**Course Name: An Introduction to Climate Dynamics, Variability and Monitoring**

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# ATMOSPHERIC PRESSURE AND MASS

Good morning class and welcome to our lectures in climate dynamics variability and monitoring class. We will continue our discussion on the various aspects of climatic variables that we have been covering. In the last class we completed our discussion on atmospheric composition and started our discussion on atmospheric pressure or the more accurately the vertical variation of atmospheric pressure with altitude. In this context, let me discuss a few worked out examples today that will help in fully understanding how these various composition and hydrostatic variable pressure relations can be used to find useful aspects of our atmosphere. So let us look at an example and let us see how that example can be solved. So the example is we have 0.934% by volume of argan present in the atmosphere. We have to ask we are asked to find out what is the total volume of sorry total not total volume its total mass total mass of argon that is present where we are told that the mass of air m air is equals to 5.14 into 10 to the power 18 kgs. So, the volume fraction of argon is given. We have to find the mass of argon that is present in the atmosphere when the total mass of air is 5.14 into 10 to the power 18 kgs. Now, using the ideal gas law, this is the answer. we have PV equals to mass of air, gas constant for air into T and P into Vi, V of argon equals to mass of argon, gas constant of argon into T. So, here we have used the partial volume approach that we discussed in the previous class.

$$\Rightarrow \frac{V_{Ar}}{V_{air}} = \frac{m_{Ar} R_{Ar}}{m_{air} R_{air}}$$

The pressure into the partial volume of argon equals to mass of argon, gas constant of argon into the temperature.

$$R_{air} = 0.287 \text{ kJ/kg K}$$

$$R_{Ar} = \frac{8.314}{39.95} = 0.208 \text{ kJ/kg K}$$

So, if we divide these two, we get V of argon by V of air is equals to the pressure cancels out. mass of argon gas constant of argon by mass of air into gas constant for air okay now gas constant for air is 0.287 kilojoules per kg kelvin or 287 joules per kg kelvin whichever unit you prefer gas constant for argon is 8.314 by the molecular mass of argon which is 39.95 equals to 0.208 kilojoules per kg kelvin okay so r of argon r of air are known mass of air is known v of argon by v of air is 0.934 percentage so the total expression becomes 0.934 into 10 to the power minus 2. This is the V of argon by V of air equals to mass of argon into gas constant of argon which is 0.208 by mass of air into gas constant of air 0.287. So mass of argon is equals to 0.934 into 10 to the power minus 2 into 0.287 by 0.208 into the mass of air which is 5.14 into 10 to the power 18 kg. Solve this and you get the mass of argon as 6.63 into 10 to the power 16 kg.

$$\Rightarrow \quad m_{Ar} = 0.934 \times 10^{-2} \times \frac{0.287}{0.208} \times 5.14 \times 10^{18} \text{ kg}$$

$$= \underline{6.63 \times 10^{16} \text{ kg}}$$

Alright, so this is how you can use the ideal gas relationships and the idea of partial volume. Similarly, we could have given partial pressure also and we would have gotten similar kind of expressions and you would get the mass of argon. Alright, now let us go back to our discussion on the hydrostatic pressure iterations. We will complete the discussion and then look at a few worked out examples for the hydrostatic pressure balance equations. So, here what we have found is we balance the static pressure with the gravitational mass for a differential volume of air and we found the hydrostatic balance relation to be equal to g which is acceleration due to gravity equals to minus 1 by rho dp dz the gradient of pressure with altitude. where dp dz will be negative because pressure is decreasing with increasing altitudes.

$$or, \ \ g = -\frac{1}{\rho}\frac{dP}{dz} \quad (hydrostatic\ balance)$$

Then we use the ideal gas constant values for air. So P equals to rho RT. So 1 by rho becomes RT by G, RT by P. And we use that expression and you define the scale height variable as R into T by G and got a new form of hydrostatic balance relation as DP by P, which is differential of log of pressure equal to minus DZ by the scale height H.

$$H = \frac{RT}{g} \quad (metres) \quad (2)$$

Now, in general, the scale height h is a variable of temperature, but we can use a mass averaged atmospheric temperature T0, which is the mean atmospheric temperature when we are taking into account the mass averaged one. So, it is basically integral rho into T into dV by m. And then the scale height H, if you give the mass average temperature of T0 here, the scale height H becomes 7.6 kilometers. So this expression can be put here directly to get an approximate expression of how the pressure is changing with altitude. This is not exact because we have assumed an average temperature instead of the actual temperature gradient with Z. But we get that the pressure at any given altitude Z must be equal to the sea level pressure PS into exponential of minus that altitude Z by the average scale height. So we will get an exponentially decreasing value of pressure as we move up in altitude. And this is seen in the altitude versus pressure plot where we have plotted this

$$P = P_s \exp\left(-\frac{z}{H_o}\right)$$

expression 19

and we see here the pressure unit is in hectopascals which is 100 pascals. So, Ps is basically 1013.25 hectopascal this is the value here and it kind of decreases steadily as we move upwards. So by 4 kilometers, it has decreased to around like 700 hectopascals. By 8 kilometers, around 500 hectopascals. And at the top of the troposphere, around 12 kilometers, it has gone up to 300, down to 300 hectopascals. Okay. So a rapid decrease in pressure.

All right. Another important parameter in this context is the mass of a column of air per unit area of the earth's surface. So you have the mass per unit area on which the mass is standing. So you can think of the air as separated into multiple columns, each which man one meter square cross-sectional area on the surface of the earth. So what is the mass per unit surface area of the earth? So that is what given by this M wavy M, M hat kind of a profile. So in this context what we see is this total mass dM is density into dy dx into dz.

If we divide it by the cross sectional area at the bottom of this volume, which is dx dy, you get rho into dz, which is the mass per unit cross sectional area on which that mass is standing. Correct? so this dm hat the differential mass of a column of differential air is basically rho dz which if you use the hydrostatic balance relationship given here rho dz is minus dp by g so we put here dm d of this hat mass hat equals to minus dp by g

$$d\tilde{m} = \rho dz = -\frac{dp}{g}$$

Now, we can integrate from certain arbitrary altitude z1 to final arbitrary altitude z2. So, this becomes mass of air per unit area at z2 minus mass of air per unit area at z1 equals to the change in pressure between z2 and z1 by g negative of that. So, this basically gives the mass of a column of air whose height is z2 minus z1 per unit area of the surface. And this value will be positive because if z2 is higher than z1 then the pressure here is lower so you get a negative value here so you get a positive.

$$\tilde{m}(z_2) - \tilde{m}(z_1) = -\frac{P(z_2) - P(z_1)}{g}$$

So now we can extend this from the sea level. So Z1 is sea level altitude which is 0. So P at 0 and the top of the atmosphere where pressure is 0 because it's at vacuum of the space. So at the top of the atmosphere the pressure is 0 and the bottom of the atmosphere the pressure is Ps. So this becomes 0 minus Ps.

And this becomes the total mass of a column of air from the sea level to the top of the atmosphere, just m. So this just becomes Ps by g, which is 1.03 into 10 to the power 4 kg per meter square. Now one useful thing that we can use here is that you can use this expression where the mass of a column of air of arbitrary height per unitary of earth's surface can be evaluated. And we can show that most of the mass of a column of air lies below the troposphere.

$$\tilde{m} = \frac{P_s}{g} = 1.03 \times 10^4\ kg/m^2$$

So we can put z2 as the top of the troposphere and z1 as the surface of the earth and use the pressure variations we have evaluated here. So this is the where we can get the Pz value. So Pz value we put here directly and we get the mass of the column of air from the surface to the troposphere and compare with the total mass of the column of air to the top of the atmosphere. And we will see that most of the mass of the column of air is actually concentrated within the troposphere itself. And this is something that we can show also.

All right. So let's do a workload example of that format here and see how this can be evaluated. All right. So. Evaluating. the mass present in a column of air extending from sea level to 5 km above the sea level per unit area of the earth's surface and what fraction is this mass compared to the total mass per area of the atmospheric column Okay, so the idea is suppose this is the surface of the earth we have the total this is the atmospheric column say 1 meter square and 1 meter on the length and 1 meter width.

So, you will have a kind of a column of air like this. This is the top of the atmosphere, top of atmosphere and here is the first 5 kilometers. So, we have to find the total mass content within this and see what fraction of this is the total mass content up to the top of the atmosphere. Now, Ps is 101.325 kilopascals. This is the pressure at the sea level. So, at z equals to 0. All right. Scale height h0 is 7.6 kilometers. Correct.

So, the pressure at z equals to 5 kilometers is equals to Ps into exponential minus z by h0, correct. So, 101.325 exponential minus 5 by 7.6, okay. This becomes equals to 52.48 kilopascals okay this is the pressure at 5 kilometers above the ceiling then mass of a column of air extending from sea level to 5 kilometer above sea level is m with z equals to 5 minus m at sea level equals to minus p at z equals to 5 minus ps by g correct this is minus 52.48 minus 101.325 we will use we will use the units of pascals because that is what we want when we have the units so be careful to use the units correct units okay So, this unit is in pascals and this unit is meter per second square.

$$P_s = 101.325 \text{ kPa at } z = 0.$$

$$\text{Scale height } H_0 = 7.6 \text{ km}$$

$$P(z = 5\text{km}) = P_s \exp\left(-\frac{z}{H_0}\right)$$

$$= 101.325 \exp\left(-\frac{5}{7.6}\right)$$

$$= \underline{52.48 \text{ kPa}}$$

$$\tilde{m}(5) - \tilde{m}(0) = -\frac{P|_{z=5} - P_s}{g}$$

$$= -\frac{(52.48 - 101.325)\times 10^{3} \leftarrow \text{Pascals}}{9.81 \leftarrow \text{m/s}^2}$$

$$= \underline{4979.1 \text{ kg/m}^2}$$

Express this formula to get 4979.1 kg per meter square. So, this is the mass per unit area of a column of air extending from sea level to the top of the atmosphere. However, total mass per unit area of the atmosphere is  m equals to Ps by g which we saw before is 1.03 into 10 to the power 4 kg per meter square. So, fraction of total mass present  Below 5 kilometers is m(5) by m, which is equals to 4979.1 by 10300. zero zero. which is equals to 0.48. So, 48% of the total mass of air is present below 5 kilometers only.

$$\tilde{m} = \frac{P_s}{g} = 1.03\times 10^{4} \text{ kg/m}^2$$

fraction of total mass present below 5km

$$\frac{\tilde{m}(5)}{\tilde{m}} = \frac{4979.1}{10300} = \underline{0.48}$$

 If you go up to 12 kilometers to the top of the atmosphere, you will see this goes up to 80%, which kind of shows that the majority of the air, mass of air at least, is present within the first layer of the atmosphere itself. And this is because of the much higher density and hence much higher pressures of the atmosphere at the bottom than as we go above in the altitude.

So while the atmosphere may extend up to 100 kilometers, within the first 10 or 12 kilometers, 80 to 85 percent of its mass is present. So, this is a very important factor when we are looking at mean quantities. We need to have mass averaged quantities because if you do just the average with respect to height, you will not get the actual mean value where the most of the air is present. So, in these examples, what we used was a average value of H0 where we have taken an average T0 value.

But we can do a little bit better than that. For example, when we were looking at the temperature variation with altitude, we saw a curve like this okay so within the troposphere for example we have a nearly linear decrease of temperature with height and we saw that delT delZ is minus 6.5 Kelvin per kilometer which was called the lapse rate of the atmosphere or the mean lapse rate within the troposphere okay So, we can use this expression to evaluate a linear function of temperature and put that linear function within the scale height H expression. So, the next kind of problem that we will do is we will not assume a mass averaged constant temperature for the atmosphere, but within the tropospheric region we will impose the temperature fall off curve gamma which is del T by del Z of minus 6.5. So, we will use that and let us see how that works out.

So, we will look into this in the next class. How we can use a variable temperature plot to get a more accurate estimation of scale height h and get a more accurate expression of pressure falloff with altitude. Thank you for listening and see you again in the next class.