Course Name: An Introduction to Climate Dynamics, Variability and Monitoring

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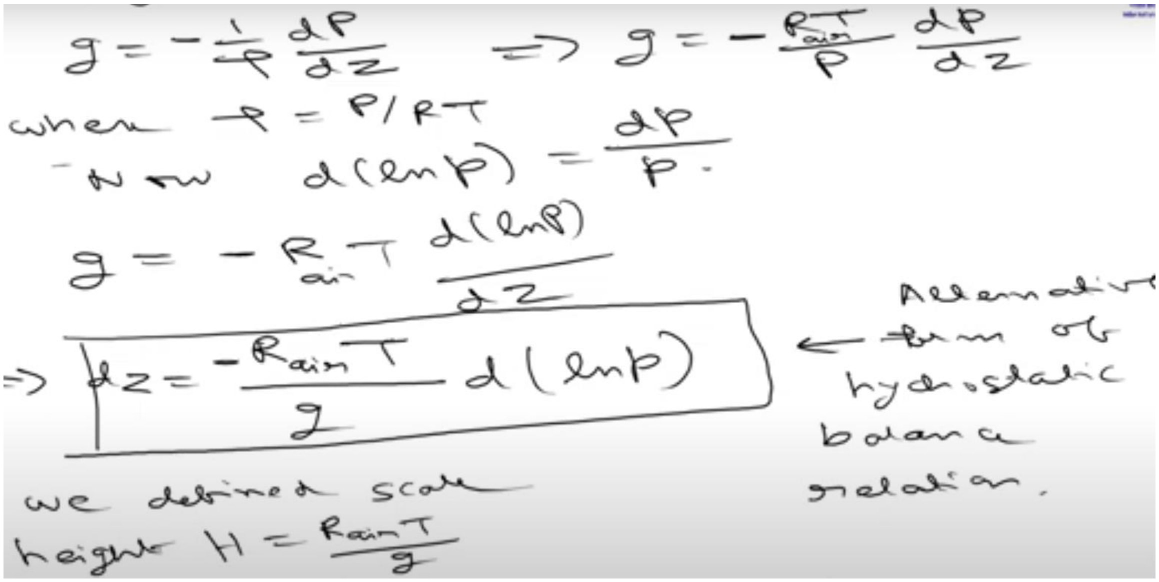
HOW TO MEASURE VERTICAL VARIATION OF PRESSURE

Good morning class and welcome to our continuing lectures on climate dynamics variability and monitoring plus. Today we will continue our discussion on some worked out examples based on the equations that we looked at in the last few classes and then we will further discuss the theory associated with the various climatic variables. So In the previous class we discussed that we have the hydrostatic balance equation which we can write as follows. So the hydrostatic balance equation So, we can write this equation as g equals to minus 1 by rho dp dz. Where by ideal gas law rho is equals to p by RT. Hence, we get g equals to minus RT by p into dp dz okay so minus r here r is the gas constant for air into temperature in kelvins by pressure by the gradient of pressure with altitude okay so if we change this so now d of log p can be written as dp by p correct so this implies we can write the hydrostatic balance equation as g equals to minus Rair into t into d of log p by dz, which implies that dz is equals to minus Rair into T by g into d of log p.

Let me write this properly. So, this implies dz equals to minus Rair into T by g into d of log p. So this is an alternative form of the hydrostatic balance relation. So alternative form of hydrostatic balance relation.

So now, what are we trying to do with this? So the idea here is, it is not necessarily the case that temperature is constant. We have used in the previous case, remember H basically is the scale height. So this expression we have written as H. So in general, we define scale height H as Rair into T by g, correct. However, if the temperature is a function of Z as is the case in actual conditions, then using the scale height relationship may not be quite accurate.



All right. So what we will do instead in this case, so for a more generalized case, T is a function of Z. So remember in this context that if we plot Z and T what we were getting before is something like this. Correct. Approximately.

So this is kind of troposphere, this region is the stratosphere and beyond this is your mesosphere and the exosphere. And each of these regions that change of temperature with gradient is more or less linear. So, let us consider that for the troposphere we have the fall of gradient or it is often also called the lapse rate. the lapse rate gamma as minus dT dZ for the troposphere. And this gamma which is the slope of the change in temperature with altitude can be considered constant in the troposphere.

We will have a different and a negative lapse rate in the stratosphere again the lapse rate will change. So for each of these zones of the atmosphere we will have different lapse rate values. But let us consider our case only for the troposphere which is approximately varied from 0 to 12 kilometers of the atmosphere. So, in this case, we have dT is equals to minus gamma dZ, where gamma is a constant positive value for the troposphere. Now if you go back, we have expressed dZ in this type of formulation.

So we can replace dZ in this expression by this expression here. So if we do that, so this implies dT equals to minus gamma into Rair into T by g into d of log p, p is the pressure of the atmosphere. Now, we can take this temperature here. So, we get dt by T is equals to minus gamma Rair by g into d(log p) but dT by T itself is d log t so we get d of log t equals to so just one correction dz is minus here Here you have minus. So, minus minus will cancel.

So, you will not have this minus sign here. So, this is a positive term. Sorry for the mistake. So, we get d(log T) equals to gamma into Rair by g d(log p).

$$\boxed{d(\ln T) = \frac{\Gamma R_{air}}{g}\, d(\ln p)}$$

Now we integrate from $z = 0$ to $z = z_1$

when z_1 is within

All right. this is the expression we will get assuming a constant lapse rate for temperature in the troposphere. So, now we integrate from Z equals to 0 Z equals to say Z1 some altitudes Z1 which is below the troposphere where Z1 is within the troposphere okay now at Z equals to 0 we set P equals to Ps the pressure at sea level and T equals to Ts the surface temperature of air ok and at Z equals to Z1 we set P equals to P at Z1 and T equals to T at Z1 all right. So, if we put this and do an integration here within the limits we will have log of T at Z1 by Ts equals to gamma gas constant for air by G log of P at Z1 by Ps alright so then we can take the take out the logarithm and get hence for any altitude Z1 within the troposphere, we have T, the temperature at the altitude Z1 by Ts equals to pressure at altitude Z1 by Ps to the power gamma Rair by g all right so this is an important expression here now the next question is can we simplify this further all right so remember that gamma is equals to minus dt dz. This implies dT equals to minus gamma dZ. Since gamma is a constant, we can integrate this term as well from Z equals to 0 to Z equals to Z1.

Integrating from Z equals to 0 which is at sea level and Z equals to Z1 we get T of Z1 minus T of s equals to minus gamma Z1 minus 0 all right which implies temperature Z1 equals to T temperature at the sea level minus gamma into Z1 this expression here okay so we have these expressions now temperature at Z1 is temperature at sea level minus gamma Z1 so we can put that expression here and get So, Ts minus gamma Z1 by Ts equals to pressure at Z1 by the sea level pressure to the power gamma Rair by g okay implies 1 minus gamma Z1 by Ts so we are just taking this common equals to P(Z1) by Ps into gamma Rair by g which implies that pressure at Z1 is equals to the sea level pressure into 1 minus gamma Z1 by Ts to the power g by gamma into Ra. So, this is the final expression that we are getting. Alright, so this is the final expression.

$$\ln\left(\frac{T(z_1)}{T_s}\right) = \frac{\Gamma R_{air}}{g} \ln\left(\frac{p(z_1)}{p_s}\right)$$

So we can compare this expression with the expression where we kept the temperature as the mass averaged mean temperature and plotted the pressure profile that way. So remember that for our atmosphere the mean lapse rate.

Hence for any altitude z_1 within the troposph
we have

$$\boxed{\frac{T(z_1)}{T_s} = \left[\frac{p(z_1)}{p_s}\right]^{\frac{\Gamma R_{air}}{g}}}$$

So, gamma mean is equals to 6.5 kelvins per kilometer in the troposphere. And Ts is approximately 15 degree centigrade or 288 kelvins and Ps is 101325 pascals. And Rair is 0.287 kilojoules per kg kelvin for air.

So, with these expressions, we can determine the variation of pressure with altitude within the troposphere. And we can compare that with the more simplified case where we would put the temperature to be a constant mean average temperature value and got an average scale height. So we can compare these two and check what are the differences. So, this is all of the worked out example for now. So, let us go forward and discuss a little bit on the next part of our discussion.

We have looked at the pressure and we have looked at how the pressure falloff can be expressed either for the case of an average temperature case or a temperature that is varying linearly with altitude. And we saw a few examples of using these two expressions. Now in atmospheric sciences several units of pressure are popularly used interchangeably. So it's good to have a table where all the pressure units are related with respect to each other. So the SI unit of pressure which is the standard international unit of pressure is 1 Pascal which is 1 Newton of force over 1 meter square of area.

It's also called 1 Pa. Now, 1 hectopascal is equal to 100 pascals. So, when you are looking at a hectopascal as a unit, it is equal to 100 pascals. Similarly, an alternative unit is 1 millibar, which is also 100 pascals. So, 1 hectopascal and 1 millibar are basically the same units expressed in terms of different descriptions. 1 kilopascals is of course 1000 pascals.

1 bar is also a very popular unit of pressure which is 10 to the power 5 pascals. So which is why millibar which is 10 to the power minus 3 bars is 100 pascals. So 1 kilopascal is 10 to the power 3 pascals. 1 bar is 10 to the power 5 pascals. one atmosphere is close to one bar which is equals to 101325 pascals so this becomes 1.01325 bars or 101.325 kilopascals okay Then you have one of the highest units of pressure 1 megapascals which is equal to 10 to the power 6 pascals. So these are all basically units in terms of pascals except the 1 atmosphere unit which is slightly different from units of orders of 10. Then you have the English unit of pressure, which is 1 psi, which is still common in

many engineering disciplines. It's pound force per square inch of area, which is equal to 6894.8 Pascals. Another popular unit is 1 Torr, which is equals to 1 millimeter of mercury column. This is used in common places in the barometric pressure readings. So this is basically 1 atmosphere is approximately 76 millimeters of mercury column. So 1 torr is 1 by 760 of that. So it is 76 centimeters, so 760 millimeters.

So 1 atmosphere is basically 760 torrs. So 1 torr is 1 atmosphere by 760 which is 133.32 pascals. So, these are the common units of pressure that we will discuss as we go along in the course. So, this kind of completes our initial discussion on pressure and the vertical variation of pressure, how to solve for it.

In the next class, we will discuss atmospheric humidity. which is another very important variable when it comes to climatology as well as weather. Because when air is hot and humid, you will have convection currents that leads to cloud formation and as well as the fact that air water vapor is a strong greenhouse gas. The extent of water vapor carrying capacity of air is also a very important criteria determining the extent of anthropogenic global warming. So we will discuss that briefly here with further discussions postponed to later in the class.

So today I am stopping here. Thank you for listening and see you in the next class.