Course Name: An Introduction to Climate Dynamics, Variability and Monitoring

Professor Name: Dr. Sayak Banerjee

Department Name: Climate Change Department

Institute Name: Indian Institute of Technology Hyderabad (IITH)

Week- 9

Lecture- 50

TEMPERATURE RESPONCE TO RADIATIVE FORCING – CONT

Good morning class and welcome to our continuing lectures on climate dynamics, climate variability and climate monitoring. In the previous class, we derived the zero-dimensional climate model and solutions to zero-dimensional climate models in terms of integration of the relative forcing term. And we looked at one or two cases of various radiative forcing functions like a constant radiative forcing and how that impacts the temperature perturbation over time and a linearly increasing radiative forcing which is similar to how the CO_2 concentration is affecting radiative forcing today. And how that is affecting the temperature, rate of temperature increase which time in the climate system. Today we will discuss another little model where we are assuming that the relative forcing increases linearly till a certain time t_1 say and then it achieves a constant value. So, this will happen in case we are able to control our emissions of CO_2 and CH_4 and other greenhouse gases and they reach an eventual new steady state concentration.

So, that the final relative forcing due to these greenhouse gases becomes a constant again say at the end of the 21st century. So, this is called a ramp flat forcing where we have a linear growth of R_F with time which eventually becomes steady after a certain specified time t_1. Such a situation can approximate a case where CO_2 concentration rises exponentially for a certain time and then stabilizes to a fixed mass fraction value as emissions decrease to 0. So, this is the best-case scenario we have for our case right now. So, then the relative forcing term will look like this. Up to a certain time between 0 to t_1, it is equal to gamma t. Then beyond t_1, it is a constant value which is equal to gamma t_1. Remember, the functional relationship of the temperature perturbation with time is $T'(t) = \frac{e^{-t/\tau}}{c} \int_0^t R_F(t') e^{-t/\tau} dt'$, this expression. So, here we will integrate this in two parts, first from 0 to t_1 where it is gamma t and then for t greater than t_1, so t_1 to t where this equals to gamma into t_1, okay. So if we do that integration and then again do an integration by parts for the various components, the final expression of the temperature perturbation with time is $\frac{\gamma\tau}{\alpha}$, remember gamma is the slope of the relative forcing term, tau is the relaxation time, alpha is the climate feedback parameter equal to $\frac{t}{\tau}$ which is our non-dimensional time $-1+e^{-t/\tau}$. And this expression is between 0 to t_1.

$$T'(t)=\frac{\gamma\tau}{\alpha}\left(\frac{t}{\tau}-1+e^{-t/\tau}\right),0<t<t_1$$
$$=\frac{\gamma\tau}{\alpha}\left(\frac{t_1}{\tau}-1-e^{-(t-t_1/\tau)}+e^{-t/\tau}\right),t\geq t_1$$

Beyond t_1, again we have $\frac{\gamma\tau}{\alpha}$, $\frac{t_1}{\tau}$, now we have a different formula. Minus $-e^{-(t-t_1/\tau)}$. So, this new expression comes. This is the time, this is the t_1, the time at which the relative forcing becomes steady by the relaxation time, exponential negative of that plus e to the power minus t by tau. This is for t greater than t_1. When t is tending to infinity, so at very, very long times, Then t minus $t_1$is much, much greater than tau. So, this t minus t_1 also tends to infinity. So, this becomes infinity power minus infinity. So, this becomes 0. This also becomes 0. So, then t prime becomes $\frac{\gamma\tau}{\alpha}\times\frac{t_1}{\tau}$.

$$T'(t)\to\frac{\gamma t_1}{\alpha}\ or\ \frac{R_F}{\alpha},t\to\infty$$

So, $\frac{\gamma t_1}{\alpha}$ or $\frac{R_F}{\alpha}$ because R_F at that point is equals to γt_1 at very long times. So, this is again if you remember is the climate sensitivity term that we defined earlier. Remember in the previous case for the constant forcing case we defined this climate sensitivity term R_{F1} by alpha. So, here Rf1 is your gamma t_1. So, this becomes R_{F1} by alpha gamma t_1 by alpha. So, we are back to the climate sensitivity term. So, t prime at very, very long times again becomes equal to the climate sensitivity function S and this we would expect because again we have a constant relative forcing and the initial difference does not make an impact on the final steady state value of the temperature perturbation. However, the growth rates are different, clear. We have this function initially and this function in the middle. And finally, it becomes R_{F1} by alpha, alright. So, what is, how does it look like? So, it kind of depends on the value of this $\frac{t_1}{\tau}$, your non-dimensional time fundamentally. So, we have here, we have plotted the dimensional cases, alright. So, this is T prime t and time and dimensional perturbation. This is your relative forcing term, gamma $t_1$up to this point here. What is your case of the temperature term for two cases? In one case, the $\frac{t_1}{\tau}$ is much much greater than 1. So, this is the time at which the radiative forcing becomes constant.

That time is much much larger than the relaxation time tau. So, the relaxation time tau is say 30 years say and t_1 is say 300 years. So, for 300 years radiative forcing was increasing linearly and at the 300th year it kind of became constant. So, in that case you have $\frac{t_1}{\tau}$ is equal to 10 which is much more than 1 for example. There the temperature response reaches steady state just beyond t_1. So, the steady state value is reached quite close to the time at which the relative forcing becomes a constant. The other case is where $\frac{t_1}{\tau}$ is equal of the order of 1. So, if tau is 30 years, t_1 is say 20 years or 40 years or 50 years or 60 years, like that. So, of the order of 1, say 0.5 or 2, whatever. In that case, we see that the temperature continues to increase for a very long time even after the relative forcing has become constant, that is the greenhouse gases have stabilized. So, this is kind of called the history effect that even after we have stabilized the greenhouse gas concentration that is we are no longer emitting any further CO_2 and greenhouse gas concentration because we have emitted over a very short period of time. which is of the order of the feedback response time. So, we have emitted say over a 90 years or something like that. There will still be a kind of pent-up effect that will cause the temperature to continue to rise till it reaches the steady state level which is $\frac{R_{F1}}{\alpha}$.

Eventually, it will reach that but it will take a long time beyond at which we have stabilized the greenhouse response time. So, as again we see that initially the temperature response lags behind the relative feedback response. So, we have seen this effect before here. In the ramp forcing case, the temperature response lags behind the climate response and then it linearizes, right? And then it kind of starts to level up. So, this difference becomes if, say for example, this difference is quite large in the initial cases and then kind of becomes stable. So, this is what is happening here. Here, this difference has kind of linearized by the time t equals to t_1 has reached. So, this has become linear. Here, the linearization has not happened. We are still in the kind of the slow rising case of the temperature response. Alright. So, it kind of has to linearize and then level off. Alright. So, as a result, we will have a long duration even beyond the stabilization of the greenhouse gas concentration for which the temperature is going to continue to rise till it reaches the $\frac{R_F}{\alpha}$ value. So, it is kind of critical to understand the value of tau, understand when we are kind of stabilizing the greenhouse gas concentration in order to fully predict how much warming is still left after the greenhouse gas concentration has failed. So, these we are going to more sophisticated models, but these very simple models kind of gives us a picture of what type of trends to expect for different cases.

One final expression that we will do is an exponentially decaying pulse forcing. So, what happens? Suppose you have a large volcanic eruption or a large meteor strike and a huge amount of aerosols get injected into the stratosphere. These aerosols will start to reflect sunlight and decrease the net incoming shortwave radiations and hence the net outgoing longwave outgoing radiation will increase. So, we have a negative radiative forcing due to a net decrease in the incoming radiation at the tropopause level, alright. So, this has, because if you have inputting a lot of aerosols in a very quick amount of time, you have a rapid negative R_{F1} and this effect as the aerosols then slowly again descend back in onto the ground, this negative radiative forcing effect kind of decays back exponentially to 0. Where t_0 is some constant based on the stabilization. So, the aerosols slowly again kind of accumulate back onto the ground. They cannot stay in the atmosphere forever. So, you have a slow decrease in this negative radiative forcing due to the aerosol cause increase in the reflection of shortwave radiation. So, relative forcing is 0 for t less than 0. At t equal to 0, it becomes minus R_{F1} and then it slowly decays as minus R_{F1} equal to minus t by t_0. This is also a very interesting case because more recently there has been discussions of artificially injecting aerosols in the stratosphere to decrease the impact of warming. So, instead of a volcano or meteor, we can use say aeroplanes or balloons to inject silicate particles in the stratosphere that are highly reflective and will reflect more of the short wave radiation. So, the albedo will increase and hence the net effective downward coming radiation will decrease and hence you have a negative radiative forcing. So, those cases also you have a pump injection human made or natural which will cause a decrease in the radiative forcing which will slowly decay back over time, alright. So, in this case we can again plot the results and get the expressions. We will not do it here. But the basic idea we can write like this. The temperature profile will look like this. While the relative forcing is exponentially decaying, The temperature will decrease, the temperature perturbation will reach a peak and then will slowly exponentially decrease as well.

So, temperature will rapidly decrease to a certain peak value and then slowly get back to its original steady state value over a certain period of time. So, because radiative forcing is decaying back to zero, temperature will go back to its original steady state mode, and this is what happens in case of a volcanic, large volcanic eruption or a meteor strike that you will have a few years to a decade of cold temperatures because of the negative radiative forcing. And as the aerosols again go back down onto the earth's surface, the temperature recovers once more. And it is proposed that an aerosol injection in the stratosphere will also do the same thing. It will give us a certain time over which the temperature is going to be lower and help us mitigate though not completely remove the impact of the positive radiative forcing due to CO_2, methane, etc. So this kind of gives us a few cases of how the various radiative forcing functions can work together to have different types of temperature responses. Now we will discuss a little bit more complex models. Models which does not just look at one greenhouse gas or one feedback parameter but multiple parameters, all of which may have an impact on the overall flux. So, in the previous simple models, we assume that the net downward energy flux is only dependent on the temperature of the climate system and the concentration of the greenhouse gas. That is, the net downward flux is the incoming absorbed shortwave radiation minus outgoing emitted long-wave radiation at the topophos level, which is the function of the temperature of the climate system and the one greenhouse gas concentration alone. All right. Now, this is not really true, as we know. In actuality, the climate system, in a climate system, the net downward energy flux will depend on three things. Firstly, the direct dependence on temperature, the temperature of the climate system itself. Then second, a set of temperature dependent climate variables which we define as V1(t), V_2(t), going up to say V_n(t). So, these are variables that depend on temperature and impact the net incoming flux. Examples of such variables include water vapour concentration, very important, we will discuss this. So, water vapour concentration in the atmosphere depends on the temperature of the climate system because how much total water vapour is present increases exponentially as the temperature of the climate increases. We have seen this from the Clausius Clapeyron relations that we discussed very, very early in our course.

Concentration of clouds. Again, a very strongly temperature dependent parameter. Higher warmer the oceans, greater is the rate of evaporation, greater is the amount of clouds. And we have not discussed cloud feedback in a lot bit of detail and we will not discuss it today, but it is a very strong area of research of how the concentration of clouds as well as where the clouds are forming has an impact on the net incoming radiation. Albedo. So, albedo is temperature dependent. If you increase the temperature, the snow cover decreases. So, albedo increases. Right, so albedo is also temperature dependent tropospheric lapse rate of course a temperature dependent term it also has a strong impact on the net incoming radiation and the radiation because it impacts the temperature at which the atmospheric gases are emitting and absorbing the greenhouse the radiation the long wave and the short wave radiation for example okay. So, these are all variables that are dependent on temperature and also impact the net incoming radiation is the tropopause level. The third set is a set of climatic variables that do not depend on temperature.

These are usually the concentration of the various gases and aerophores. The concentrations of various gases like CO_2, methane, NOx, CFC, concentration of ozone, as well as solar insulation. The sun itself has its own cycle, so that also is a variable that needs to be taken into account in this set. So, a set of variables that impact the net incoming radiation without being dependent on temperature of the system, a set of variables that are dependent on the temperature and impacts the net incoming radiation and finally the temperature itself. So, then the actual net incoming energy flux at the tropopause is F downward absorbed minus F upward is a function of temperature, the set of variables V_1 to Vn which depend on temperature and the set of variables X_1 to X_m that do not depend on temperature. And this helps us to define, A generalized formulation of both the climate feedback parameter and the radiative force term. The climate feedback parameter, the complete expression then becomes $\frac{-\partial f}{\partial T}$, the gradient of this net incoming flux with temperature at the steady state greenhouse gas concentration minus summation i equals to 1 to n, that is over all these variables, $\frac{\partial f}{\partial V_i}\frac{\partial V_i}{\partial T}$. the gradient of the net incoming flux with the temperature determinant variables vi into the gradient of these variables with temperature, summation of that.

$$\alpha = \frac{-\partial f}{\partial T} - \sum_{i=1}^{N} \frac{\partial f}{\partial V_i}\frac{\partial V_i}{\partial T}$$

So, this extra term as a summation for all of these variables are put into and becomes the actual climate feedback parameter we have to deal with in a more complicated model. And all of these derivatives are evaluated steady state value of temperature, V_{ss} and X_{ss}. So, as we remember, $\frac{-\partial f}{\partial T}$ at X_{ss}, right? Similarly, $\frac{\partial f}{\partial V_i}\frac{\partial V_i}{\partial T}$ at X_{ss}. So, these X values are being held constant, okay? We can define the feedback parameter for the ith variable V_i as α_i as $-\frac{\partial f}{\partial V_i}\frac{\partial V_i}{\partial T}$, i going from 1 to n.

$$\alpha_i = -\frac{\partial f}{\partial V_i}\frac{\partial V_i}{\partial T}\ , i = 1,2,3 \ldots\ldots N$$

And the direct temperature feedback value as $\alpha_0 = -\frac{\partial f}{\partial T}$. So, this is $-\frac{\partial f}{\partial T}$ where all the other terms V_i's X_i's are held constant. For alpha i all the other terms are held constant. So, v not equal to V_i is held constant, X, all the X terms are held constant. So, the total feedback then becomes the direct temperature feedback alpha 0 plus summation over all the variables i to n which depend on temperature and also impact the incoming radiation. So, α_i this is $-\frac{\partial f}{\partial V_i}\frac{\partial V_i}{\partial T}$. So, this is the total feedback.

$$\alpha = \alpha_0 + \sum_{i=1}^{N} \alpha_i$$

So, this expression is basically this expression. So, here then we can model and evaluate the values of each of this alpha, alpha 0, alpha 1, alpha 2, alpha 3 for all the variables. Alpha 0 is the direct temperature feedback, alpha 1 may be the water vapor feedback, alpha 2 may be the cloud feedback, alpha 3 may be the radiation balance feedback etcetera. The radiative forcing term can also be generalized in that format that the total radiative forcing is summation over all the non-temperature dependent variables X_1, X_2, X_3, X_4 till X_m, summation j equals to 1 to M, R_{Fj}; where, $R_{Fj} = \frac{\partial f}{\partial X_j} X_j'$. the incoming radiation flux, the gradient of that with respect to the concentration of the mixing ratio of the greenhouse gas species X_j into the perturbation of that species X_j prime, the concentration, perturbation of concentration of that species X_j prime. This one we only did for CO_2, now we can do for CO_2, methane, aerosol particles, NOx, CFC, all of that summation becomes the total radiative forcing for our climate system, all right.

We can still use the zero-dimensional energy balance model, $C = \frac{dT'}{dt} + \alpha T' = R_F(t)$, just that the R_F becomes this expression here and alpha becomes this expression here, all right. So, we have just expanded that reach of R_F and alpha to contain all possible variables temperature dependent or temperature independent and we are using the zero-dimensional energy balance model. So, we will stop here today. In the next class we will try to evaluate alpha 0 and alpha 1 and then we will evaluate R_F for CO_2.

So, that will give us a very good idea of how to do these analytical expressions and how to model them before we move on to the next section of our course. So, thank you for listening and see you again in the next.