Course Name: An Introduction to Climate Dynamics, Variability and Monitoring

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ADVANCED ATMOSPHERIC HEATING AND HEAT BALANCE

Good morning class and welcome to our continuing lectures on climate dynamics, climate monitoring and climate variables. In the previous class, we discussed the general principle based on which we want to proceed. It is how each layer of the atmosphere is absorbing and radiating various fractions of short wave and long wave radiation. how that is heating or cooling that atmospheric layer and we want to finally do an energy balance that tells us based on that heating and cooling rate what is the equilibrium temperature of each layer of the atmosphere which in turn will decide how the temperature varies with altitude. We started our discussion in the previous class on the short wave heating and we saw that the spectral heating rate per unit volume which is the rate at which an atmospheric layer is heating up due to short wave radiation absorption at a given frequency is given by the density of that atmospheric layer into the atmospheric heating rate per unit mass which is watt per kg and this is equals to negative of the short wave radiation flux, negative of the gradient of the short wave radiation flux at that frequency with respect to altitude. So, dF Z nu dz and this gradient we derived in the previous class is equal to the radiative flux dense, the spectral radiative flux density at the top of the atmosphere for solar radiation in that frequency into an exponentially decaying term in terms of the optical depth of the atmosphere at altitude z into the partial density of the absorbing species.

It can be ozone, it can be water vapor at that given altitude z into the mass based absorption coefficient K nu absorption I. Note here that we are using some simplified approximations that the sun is directly overhead and the all the rays are coming normal with respect to the ground. So there is no cos theta term for example in the exponential term here. Also, we are assuming a single mass absorbing species.

Here we will look at a simple case where **the sun is directly overhead**. We will also neglect scattering. The optical depth for a SW radiation of frequency v at an altitude z is given by,

$$\tau_v(z) = \int_z^{\infty} \rho_i k^i_{v_{abs}} dz'$$

The downward irradiance of solar radiation at frequency v is

$$F^{\downarrow}_{z,v}(z) = F^{\downarrow\infty}_{z,v} e^{-\tau_v(z)} \quad (113)$$

The atmosphere does not radiate energy at the shortwave wavelengths. Hence $F^{\uparrow}_{z,v}(z) = 0$. Hence the net upward spectral irradiance is

$$\boldsymbol{F_{z,v}(z) = -F^{\downarrow\infty}_{z,v} e^{-\tau_v(z)}} \quad (114)$$

The **spectral heating rate per unit volume** at height z due to shortwave radiation is given by,

$$\boldsymbol{\rho(z)\dot{q}^{SW}_{rad,v}(z) = -\frac{dF_{z,v}}{dz} = F^{\downarrow\infty}_{z,v} e^{-\tau_v(z)} \rho_i(z) k^i_{v_{abs}}} \quad \mathbf{(115)}$$

Usually, mass absorption coeff. is independent of z and for many species the partial density can expressed as

Of course, if you have multiple species, then this density will have a summation over the multiple partial density values with their individual contributions. Now, usually the mass absorption coefficient does not depend on the altitude z. It is a function of the molecular nature of that species only. So, this term can come out of the z dependent term. And for many species the partial density can be modeled using a very simple exponential relationship that the partial density of the absorbing species at point z is the partial density at sea level into an exponential function of minus z by h i where h i is related to the scale height.

related in some way to the scale height, it may not exactly be the scale height. So, this is kind of considered the scale height for that absorbing species. So, that is how it is said. So, if this simple analytical model holds good, then we can replace rho i z in terms of rho i naught e to the power minus z by h i here. Also, the tau nu term if you see also has the rho i z term.

$$\rho_i(z) = \rho_i(0)e^{-\frac{z}{H_i}} \quad (116)$$

Where H_i is a constant related to the scale height. Then

$$\tau_v(z) = \int_z^{\infty} k_{v_{abs}}^i \rho_i(0)e^{-\frac{z'}{H_i}}\,dz'$$

$$= H_i k_{v_{abs}}^i \rho_i(0)e^{-\frac{z}{H_i}}$$

$$= \tau_v(0)e^{-\frac{z}{H_i}} \quad (117)$$

Where

$$\tau_v(0) = H_i k_{v_{abs}}^i \rho_i(0) \quad (118)$$

Hence (113) becomes

$$F_{z,v}(z) = -F_{z,v}^{\downarrow\infty} exp\left[-\tau_v(0)e^{-\frac{z}{H_i}}\right] \quad (119)$$

And the **volumetric heating rate for SW radiation of frequency v at altitude z w**hen sun is directly overhead is

$$\boldsymbol{\rho(z)\dot{q}_{rad,v}^{SW}(z) = F_{z,v}^{\downarrow\infty} k_{v_{abs}}^i \rho_i(0) exp\left[-\frac{z}{H_i} - \tau_v(0)e^{-\frac{z}{H_i}}\right] \frac{W}{m^3} \ (120)}$$

So, here also you can put rho i naught into exponential minus z by h i into k into dz prime. So, you can put these values here. Remember the mass absorption coefficient is not a function of z. So, that can go out. Rho i naught is the partial density at sea level which is also not a function of z.

So, that can go out. Then you can just integrate this from z to infinity. And you will get the scale height, quarter absorbing species, mass absorption coefficient of the species, partial density of the species at sea level, all of these can be easily evaluated using either known data or instrumental experimentation and the exponential term minus z by h i which is the only z dependent term. So, the first three terms we can bring together and it is called Tv0. Tv0 is the constant in front. scale height at z is scale height at 0. Why is it scale height at 0? Because if z equals to 0, this becomes e to the power 0, so it is 1. So, the scale height at the ground level is this value h i k into rho i. So, that is, this is the, sorry, this is the optical depth at the ground level. So, optical depth at a given altitude z is the optical depth at the ground level e to the power minus of this term can be replaced by this term here.

So, then we can get fz nu at z. So, this expression here. equation 1114, fz nu at z is the fz nu at infinity that is the downward shortwave solar radiation at infinity in that frequency nu into e to the power minus of this optical depth and we can write that as exponential minus Tv naught e to the power minus zi. So, there is a double exponential term. So, exponential and within that is another exponential.

So, just be a little bit careful about this explanation. Now that we have, we know this expression, we can evaluate d dz of this expression using this expression here. rho z, the volumetric heating rate of shortwave radiation, this expression is the radiation flux density at the top of the atmosphere for shortwave radiation. The mass based absorption coefficient rho i naught into exponential, this term here, how do we come into this expression? So, rho i z is rho i naught e to the power minus z by h i. So, we put this here. e to the power minus tau nu z is e to the power minus tau nu 0 into e to the power minus z i. So, this expression comes here. So, this two together is put into the single exponential term. So, e to the power minus z by h i into e to the power minus tau nu at sea level 2 and e to the power minus z by h i. So, this expression is combined to it.

So, this is the volumetric heating rate of shortwave radiation at any altitude z when the sun is directly overhead. So, certain approximations have been made, but this we get an analytical expression in terms of the various functions that we already know. The only unknown we have to evaluate is the scale height for that absorbing species. So, that is the only function that we have to evaluate. Other things we can directly obtain for from data or from experiments.

So, now we can plot this expression. So, we can plot the volumetric heating rate rho z into the mass based heating rate with respect to the z by h i as the equal, as the axis. So, z by h i and volumetric heating rate is on this side. And the optical depth has been given. So, optical depth at the top of the atmosphere is 0 and the optical depth at the ground level here is assumed to be 3.

(6.6.2) Long Wave Heating and Cooling

In this case the expression is more complicated. It can be shown that by solving the radiative transfer equations (101) and integrating over the solid angles, the upward thermal irradiance at a Long-Wave frequency v is given by,

$$F_{z,v}^{\uparrow}(z) = \pi \int_0^z B_v(T_{z'}) \frac{\partial \Gamma_v^*(z',z)}{\partial z'} dz' + \pi B_v(T_s) \Gamma_v^*(0,z) \quad (122)$$

Where $B_v(T_z)$ is the spectral blackbody radiation intensity at frequency v and at temperature T_z corresponding to the atmospheric layer. $B_v(T_s)$ is the spectral blackbody radiation intensity at the ground with temperature T_s . $\Gamma_v^*(z',z)$ is the spectral transmittance averaged over the upward hemisphere to take into account of all the slanting paths between z' and z.

For downward spectral irradiance we have,

$$F_{z,v}^{\downarrow}(z) = -\pi \int_z^{\infty} B_v(T_{z'}) \frac{\partial \Gamma_v^*(z',z)}{\partial z'} dz' \quad (123)$$

The **net upward longwave radiation** at frequency v is given by,

$$F_{z,v}(z) = F_v^{\uparrow}(z) - F_v^{\downarrow}(z) \quad (124)$$

And the volumetric heating rate is given by

$$\rho(z) \dot{q}_{rad,v}^{LW}(z) = -\frac{dF_{z,v}}{dz} \quad (125)$$

The solution to this is complicated and not discussed here. However, the value will be negative, *hence the atmospheric layers on average lose heat in the longwave spectrum*.

The plots for global mean shortwave heating rate (over all frequencies) divided by the specific heat, $\frac{\dot{q}_{SW}}{C_P}$ in the units of Kelvins/day are plotted below. Both the total heating rate and the contributions of the individual gases are shown. Similarly, the average global Longwave cooling rate expressed as $-\frac{\dot{q}_{LW}}{C_P}$ are also plotted.

If I, if I, so tau nu 0 value has been taken to be 3. So, this is a typical optical depth for a certain types of absorbing species and z by h i has been shown here. And we are plotting the volumetric heating rate as well as the negative of the net upward vertical irradiance. So, we are plotting.

So, this is fz nu. Negative of this is fz infinity e to the power minus tau nu z. So, the negative of the net upward irradiance. So, basically the net downward irradiance we are plotting when we are doing the negative of the upward irradiance. And the absorber partial density rho i has also been plotted. So, here for these expressions we can analytically plot the solid line is the heating rate, volumetric heating rate.

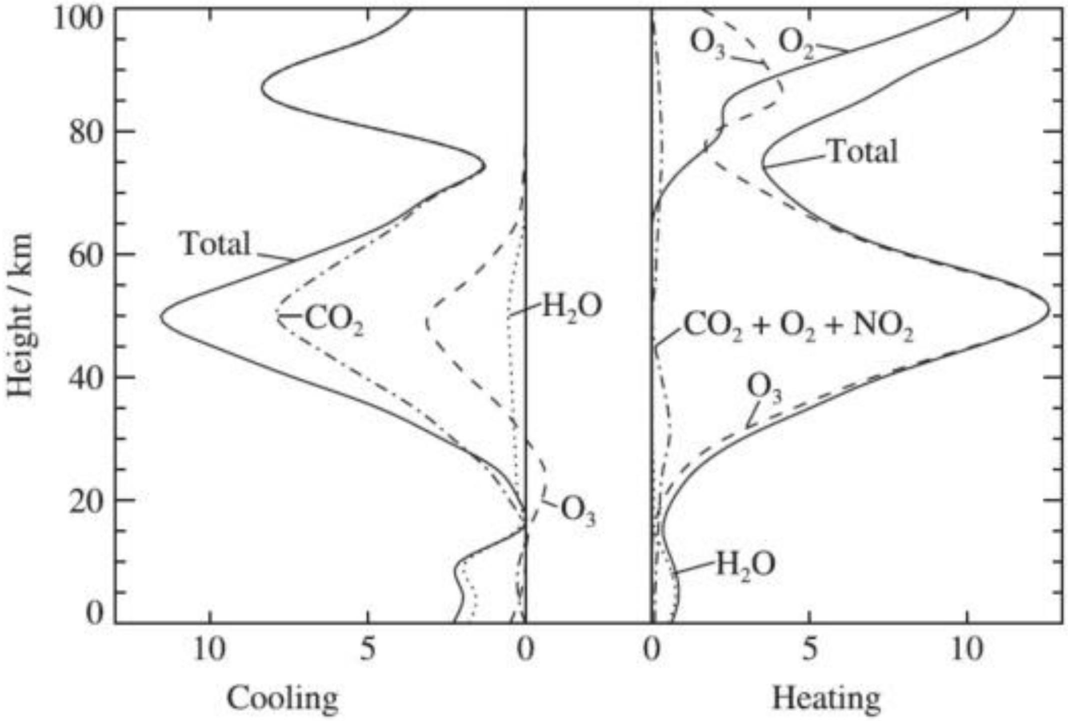
The dashed line is the negative of the upward irradiance. This is the upward irradiance. Basically, the net downward irradiance here. So, here it is the maximum value. It goes down to a small value at near the ground.

And the dotted line is the exponentially decaying partial density. Starts from a certain value here, rho i 0 and it decays exponentially. Alright. So, what you see here and this is the optical depth has been plotted on in this axis.

Okay. What is the optical depth of the atmosphere? Remember the optical depth is given by this expression here. So, you can and tau nu 0 here is 3 and it exponentially kind of decays. So, this is a logarithmic graph. So, based on this expression you see that the maximum heating rate due to shortwave radiation is happening at z by h i equals to 1. So, the maximum volumetric heating rate is given as where the altitude is equal to the optical the scale height for that specific absorbing species.

We can also show this we can do a DDZ of this expression here, DDZ of this expression here and set it to 0 and you will see when you do this it will you can derive that the optical depth is equal to 1. So, analytically the heating rate will always be maximum where the optical depth for the atmosphere is equals to 1, optical depth is equals to 1. Here incidentally it is also close to where z by h i is equals to 1, but that need not be the case. So, you can see optical depth equals to 1, then This is 1, whatever is the optical depth here, so you can get the expression for z by h i at which the optical depth equal to 1 is valid. Here because it is 3 and here it is 1, it is one-third equals to e to the power minus z by h i.

So, e to the power minus 1 is almost one-third. So, that is why coincidentally the z by h i equals to 1 and optical depth equals to 1, both are corresponding closely to the maximum heating heat, but this need not be the case. Fundamentally, analytically the maximum heating rate is equals to where the optical depth of the atmosphere is 1. So, this kind of gives us a very nice way to track the shortwave heating rate and gives us the expression of where the shortwave heating rate is going to be maximum. Next, we look at the longwave heating and cooling.



We can see that **most of the SW heating in the stratosphere is dominated by O_3**. In the troposphere, **SW heating is caused by absorption of solar radiation (near infrared range) by water vapor**. For **Longwave heating**, CO_2 and O_3 both contribute significantly in the stratosphere, whereas **water vapour again dominates the LW cooling in the troposphere**. Note that while the heating rate in terms of K/day are larger in the stratosphere, most of the mass of the atmosphere is in the troposphere. Hence in terms of total heat added (after multiplying by mass), the tropospheric values would completely dominate over stratospheric values. $\dot{Q} = mC_p \frac{dT}{dt}$

So, here the radiative balance equation at any layer of atmosphere will include not only the absorption of the radiative flux coming to that layer, but also radiative cooling due to emission from that layer. and we have to integrate that over the all the solid angles. So, the derivation is more complicated. So, we are not going into this, but we can find the net upward irradiance that the sorry it is not the net the The total hemispherical spectral radiative flux density f z nu upward at any altitude z. So, this is the atmospheric layer whatever it is emitting upwards at z.

The upward thermal irradiance at the long wave frequency, so this is the thermal radiative flux intensity. at a frequency mu is given by this expression here, where this expression has two components. The first component, the second component is basically giving us the blackbody emission value from the ground and the optical and what is called the spectral transmittance averaged over the upward hemisphere. So, before we go into this expression, let me just quickly go over something that I missed in the previous class, so that you know what the expressions mean. So, here is an expression of transmittance and absorptance.

So, we have discussed optical depth and optical path length, transmittance and absorptance are very closely related to these two concepts as well. So, the spectral transmittance tau nu is defined as the fraction of the spectral radiation intensity leaving from a certain point S1 that arrives at another location S2 along the direction of the ray S. So this is basically that ratio, the radiation intensity at S2 divided by the initial radiation intensity at S1 along the direction of S. The fraction of the radiation intensity that is reaching S2 with respect to how much is being incident at S1 is defined as the spectral transmittance. And this we have already explained before.

So, if this material is a non-absorbed, non-emitting, so if this material is not emitting anything in this wavelength, then this becomes transmittance S1 to S2 exponential of minus integral of the optical path length along s, the integral of the optical path length s1 to s2, the mass absorption coefficient rho i into ds. And this expression just becomes exponential of minus of the optical path length at S2 minus optical path length at S1, where optical path length X nu is the mass absorption coefficient rho i into dS from S0 to S, where S0 is certain initial point from which the radiation is emerging. So, the transmittance function between S1 and S2 is exponential of minus the optical path at S2 minus optical path at S1 and this is the modulus of that. So, this is the modulus function.

So, this has to be always positive. For the special case where the partial density of the absorber species, this rho i and the mass absorption coefficient is constant. So, suppose rho i and k are not dependent on S, then we can take this out of the system and we just get transmittance is exponential minus the absorption coefficient into rho i into the distance between S1 and S2, which is just delta S, where delta S is the distance along the path of the beam from S1 to S2. So this is the transmittance of radiation from S1 to S2. The absorptance is 1 minus transmittance.

So those are the two definitions. What fraction is being transmitted and what fraction is being absorbed. In specific cases, if we want to find the transmittance over a range of frequencies, then we can call, we define band transmittance and band absorptance. Band transmittance is one by the entire frequency range, okay. So, suppose we want a frequency range from say 300 nanometer to 600 nanometer. The corresponding frequencies say, So, this becomes 30000 minus 20000 hertz the frequency band, then 20000 to 30000 the transmittance over that frequency into dd.

So, this is kind of the average transmittance over a frequency band. Similarly, the alternate is average absorptance over the frequency band. So, this kind of defines what transmittance and absorptance is. So now let us go back.

So we have two expressions here. The first expression, the second expression has this transmittance term z prime to z. T now star z gamma now nu star z prime to z is the spectral transmittance averaged over the upward hemisphere to take into account all the slanting plots between z prime and z. This term here, tau nu star z prime and z is the spectral transmittance averaged over the entire upward hemisphere. So, this is not just any direction, it is averaged over the entire upward hemisphere and taking into account all the slanting paths between any altitude z prime to another altitude z because there are many slanting paths also rather than the normal paths. The gradient of this transmittance with respect to z prime is one term, then dz prime is another time and then the blackbody spectral radiation intensity at the temperature corresponding to z prime.

This is the first expression, it is going from 0 to z and the pi term is basically, because it is over the entire hemisphere basically. And it is diffuse radiation so if you integrate you will get the pi expression to move from radiation flux radiation intensity to radiation flux density the pi term comes into the picture. And the second term is the blackbody radiation intensity at the ground. So, B nu Ts into pi, which is the blackbody radiation flux density, the upward blackbody radiation flux density at the ground and the transmittance between 0 and z. So, this is basically how much of the surface blackbody radiation is reaching the altitude z.

This is given by the transmittance function from 0 to z. into the total blackbody radiation flux density which is pi B nu T s. And this is, this expression is giving us all the contributions from the intervening layers of the atmosphere and their transmittance gradients. So, this is the contribution of all the atmospheric layers and this is the contribution of the ground, basically that is it. The downward spectral irradiance The downward flux density for us specific frequency of long wave radiation.

Here only this term is there. This term is of course not there because infrared radiation is not coming from the sun. So this is minus pi of the same term here. Here it is z to infinity.

So it is plus pi infinity to z. So it is downward. So that is why it is the negative term. Z is going up whereas the downward is going down. So, the directions are opposite that is why the negative term is coming. And this is the contribution of all the downward going radiative intensities multiplied by the transmittance function.

So, these are the two expressions. Then the net upward long wave radiation is the difference between these, the net upward radiation at the net upward radiation flux minus the net down going radiation flux at any station z in the altogether that is the net upward long wave radiation. And then as before the volumetric heating rate for the long wave radiation is q dot radiation nu long wave into rho z is the gradient of this term minus df z nu dz negative of the gradient of this term fine. So, we again have to differentiate this term here and get a, so again analytical solutions are possible, but here the derivation is very difficult because we have a lot of terms here to take into account. what we will plot them and what we will see here is that the volumetric heating rate is actually negative.

So, that df Z nu dz term is positive. So, this term is actually a negative term. So, the atmospheric layers are actually losing heat. So, it is the cooling rate is positive. So the atmospheric layers on average lose heat in the long wave spectrum. So the atmospheric layers gain heat in the short wave spectrum and lose heat in the long wave spectrum.

And we can find the exact expressions for the heating rates through short wave radiation and the cooling rates in the long wave radiation. And from the radiation balance equation, it means these two must be balancing each other out. Correct. So, below we will plot the heating rates and the cooling rates for the total as well as the contribution of the individual absorbing species. So, we can get the heating rate and the cooling rate for individual absorbing species, add them up to get the total heating and cooling rates.

So, what is being plotted is the shortwave heating rate. divided by the specific heat in terms of Kelvins per day. So, this is one term here. This is for a given frequency nu. You can integrate this over all frequencies to get the total heating rate per unit mass. Now, if you divide that by Cp, then you get the heating rate per unit mass And you divide it by the specific heat, you get it in terms of temperatures.

So, the unit becomes Kelvins per day. So, here we use the appropriate units. Remember, heating rate, total heating rate is equals to the mass into Cp into change in temperature. So, heating rate by mass and Cp gives you the change in temperature. So, that is what it is giving, Kelvins.

And the heating rate, so there is a time unit also. So, it is Kelvins per day. So, that is the plot. So, we are plotting the heating rate and cooling rates in terms of Kelvins per day and the total heating rate contributions are shown. Similarly, global cooling, long wave cooling rate is also expressed.

Now, here is something that you need to understand. So, you typically 0 to 15 is the troposphere, then 15 to 60 is the stratosphere and over that is the mesosphere and everything. If you look at the Kelvin's per day values, it seems that the tropos, stratospheric values are much larger. This does not mean that the heating rates in the stratosphere are much larger. Let me explain why. This is heating rate per unit mass, Q dot SW is watt per kg divided by the CP term, okay, which is joules per kg Kelvin, which gives you Kelvin per take.

So, it is per unit mass. The total heating rate is mass into CP, correct? And the mass in the troposphere is much higher than the mass elsewhere. So, clearly even if the heating rate per unit mass per unit Cp is much greater in the stratosphere because 80 to 90 percent of the total mass is in the troposphere, the total heating rate in terms of watts is much larger in the troposphere than in the stratosphere. It is only because we are dividing by mass, it appears that the heating rates are higher in the stratosphere than in the troposphere. This point you need to remember.

Given that, let us look at the individual cases. Firstly, this is the short wave case. The total is the solid line. The dotted is water vapor contribution, the dashed is the ozone contribution and the dashed dot is basically CO2 and NO2 contribution. Oxygen does not contribute significantly, but it does in some cases. So in the troposphere, most of the shortwave heating effect is coming from the water vapor contribution.

So water vapor is the primary absorbing species of shortwave radiation in the troposphere. In the stratosphere ozone as you would expect is the primary absorbing species of shortwave radiation. Remember water vapor is absorbing the near infrared region of the solar spectra, ozone is absorbing the ultraviolet region of the solar spectra and these are the dominant absorbers in the troposphere and the stratosphere respectively. CO2 has some contributions. because it also absorbs some of the IR radiation coming from the sun, but it is lesser than these two dominant modes.

Now, we are looking at the cooling section. Here again, this is the total solid line, the dashed dot is the CO2, the dashed is the ozone and this is the water vapor. Here we see that in the troposphere, again water vapor is the largest contributor. CO2 has some contribution after that. So, water vapor, then CO2. In the stratosphere, CO2 is the primary contributor to cooling, followed by ozone and then water vapor.

So, CO2 dominates the long wave pooling in the stratosphere while water vapor again dominates the long wave pooling in the troposphere followed by CO2 as a second important constant. So, what we see is overall water vapor is the primary gas which is both absorbing short wave radiation and emitting long wave radiation in the troposphere. with CO2 as a secondary emitter of long wave radiation in the troposphere. And in stratosphere ozone is the primary short wave absorber and CO2 is the primary long wave emitter with ozone being also a secondary long wave emitter.

So, that is the contribution of the different gases at different places. We will continue this discussion in the next class. Thank you for listening and have a good day.