Course Name: An Introduction to Climate Dynamics, Variability and Monitoring

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Week- 07

Lecture- 37

ATMOSPHERIC CIRCULATION SYSTEMS - CORIOLIS FORCES

Good morning class and welcome to our continuing lectures in climate dynamics, climate monitoring and climate variability. In the previous class, we were discussing the x, y and z momentum equations that governs the change in the zonal, meridional and vertical component of velocity of wind velocity in the atmospheric circulation system. And we saw that there are three or four forces that govern these wind velocities. The first is the Coriolis force, which is the apparent or the pseudo force that arises because of the non-inertial nature of Earth's reference space. and we said that it is coming from two sources. First is the concept of the conservation of angular momentum and the second is the centrifugal pseudo force that is arising due to the rotation of the earth.

So, we will discuss these two points today. So, here let us consider a parcel of air that is at a certain altitude z from the ground where the ground is located at a certain latitude phi with respect to the equator. This is the latitude angle phi. The ground is located at a certain latitude angle and z is the height of the air parcel above this point of the ground.

Okay. The radius of the earth is a. Then the distance of this air parcel from the center of the earth is a plus z. Clear? We can drop a perpendicular to the axis of the earth from the centroid of this air parcel. And we can call the distance of the air parcel from the axis of the earth to be r.

Then since this is phi, this is 90 minus phi. So by right triangle equation this angle is phi. So this distance r is nothing but a plus z cos phi. Clear? Now the earth is rotating at an angular speed omega around the axis. This omega is in radians per second and based on the idea that the earth completes 360 degree in a 24 hour time, this angular speed omega is given to be 7.29 into 10 to the power minus 5 radians per second. You can calculate it yourself because 360 degrees is 2 pi radians, 24 hours is 24 into 60 into 60 seconds. So 2 pi divided by that will give you this angular speed omega of earth's rotation and earth is rotating from west to east. So now, so till now we have gone up to this point. We assume first that the parcel of air is at rest in that location.

So the parcel of air is stationary with respect to the atmospheric column at that location. However, note here that the earth itself is rotating at a certain velocity with respect to the axis of the earth. So what it means when we say that parcel of air above me is stationary with respect to the ground is it is rotating along with the ground with the same angular velocity. So, it is actually moving though with respect to the ground it looks stationary and that is where this conservation of angular momentum comes into the picture. So the rotational velocity of this stationary parcel of air is given as u rotation equals to omega into r where r is the distance between the axis of the earth and the center of this parcel of air.

This omega into r is the rotational velocity of this parcel of air in the west to east direction. And this is omega into a plus z cos phi. Remember a is the radius of the earth, z is the altitude of this parcel of air with respect to the ground. And this velocity is in the zonal west to east direction because that is the direction in which the earth rotates. Suppose now that in addition to this the parcel of air has a zonal velocity small u relative to the ground.

$$r = (a + z)cos\phi$$

The parcel of air is no longer stationary but it is moving in the west to its direction relative to the ground from where we are observing it. And this velocity in the west to its direction is a zonal velocity and which we call it small u. So the absolute zonal velocity that this parcel of air has is the zonal velocity imparted to it by the rotation of the earth plus the relative zonal velocity it has with respect to the ground, which is u rotation plus small u. Then the magnitude of the absolute angular momentum L of this parcel of air in the zonal L absolute in the zonal direction, so in the x direction, right, is mass of this parcel into the absolute zonal velocity into the radius, radial distance of this parcel of air with respect to the axis of the air. So, mass and u absolute is u rotation plus small u where u rotation is sigma a plus z cos phi.

$$U_{abs} = U_{ROT} + u \quad (7.7)$$

Then the magnitude of the **absolute angular momentum** (L) of this parcel of air in the zonal direction is,

$$L_{abs} = mU_{abs}r = m[\Omega(a + z)cos\phi + u](a + z)cos\phi \quad (7.8)$$

So, we can put all of this part here, u rotation plus small u and r is a plus z cos phi. This is the absolute angular momentum and this absolute angular momentum is conserved by the law of conservation of angular momentum. So, the total derivative of absolute angular momentum in the zonal direction with respect to time is equal to 0.

$$\frac{DL_{abs}}{Dt} = 0$$

Now, let's understand how the Coriolis force comes into the picture. Suppose that this parcel of air also has a meridional south to north velocity relative to the ground. That is, this parcel of air is also moving in the south north direction, along the meridian. So, if it moves northward, it is moving towards a higher latitude. So the phi is increasing. As phi is increasing cos phi in this equation is decreasing. Cos of an angle as the angle increases 90 degrees it is going to decrease.

So the L absolute should be decreasing. However the L absolute cannot decrease. There is a law of conservation of angular momentum. As a result there has to be a balancing increase in the zonal velocity of this air parcel which compensates for the decrease in the cos phi term so that L absolute remains a constant. In order to keep the angular momentum conserved in the zonal direction, its relative velocity u must be increased.

So, because cos phi is decreasing here and here, the relative velocity u must be decreasing to compensate for this decrease in the cos phi term to keep L absolute constant. Thus, it appears that a north moving parcel of air gains in its eastward moving velocity, eastward moving west to east, zonal eastward moving velocity magnitude in the northern hemisphere. So as the wind parcel moves northwards its new velocity is increasing and it is increasing such that its absolute angular momentum remains constant. So this increase is basically being seen in terms of the Coriolis force. So if you go back down here. This u is increasing, small u, the zonal velocity with respect to the ground as the parcel moves northwards.

And this is explained in terms of a pseudo course which is the Coriolis course. So, the m d u dt term because of this northward moving parcel is given by the value of this Coriolis term. That is why this is called a pseudo force because actually there is not a force. It is just a concept coming from the conservation of angular momentum. But since we are balancing the forces in terms of the ground as a stationary object which is a non-inertial frame, we will see an increase in the velocity which we will infer in terms of a pseudo force which is the correlate force in the x direction, fine.

So, now in the southern hemisphere, the case is reversed. you have a north moving parcel in the southern hemisphere. As it moves north in the southern hemisphere, its latitude angle is decreasing. So, phi is decreasing. So, cos phi is going to increase.

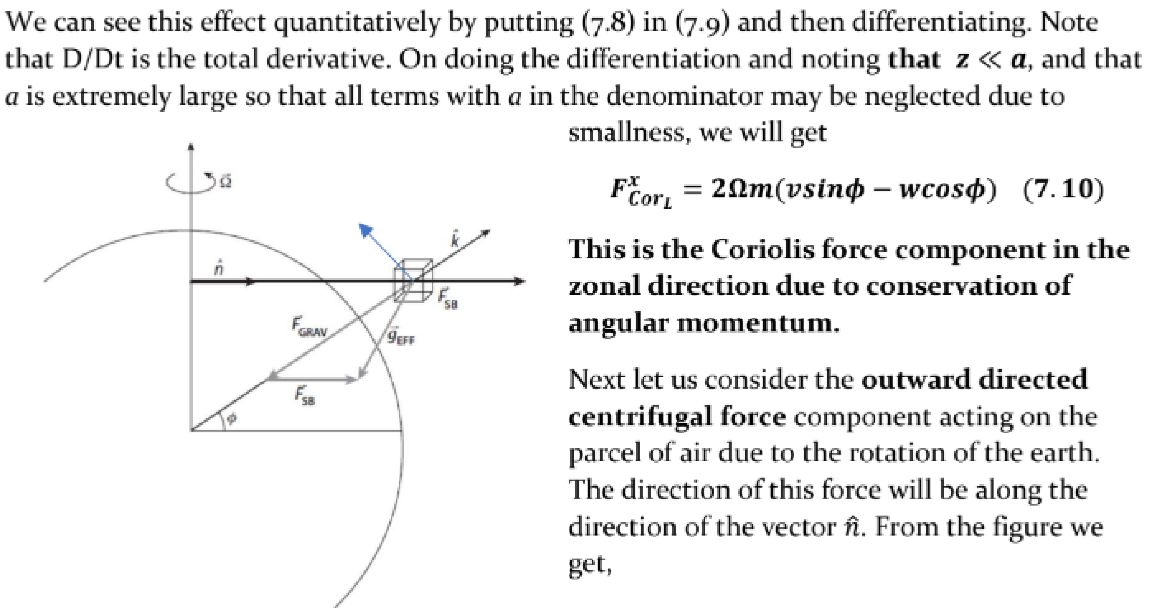
Hence, to balance the angular momentum, its zonal velocity must be decreasing. So, what is zonal velocity here? The positive zonal velocity is west to east. So, what we will see is, we will see an acceleration of this north-south moving parcel in southern hemisphere in the western direction. In the northern hemisphere, it will be in the eastern direction the acceleration. In the southern hemisphere, there is an acceleration in the western direction.

So, what this means is, sorry, yes. In the southern hemisphere, a north moving parcel of air gains in the westward moving velocity magnitude. Hence, north moving wind in northern hemisphere inclines towards the east naturally, while a north moving wind in the southern hemisphere inclines towards the west. So, northern hemisphere, wind moving northwards goes towards the east. In the southern hemisphere, wind moving northwards goes towards the west.

We can do this quantitatively by putting this expression 7.8 l absolute value into 7.9 and differentiating. Remember d dt is del del t plus u del u del x plus b del v del y plus w del del z. So, u del l del x, v del l del y, w del l del z and we can differentiate this and set it to 0.

If we do that and we will make some simplifying assumptions here, noting that Z is much much smaller than A. A is the radius of the earth and Z is the altitude of the air parcel which relates to the ground. Clearly the radius of the earth is much larger. So, A is extremely large. So, that all terms with A in the denominator we can neglect due to smoothness.

If we do this, then the Coriolis force in the x direction due to this conservation of angular momentum principle, so Fx Coriolis. be shown to be equal to twice the angular velocity of earth into the mass of the air parcel into V sin phi minus W cos phi. V is the meridional velocity, the south to north moving velocity. phi is of course the latitude angle. So, this meridional velocity into sine of the latitude minus the vertical velocity w into cos of the latitude, all of this multiplied by twice sigma into m.



We can see this effect quantitatively by putting (7.8) in (7.9) and then differentiating. Note that D/Dt is the total derivative. On doing the differentiation and noting **that** $\boldsymbol{z \ll a}$, and that a is extremely large so that all terms with a in the denominator may be neglected due to smallness, we will get

$$F^x_{Cor_L} = 2\Omega m(v sin\phi - w cos\phi) \quad (7.10)$$

This is the Coriolis force component in the zonal direction due to conservation of angular momentum.

Next let us consider the **outward directed centrifugal force** component acting on the parcel of air due to the rotation of the earth. The direction of this force will be along the direction of the vector $\hat{n}$. From the figure we get,

So, this is the core eddy force component in the zonal direction x component due to the conservation of angular momentum. So, note that the x component of the force which is causing the change in the u component of velocity is caused by velocity components in the v and w direction respectively. This is very important. Next we will look at the other component of the Coriolis force which is coming from the centrifugal force. So the centrifugal force is a pseudo force coming due to the rotational motion of the earth.

How is the centrifugal force oriented? Clearly this parcel of air is rotating around the axis of the earth. The centrifugal force, the pseudo force is directed away from the axis of the earth. directed away from the axis along the vector which is connecting the axis of the earth to the air parcel. So, we drop a 90 degree vector from the air parcel centroid to the axis of the earth and the direction of the centrifugal force is in this direction.

So, in this n direction. What is this n direction? This is the vertical, vertical direction with respect to the ground, K. This is the north-south direction, the J direction. So, because with respect to the ground, this is the north-south, this is vertical and perpendicular to the plane of paper is the meridional direction. Clear? So, these three are in the same plane. So this n vector can be expressed as a component of this k vector and this j vector.

And this we can do quite easily. The n vector is minus sin phi j. How does this work? You drop a minus j on this side. So this angle is cos phi and this angle is sin phi. So how does this look like? this is phi. So, if you draw this angle here, k and j are perpendicular.

So, this is 90 degrees. This angle becomes phi because this line and this line are parallel to each other. So, this phi is equal to this phi here. Because j is 90 degrees, this is 90 minus 5, right. So, this n, if you drop in the minus j direction, this becomes cos of 90 minus 5, so minus sin phi. So, minus sin phi into j and this side is cos k, right.

So, you get the n vector as minus sin phi j plus cos phi k, where j is the south north direction, k is the vertical direction. And the centrifugal force is along this direction. And what is centrifugal force? Centrifugal force per unit mass is the absolute zonal velocity. So, the tangential velocity of this air parcel with respect to the axis of the earth which is the zonal velocity. The tangential velocity with respect to the axis of the earth is the west to east velocity which is the zonal velocity.

square of the absolute zonal velocity by the distance separating the center of this mass of air with the axis of the earth. So, it is rotating like this u absolute square by r. So u absolute square by r and the orientation is n. And what is u absolute? The rotational velocity of earth plus any additional zonal component of velocity that this parcel of air has.

u rotation plus u volt square into n hat by r. So you can separate out into these two sections. u square r rotation by r n hat plus 1 by r twice u rotation u plus u square n hat. The first term is the centrifugal force imparted by the rotation of the earth itself, alright. While the second term is the additional centrifugal force component relative to the ground due to the zonal velocity u of the parcel of air. So, if the parcel of air was stationary this term will go to zero, right.

And this term, this centrifugal force term has no either as the altitude z or the zonal velocity u. So, this is the centrifugal force imparted by the rotation of the earth itself. So, the centrifugal force on the parcel of air relative to the ground is the second term only. what is also being suffered by the ground. So, both the ground and the parcel of air is having a centrifugal force term of this term u square rotation by R n hat.

So, centrifugal force relative to the ground is this term minus the first term and this is due to the zonal velocity that this wind may have. The centrifugal force relative to the ground we call f prime cf is m by r into this term here. Okay. What is u rotation? Omega into r. So 1 by r omega r is twice omega u plus u square by r and n hat is minus sin phi j plus cos phi j.

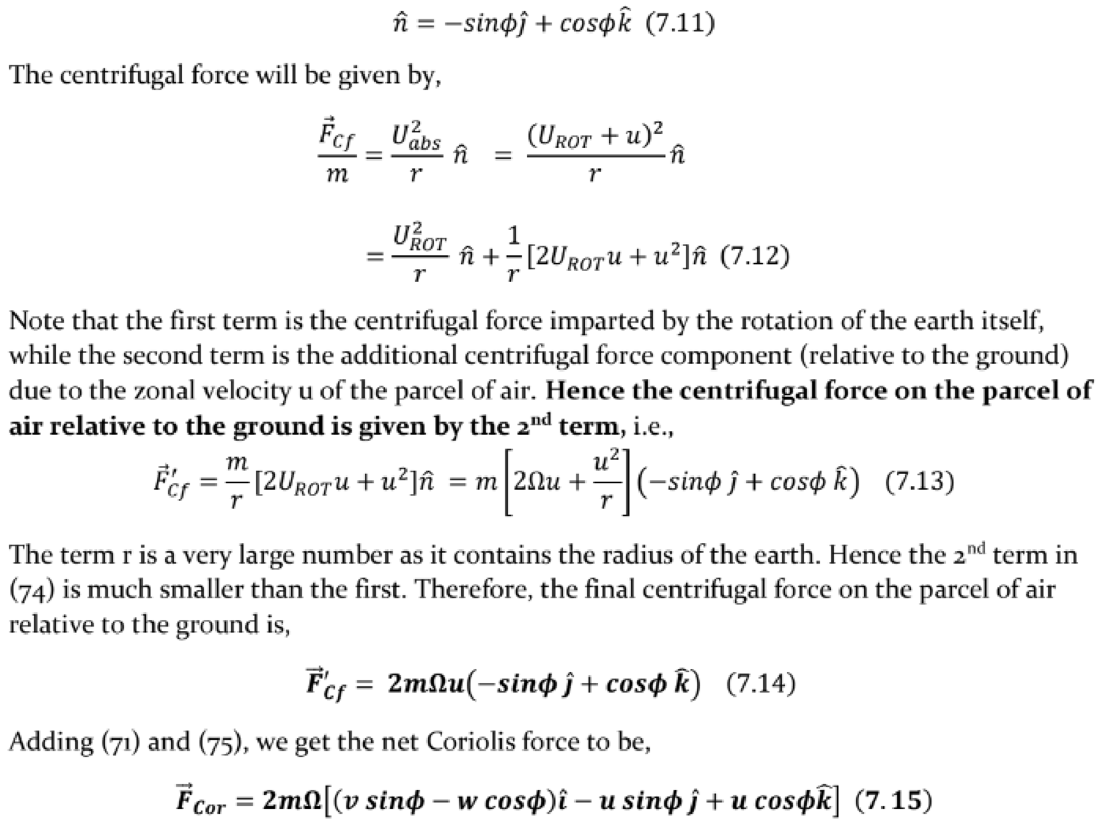
Okay. Now, the r is again the radius of the earth which is very large. So, this second term can be neglected. So, you are just getting m into twice omega u into minus sin phi j plus cos phi j as you can see here. Note here, this centrifugal component is in the j and k direction.

meridional direction and vertical direction. So, the y component of centrifugal force is this one, z component of centrifugal force is this one. So, now we can add 7.14 with respect to 7.10 which is in the i direction. So, this is in the i direction zonal This is meridional, this is vertical.

So we get twice m sigma. Remember this is also twice m sigma. B sin phi minus omega w cos phi i. This is the first term due to conservation of angular momentum. This is the zonal component.

Plus u sin phi j, this term here. u cos phi k, this term here. These are the x component of centrifugal force, y component of centrifugal force, z component of centrifugal force. In the vertical direction, the gravitational force magnitude is much larger than the Coriolis force. So, if we look at here, this is in the k direction, right? So, here you see This F Coriolis Z is much, much smaller than F GZ, the M GZ term. The downward gravitational force is much larger than the upward Coriolis force.

So, we usually neglect the K component of Coriolis force because gravitational force overwhelms that. So, we can neglect this term. Further, in the Earth's atmosphere, the vertical component of wind velocity W is significantly smaller than the horizontal components. So, while there is a significant upward and downward moving convection currents, the velocities associated with them is actually 10 to 100 times smaller than the velocities associated with the zonal and the meridional wind circulation systems.As a result, omega is much much smaller than V. As a result, you can neglect omega cos phi, W cos phi in most circumstances.



$$\hat{n} = -sin\phi\hat{j} + cos\phi\hat{k} \quad (7.11)$$

The centrifugal force will be given by,

$$\frac{\vec{F}_{Cf}}{m} = \frac{U_{abs}^2}{r}\hat{n} = \frac{(U_{ROT} + u)^2}{r}\hat{n}$$

$$= \frac{U_{ROT}^2}{r}\hat{n} + \frac{1}{r}[2U_{ROT}u + u^2]\hat{n} \quad (7.12)$$

Note that the first term is the centrifugal force imparted by the rotation of the earth itself, while the second term is the additional centrifugal force component (relative to the ground) due to the zonal velocity u of the parcel of air. **Hence the centrifugal force on the parcel of air relative to the ground is given by the 2nd term**, i.e.,

$$\vec{F}'_{Cf} = \frac{m}{r}[2U_{ROT}u + u^2]\hat{n} = m\left[2\Omega u + \frac{u^2}{r}\right](-sin\phi\,\hat{j} + cos\phi\,\hat{k}) \quad (7.13)$$

The term r is a very large number as it contains the radius of the earth. Hence the 2nd term in (74) is much smaller than the first. Therefore, the final centrifugal force on the parcel of air relative to the ground is,

$$\boldsymbol{\vec{F}'_{Cf} = 2m\Omega u(-sin\phi\,\hat{j} + cos\phi\,\hat{k})} \quad (7.14)$$

Adding (71) and (75), we get the net Coriolis force to be,

$$\boldsymbol{\vec{F}_{Cor} = 2m\Omega[(v\,sin\phi - w\,cos\phi)\hat{i} - u\,sin\phi\,\hat{j} + u\,cos\phi\hat{k}]} \quad (7.15)$$

W is like 0.1 V or 0.01 V like that. There are certain exceptions. Because this is a cos phi term, As phi decreases, the latitude goes towards the equator. This term becomes large cos term and this sin phi term becomes small. As a result, there will be a region where w cos phi will be larger than v sin phi only because sin phi has become much smaller and cos phi is close to 1. So, close to the equator, this simplification will not work and we have to use the entire expression.

This is one another important point. However, in all other places, we can neglect the k term and the w cos phi i term. So, we get the Coriolis forces m into the Coriolis constant F into v i minus u j. where f is twice omega sine phi, twice into the angular velocity of earth into sine of the latitude angle, which is called the Coriolis parameter. Clear? So, this becomes the complete Coriolis force under the above simplifications. We are only looking at the zonal and the meridional component of Coriolis force.

$$\boldsymbol{\vec{F}_{Cor} \cong mf(v\,\hat{i} - u\,\hat{j})} \quad (7.16) \textit{ where}$$

$$\boldsymbol{f = 2\Omega\,sin\phi} \quad (7.17) \textit{ is called the Coriolis parameter}$$

Also to note is that This is the x component or the zonal component of Coriolis force which is arising due to the meridional velocity v, the velocity in the south to north area. Similarly, the y component or the meridional component of Coriolis force is appearing due to the zonal velocity v. So, the north-south velocity creates the x component of Coriolis force East-west east velocity creates the y component of Coriolis force. The Coriolis force is generated perpendicular to the wind direction.

This is very important also. Note that a meridional velocity will cause the Coriolis force in the zonal direction causing the air parcel to accelerate in the west-east direction. On the other hand, a zonal velocity will create a negative Coriolis force in the meridional direction causing the wind to decelerate in the south-north direction. So, suppose you have a positive view. So, there is a west to east velocity.

you have a negative Coriolis force in the south to north direction. So, the wind will be going towards the south. So, any wind like this will be drifting southwards. Whereas, if you have a positive V velocity like this, the Coriolis force will be positive in the zonal direction.

So, any wind like this will be drifting eastwards. Clear? One other point. Phi is 0 to 90 in the northern hemisphere, 0 to minus 90 in the southern hemisphere. The F term is negative in the southern hemisphere. So, we have the exact opposite case in the southern hemisphere because you have a minus term here because of the sine phi. So, in the southern hemisphere, any wind moving like this will be drifting not eastward but westward. and any wind moving this direction will have a positive Coriolis force in the northward direction and so it will be drifting northwards.

Deflection due to Coriolis Force in NH

So, that is very important. So, here is a deflection of due to polarity force in the northern hemisphere, northward moving wind drifting eastwards, southward moving wind drifting westwards. Similarly, eastward moving wind drifting southwards, westward moving wind drifting northwards. And the reverse case in the southern hemisphere. Note further that Coriolis component depends on sine 5.

At the low latitudes, the latitude angles are small and hence the Coriolis force is weak.

Coriolis force becomes increasingly dominant in the high latitudes as the latitude angle increases being highest near the force. So, Coriolis force is extremely strong at high latitudes and weak in the low latitudes. So that completes our discussion of the Coriolis force. In the next class, we will discuss the pressure gradient force and the friction force. Thank you for listening and see you in the next class.