Course Name: An Introduction to Climate Dynamics, Variability and Monitoring

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ZERO DIMENSIONAL ENERGY BALANCE MODEL - CONT

Good morning class and welcome to our continuing lectures in climate variability, climate monitoring and climate dynamics. In the previous class we started our development of a simple analytical model called the zero-dimensional energy balance model in order to quantify what effect observed in a climate system when a forcing parameter like a greenhouse gas concentration of a certain species is changing within the climate system. We assumed the climate system to be made up of the troposphere, the surface of the earth and the top 100 meters or the mixed layer of the ocean. We assume for simplicity that the mixed layer of the ocean does not interact in terms of energy transfer or mass transfer with the deep layers of the ocean and that the only energy transport from this climate system is through the incoming short wave radiation that is being absorbed and the outgoing long wave radiation that is being emitted at the tropopause. So, the net incoming short wave radiation and the net outgoing long wave radiation. The net incoming shortwave radiation is given by F downwards, the radiation flux downwards and the net outgoing longwave radiation is given by the flux upwards.

This difference is the net energy flux that is accumulating within this climate system and that would cause a change in the mean temperature of this climate system and is given by the heat capacity of the system C into the rate of change of temperature with time, d capital T dt. We also assume that the temperature total that the stratosphere is not absorbing any shortwave radiation which is of course inaccurate because of the absorption of EU radiation by the ozone layer. And so the entirety of the absorbed shortwave radiation is occurring within our designated climate system that is below the tropopause. And this amount as we have seen is 240 watt per meter square where we have taken the albedo value alpha to be a constant.

Of course, if the albedo also is changing then this incoming absorbed shortwave radiation will be a function of this alpha and that will be a parameter in this differential equation. However, the outgoing shortwave radiation will depend on the temperature of this climate system because it's a radiation, so sigma T to the power 4 kind of a dependence is there as well as the concentration of the greenhouse gases like water vapor, CO_2, etc. in the troposphere will impact the net outgoing longwave radiation. Now, how do we measure the heat capacity C? The heat capacity of sea is made up of three components, the heat capacity of the troposphere, the heat capacity of the land surface and the heat capacity of the oceanic mixed layer. In general, the heat capacity of the troposphere and the land surface is much smaller than the heat capacity of the oceanic mixed layer.

Also, the oceans cover around 70% of the earth's surface. Hence, as a simplifying approximation, we can approximate the value of the heat capacity as the heat capacity per unit area of the ocean mix layer multiplied by 0.7. So, we are assuming that the land and the troposphere contributes very small amount to the total heat capacity and hence it is the oceanic mix layer which is 70 percent of the surface is contributing the entirety of the total heat capacity. With these approximations, we can measure the heat capacity per unit area of the mixed surface to be 1 gigajoule per meter square Kelvin.

So the value of C we can put as 1 gigajoule per meter square Kelvin where gigajoule is 10 to the power 9 joules. Clearly this is a very large heat capacity as we would expect for a planet of our size. Let us do first an extremely simple analysis which we will expand in later times. We assume that the net heat flux into the climate system which is short wave radiation downwards minus long wave radiation upwards depends only on the temperature T and the concentration X of a certain greenhouse gas. So here the concentration can be the molar mixing ratio or the mole fraction.

So, we will only consider one greenhouse gas as a parameter and temperature as the second parameter. So, in that case because the incoming shortwave radiation is taken to be a constant of 240 watt per meter square, the functional dependence of this net incoming radiation is only based on the functional dependence of the outgoing OLR. So, this will be a function of temperature and the molar mixing ratio of this greenhouse gas which is given as chi or X. So, here then what we see is that This is the expression and this expression here f downwards minus f upwards is a function of temperature of the system into the molar mixing ratio of the greenhouse gas in question. In a steady state climate at a constant temperature, let us call the steady state temperature to be T suffix ss and a constant greenhouse gas concentration of X equals to Xss, then for a such a system, because it is a steady state, the F downwards minus F upwards, this value will be equal to 0.

There will be no net incoming or outgoing energy being accumulated or lost from the system because the system is at steady state with a constant temperature equal to the steady state temperature. So, the functional dependence of this term as a function of T steady state and x steady state and this value will be equal to 0. Hence, your expression, the differential equation becomes C into D Tss DT which is equal to 0. So, Tss is a constant. So, this is what happens when the climate system is at a steady condition.

It has a steady state temperature Tss, a constant concentration of the greenhouse gas Xss and Incoming short wave minus net outgoing long wave balance each other and is equal to 0 and hence your C dtss dt is also equal to 0. So this is the steady state or the default condition. Now suppose this climate system is slightly perturbed so that we have a new temperature T equal to Tss plus T prime. So the new temperature T is a slightly perturbed variant from the steady state temperature. So, the new temperature is time dependent and is equal to the steady state temperature plus a perturbation term T prime which is dependent on time.

And the concentration of the greenhouse gas has also changed from to X which is Xss plus a perturbation value x prime which is also a time dependent value. So, what is happening? Effectively we are changing the greenhouse gas concentration by X prime. So Xss plus X prime and this X prime can be a time dependent value. So with time it may be increasing or decreasing. As a result the climate system is now accumulating heat because the outgoing long-wave radiation has decreased because of the greater heat trapping and hence the temperature of the system will also be changing with time.

So the temperature at any given time is the steady state temperature plus the perturbation term T prime which is the function of time T. We assume that the perturbation parameter T prime and X prime is much much lower in magnitude than Tss and Xss. So the values of the differences, the delta T or the delta X, T prime or X prime is much much lower than the magnitudes of Tss and Xss. Hence we can do a linear perturbation analysis. If these values are not small, we cannot do a linear perturbation analysis.

So here we are assuming that the change is quite small. So, now the differential equation becomes C into d of temperature d of time is equals to the difference between the incoming short wave minus outgoing long wave which is a function of the temperature and the greenhouse gas concentration and then the capital T is we are expressing it as Tss plus the perturbation temperature T prime and the function itself also becomes We are expanding the T and the X as Tss plus T prime Xss plus X prime. Now, the steady state temperature is a constant. So, D Tss DT is 0. So, the first term cancels out and you only have C into D of the perturbation temperature T prime by time.

So, D T prime DT. The right hand side function we can now do a Taylor series expansion. So, the first term is the base value which is the function Tss Xss whose value we already know is 0. this term is 0. So, that will be the base term plus the first partial derivative of T prime and the first partial derivative with respect to X prime. So, if we do a Taylor series expansion and we take only the first derivative of this functional term with respect to T and X, then we get this expression here.

So, C dt prime dt equal to del f del T at a constant at the steady state greenhouse gas concentration Xss into the perturbation temperature T prime plus del f del x at the steady state temperature Tss into the perturbation in the greenhouse gas concentration x prime. Basically, if you remember Taylor series, it is basically del F del X into delta X plus del F del Y into delta Y. The first order terms in the Taylor series expansion. So, this is the partial derivative of the net incoming radiation with respect to temperature at the steady state greenhouse gas concentration multiplied by the perturbation in temperature plus the partial derivative of the net incoming radiation with respect to greenhouse gas concentration at the steady state temperature So, these two terms and we are neglecting the second order and third order terms assuming that T prime and X prime values are small. So, the second order and the third order term will be quite small in magnitude.

So, this is the expression that we have to solve. And here we will define these two derivatives separately. We will define first what is called the climate feedback parameter which is negative of this term here, negative of del F del T at a constant greenhouse gas concentration and its unit is watt per meter square Kelvin. The flux term is watt per meter square by delta T, so it is watt per meter square Kelvin. The climate feedback parameter alpha is minus del F del T at the steady state greenhouse gas concentration.

Now, what is the physical significance of this climate feedback parameter alpha that we have defined just now? The climate feedback parameter alpha quantifies how the net downward radiation flux F varies with the temperature of the climatic system as you mean that other parameters like concentration are fixed. Remember del F del T is del T of f downward minus f upward. Net absorbed shortwave radiation minus the net outgoing longwave radiation. So the Variation of the net downward radiation, short wave minus long wave, with system temperature, assuming that the greenhouse gas concentrations on every other portion parameter is held fixed. That is what the climate feedback parameter tells.

Here, we have negative of this. So, the negative of that is the climate feedback parameter. So, if the flux, net incoming flux decreases with temperature, that is del F del T less than 0, then alpha which is minus del F del T is greater than 0. So, we have a positive climate feedback parameter. Physically this means that an increase in the temperature results in a decrease of net incoming radiative flux from the troposphere and hence the climate system tends to lose energy as its temperature increases.

Basically your del F del T is less than zero. So what is happening is if your system temperature is increasing assuming everything else is held constant, the difference between the incoming shortwave and the outgoing longwave radiation, that value is decreasing. As a result, you have less energy coming into the system and more energy going out of the system. So, the climate system is losing heat as the temperature of that system is increasing. So, this is a negative feedback.

So, the feedback parameter is, if the feedback parameter is positive, then we have a negative feedback. So, it is kind of counterintuitive. Basically, if alpha is greater than 0, then del F del T is less than 0. The climate is cooling down because it is losing energy when its temperature increases. So this is very close to the kind of conditionally stable, unconditionally stable conditions that we discussed when we are looking at atmospheric stability.

Here we are looking at climatic stability. What happens if for whatever reason the temperature of the system is increasing? Here under this condition when the feedback parameter value is positive, as the temperature of the system increases, the system tends to lose more heat. And so the system will tend to come decrease in temperature. So the temperature will fall back down to its steady state value. So this represents a stable climate as increasing temperature enhance the outward heat loss and the system tends to cool back down.

So alpha greater than zero means a stable climate with respect to temperature. In the current climate system that we have on our planet, alpha is positive and hence temperature fluctuations tend to get dampened down. So, change in temperature causes a net cooling effect which in turn decreases the temperature downwards. On the other hand, If F increases with temperature, so del F del T is greater than 0, then your alpha which is minus del F del T will become less than 0, negative. And hence, we have a negative climate feedback parameter.

In this case, an increase in temperature cause an increase in the net incoming irradiance. The shortwave minus long wedge, that difference becomes negative. greater than 0, becomes positive. So, the F becomes positive and as a result, as temperature increases, the system tends to gain heat and heats up further. That is the outgoing OLR falls with respect to the incoming absorbed shortwave radiation.

So, the climate system gains more energy as it increases in temperature or loses energy as its temperature falls. So, either can be happening. If the climate system increases in temperature, it is gaining more energy because F term increases. So, it is going to heat up further. if the temperature falls for whatever reason, so the del t term becomes negative, so del f term will also become negative, so that this, the gradient still remains greater than zero.

So, in that case, if the temperature falls, then the outgoing radiation will increase with respect to the incoming shortwave radiation and hence the climate system will tend to lose more energy and hence it will cool further. So such a climate system is unstable because any perturbation of temperature either in the positive direction or in the negative direction will cause an even further heating or cooling effect which is going to increase the temperature deviation beyond over the steady state value. So if the climate heats up, if the climate's temperature increases, gains energy and the temperature increases further, it gains further energy and the temperature continues to increase more and more. So, we have a runaway greenhouse effect. On the other hand, if the temperature falls, then it loses more energy.

So, temperature falls further and we have a runaway ice house effect. And so, we have an unstable climate. So, this alpha value is extremely important because if alpha is greater than 0, we have a stable climate where the temperature, any deviation tends to get balanced out and gets dampened and we tend to go back to the steady state. Whereas, if alpha less than 0, we have an unstable climate and we have a runaway greenhouse or ice house condition. So, this is the significance of the first term here.

Now, we will look at the second term del F del X. This del F del X is the radiative forcing term Rf that we have been discussing. It is the gradient of the net incoming radiation with change in greenhouse gas concentration multiplied by the absolute perturbation in the greenhouse gas concentration. So here this entire term into x prime is considered radiative forcing and its unit is watt per meter square. Then the change in the net incoming radiation flux with respect to greenhouse gas concentration multiplied by the perturbation in the greenhouse gas concentration.

So, the radiative forcing term Rf quantifies the change in the net inward heat flux at the top of the troposphere due to a small change of concentration x prime of a greenhouse gas away from its steady state value for a given temperature. So, here it is the partial derivative. So, we are keeping the temperature fixed. So, for a given temperature of the climate system. So, if and this radiative forcing term is something we have calculated for the various greenhouse gas concentrations, CO2, methane, etcetera in the plot that we looked earlier.

So, using these expressions the zero dimensional energy balance model becomes C dT prime dT plus alpha T prime plus radiative forcing term which itself can be a function of temperature. So, how does this work? If you go back to the main expression, this becomes minus alpha T prime and this is becomes the radiative forcing term Rn. So, C dT prime dT plus alpha T prime equal to the radiative forcing and this is what we are seeing here. So, for most greenhouse gases the radiative forcing term is positive.

So, what that means is, we go back here. As the greenhouse gas concentration increases, the net incoming radiation also increases. That is the OLR is decreasing with respect to the shortwave radiation and as a result the system is gaining more heat with increasing the greenhouse gas concentration. Whereas if this X is the concentration of soup or suspended matter in the atmosphere then the relative forcing term will be negative because there the increasing concentration will decrease the absorbed shortwave radiation and so the F will fall as X increases. So, in that case the relative force in term will be negative. So, RfT can be positive or negative depending on which type of species we are looking at.

If we are looking at CO2, methane etcetera then this will be positive. If we are looking at something like ozone or the soot suspended particles then it will be negative. Alpha will be positive if it is a stable climate system and alpha will be negative if it is a unstable climate system. and we have this final first order differential equation in terms of the temperature perturbation expressed in this form. So, this is the zero dimensional energy balance model and the perturb form of that model when we have a small change in one of the forcing parameters x prime and corresponding change in the temperature T prime.

So if we solve this, we will get the change in the temperature with time due to the radiative forcing. Okay. And this is what we want. How the climate systems mean temperature is changing with time due to a certain radiative forcing by a certain greenhouse gas.

Okay. Now, precise values of C and alpha are difficult to evaluate, but we have shown using the mixed layer approximation we discussed earlier that C is around 1 gigajoule per meter square Kelvin. For our climate system, alpha is 1 watt per meter square Kelvin. So, it is a positive term.

We have a stable climate. So, that is good to know. Another important parameter here is that we can derive is called the relaxation time or feedback response time and this is the time tau is C by alpha. So, here this C the heat capacity and the climate feedback parameter alpha if you divide these two you will get a time value C by alpha. Remember alpha is 1 watt per meter square and C is joule gigajoule per meter square So if you do it in joules, it's 10 to the power 9 joules per meter square Kelvin. This is joules per second.

So if you divide C by alpha, you get seconds. The feedback response time for our planet is on the order of 10 to the power 9 seconds. See, Giga is 10 to the power 9. So it is 10 to the power 9 seconds. So that is very clear from here.

And 10 to the power 9 seconds is of the order of 30 years. So, if you see how many seconds a year has, 10 to the power 9 seconds is of the order of 30 years. So, what is this? relaxation time or feedback response time. You will see how we get the physical idea of what this tau represents, but it can be defined as the approximate time scale for a perturbed climate system to reach a new steady state condition with a new steady state temperature. So, what this means is suppose your greenhouse gas concentration has been perturbed by X plus XSS plus X prime. As a result, the temperature is going to slowly increase and you have a T prime, TSS plus T prime.

Eventually, the climate system will going to go back to a new steady state value over a certain period of time. If there was an old steady state value, greenhouse gas concentration has increased. So, the temperature is slowly going to increase in time and slowly reach to another new steady state value where it will again be time independent. The time it takes for the temperature to slowly rise to this new steady state value is the value of T tau which is approximately 30 years. This is also the reason why in the IPCC reports for example, 30 year averages are taken because we know given our planet's various values and magnitudes that the relaxation time or the feedback response time, the response of our climate system to a perturbation of a greenhouse gas concentration or another forcing parameter is approximately 30 years.

So 30 years is a convenient time frame to define the response of the climate system. So, given an instantaneous perturbation, it takes the climate system approximately 30 years to reach a new steady state. So, that is the expression of tau. So, we will stop here. In the next class, we will try to solve this energy balance model and see how and look at very specific solutions for this energy balance model as well. Thank you for listening and see you in the next class.