Course Name: An Introduction to Climate Dynamics, Variability and Monitoring

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TEMPERATURE DISTRIBUTION FOR A CONTINUOUSLY STRATIFIED ATMOSPHERE IN RADIATIVE EQUILIBRIUM - Part 1

Good morning and welcome to our lectures in climate dynamics, climate variability and climate monitoring. In the previous class, we have discussed heating rates for shortwave and longwave radiation for various atmospheric layers and evaluated the analytical functions as well as plotted the total heating rate as well as the heating rate contributions of the various absorbing species in the troposphere and the stratosphere. Now that the heating and the cooling rates are known, we can finally go and derive the temperature structure of the atmosphere assuming that the atmosphere is at radiative equilibrium. That is, whatever heat it is gaining through absorption of radiation, the same heat it is losing through emission of that radiation. The problem that we will be solving is the temperature distribution for a continuously stratified atmosphere in radiative equilibrium. So, this is a more sophisticated way of understanding the temperature distribution of the atmosphere rather than looking at an arbitrary set of isothermal atmospheric layers which are radiating and absorbing heat from each other as we did in the previous global warming model.

So, the main assumptions here are for simplicity here and this we can relax in a more sophisticated model, but here we will assume that the atmosphere is transparent to short wave radiation. So, this is the main assumption which is obviously not true, but we will be doing this for to show a simple derivation, and all short wave radiation that is not being reflected is being absorbed by the ground and then re-radiated as long wave terrestrial radiation. So, we will be neglecting the atmospheric heating caused by the shortwave radiation directly. Instead, all the shortwave radiation that is not being reflected by albedo effect is absorbed by the ground and then it is being reflected, re-radiated back as longwave terrestrial radiation.

So, we will just look at the longwave radiation case and how the atmosphere is being equilibrated by absorption and emission of this radiation. For the atmosphere and ground we have the diffuse approximation that is the radiation intensity is independent of direction of emission θ, φ. So, whichever direction the radiation is being emitted, it is the same value regardless of the direction that is the diffuse approximation. Second and this is also a strong approximation that we will be using that, this is called the gray approximation that is the mass extinction coefficient is independent of frequency nu in the IR range. Now, the question comes of why this is happening, alright.

So, obviously the mass extinction coefficient is dependent on the frequency. We have seen the large scale variations of the mass extinction coefficient value or the absorption cross section value. Mass absorption or mass extinction coefficient value with frequency before. So, what is the justification of this approximation? We can say, this approximation is reasonably valid for lambda greater than say 14 micrometers where H2O based absorption is more or less universal. So, if you remember how the absorption coefficient is for water vapor, so, whatever is happening before 14, after 14 the absorption coefficient of water vapor increases to a certain high value and it remains more or less constant up to 100 micrometers.

So, between 14 and 100 micrometers this water based absorption coefficient makes the absorption coefficient more or less independent of wavelength lambda. So, this approximation is reasonably valid if the lambda is greater than 14 micrometers. So, we will make this approximation for the entire IR range, which is of course inaccurate, but that will help us develop an analytical solution to get the basic trends, okay. Of course, if you actually want the evaluation, you have to find the k with respect to nu and use that, but that will of course complicate the problem significantly. So, using the diffuse approximation, we have got minus $dF_v^{\uparrow}$ at a given altitude z.

$$-\frac{dF_v^{\uparrow}(z)}{d\Psi_v^*} + F_v^{\uparrow}(z) = \pi B_v(T)$$

This is the spectral radiation flux density or spectral irradiance upward. that is by delta tau nu star, remember tau nu star is related to the optical gap tau nu by the function 1.66, which is kind of looking at the effect of, so tau nu is only along vertical direction. Tau nu star also takes into account that radiation is emitted in angular directions and thus there is a 1.66 term that comes to the optical depth term plus $F_v^{\uparrow}(z)$ equals to $\pi B_v(T)$.

This is the spectral blackbody radiation and for the downward irradiance, hemispheric irradiance, we have $\frac{dF_v^{\downarrow}(z)}{d\Psi_v^*} + F_v^{\downarrow}(z) = \pi B_v(T)$. These two expressions we have derived before where, $\Psi_v^* = 1.66\Psi_v$. Because we have using the gray approximation we can integrate over the frequencies. Now here I want to note two things.

We have said the gray approximation is valid only for water vapor and it is for above 14 micrometers. But if you look at the atmospheric heating and the cooling rates in the troposphere, they are dominated primarily by water vapor. So, this approximation works reasonably well within the troposphere primarily because water vapor contribution overwhelms the contribution of all other gases whose frequencies absorption mass coefficients or extinction mass coefficients do depend on the frequency nu. So, it is a reasonable approximation for the troposphere over the frequencies of IR spectrum we get.

So, now because we are integrating and because we have the gray approximation, the only thing that happens is the nu term goes away.

So, we have, $-\frac{dF_v^{\uparrow}(z)}{d\Psi_v^*} + F_v^{\uparrow}(z) = \pi B\,(T)$, $\frac{dF_v^{\downarrow}(z)}{d\Psi_v^*} + F_v^{\downarrow}(z) = \pi B\,(T)$, where, $\pi B\,(T) = \sigma T^4$. σT^4 is the blackbody irradiance, this is equals to $\pi B\,(T)$. So here, basically what we have done, we have integrated the spectral blackbody radiation intensity over all frequencies to get the blackbody radiation intensity which we are calling as the $B\,(T)$. So $B\,(T)$ is the total blackbody radiation intensity. And this σT^4 is the blackbody irradiance or radiation flux density and this is equals to pi times the blackbody radiation intensity by the diffuse approximation.

So, this is equation 3, this is equation 4. Okay, we have previously assumed that atmosphere is transparent to shortwave radiation, correct? That is there is no shortwave heating effect. So, shortwave heating of the atmosphere $\dot{Q}_{SW} = 0$. Now, volumetric heating rate is given by

$\rho_{(z)}\dot{Q}_v^{LW} = -\frac{dF_{z,v}}{dz}$, where, $F_{z,v} = F_v^{\uparrow}(z) - F_v^{\downarrow}(z)$, correct? So, this is the volumetric heating rate and we do not have any other volumetric heating rate because the shortwave heating is 0 in this specific case, ok. But, since the atmosphere is at radiative equilibrium, that is at steady state, so, there cannot be any net heating or cooling of its layers.

So, if you average over say an entire year and for all times of day and night and assume that atmosphere is at a radiative equilibrium that is the mean temperature, say mean annual temperature or the mean temperature over multiple years or something like that is constant at any layer of the atmosphere, then it must be that there is no heat entering that atmospheric layer or leaving the atmospheric layer. Such that its temperature is changing, because as soon as there is a unbalanced heat flux into or out of that atmospheric layer its mean temperature is going to change. So, we are looking at the mean values and assuming that because of radiative equilibrium the radiation incoming and the radiation outgoing must be matching. So, this means this expression here must be equal to 0 by radiative equilibrium assumption ok. So now, within the anthropogenic global warming perspective, this term is not equal to zero.

Because of the increasing in the absorber gas concentration, there is a net heating of atmospheric layers, which is why this term is non-zero. As a result, the temperature is rising over the decades. But here we are first taking the steady state case, assuming that all of these things are constant and so on average the temperature is not changing. If this is accepted for now, then this means that $\frac{dF_{z,v}}{dz} = 0$ implies $F_{z,v}$ is constant. Only if the net radiative flux at any given z, v is constant, it is not changing with z, only then is this gradient 0.

What this means then is $F_{z,v} = F_v^{\uparrow}(z) - F_v^{\downarrow}(z) = constant$ or over the IR frequency band $F_{z,v} = F_v^{\uparrow}(z) - F_v^{\downarrow}(z) = constant$, independent of z. This is the spectral flux density, this is the total upward flux density equals to the net upward flux density equals to total upward flux density minus total downward flux density and this is independent of z in the IR spectrum. At the top of the atmosphere, is z tending to infinity, Ψz = 0, optical depth is of course 0 at the top of the atmosphere. Hence, Ψ_z^* is also equal to 0 and $F_v^{\downarrow}(z \rightarrow \infty) = 0$ because, there is no long wave radiation coming from space. So, $F^{\downarrow}(\Psi^* = 0) = F^{\downarrow}(0)$.

So, here we are saying that the variable with which we are evaluating the upward and the downward radiation flux density is the modified optical depth Ψ^* which is of course a function of z rather than z itself. So, $\Psi^* = 0$ is z tending to infinity which is f downwards at the top of the atmosphere is equals to 0. This is at the top of atmosphere. However, this term will always be a constant. So, $F_z(\Psi^* = 0) = F^{\uparrow}(\Psi^* = 0) = F^{\uparrow}(0) = constant$.

So, at the top of the atmosphere basically since the downward long wave radiation flux is 0, the net upward radiation flux is basically the total upward long wave radiation flux at the top of the atmosphere which is F upward 0. So, just to note Z is in this direction from 0 to infinity. This is the direction of tau star which is at 0 and whatever is tau ground, tau star ground. So, the variable z is going this way, the variable tau star is going this way. Now, The earth is at radiative balance.

What this means is the total outgoing long wave radiation flux must be equal to the total incoming short wave radiation flux. Since the earth as a whole is in radiative equilibrium, hence, $\uparrow F_z^{LW} = \downarrow F_z^{SW} = \frac{S_0}{4}(1-\propto)$, correct? Yes, which is equals to around 240 watt per meter square. So, this F_z which is the net upward moving long wave radiation must be equals to the net downward moving short wave radiation which at the top of the atmosphere is basically $\frac{S_0}{4}(1-\propto)$ where alpha is the albedo value, S_0 is the solar radiation constant, radiation flux constant which gives you 240 watt per meter square. So, $F_z(\Psi^* = 0) = F_0 \cong \frac{S_0}{4}(1-\propto) \cong 240\ w/m^2$, which is the value of the constant.

So, at any given z or tau star value,

$$F_z(\Psi_z^*) = F^{\uparrow}(\Psi_z^*) - F_v^{\downarrow}(\Psi_z^*) = \frac{S_0}{4}(1-\propto) \cong 240\ w/m^2$$

So, this must be the net upward moving radiation flux density or irradiance at any location z or at any modified optical depth tau star z, these two expressions here may be changing, but the difference must be a constant. So, this simplification we can use.

We will stop here right now. We will continue the derivation in the next class. Thank you for listening and see you in the next class.