Course Name: An Introduction to Climate Dynamics, Variability and Monitoring

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Lecture 51

PLANCK FEEDBACK OR BLACKBODY RADIATIVE FEEDBACK, SIMPLE RADIATIVE BALANCE MODEL

Good morning class and welcome to our continuing lectures on climate modeling, climate variability and climate dynamics. Today we will continue our discussion on a more generalized version of the zero dimensional energy balance model. In the more generalized version we saw that apart from the direct dependency of the feedback parameter on temperature we can also have contributions from variables which are which impact the net incoming or outgoing solar radiation, but which are dependent themselves on temperature. So, in that context then we can express the total climate feedback parameter alpha as negative of the partial derivative of the net incoming energy flux with temperature minus summation of the individual components of derivatives of the flux with the temperature dependent variables in the form of del F del Vi del Vi dVi dT. And these are being evaluated at constant values of the other parameters when we are looking at a partial derivative. So, when we are taking a derivative with respect to Vi, all V's other than Vi are held constant at their steady state value.

$$\alpha_i = -\frac{\partial \Psi}{\partial v_i}\frac{\partial v_i}{\partial T} \quad i = 1, 2, \ldots, N$$

$$\alpha_0 = -\frac{\partial \Psi}{\partial T}$$

$$\alpha = \alpha_0 + \sum_{i=1}^{N} \alpha_i$$

Similarly, all x's other are being held constant at their steady state value. So, the total feedback becomes the initial feedback, direct feedback due to temperature plus summation of the feedback due to individual contributions from the various variables I. Similarly, the radiative forcing term is a contribution from all the individual greenhouse gas and aerosol concentration values which impact the net incoming flux at the tropopause level, but which are not dependent directly or indirectly on the temperature of the planetary system. All the variables that impact the net incoming flux which are dependent on temperature are part of the climate feedback parameter and all variables which are not dependent on temperature but also impact the net incoming flux are part of the radiative force impact.

And together they complete this differential equation where C is again the total heat capacity, the gradient of the temperature perturbation with time plus feedback parameter into temperature perturbation equal to the radiative force meter. So, now we will look more closely at the contributions of alpha here. So the first term in the feedback parameter is alpha 0 which is negative of the gradient of the net incoming flux with respect to temperature. This is called the Planck feedback or blackbody radiative feedback. How the net incoming flux at the trocopause level is dependent on the change in temperature of the planetary system directly. How do we evaluate this? Firstly, notice that the incoming solar radiation at the top of the atmosphere that is being absorbed by the planet, which is F downwards, is the solar constant S0 by 4 into 1 minus alpha, where alpha here is the albedo. this is not the feedback. So, just you remember that and this is of the order of 240 watt per meter square and this is we can assume to be constant. Of course, albedo itself is a temperature dependent parameter. So, in a more complex model the net incoming flux will have a temperature dependency as well.

$$F^{\downarrow} = \frac{S_0}{4}(1-\alpha) \approx 240\,W/m^2 = F_0$$

Absorbed shortwave radiation will have a temperature dependency as well. Now, if earth had no atmosphere, so you can think instead of earth you can consider the moon which is at the same distance as the earth is from the sun. So, in terms of the incoming solar flux S0 that is still the same for the earth as well as the moon, albedo is of course different. So, assuming that earth had no atmosphere, then the temperature of the climate system will simply be the blackbody emission temperature T e, ok. So this is a case where we can assume that earth has no atmosphere or whatever atmosphere it has is completely transparent to both short wave and long wave radiation.

$$T_{ss} = T_e \quad \text{(for no atmosphere case)}$$

Then outgoing longwave radiation (OLR):

$$F^{\uparrow} = \sigma T_e^4 = \sigma T_{ss}^4$$

At steady state:

$$\Psi = F^{\downarrow} - F^{\uparrow} = F_0 - \sigma T_e^4 = 0$$

So you can think of a planet with complete nitrogen atmosphere. Such an atmosphere will never interact with and no water vapor for example. Then such an atmosphere will never interact with either the short wave radiation or the long wave radiation. And so that is almost equal to having no atmosphere. Then the climate system's temperature will simply be the back body emission temperature Tb.

So, in that case then Tss is equals to Te for the no atmosphere case. The steady state planetary temperature will be the blackboard emission temperature if there is no atmosphere. The outgoing long wave radiation under steady state condition will be equal to the incoming absorbed shock wave radiation. So, F upwards which is equals to the sigma into Te to the power 4, Te is the blackboard emission temperature, will be equal to sigma Tss to the power 4. So, here what we are saying is in the no atmosphere case, the steady state temperature of the planet will be the blackboard emission temperature Te. And the outgoing long wave radiation is equal to sigma into the steady state temperature to the power 4, which in this special case will be equal to sigma Te to the power 4. Okay. And at the steady state condition, the incoming absorbed flux and the outgoing long wave flux will be equal. So, F downwards minus F upwards, which is F0 minus sigma Te to the power 4 will be equal to 0. So, now let us under this no atmosphere condition, we create a temperature perturbation to the planetary system.

Then the climate feedback component alpha 0 for this no atmosphere case is minus del F del T where we are taking the gradient around Tss. So, this is minus del del T of this F term here, this F term here. So, minus del del T of F0 minus sigma Te to the power 3. Now, F0 we are assuming to be constant. So, then what we are getting is a derivative of this term. So, this becomes minus minus plus 4 sigma Te whole cube. So, for the no atmosphere condition, the alpha 0 term, the direct dependence on the net incoming flux with respect to the planetary temperature is equals to 4 times the Stefan Boltzmann constant into the blackbody emission temperature whole cube. Now, when F0 is 240 watt per meter square, Te, the blackbody emission temperature of earth is 255 Kelvin.

$$\alpha_0 = -\left.\frac{\partial \Psi}{\partial T}\right|_{T_{ss}} = -\frac{\partial}{\partial T}\left[F_0 - \sigma T_e^4\right]$$
$$= 4\sigma T_e^3$$

Hence:

$$\alpha_0 = 4\sigma T_e^3$$

With $F_0 = 240\,W/m^2$, we have:

$$F_0 = \sigma T_e^4 \quad \Rightarrow \quad T_e = 255\,K$$

So, with Te as 255 kelvins, alpha 0 term for the no atmosphere case would be 3.8 watt per meter square kelvin. So, we put 2 Te to the cube here, the Stefan-Boltzmann's constant here and multiply it by 4, we will get 3.8 watt per meter square kelvin as the climate feedback parameter alpha 0 term, the direct climate feedback parameter with temperature. Now, del F del T is minus alpha 0. So, as you can see here, okay. So, del F del T is minus 3.8 watt per meter square Kelvin.

$$\alpha_0 = 4\sigma T_e^3 \approx 3.8\,W/m^2K$$

$$\frac{\partial \Psi}{\partial T} = -\alpha_0 = -3.8\,W/m^2K$$

What this means is for each Kelvin increase in temperature of the planetary system, the net incoming radiation will decrease by 3.8 watt per meter square. Hence, if Te is increasing by 1 Kelvin, the net incoming energy flux will decrease by 3.8 watt per meter square at the troposphere level. And hence, an increase in temperature will cause a feedback that will in turn cause the planet to lose energy to increase outgoing long wave radiation and hence the planet will tend to cool back to its equilibrium value. So, alpha 0 greater than 0 in this case. So, we have a stable equilibrium climate system and this is the case of a negative feedback. So, the Planck feedback is a negative feedback parameter. And this is clearly seen in this case specifically for a planet with no atmosphere or no absorbing atmosphere. Now, earth is of course, has an atmosphere and that does absorb long wave and short wave radiation. So, how do we model that? Alright. In presence of an atmosphere containing greenhouse gases, the steady state temperature of the planetary system will not be equal to the blackbody emission temperature. But it will be some function of the blackbody emission temperature. Correct. So, the steady state temperature of earth is not equal to the blackbody emission temperature, but it is some function of that. So, in this case alpha 0 is minus del F del T around Tss. So, gradient of the net incoming flux with change in the steady state temperature. But the steady state temperature itself is a function of the blackboard emission temperature. So, what we can write is minus del F del Te into d Te d Tss. So, this expression we have already evaluated earlier. This is at Tss equal to Te, correct? So, this minus del F del Te is this term here, alright. But we have an additional term d Te dT ss, the gradient of the blackbody emission temperature with respect to change in the steady state temperature of earth. So, alpha 0 in this case becomes 4 sigma Te to the cube d Te d Tss. This extra parameter comes into the picture. A simple functional relationship between the blackboard emission temperature and the steady state temperature can be written as Te to the power 4 equals to some factor which is an equilibrium emittance factor epsilon into Tss to the power 4.

$$\alpha_0 = -\frac{\partial \Psi}{\partial T_{ss}} = -\frac{\partial \Psi}{\partial T_e}\frac{dT_e}{dT_{ss}}$$

$$\alpha_0 = 4\sigma T_e^3 \frac{dT_e}{dT_{ss}}$$

Where epsilon is less than 1 and is equal to the equilibrium emittance for the climate system. And here we are assuming that this emittance is not a direct function of temperature.

So, if this type of functional form is valid, then the emission temperature of earth is equals to emittance to the power one-fourth into the solid state temperature of earth. So, d Te d Tss is emittance to the power one-fourth.

$$T_e = \mathcal{E}^{1/4} T_{ss}$$

$$\frac{dT_e}{dT_{ss}} = \mathcal{E}^{1/4}$$

How do you calculate the value of this emittance? We will discuss that, but first you see Alpha 0 is 4 sigma Te whole cube emittance to the power 4. So, it is equal to emittance to the power 4 alpha BB. So, alpha BB is the no atmosphere case. So, alpha blackbody emittance temperature base case. This is 4 sigma Te whole cube which is minus del F del Te.

$$\alpha_0 = 4\sigma T_e^3 \mathcal{E}^{1/4} = \mathcal{E}^{1/4} \alpha_{BB}$$

$$\alpha_{BB} = 4\sigma T_e^3 = -\frac{\partial \Psi}{\partial T_e}$$

And we have already evaluated alpha BB for our case as 4. 3.8 watt per meter square Kelvin. So, this is the value here into emittance to the power 1. How do we calculate this emittance value? We can use for example, the simple radiative balance model for a continuously stratified atmosphere. derived this in several weeks earlier for a continuous stratified atmosphere climate, we evaluated how the ground temperature, the air temperature just above the ground T0 is related to the blackbody emission temperature. We fully derived the radiative equilibrium for a continuous stratified climate for that case and we found that the temperature of air near the ground T0 is equal to the blackboard emission temperature Te * (1 plus tau star g by 2) to the power 1/4.

$$T(0) = T_e \left[\frac{1+\tau_g^*}{2}\right]^{1/4}$$

So, you can go over previous notes and you can see this derivation done. So, here we have a relationship with the surface air temperature which is usually the default temperature we consider as the steady state temperature of interest is equal to the blackboard emission temperature into a certain emittance like term, where this tau g star is basically 1.66 into tau g, the optical depth for a gray atmosphere at the ground.

$$\tau_g^* = 1.66\tau_g$$

$$\tau_g = \int_0^\infty k_{\text{abs}}^i p_i \, dZ$$

So, tau g is the optical depth of the atmosphere at the ground which is 0 to infinity. The absorption coefficient for the species i into the partial density of the species i into dz where z varying from the sea level to the top of the tropopause for example. Where k absorption i is the mass absorption coefficient which we can assume to be independent of altitudes. So, we have the model Te equal to e to the power one-fourth Tss and from the radiative equilibrium model we have Te equal to 1 plus tau g star by 2 to the power minus one-fourth Tss. This is Te into this is equal to T0. So, this is Tss. So, Te is equals Tss by this term. So, this is the expression. So, the blackboard emission temperature equals to 1 plus the equivalent optical depth by whole by 2 to the power minus 1/4 Tss. So, looking at these two expressions then the emittance of the atmosphere under this simple stratified climate atmosphere model is given by 2 by 1 plus tau g star.

$$T_e = \mathcal{E}^{1/4} T_{ss}$$

$$T_e = \left(\frac{1+\tau_g^*}{2}\right)^{-1/4} T_{ss}$$

$$\mathcal{E} = \frac{2}{1+\tau_g^*}$$

This becomes the emittance, equivalent emittance of the atmosphere under this simplified model. So, now you can find the optical depth of the atmosphere for various greenhouse gases, add them together to get the actual optical depth and hence get the emittance value and hence you get the emittance. Thus, we can get the values of epsilon through radiative balance models of the atmosphere, making it possible to relate Te with Tss. A more detailed climate models, and we can do this not only with radiative, but we can have radiative convective equilibrium models as well. So, a more fleshed out climate model gives the emittance to be around 0.5. So, the alpha 0 term for our atmosphere is 0.5 to the power 1 fourth alpha blackbody. We have already evaluated the alpha blackbody term before, this value here and we have 0.5 to the power 1/4 into that value, so it is equal to 3.2 watt per meter square Kelvin.

$$\alpha_0 \approx (0.5)^{1/4}\alpha_{BB} \approx 3.2\, W/m^2 K$$

So, 1 Kelvin increase in the planetary atmosphere, planetary temperature will cause a decrease in the net incoming flux by around 3.2 watt per meter square. The climate remains stable though due to the presence of the greenhouse gases the positive value decreases somewhat because of this emittance term. So, this gives alpha 0. The next most important contribution is del F del V1 into dV1 dT, the variable which depends on temperature and also has an impact on the net incoming radiation.

And the most important variable for our case is water vapor, which is a strong greenhouse gas and whose concentration is strongly dependent on the mean temperature of the atmosphere. So, water vapour feedback is alpha 1 term for our climate system. From previous discussion and we discussed relative humidity, water vapour, specific humidity, mixing ratios, etcetera in some of the first few weeks of our class. So, notice how many of the concepts that we have discussed earlier are now coming back and fleshing out some of the climate models that we are developing. So, this is kind of how all the concepts that we have developed independently are coming together now in developing this model.

So, in previous discussion the saturation vapour pressure is given by the Clausius Clapeyron equation where the saturation vapour pressure es as a function of temperature at 1 bar is given as 611 into exponential latent heat of vaporization by the ideal gas constant for water 1 by 273 into 1 by T minus 1 by T and this is a very strong function of temperature.

$$e_s(T) \approx 611 \exp\left[\frac{L_v}{R_{H_2O}}\left(\frac{1}{273} - \frac{1}{T}\right)\right]$$

We define the mass mixing ratio for water vapor omega as the mass of vapor by mass of dry air which is equal to 0.622 the partial pressure of water vapor which is the function of temperature by the partial pressure of dry air. And the saturation mass mixing ratio is of course 0.62 es(T) which is given by this expression here by partial pressure of air.

$$\omega = \frac{m_{\text{vap}}}{m_{\text{dry air}}} = 0.622\frac{e(T)}{P_{\text{air}}}$$

where $e(T)$ is the partial pressure of water vapor in air.

The saturation mass mixing ratio is:

$$\omega_s = \frac{m_{\text{vap}}}{m_{\text{dry air}}} = 0.622\frac{e_s(T)}{P_{\text{air}}}$$

So, this is the mass of water vapor under saturation condition by mass of dry air. And relative humidity is e by es. This term we can write as e by es * es. So relative humidity * es. Okay. So omega is relative humidity * omega s.

$$\text{RH} = \frac{e}{e_s}$$

$$\omega = \frac{m_{H_2O}}{m_{\text{dry air}}} = 0.622\frac{e}{P_{\text{air}}}$$
$$= 0.622\,\text{RH}\frac{e_s}{P_{\text{air}}}$$
$$= \text{RH}\,\omega_s(T)$$

Alright. For water vapor feedback, we want to use some measure of the water vapor content in the atmosphere. we find that relative humidity is not a strong function of the change in the mean surface temperature. So, relative humidity is relatively independent of the temperature of the climate system. However, the saturation mass mixing ratio is a strong function of temperature of the climate system because it depends on es(T). We can write the mass mixing ratio, mass of water vapor by mass of dry air equals relative humidity into saturation mass mixing ratio, which is the strong function of temperature. Hence, omega s(T), the saturation mass mixing ratio is a good choice of variable for water vapor feedback into the climate system. So, we take the saturation mass mixing ratio at C level as our variable of interest. The saturation mass mixing ratio at pressure of 1 bar C level as our variable of interest. So, Vt, our variable, temperature dependent variable is the saturation mass mixing ratio at C level which is a function of temperature. Clear? So this is called omega sg(T) which is 0.622 des(T) by p0 where p0 is approximately 1 bar.

$$\omega_s(T) = 0.622\frac{e_s(T)}{P_0} \quad ; P_0 = 1\,\text{bar}$$

So, the water vapor feedback parameter:

$$\alpha_{H_2O} = -\frac{\partial\Psi}{\partial\omega_s^g}\frac{\partial\omega_s^g}{\partial T}$$

$$\frac{\partial\omega_s^g}{\partial T} = \frac{0.622}{P_0}\frac{\partial e_s(T)}{\partial T}$$

And water vapor feedback then is del F by this variable del F del omega s, it is again a negative value, del F del omega s * d omega s at the ground dT. This is our alpha H_2O, the water vapor feedback. d omega s d T, remember omega s is this term here, pressure of air does not change 0.622. So d omega s d T is 0.622 by the pressure at the sea level * d es(T) dT. The gradient of the saturation vapor pressure by temperature and this is given by the Clausius-Clapeyron relation. By the Clausius-Clapeyron equation, d es d T is the latent heat of vaporization of water into es(T) by the RH2O by T square, ideal gas constant for water * T square, fine.

$$\frac{de_s}{dT} = \frac{L_v e_s(T)}{R_{H_2O}T^2}$$

$$R_{H_2O} = \frac{R}{M_{H_2O}} \quad (R = 8.314\,\mathrm{J/mol\,K},\ M_{H_2O} = \text{Molecular weight of water})$$

$$\frac{d\omega_s^g}{dT} = \frac{\omega_s^g L_v}{R_{H_2O}T^2}$$

$$\Rightarrow \frac{d\omega_s^g}{dT} > 0$$

So, we can put this expression here. to get d omega s g d T as equal to 0.622 by T, T_0 into this expression which is, sorry, Lv es(T) RH_2O T square. You can combine that with 0.622 T_0 to get back this expression here, alright. So, we get back omega sg into Lv by RH2O T square.

How do we do that? What is omega sg? Omega sg is 0.622 es(T) by P0. 0.622 by P0 is already here, d es(T) d T is Lv es(T) RH2O T square. So, we take es(T) on the side to get omega sg, then you multiply by the latent heat of vaporization, gas constant for water and T square. So, this becomes our expression here. The saturation mass mixing ratio is a positive term. The latent heat of vaporization of water is a positive term. Ideal gas constant for water is a positive term and temperature as it is expressed in Kelvins is always a positive term. So, the gradient of the saturation mass mixing ratio with temperature is positive. This we had expected because as temperature rises the amount of saturation amount of water vapor that the saturated parcel of air can hold is going to increase. So, this gradient is expected to be positive.

We can also do some further expressions here. So, we can take d omega s g by omega s on this side and dT by T square on this side to get this expression here. So, for T equals to 298 kelvins, Lv Rh2o * T is of the order of 20. So, you can put the latent heat of vaporization, gas constant of water and the temperature is 298. So, this expression is around 20. So, what this means is a 1 percent change in air temperature, so if d omega sg, omega sg is 0.01. 1 percent change, that becomes equals to, sorry, d T by t is 0.01, 0.01, this is 20, so this becomes 0.2. So, 1 percent change in air temperature leads to an over 20 percent change in the saturation mass mixing ratio, which kind of shows how sensitive the saturation mass mixing ratio variable is to the climate temperature, all right.

$$\frac{d\omega_s^g}{\omega_s^g} = \left(\frac{L_v}{R_{H_2O}T}\right) \cdot \frac{dT}{T}$$

$$\frac{L_v}{R_{H_2O}T} \approx 2.0$$

Anyways, let us go back to our water vapor feedback parameter, alpha H_2O. This becomes minus del F del omega sg into d omega sg dT. And we have expressed d omega sg dT as this term here. This is a positive term. What is del F del omega sg? Del F del omega sg is del omega... del omega sg into downward absorbed shortwave radiation flux minus upward emitted longwave radiation flux at the tropoplast. The absorbed shortwave radiation is being held constant F0 240 watt per meter square. So, this term vanishes in the differential. So, you get minus del F upward by del omega sg.

$$\alpha_{H_2O} = -\frac{\partial \Psi}{\partial \omega_s^g}\frac{d\omega_s^g}{dT}$$

$$\frac{\partial \Psi}{\partial \omega_s^g} = \frac{\partial}{\partial \omega_s^g}\left(F^{\downarrow} - F^{\uparrow}\right)$$

$$= -\frac{\partial F^{\uparrow}}{\partial \omega_s^g}$$

This term only. Now, since water vapour is a strong greenhouse gas, increasing water vapour fraction will decrease the outgoing long wave radiation from the top of the tropical. How much will it decrease? We will show that when we are looking at CO_2 pace of how do we express this term, ok. How does we, how do we evaluate how much change in the outgoing long wave radiation flux will there be for a change in the concentration of a gas, be it water vapor, be it CO_2. We will derive this explicitly when we look at CO_2. So, this expression how to solve this we will see later. Here we are more interested in the sign. If water vapor concentration increases, clearly the outgoing long run radiation of the tropopause is going to decrease. So, del F upward del omega sg, this term must be less than 0. The outgoing flux will decrease with increase in water vapor concentration. So, the negative of this term is positive. So, here this term will be positive. Right. This term will be positive. Okay. And del F del omega sg is this term here. Right. So del F del omega sg will be positive in this context. Correct. So del F del omega sg is positive. Del omega sg d T is also positive. We have shown this here. So, this term is positive. Del F del omega sg itself is also positive. So, alpha which is negative of del F del omega sg * d omega sg d T. This is positive, this is positive, this is negative. So, the alpha H2O is negative. The water vapor feedback term itself is negative. So, we will stop here because we are running short on time. In the next class, we will look at what this means, the total climate feedback that the water vapor feedback term is negative, less than 0.

$$\alpha_{H_2O} = -\frac{\partial \Psi}{\partial \omega_s^g}\frac{d\omega_s^g}{dT}$$

$$\frac{\partial \Psi}{\partial \omega_s^g} = \frac{\partial}{\partial \omega_s^g}\left(F^{\downarrow} - F^{\uparrow}\right)$$

$$= -\frac{\partial F^{\uparrow}}{\partial \omega_s^g}$$