Course Name: An Introduction to Climate Dynamics, Variability and Monitoring

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Lecture 35

TEMPERATURE DISTRIBUTION AND GREENHOUSE EFFECT IN A CONTINUOUSLY STRATIFIED ATMOSPHERE

Good morning and welcome to our continuing lectures on climate dynamics, climate variability and climate monitoring. In the previous class, we were finally able to derive the expression of the temperature of the atmospheric layers as they vary with altitude z in terms of the blackbody emission temperature of earth and this was found to be a function of the modified optical depth of the ground tau g star and an exponential function of minus z by H i where H i is the mass absorption, H i is the scale height for the absorbing species. Note that if you have multiple absorbing species, there will be additional terms for each of these absorbing species. Here we are looking at only the case of a single absorbing species. And we were able to also derive the upward and the downward radiative hemispheric fluxes in terms of the ground optical depth. modified ground optical depth and the exponential terms.

$$F^{\uparrow}(z) = \frac{1}{2}F_0\left(2 + \tau_g^* e^{-z/H_i}\right) \qquad \text{(XVI)}$$

$$F^{\downarrow}(z) = \frac{1}{2}F_0\tau_g^* e^{-z/H_i} \qquad \text{(XVII)}$$

$$\tau_g^* = 1.66\tau_g$$

$$\frac{T(z)}{T_e} = \left[\frac{1}{2}\left(1 + \tau_g^* e^{-z/H_i}\right)\right]^{1/4} \qquad \text{(XVIII)}$$

And note that F0 is the S0 by 4 1 minus alpha which is the amount of solar radiation that is being absorbed by earth. This much we have derived previously. So, now two important points we will note in this class. We can define something called emission height. Ze as the height where T of z is equals to the blackbody emission temperature T. So, the altitude at which the atmospheric temperature equals the blackbody emission temperature of earth that is called the emission height. So, if you look at the previous example, then this is 1 equal to that expression. So, we get 1 equals to half 1 plus tau g star e to the power minus ze by H i this is expression implies tau g star exponential minus ze by Hi equals to 2 minus 1, so equals to 1. So, first point is this expression is basically tau star, this implies tau star equals to 1, correct. Because tau star is tau g star e to the power minus ze by Hi. This also means e to the power ze by Hi, so we are taking this on this side equals to tau g star implies the emission height ze is Hi log tau g star. Now remember what tau g star is. tau g is this expression and tau g star is 1.66 times this expression. So, tau g star is 1.66 times the absorption or the extinction coefficient of the ith absorbing species, the scale height for that absorbing species and last term is the density partial density of the absorbing species at the ground y g. So, these are the three expressions here. So, let me call this as 8, this as So, clearly firstly the atmosphere will have the temperature equal to the blackbody emission temperature where the scaled or the modified optical depth is equals to 1 and the corresponding emission height the altitude ze is given by the scale height of the absorbing species into log of the modified optical depth at the ground. Another important point and this will show some of the limitations of the radiative equilibrium theory is to see what happens as we are reaching the ground.

$$1 = \frac{1}{2}\left[1 + \tau_g^* e^{-Z_e/H_i}\right]$$

$$\tau_g^* e^{-Z_e/H_i} = 1 \quad \Rightarrow \quad \tau_g^* = 1 \qquad \text{(XIX)}$$

$$e^{Z_e/H_i} = \tau_g^* \quad \Rightarrow \quad Z_e = H_i \ln(\tau_g^*) \qquad \text{(XX)}$$

$$\tau_g^* = 1.66 k_{\text{abs}}^i H_i p_i \qquad \text{(XXI)}$$

At the ground, z is tending towards 0. If you go to this expression here, z is tending to 0, so this becomes 1. The temperature of the layer of air above the ground tau z equals to 0 by Te is equals to sorry about that. No, because this is 1, the one-fourth term cancel out. So, this is correct. This expression, this is 1. So, 1 plus tau g star half. So, we get 1 plus tau g star by 2 to the power 1. So, this is the temperature of the layer of air above the ground. This is 22 equation.

$$\frac{T(z=0)}{T_e} = \left[\frac{1+\tau_g^*}{2}\right]^{1/4} \qquad \text{(XXII)}$$

If you look at the ground, the net, so by radiation balance at the ground, the net blackbody radiation emitted by the ground has to be equal to the upward radiative flux density. So, if Tg is the ground temperature, then assuming that the ground is a black body, we get sigma Tg to the power 4 equals to F upwards at Tg star, ok. Now what is F upwards? F upwards is this expression here, all right. So F upwards at Tg star is equal to, if you look at this expression here, 2 plus Tg star this expression is 0. So, half F0 2 plus Tg star e to the power 0 since z equals to 0 and this we are getting from equation 13. equation 13. So, we get sigma Tg to the power 4 equals to half F0 2 plus Tg star. And what is F0? It is half sigma Te to the power 4 2 plus Tg star. So, sigma sigma cancels out. So, we get the temperature of the ground, so this is ground temperature is equals to Te 1 plus Td star by 2 to the power 1 by 4. This is the temperature of the ground under the radiative equilibrium approximation. So, this is the temperature of the air above the ground which is Te into 1 plus Tg star by 2 to the power 1. So, here it is 1 plus Tg star by 2. See, there is a slight difference. So, here it is half plus Tg star by 2, here it is 1 plus Tg star by 2 to the power 1 by 4. Everything else is the same. basically expression 23. So, what this kind of means is under radiative equilibrium temperature of the ground is greater than temperature of the air just above the ground and hence we have a thermal discontinuity. This is basically due to the incompleteness of the radiative equilibrium theory. This discontinuity will disappear once we take convective heat transfer from ground to air layers into account. So, we would not have this sharp discontinuity because we have to take into account the convective heat transfer as well between the land and the air layers above it in order to, so if we take that into account then this thermal discontinuity kind of disappears.

$$\sigma T_g^4 = F^{\uparrow}(\tau_g^*)$$

$$F^{\uparrow}(\tau_g^*) = \frac{1}{2}F_0\left(2+\tau_g^* e^0\right)$$

$$\sigma T_g^4 = \frac{1}{2}F_0(2+\tau_g^*)$$

$$\sigma T_g^4 = \frac{1}{2}\sigma T_e^4(2+\tau_g^*)$$

$$T_g = T_e\left[\frac{1+\tau_g^*}{2}\right]^{1/4} \qquad \text{(XXIII)}$$

So, this kind of gives us a simplified view, but an interesting view of what is going on in terms of how to identify the temperature of air layers above the ground. Here we have taken just radiative equilibrium. When we are considering the full model of evaluating the temperature above the ground, we also have to do the convective radiative equilibrium where convection currents and the associated heat transfer due to the movement of air masses as well as evaporation and condensation forces also have to be taken into account to evaluate the vertical temperature gradient. That is the task of a more sophisticated model which we will not discuss here. So, now let us look at some of the outcomes of this system.

So, note we have expressed, we have derived this expression Tz by Te as 0.51 plus the optical depth, modified optical depth at the ground e to the power minus z by Hi, where Te is the blackbody emission temperature which is 255 kelvins, Tg star is the equivalent optical depth of the ground and Hi is the scale height of the absorbing species. So now we can plot these expressions for various values. So here we are again plotting this with respect to z by Hi. The temperature profile is given on the right plot and the upward and the downward hemispherical fluxes which are also dependent very similarly on those expressions that we showed have also been plotted. So, this is the irradiance in watt per meter square is hemisphere flux density in watt per meter square. This is temperature in kelvins. When you look at the temperature in kelvins, so firstly here the assumption is the modified optical depth at the ground is 2 and F0 is of course 240 watt per meter square, 240 watt per meter square. And you can see how the downward hemispherical flux is exponentially increasing as we come closer towards the ground, whereas the upward hemispherical flux is kind of decreasing and reaching a steady value of 240 watt per meter square at a sufficiently high altitude. The corresponding temperature values are shown.

So, this is the, this is the temperature as z tends to infinity, it is also called the skin temperature, sometimes also called the stratospheric temperature because usually most of these heat balance expressions are valid in the troposphere only where the shortwave heating is being is being neglected. So, this is kind of the temperature of the stratotropopause for example. So, that is what usually the it matches best with the skin temperature. And it kind of also has decreases exponentially. Initially, it is linear, then it has an exponentially decreasing function till it reaches a constant value. And this is Tb, the temperature of the air layer just above the ground and Tg is the temperature of the ground. So, Tstrat here is 215 kelvins, T0 the air layer above the ground is 282 kelvins and Tg is 303 kelvins. Using these values you can evaluate these things. So, a more detailed radiative equilibrium based temperature profile will also include shortwave heating fluxes. In such case, this temperature is going to change as we reach the stratosphere.

It is not just going to become stable, just. So, this is more or less valid till the stratopause, after that it changes. The radiative equilibrium theory based prediction of the temperature profile with altitude is given where all the absorbing species have been taken into account and the shortwave heating has also been taken into account. So, when that is done, this is the purple line that we are seeing. So, it starts at around 320 kelvins. This is the ground temp, so this is 300, 320, so this is 310 kelvins is the temperature of the air layer above the ground, 320 kelvins is the temperature of the ground. It goes, decreases rapidly to around 10 kilometers where the shortwave heating starts to take effect and in the troposphere it kind of increases temperature. starts to go back up. So, this is the actual radiative equilibrium temperature profile when the all the absorbing species are taken into account and shortwave heating is also taken into account. Now, what you notice the dry it is actually the slope is much steeper than even the dry adiabatic lapse rate.
So, the radiative equilibrium based temperature profile in the troposphere is unconditionally unstable because the d T d z value is more negative than even the dry adiabatic lapse rate and is much larger than the mean lapse rate which is closer to the moist adiabatic lapse rate. So, why is the prediction so different? It is different because we have neglected convective and latent heat transports that are transporting a significant amount of the heat from the lower atmospheres to the upper layers of the atmosphere within the troposphere, which is why the actual mean lapse rate is much less steep and are closer to the moist adiabatic lapse rate. So, this is the moist adiabatic lapse rate, the dash dot blue line and this is the global average which is 6.5 kelvins per kilometer for the global mean lapse rate. Now, this is not actually something that is of concern because what this shows is that because the radiative equilibrium theory is predicting such a large gradient of temperature variance with altitude, which is unconditionally unstable, that high gradient is actually causing the instabilities that are leading to the development of convective overturning forces. So, convective overturning is a direct consequence of the fact that under radiative equilibrium the atmospheric lapse rate makes it unconditionally unstable. So, convective overturning and movement of latent heat and heat due to direct transport is a direct outcome of this unconditional instability. Another important use of the radiative transfer model is to understand the impact of the individual absorbing gases. So, we have derived it for a single absorbing gas. The total one is the overall contribution.

But this helps us also to identify how the individual gases are contributing to the overall radiative lapse rate. So, here is the following figure. of looking at the lapse rate for all the gases together which includes water vapor, CO_2 and ozone all together. The blue is the water vapor only, the red is water vapor plus CO_2 and the black is water vapor CO_2 plus ozone. You can see that the gradient in the troposphere for water vapor only case is almost the same as the total lapse rate gradient.

However, CO2 makes it more accurate is that the absolute temperature values are better predicted when you put water vapour and CO2 combined contribution. So, with only water vapour, the temperature at the surface and at any altitude above the surface till the tropopause will be slightly lower than what is actually being observed. With the presence of the CO2, while the gradient does not change, the temperature increases. So, from in the ground the temperature increases from say around 285 Kelvin. to around 300 Kelvin with the presence of CO_2. And so this 15 degree change, 10 to 15 degree change is being caused by the presence of CO_2. In the stratosphere, neither water vapor nor CO_2 is a good predictor of the lapse rate. Only when you put in the short wave absorbing ability of ozone that we get the true lapse rate when ozone is taken into account. Note that CO_2 adds a small but important correction to the profile of both the layers, increasing the temperature by about 10 kelvins in the troposphere and increasing it by about 20 kelvins in the stratosphere. So, CO_2 has a small but important contribution which is more around 10 kelvins in the troposphere and 20 kelvins in the stratosphere.

So, water vapor controls the tropospheric lapse rate gradient. CO_2 adds 10 Kelvin to make it more accurate. Ozone controls the stratospheric lapse rate gradient. However, both water vapor and CO_2 makes some important contributions in the overall picture in the stratosphere. So, that much we understand. So, this kind of gives us a snapshot of how the temperature profile, the vertical temperature profile of earth is arrived at. Radiative equilibrium of course overestimates, hence makes it unconditionally unstable. So, convective forces have to be taken into account. So, when the full energy balance is taken, when the convective energy transport is also taken into account, then we get the overall mean lapse rate. Now, if you look back, we also discussed things like the differences in the fluxes from one location to another location or from one season to another season.

So, local radiative equilibrium based lapse rates can also be evaluated. And if you do a radiative convective equilibrium for a specific region of the earth for a specific period of time, the corresponding vertical temperature gradient can also be deduced from these models. So, a more sophisticated climate model will include radiative equilibrium, radiative convective equilibrium and latent heat effects in evaluating the vertical profile of the temperature in a certain region of space. We will go back to these ideas of lapse rate temperature gradients and how they are perturbed when the concentration of an absorbing species like say CO_2 is changing due to anthropogenic effects. So that is something we will do later in the class.

Over the next few classes, we will have a small tutorial session in the next lecture discussing some of some ways to practice some of the numerical problems here, short one. After that we will switch gears somewhat. All of these energy balance models have primarily been looking in the vertical equilibrium or vertical transport of heat, radiation and convection we have mentioned, but we have not evaluated. Of course, as we have shown previously, horizontal transport of energy through air and wind currents is also a very important parameter and is leading to all the weather related phenomena. like global circulation systems as well as large scale variabilities like the El Nino, La Nina cycles.

So, horizontal transport of energy through wind and ocean currents, their mean values and their variations with seasons or decades is something that we have not covered yet. Yet this is a very important criterion as well. So both the vertical transport is of course important, very important for global warming considerations. Horizontal transport is important as it determines the wind patterns, the weather patterns that are happening in the world. So, over the next few classes, we will shift our gear somewhat and focus on the horizontal flux of energies and the kinds of currents and wind patterns and ocean current patterns it generates. So, thank you for listening and see you in the next class.