**Course Name: An Introduction to Climate Dynamics, Variability and Monitoring**

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# INFRARED RADIATIVE TRANSFER IN THE EARTH'S ATMOSPHERE

Good morning class and welcome to our continuing lectures on climate dynamics, climate monitoring and climate variability. In the previous class, we were discussing how we can measure the amount of radiative flux that is being absorbed by the atmosphere, radiative solar flux that is being absorbed by the atmosphere as solar energy comes from the top of the atmosphere and hits the ground eventually. We saw that the expression for the solar radiation intensity in a given direction theta with respect to the ground normal at a given altitude z above the ground can be expressed as the radiation intensity at this direction theta at an altitude z. So, the value of the intensity at an altitude z coming from a certain direction theta for a given frequency nu. that is I nu z radiation frequency at z for frequency mu for the solar shortwave radiation divided by the radiation intensity at the top of the atmosphere. So, this is infinity for that given frequency nu.

So, the ratio of the radiation intensity at a given frequency at altitude z by the radiation intensity at that given frequency at the top of the atmosphere that is equals to exponential of minus of the optical depth tau nu at that altitude z by cos theta, okay. Here note that optical depth tau nu is given by minus of infinity to z, the mass absorption coefficient k into the partial density of the absorbing species into dz. This is defined as the optical depth tau nu z. Note that the optical depth will be different for radiations of different frequencies because the mass absorption coefficient will be different at different frequencies.

$$\frac{I_v(z)}{I_v^{\infty}} = exp\left(-\frac{\tau_v(z)}{cos\theta}\right) \quad (84)$$

***where optical depth at z is given by,***

$$\tau_v(z) = -\int^{z} k^{i}_{v_{abs}} \rho_i dz \quad (85)$$

This ties us back to the idea that certain gas molecules will be strongly absorbent at certain frequencies while it will not be absorbent at certain other frequencies. So the mass absorption coefficient K changes with the frequency. And it also changes, of course, with the identity of the absorbing species. So, in this case, if we assume a single species is absorbing, then this will be the expression here. If you have multiple species that are absorbing, then there will be a summation over all k i rho i from infinite to z. So, that generalization you should keep in mind. So what you see here is that if the mass absorption coefficient is large, that is a species is strongly absorbent at a given frequency, then the optical depth at the altitude z will also be large because this integral will be large. Hence, the exponential term will be greater and this expression will be smaller. Hence, what you will get then in that case is that I nu z will be significantly lower than the case when the mass absorption coefficient is larger. Note also that the partial density of the absorbing species can be expressed in terms of the mass mixing ratio yi which is also the mass fraction yi of that absorbing species into the density of the atmosphere at z. Where the mass mixing ratio can also be related to the molar mixing ratio in terms of the molecular weight of the absorbing species i by the molecular weight of the atmosphere at that altitude z. We also saw that the molar mixing ratios are plotted with respect to altitude for the different gases and some gases like CO2, O2, have reasonably same values of molar mixing ratios with altitude and hence their variation with altitude can be neglected. Whereas other species like water vapor has very strong variation of molar mixing ratio with altitude in the troposphere at least and hence there we cannot we have to use the explicit Z dependence of the molar mixing ratio. The density of the atmosphere can be expressed in terms of the hydrostatic balance relation where h is the average scale height. Of course if you do not use the like the average temperature approximation then scale height itself will be a function of z in which case as we saw in our early classes you can take the mean atmospheric lapse rate and hence evaluate the density at any given z of the atmosphere. So, this density of the atmosphere at z can be put in this expression here to get the rho i z value.

$$\boldsymbol{\rho_i(z) = y_i(z)\rho(z)} \ \ (\mathbf{86}) \ \textit{where,}$$
$$\boldsymbol{mass\ mixing\ ratio\ y_i = \frac{m_i}{m} = x_i\frac{\mathcal{M}_i}{\mathcal{M}}} \ \ (86)$$

In the next class, we will have a discussion of how to do this in a worked out example. We also saw that Another way to measure the mass absorption coefficient is in terms of the absorption cross section sigma i nu absorption where the absorption cross section is nothing but the mass absorption coefficient into the molecular weight of the absorbing species by the Avogadro number which unit will become centimeter square per molecule. And for many gases like ozone, the absorption cross section with wavelength is plotted from which you can get the absorption cross section and hence the mass absorption coefficient which you can put in the expression to get the degree of attenuation of the radiation intensity. So, this much we have covered in the previous class. I am just repeating this here because the next set of derivations will use these concepts a lot. So, it is good to be refreshed about this. So the next logical step is to understand the transfer of terrestrial radiation, the radiation that is being emitted from the ground and going into the atmosphere. What is happening basically in the far infrared region where the earth and the atmosphere are both emitting infrared radiation. So, as we saw unlike solar radiation which is happening at a between say 0.3 to around 3 micrometers, the terrestrial radiation happens between 5 and 100 micrometers and in this region the atmospheric layers also emit a lot of terrestrial radiation at this range. the atmospheric layers will not only absorb far infrared radiation, it will also emit a significant amount of infrared radiation. So, each of these layers here are not only absorbing the radiation coming from say to from the ground, they will also be emitting radiation in these frequencies both upwards and downwards, okay. So, here we consider the case of infrared radiation at a given frequency nu that is emitted upwards from the surface of the earth as it travels to the top of the atmosphere. The radiation is being emitted at an angle theta again with respect to the ground. So, the figure is very similar to the figure in the earlier case except now the radiation is traveling upwards. So, we are looking at the upward moving radiation flux, okay. And this radiation intensity we are calling I nu once more. So, it is a similar terminology we are using. The radiation being emitted from the ground at this frequency is I nu S, S meaning surface. However, this time the emission of radiation by the gas layers themselves will also contribute to I nu since the atmospheric gases also emit in this IR range. So, the gases will absorb some of the radiation at this frequency but will emit some of the radiation also. So the radiation amount going outward may be lessened or increased depending on the ratio of the absorption and emission. So the increase in the spectral intensity in the upward direction as the radiation passes through a gas layer is given by the amount being emitted upward by the gas minus the amount being absorbed by the gas layer. So let us assume that in altitude z, the radiation passes through a layer of thickness dz. The slanted path length of the radiation through the gas layer is ds. So this is ds here as before. And because here dz and ds are in both the same direction, dz is also increasing upwards, ds is also increasing upwards. Here dz equal to ds cos theta by looking at this right triangle here. Let d nu be the differential amount of radiation intensity.

$$dz = ds\, cos\theta \quad (89)$$

Just remember this all of this is radiation intensity. Watt per meter square stay radians. Okay. Absorb watt per meter square stay radians hertz or nanometers depending on whether you are using frequency or wavelength structure. Okay. Then by the Lambert-Bouger-Beer law, we have, so if dA nu is the amount being absorbed by the gas layer, then this dA nu is equals to the mass absorption coefficient of the absorbing species I, the partial density of the absorbing species I, the radiation intensity at Z into dS, okay. So, this is straightforward.

$$dA_v = k^i_{v_{abs}} \rho_i I_v ds \quad (90)$$

But let dE nu be the differential amount of radiation being emitted upwards by the layer of gas. So, as dNu is getting absorbed, dE nu is getting emitted. And this gas is assumed to be at a certain temperature Tz at the altitude z, which we can find again using the environmental lapse rate. Now, because it is being emitted through radiation, there is a emissivity for this gas layer. So, let E nu be the spectral emissivity of the medium, then the upward emitted radiation is dE nu into the emission emissivity, spectral emissivity into the blackbody radiation intensity d nu at the temperature Tz into ds. ds is the total length, correct? So, the total energy being radiated is this one into ds, alright. The emission, emissivity of this medium into the blackbody radiation intensity into ds. where B nu is the spectral blackbody radiation intensity that we have discussed earlier.

$$dE_v = \varepsilon_v B_v(T_z) ds \quad (91)$$

Thus, the net increase in spectral radiation intensity as the beam passes through this gas layer is dI nu equals to dE nu minus dA nu.

$$dI_v = dE_v - dA_v \quad (92)$$

Note in the previous case for shortwave radiation, it was just minus dN, minus this expression here, okay. Here we have the emission added, okay. Now, how do we evaluate this aspect? So, there is further simplification that we can do. For gases that are at thermodynamic equilibrium which is a good assumption for most of the atmosphere we have the Kirchhoff's law of radiation which holds that the fraction of radiation absorbed by a gas will be equal to the fraction of the total blackbody radiation emitted by the same gas.

So suppose you have a gas which is emitting say 50% of what a black body would have emitted. Then Kirchhoff's law of radiation will say that it will also absorb 50% of the total radiation that is passing through it. The same 50% ratio will be applied for both the absorption and the emission. This is called the Kirchhoff's law of radiation. What this means is this K nu absorption I into rho I.

This is basically the fraction of I that is being absorbed. And this E nu is the fraction of the total radiation intensity. black body, that is the total fraction of the black body radiation intensity. The fraction of the radiation passing through that is being absorbed is equal to the fraction of the black body radiation intensity. So, this means K nu absorption I into rho I is equals to E.

$$k^{i}_{v_{abs}}\rho_i = \varepsilon_v \quad (93)$$

Okay. So, if we put this expression here, then we get di nu equals to rho i k nu absorption i. So, now these two terms basically this minus this, but e nu and k nu absorption i rho i are the same. So, we can take that as a common. into the blackbody radiation intensity minus the intense radiation, the intensity of radiation entering this layer of atmosphere at altitude z and Bs is dz by cos theta from this expression. So, this becomes the final expression which we can also simplify as di nu by dz equals to partial density of absorption species into absorption coefficient, mass absorption coefficient into blackbody radiation intensity minus intensity of incident radiation at z by cos theta where theta is the angle of incidence. So, this we can integrate directly with respect to Z if you know all the explicit formulations or we can use the concept of optical depth again for simplicity, ok. So, introducing the concept of optical depth of IR radiation we have. The optical depth at z for anything coming from the surface upwards is from the surface where z equal to 0 to that given altitude z equal to z rho i k nu absorption i dz. So, this is the optical depth.

$$dI_v = \rho_i k^{i}_{v_{abs}}[B_v(T_z) - I_v(z)]\frac{dz}{cos\theta} \quad (94a)\ or$$

$$\frac{\boldsymbol{dI_v}}{\boldsymbol{dz}} = \frac{\boldsymbol{\rho_i k^{i}_{v_{abs}}[B_v(T_z) - I_v(z)]}}{\boldsymbol{cos\theta}} \quad (94b)$$

$$\boldsymbol{\tau_v(z) = \int_0^z \rho_i k^{i}_{v_{abs}} dz} \quad \boldsymbol{(95\ a)}\ and$$

$$d\tau_v = \rho_i k^{i}_{v_{abs}} dz \quad (95\ b)$$

So, d tau nu is basically this expression here. This optical depth is along the altitude. Now, this is again related to the optical path length which is along the path of the ray of radiation which is at an angle theta. This is given by So, this is tau nu, this is x nu at S which is S0 to S, where S0 is the earth surface in this case. In the previous case S0 was the top of the atmosphere, here S0 is the beginning point of the radiation which is the earth surface to the final S value and this is k nu absorption I rho I dS prime where S prime is basically the dummy variable. These two are again related because ds prime is equal to dz cos theta.

So, you can express this in either case. However, so we will do it using the optical depth position. So, here dz is being replaced by d tau nu by rho i k nu absorption i, this expression. So, di nu d tau nu equals to the black body radiation intensity, black body spectral radiation intensity minus the actual intensity at altitude z by cos theta. where the blackbody radiation intensity as we discussed earlier is expressed by this expression here, watt per meter square stay radiation hertz.

Now, this is a fairly complicated integration, so I will skip the steps. This is the final expression if we integrate this and try to find the intensity of radiation, intensity of infrared radiation at the altitude z, okay, for frequency nu at the given direction theta. So, theoretically I should also write this as of actually a function of both theta nu and z. This is the intensity of radiation being emitted by the surface exponential minus the optical depth by cos theta.

$$I_v(z) = I_v^s exp\left(-\frac{\tau_v(z)}{cos\theta}\right) + \int_0^{\tau_v(z)} \frac{B_v(T_{\tau'})}{cos\theta} exp\left[-\frac{\tau_v(z) - \tau'}{cos\theta}\right] d\tau' \quad (99)$$

So, this is again as before. plus this complicated expression. Firstly, this expression is very easy to understand. This is the amount of the surface radiation that is actually reaching the altitude z after it is being progressively absorbed by the intervening atmospheric layers up to the altitude z. And this extent of absorption is given by this exponential minus optical depth by cos theta term. Now, on this side, we have the contributions of a series of emissions in the frequency nu and along the theta direction by the atmospheric layers below the altitude z.

So, basically what we are saying is Below this layer is another layer, then another layer and many, many, many, many layers till you get to z equal to 0. All of these layers are adding some radiation in the direction theta in the frequency nu. The sum of these contributions is this integral going from the optical depth of 0 to optical depth at z. of the blackbody radiation intensity by cos theta into this exponential term minus tau nu z minus tau prime by cos theta. So, the first term represents the fraction of the terrestrial radiation that reaches the height z after being absorbed by the intervening medium between z equal to 0 and z equal to z. I nu s is the radiance or the radiation intensity from the surface in the direction theta from the normal. The second integral term represent the upward emission contributions from all the atmospheric layers between the surface and the altitude z with each contribution being attenuated by the exponential factor due to absorption by the thickness between the emission layer and the final z location. This is very important. This is the emission of any blackbody radiation term at a certain altitude. This is the attenuation that emission from a certain atmospheric lens happening has experienced till it reaches the altitude z.

What do we mean? Suppose there is an atmospheric layer here at certain location, it is emitting in this direction. Of course, between this layer and this final layer, there is a series of atmospheric layers and these will absorb some of this emitted radiation from this atmospheric layer as it moves upward. So, the contribution is an integral of all the emissions by the atmospheric layers below Z and all the attenuations of all of these radiations due to the intervening atmospheric layers. So, that is what makes this integral have both this exponential term, this is basically the measure of the distance between the actual optical depth and the optical depth at altitude z and this is the B nu term. Remember this is B nu t at tau prime actually. So, this is B nu T at the optical depth tau prime which is the dummy variable here. So, the temperature is also changing for each of these optical layers, correct. A certain, at a certain location the atmospheric layer will have a certain temperature. at that given optical depth and that temperature comes here. So, this is the emission from an atmospheric layer which is at optical depth tau prime.

$$\chi_v(s) = \int_{s_o}^{s} k_{v_{abs}}^{i} \rho_i ds' \qquad (96\ a)$$

$$so, \qquad d\chi_v = k_{v_{abs}}^{i} \rho_i ds \quad (96\ b)$$

$$\frac{dI_v}{d\tau_v} = \frac{B_v(T_z) - I_v(z)}{cos\theta} \quad (97)$$

This is the degree of attenuation that emitted radiation undergoes as it moves from depth tau prime to the final tau. So, this is the very important point. In many cases the earth surface can be assumed to be a black body and hence I nu s is equals to the black body radiation intensity at T s where T s is the surface temperature. This simplification we can easily do. Now in both the case for the terrestrial radiation as well as the solar radiation we have looked at the radiation intensity that is going at a particular direction at a particular frequency. However, what we want to know is usually the total radiation intensity over the entire short wave range and the entire long wave range separately, of course, and throughout the entire hemisphere, not just at any particular direction. So, we have to integrate these over all frequencies and throughout the hemisphere to get the radiation flux density F. So, for the total upward radiation transfer, we have to integrate over all directions and over all frequencies, which will give you the upward infrared radiation flux, which is called F upward Z in watt per meter square. The atmosphere also emits IR radiation downwards towards the ground. So this is upwards, but there is a downward moving infrared radiation intensity also because the atmospheric layers will emit upwards and downwards as well.

Thus the net downward IR irradiance incident on a gas layer at altitude z is the integral of the downward emission contributions from the all the gas layers between z and the top of the atmospheres. So at any point a downward moving radiation is the integration of all the radiations emitted for all the atmospheric layers from the top to the atmospheric layer at altitude z. minus the attenuation happening in the intervening atmospheric layers, ok. So, this term will be there, this term won't because there is no infrared, far infrared source of radiation at the top of the atmosphere. So, this term will go out, this term will exist, here it will be infinity at the top of the atmosphere, ok, going to this point, ok.

The integrating this irradiance over all downward angles and over all frequencies will give you the downward IR radiative flux by the atmosphere towards the ground. This we represent as F downward z in watt per meter square. So, for this case, we have a expression very similar to the second term here, the first term does not exist. At any given altitude, the infrared radiation flux is the difference between the upward going flux and the downward going flux. So, F net, the net emission, atmospheric emission at a level Z is the upward going hemispherical flux minus the downward going hemispherical flux and this is the net infrared flux at any given altitude.

Okay. Now here also we can make a very useful approximation called the diffuse approximation where we can assume that the atmospheric layer emits  equally in all directions. So, there is no directional dependence of I nu. So, I nu is not a function of theta. So, if the diffuse approximation is valid, which it is for atmospheric layers, then we get a very simple set of equations relating the flux and the blackbody radiation intensity. So, the change in the upward flux for a given frequency divided by the by a modified optical depth term.

So, the gradient of the upward moving flux for at a given frequency with respect to a modified optical depth term plus the upward moving flux equals to pi times the spectral blackbody radiation intensity. pi B nu t is equals to this ordinary differential equation in terms of the upward going spectral flux density with respect to the modified optical depth term tau star nu. Similarly, the downward moving flux also has a similar ODE expression in terms of the black body spectral radiation intensity, where this tau star is basically 1.66 times the optical depth tau nu. Where the optical depth tau nu is from the z to the top of the atmosphere. So, here remember this is the expression that you have to use if you use these expressions. The optical depth is z to the top of the atmosphere rho i k nu absorption i dz prime. Note that here the optical depth tau nu is being measured from z, altitude z to the top of the atmosphere. And this, if you solve this expression, you get the upward moving flux and the downward moving flux in the infrared zone. The 1.66 term is obtained upon integrating over radiations coming from all directions with different optical path lengths. So, here the idea here is if there is a correction to be made because the earth surface is spherical instead of flat. If it was flat, all the radiation is going parallely upwards and parallely downwards. Okay, that was the assumption. But here because it is spherical, the angles change slightly and that effect comes by this 1.66 tau nu. So, this is the final set of expressions that will give us the upward and the downward going total hemispherical fluxes for the infrared zone for the atmosphere.

$$-\frac{d\mathcal{F}_v^{\uparrow}(z)}{d\tau_v^*} + \mathcal{F}_v^{\uparrow}(z) = \pi B_v(T) \quad (101)$$

$$\frac{d\mathcal{F}_v^{\downarrow}(z)}{d\tau_v^*} + \mathcal{F}_v^{\downarrow}(z) = \pi B_v(T) \quad (102)$$

$$\tau_v^* \approx 1.66\tau_v \quad (103)$$

$$\tau_v = \int_z^{\infty} \rho_i k_{v_{abs}}^i dz' \quad (104)$$

In the next class, we will have a brief discussion on the physical significance of these expressions as well as a few worked out examples. Thank you for listening and see you in the next class.