 0.5 \times 18.5 \times 4.5 \times 26.55 \times 0.63 \times 1 = 2780 + 696 \\ &= 3476 \text{ kN/m}^2 \end{aligned}$$

$$Q_{ult} = A' \times q_{ult} = 21.6 \times 3476 = 75082 \text{ kN}$$

For phi equal to 32 degree 33 degrees we get Nq and N gamma. And using Brinch Hansen equation we can find out sq s gamma dq d gamma. And when we substitute these

values in we will get ultimate bearing capacity as 3476 kilonewton per meter square. And hence the ultimate load multiplied by the effective area will be 75082 kilonewton. So, through this lecture we have discuss the modifications made by Meyerhof Vesic and Hansen. They have given safe factors inclination factors and depth factors. And we have also discussed the cases of eccentric loads in which eccentricity is either in one direction. Or maybe two way eccentricity when the footing is subjected to the moments like a ven force. We have also tried to discuss solved problems which will take into account all these factors and also the eccentric loads.

Thank you.