**Course Name: An Introduction to Climate Dynamics, Variability and Monitoring**

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# RADIATIVE FLUX AND OPTICAL DEPTH

Good morning class and welcome to our continuing discussions on climate dynamics, climate monitoring and climate variability. In the previous class we had looked at the absorption of either short wave radiation or long wave radiation as they pass through the atmosphere and derived expressions for radiation intensity first and then radiation flux density second for both the cases. We saw that the radiation flux density values for a given frequency f nu upwards, upward moving radiation, hemispheric radiation flux density is related to the blackbody radiation intensity using this ordinary differential equation 101 and the downward moving radiation flux density is related to the blackbody radiation intensity at a given temperature using expression 102. Remember here that T itself is a function of z. So, this expression also is a z dependent function and is not a constant. Here tau star is related to the optical depth tau nu, where the 1.66 ratio is obtained upon integrating radiations coming from all directions with different optical pathways. Now, given these expressions, we can now solve for these expressions and plot them for our case. So, we can now plot the upward and downward fluxes of infrared radiation with altitude. This figure is what is plotted here.

The f upward z is shown by this dotted line and f downward z downward moving in long wave flux density is given by this line. And the temperature as a function of z is given by this line. So this is like the top of the troposphere. So you have the atmospheric lapse rate and then you have the stratospheric condition where it is more or less constant. Here note that the flux densities are being normalized.

So the upward flux is normalized against the net flux escaping the atmosphere from the top. So, what you are giving here is f upward z by f upward at infinity. So, what is the outgoing flux from the top of the earth's atmosphere? That is what is in the denominator and so this is the normalized variant of the upward radiative flux density or upward irradiance. Similarly, the downward moving flux has been normalized by the net flux hitting the net infrared flux hitting the earth surface F downwards at zero. So, the downward irradiance, radiative irradiance act for the long wave region is given by this expression by F up downward zero and this normalized value is plotted using this line.

That is why these values are varying from 0 to 1. So, clearly as we move to the top of the atmosphere, the upward flux density approaches the flux density at the top of the atmosphere, so it goes to 1. Similarly, as we reach the ground, the downward flux density approaches the ground flux density, so it goes to 1. Now, based on this plot, what do we see? We see that by 10 to 11 kilometers above the top of the atmosphere, 90% of the total flux that is going to move, escape from the top of the atmosphere is already going upwards.

Okay. So the net upward flux reaches to 95% of its total value at the top of the troposphere. So at 11 kilometers, this reaches up to 95% of the value that it will have at the top of the troposphere. Similarly, if you go near the ground, this value is just 10%. So what this means is, only 10 percent of the total upward flux that would be escaping the atmosphere is accounted for at the ground level. So, the ground level is only contributing 10 percent of the total infrared flux going up to the top and escaping at the top of the atmosphere.

Similarly, if you look at the downward flux, of course, the downward flux is 1 at the ground level. However, it goes down very quickly. So, by around 2 kilometers above the atmosphere, it has gone down to 50 percent. So, this downward infrared flux is basically atmospheric layers radiating infrared radiation towards the ground. So, 50 percent of all the infrared radiation hitting the ground is coming from the first 2 kilometers of the atmosphere, okay.

Similarly, if you see here at around 5 kilometers from the ground, 50 percent of the total outgoing radiation is accounted for, okay. So, main conclusions, the troposphere emits most of the outgoing long wave radiation because by the end of the troposphere 11 kilometers, 95 percent of the outgoing long wave radiation has already been accounted for. So, when it comes to the outgoing radiation to the space, troposphere plays the dominant role. 50% of the outgoing long wave radiation is emitted by 5 km altitude. So, by 5 km above the ground, 50% of the outgoing long wave radiation has been emitted.

And at 50%, the temperature is around 255-250 K, which is close to the emission temperature of the earth. So, this kind of makes intuitive sense. The mean emission temperature is where the average outgoing long wave radiation is being emitted. And that average height will be approximately where 50% of the outgoing long wave radiation has been emitted. So, what we see here is that the temperature that we are, the emission temperature of the earth is basically the temperature of the atmosphere around 5 kilometers from the surface of the earth, which is 255 Kelvin.

The surface temperature of the earth is much higher which is around 285 Kelvin as we see here. The difference between the mean emission temperature of the OLR and the actual surface temperature is one of the major aspects of the greenhouse effect and is caused by the heat absorbing gases. So why is this difference? We have seen this difference is because the atmosphere is absorbing the surface radiation and re-radiating it back to space. And this effect becomes more clear when we see this functional dependence of the outgoing and the upward moving and the downward moving long wave radiations. Also, 80% of the downward infrared radiation originates below 5 kilometers. So, at 5 kilometers, if you see here, this around 0.2. So, 80% of the downward radiation has been emitted by 5 kilometers. And 50% is emitted by around 2 kilometers from the surface, where the temperature is around 275 Kelvin.

So, this is 275. So, by this time 50 percent of the, so at 2 kilometers from the surface it is around 275 Kelvin. So, the heat hitting the surface from the infrared in the infrared zone is coming from about 2 kilometers from at the top of the atmosphere from the bottom from the ground. And this downward emission originates in the lower troposphere because most of it is emitted by water vapor and most of the water vapor is concentrated in the lower troposphere. So, most of the downward emission is being emitted by water vapor. So, this water vapor is a strong emitter and absorber of infrared radiation and the first 2 kilometers of the atmosphere most of the water vapor is present.

Hence, it is water vapor which is causing most of this emission to occur, which is why the downward emission kind of decreases very rapidly with altitude as most of the water vapor is concentrated near the surface only. All right. Before we begin the next section, let us look at a small example to see how we can use these kinds of expressions. So many of these expressions are quite complicated. So we will not use a lot of complicated integrations or anything like that.

But let us do one example where we will use the expressions for the absorption coefficients for ozone to estimate what is the fraction of the UV radiation that is being absorbed by a layer of ozone in the stratosphere. And we will use the most complicated expression here primarily to show as an example, but much simpler examples also we will show later in exercises and tutorials. Suppose this is the ground and we have a 10-kilometer layer in the stratosphere which is 20 kilometers above the ground. This is 20 kilometers, this is 10 kilometers. Now, what is being shown here? So, suppose this is the normal to the ground and UV radiation is passing through this layer at an angle of 30 degrees with respect to the ground downward.

So, this is I nu at 30, so it is hitting this layer 30 kilometers above the ground with a certain value I nu at 30 is the radiation intensity. What we have to find is the value of I nu at 20. So, the ratio I nu at 20 by I nu at 30 is what we want to find. Clear? Now, we know that the frequency here is that of ozone. So, the frequency, so the wavelength of radiation lambda is 255 nanometers which is in the UV range and this layer contains ozone with mole fraction x O3 as 10 parts per million. So, the ozone mole fraction in this 10 kilometer layer is 10 parts per million. Now, we can we can assume that scale height h is 7.6 kilometers. Density of air at z equal to 0 that is at sea level is 1. 2 kg per meter cube. Molecular weight of ozone is 48 grams per mole. and molecular weight of air is 28.97 grams per minute. So, this is what we have. The first thing we want to know is the optical depth of this 10-kilometer layer. So, first step is find the optical depth of the 10 kilometer stratospheric layer for the wavelength lambda 255 nanometer radiation. Now, the optical depth is given by minus of the beginning point here, the end point here, here it is negative because z is moving in that direction, but we are starting from the top and going towards the bottom. The mass absorption coefficient, K nu absorption of ozone. partial density of ozone into dz. How do we find all of these values? So, that is the next question.

$$\tau_\nu = -\int_{30}^{20} k_{\nu,abs}^{03} \rho_{03}\, dz$$

1) Find the optical depth of the 10km stratospheric layer for $\lambda = 255$ nm radiation.

$$\tau_\nu = -\int_{30}^{20} k^{O_3}_{\nu,abs}\, \rho_{O_3}\, dz$$

From the absorption cross section graph, so this graph is given in the  I will show you the graph after this problem is over. Sigma O3 for lambda equals to 255 nanometer is around 1.1 into 10 to the power minus 17 centimeter square per molecule. So, this value we are getting from directly from the ozone absorption cross section graph which has been plotted with respect to wavelength. Now that we know the absorption cross section, the mass absorption coefficient K absorption for ozone is equals to sigma O3 into  the Avogadro number n by the molecular mass of ozone. So, 1.1 into 10 to the power minus 17 into 6.022 into 10 to the power 23 divided by 48, which is equal to, if you solve all of this, this is grams per mole centimeter square per molecule. So, this becomes 1.38  into 10 to the power 5 centimeter square per gram. We will change this into kilometer square per kg, 1.38 into 10 to the power 5. So, centimeter square going to kilometer square is 10 to the power  Sorry, centimeter square per gram, we will go, move it into meter square per kg, not kilometer square. Sorry about that. Sorry, kilometer square per kg, okay. So, centimeter is 10 to the power minus 2 meters and 1 meter is 10 to the power minus 3. So, this becomes 10 to the power minus 5, okay, whole square by kg, so this is 10 to the power minus 3, this becomes kilometer square per kg. Because 1 meter is 10 to the power minus 3 kilometers and 1 centimeter is 10 to the power minus 2 meters, so 1 centimeter is 10 to the power minus 5 kilometers. That square becomes kilometer square and kg gram to kg 10 to the power minus 3. So, then  This becomes, so if you do the solution, 10 to the power 5 and 10 to the power minus 5 cancels out. So, this becomes 10 to the power minus 5, this becomes 10 to the power minus 3. So, this becomes 1.38 into 10 to the power minus 2 kilometer square per kg.

$k^{03}_{\nu,abs} = \sigma_{O3}\times\frac{N}{M_{03}} = (1.1\times10^{-17}\text{cm}^2/\text{molecule})\times(6.022\times10^{23}\text{molecules/mol})/(48\text{g/mol})$

$k^{03}_{\nu,abs} = 1.38\times10^{5}\text{cm}^2/\text{g} = 1.38\times10^{5}\times(10^{-5})^2\text{km}^2)/(10^{-3}\text{kg})$

$=1.38\times10^{-2}\ \text{km}^2/\text{kg}$

From the absorption cross-section graph

$$\sigma_{O_3}(\lambda = 255\text{nm}) \cong 1.1\times10^{-17}\ \text{cm}^2/\text{molecule}.$$

$$k^{O_3}_{abs,\nu} = \sigma_{O_3}\times\frac{N}{M_{O_3}} = 1.1\times10^{-17}\times\frac{6.022\times10^{23}}{48}$$

$$= 1.38\times10^{5}\ \frac{\text{cm}^2}{\text{gm}} = 1.38\times10^{5}\times\frac{(10^{-5})^2}{10^{-3}}\ \frac{\text{km}^2}{\text{kg}}$$

$$= 1.38\times10^{-2}\ \frac{\text{km}^2}{\text{kg}}$$

With this value known, now we will do the partial density of ozone. The partial density of ozone is the mole fraction of ozone into molecular mass of ozone by molecular mass of air into density at the given altitude z, ok. The mass, the mole fraction of ozone is 10 ppm, 10 parts per million. So, this is 10 into 10 to the power minus 6, which is 1 in 1 million. Mass of ozone is 48, mass of air is 28.97 and density we will discuss later, we will just put it at rho Z.

$\rho_{O3} = x_{O3} \times M_{O3}/M_{air} \times \rho_z$

$= 10\times10^{-6}\times48/28.97\times\rho_z$

$$\rho_{O_3} = x_{O_3} \times \frac{M_{O_3}}{M_{air}} \times \rho_z$$
$$= 10 \times 10^{-6} \times \frac{48}{28.97} \times \rho_z$$

So, this expression becomes using hydrostatic balance relation. density at any altitude z is equals to density on the ground exponential minus z by h where h is the scale height. So, the final expression for rho O3 becomes this expression equals to 48 by 28.97 into 10 to the power minus 5 into density here rho s is 1.2 kg per meter cube. So, this becomes 1.2 exponential minus z by h and this expression is in kg per meter cube. this is the density of ozone in the, in that stratospheric layer. So, now tau nu z becomes, so minus 30 to 20 you can change it to become 20 to 30, ok. This expression is 1.38 into 10 to the power minus 2. So, 1.38 into 10 to the power minus 2 into 10 to the power minus 5 into 48 by 28.97 into 1.2 into exponential minus z by 7.6 dz. Note here, the unit here is kg per meter cube and the unit of this is kilometer square per kg. So, we have to get the units correct, this also has to be in kg per kilometer cube, otherwise the values will not match. has to go into kg per kilometer cube. Now, 1 meter is 10 to the power minus 3 kilometers, correct? So, meter cube is 10 to the power minus 3 into cube of that, alright. So, this becomes into 10 to the power 9 because it will be 1 meter is 10 to the power minus 3. So, 10 to the power minus 3 to the power 3 is 10 to the power 9, minus 9 it goes up.

$\rho_z=\rho_s.\ \exp(-z/H)$

$\rho_{O3}=48/28.97\times10^{-5}\times\rho_s\times\exp(-z/h)kg/m^3$

$\tau_v(z)=\int_{30}^{20} 1.38 \times 10\hat{} - 2 \times 10\hat{} - 5 \times 48/28.97 \times 1.2 \times \exp(-z/7.6z)dz \times 10\hat{}9$

using hydrostatic balance relation

$$\rho_z = \rho_s \exp\left(-\frac{z}{H}\right) \qquad \rho_s = 1.2\ kg/m^3$$

$$\rho_{O_3} = \frac{48}{28.97} \times 10^{-5} \times 1.2 \exp\left(-\frac{z}{H}\right) \frac{kg}{m^3}$$

$$\tau_\upsilon(z) = \int_{20}^{30} \underbrace{1.38 \times 10^{-2}}_{km^2/kg} \times 10^{-5} \times \frac{48}{28.97} \times 1.2 \times \exp\left(-\frac{z}{7.6}\right) dz \times 10^9$$

km.

So, this becomes 10 to the power 9. So, then this expression then it becomes in kilometers. This is kilometer square per kg and this expression here is kg per kilometer cube. So, this is kilometer square per kg, this is kg per kilometer cube. So, it becomes 1 by kilometer And this is in kilometers. So, this becomes unit less as we want it to be. So, this unit conversion is very important. So, now you can solve for this expression. So, this is 10 to the power 7 and this is 10 to the power minus 5. This becomes, if I am correct. becomes 2.08 into 10 to the power 3 sorry let me just put this term 1.38 into 48 into 1.2 divided by 28.97. This is 0.0274 and this term minus 7 into 9, so 10 to the power 2 into 100. This becomes 2.74 minus 7, 9, so 100 and this expression is 2.7. So, this becomes 2.74 exponential of 20 to 30, exponential minus z by h dz. Now you can just integrate this expression, h is given as 7.6 kilometers. So, remember e to the power mx is equal to e to the power mx by m. Here m is minus 1 by h. So, minus 2.74 into h, this expression becomes exponential minus 30 by h minus exponential minus 20 by h, where h is 7.6 kilometers. H is 7.6 kilometers. So, you can solve this expression. If you, once you solve this expression, then the radiation intensity ratio I nu 20 by I nu 30 is just exponential minus tau nu z by cos 30 degrees. So, this expression here and this expression here will give this ratio. So, you can evaluate the answer. I will leave this answer to you at homework, just check the calculations and remember that matching units is a very important aspect of this problem.

$\tau_\nu(z)=274\int_{30}^{20} \exp(-Hz)dz$

$=-274\times H[\exp(-30/H)-\exp(-20/H)]$

$I_\nu(20)/I_\nu(30)=\exp(-I_\nu(z)/ I_\nu(30))$

$$\tau_\nu(z) = 274 \int_{20}^{30} \exp\left(-\frac{z}{H}\right) dz .$$

$$= -274 \times H \left[\exp\left(-\frac{30}{H}\right) - \exp\left(-\frac{20}{H}\right)\right]$$

$$H = 7.6 \text{ km}.$$

$$\frac{I_\nu(20)}{I_\nu(30)} = \exp\left(-\frac{\tau_\nu(z)}{\cos 30^\circ}\right) \leftarrow \text{Answer}.$$

$$\tau\nu(z)=-274\int_{30}^{20} \exp\left(-\frac{z}{7.6}\right) dz$$

$$=274\int_{20}^{30} \exp\left(-\frac{z}{7.6}\right) dz$$

$$\tau_\nu(z) = -274 \int_{20}^{30} \exp\left(-\frac{z}{7.6}\right) dz$$

$$= 274 \int_{30}^{20} \exp\left(-\frac{z}{7.6}\right) dz$$

$I_\nu(20)/I_\nu(30)=\exp[-\tau_\nu(z)/I_\nu(30)]=\exp[-126.9]\approx 7.7\times 10^{-56}\approx 0$

$$\frac{I_\nu(20)}{I_\nu(30)} = \exp\left[-\frac{\tau_\nu(z)}{(\nu)30}\right) = \exp[-126.9] \approx 7.7\times 10^{-56} \approx \underline{\underline{0.}}$$

So, see you in the next class and I will continue our lectures. Thank you for listening and have a good day.