Course Name: An Introduction to Climate Dynamics, Variability and Monitoring

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RADIATIVE TRANSFER IN THE ATMOSPHERE KEY CONCEPTS

Good morning class and welcome to our lectures on climate dynamics, climate variability and climate monitoring. In the previous class, we looked at how various atmospheric gas constituents selectively absorb various frequency bands between the shortwave radiation spectra of sunlight and the longwave radiation spectra of terrestrial radiation. In today's class, we will be discussing how we can model the transmission of solar radiation from the top of the atmosphere to the ground as the various atmospheric layers absorb parts of that radiation as the sunlight passes through them. So, we are discussing the transmission of solar radiation through the atmosphere. This is what we will discuss today. In the next class, we will be discussing the transmission of terrestrial radiation through the atmosphere or the long wave radiation through the atmosphere.

So, here we will be assuming that the ground is a flat surface so we are neglecting the curvature effect of the earth. Second, we are assuming that the sun is at a certain angle theta with respect to the ground normal as you can see here. Initially, the solar, the spectral solar radiation intensity at the top of the atmosphere is given as I_v^∞. So, this is sunlight at a specific frequency that we are looking at, and the intensity of radiation at this specific frequency is I_v^∞ at the top of the atmosphere.

As it passes through the atmosphere, part of this radiation gets absorbed and eventually, a radiation intensity of I_v^s hits the surface at sea level. Let us assume that we have a thin atmospheric layer of thickness dz at an altitude z from the surface. And the spectral radiation intensity of sunlight at this altitude location Z is given by I_v which is a function of Z. We will ignore the scattering and reflection effects of clouds and aerosols from this model. So, a more complete model will also have to incorporate these because these are quite significant.

However, here we are assuming those are negligible and the solar radiation is either getting absorbed or transmitted as it passes through the atmosphere. One of the simplifications that we can make when we are discussing the transmission of an absorption of solar shortwave radiation is that we can ignore the radiation from the atmospheric layers or the surface layers at this frequency range. We have seen that atmosphere in the surface are at temperatures that are significantly lower than the sun and hence the emission spectra of the atmosphere or the surface is quite different and is at much lower frequency ranges than the solar radiation spectrum. Hence, we can say that at this frequency band I_v there is no emissions from either the atmosphere or the surface. So, it is a clean case of just solar radiation getting either absorbed or transmitted to successive layers of the atmosphere with these layers of atmosphere not radiating any radiation at this frequency.

This will not be true in the terrestrial radiation where the expression will therefore be more complex. Now, the distance z is from the sea level upwards. So, z is increasing from z equal to 0 at sea level to z tending to infinity at the top of the atmosphere. We can create another coordinate s which is along the direction of the solar rays which is starting at s equal to 0 at the top of the atmosphere and s equal to certain value s as it hits the ground, s_0 or something, s equal to s_0 at the level of the ground. So, the coordinate s is along the rays of the sun, while the coordinate z is along the ground normal increasing upwards, whereas coordinate s starts at s equal to 0 at the top of the atmosphere and increases downwards till it hits the ground level.

What this means is the differential dz and ds are negative of each other and if you see in this right triangle, this length is ds, this length is dz and this angle is theta. So, ds cosθ is -dz, because ds is directed in this direction the vector and dz is vector in this direction. So, -ds cosθ will be dz based on this right triangle. And this is what we are seeing here that dz the differential change in the z direction is the negative of the differential change along the solar ray direction into cosθ where theta is the angle of incidence. Since, this radiation is being absorbed in the downward direction, we expect the intensity to increase as we gain altitude.

So, the lowest intensity is at sea level where z equal to 0 and intensity will increase at z becomes larger and larger. So, $\frac{dI_v}{dz}$, the rate of change of solar radiation intensity at this frequency with respect to z is greater than 0, whereas $\frac{dI_v}{ds}$, is less than 0 because radiation is decreasing along the s direction. Next, we introduce the main law out of which all these derivations will be done. This is called the Lambert-Bouger-Beer law of extinction. So, Lambert-Bouger-Beer law of extinction, which often is also called the Lambert-Beer law or just the Beer's law.

Which states that the absorption by a layer of depth ds, so this layer ds, along the path of the radiation beam is proportional to the spectral intensity I_v incident on that layer times the mass density of the absorber along the path of the radiation. So, the amount of absorption that is happening in this layer is dependent on firstly the spark length ds along this layer, next proportional to the spectral intensity I_v at this layer and thirdly, the mass density of the absorber material in this layer. Here, the absorber material is the mass density of the absorbing trace gases like CO2, water vapor, etc. in this layer. This is a proportional.

The proportionality constant is called the spectral mass absorption coefficient. Spectral mass absorption coefficient, which I am writing as $k_{v_{abs}}$ for the absorbing species I. So, here it is an important point. You have multiple species that can be absorbing CO2, ozone, water vapor etc. Each will have its own proportionality constant which is dependent on the molecular structure of that material because obviously at certain regions CO2 will be more absorbing, at other regions H2O will be more absorbing.

So, it obviously depends on the identity of the material of that species I for which we are trying to find out the mass absorption coefficient. It is spectral because it is dependent on the frequency and its absorption coefficient. So, $k_{v_{abs}}$ for species I. Its unit is meter square per kg or centimeter square per gram. You will see how the units look like when we look at the Lambert-Bouger-Beer law expression.

And its value depends on the frequency of radiation, the identity of the absorbing species as well as the temperature and pressure of the medium. So, if you remember due to the pressure effect, there is a pressure broadening effect, right? Temperature also has an impact because it increases the rate of pollution. So, increases the impact of pressure broadening. So, the temperature and pressure of the medium is important, the identity of the absorbing species is important and the frequency of radiation that is passing through that layer is important. So, if we express this then, the amount of radiation intensity that is being absorbed dI_v is equal to, so, firstly the change in the intensity, change in intensity dI_v will be negative in the ds direction along the path of radiation and will be positive in the dz direction which is along the altitude, infusing with altitude.

$$dI_v = -k^i_{v_{abs}}\rho_i I_v(z)ds = k^i_{v_{abs}}\rho_i I_v(z)\frac{dz}{cos\theta}$$

So, if you think in terms of the ds direction, then the Lambert-Beer law is dI_v equals to minus the mass absorption coefficient for that absorbing species, the partial density of that absorbing species ρ_i, the actual radiation intensity at this location s or z×ds. So, it is negative because absorption will decrease the amount of radiation into the mass density of the absorbing species, the radiation intensity, that path length ds and the mass absorption coefficient K nu absorption for that species i. If you replace ds with minus dz by cos theta then you get it in terms of dz which is $k^i_{v_{abs}}\rho_i I_v(z)\frac{dz}{cos\theta}$, ok. Now, if you have multiple species you have to add them up. So, this $I_v(z)\frac{dz}{cos\theta}$ is constant, you will have a summation over $k^i_{v_{abs}}$ for all the absorbing species like H2O, CO2, N2O etcetera, to get the total decrease in this ds location.

ρ_i is the partial density of the absorbing species at altitude z. We will discuss explicit expressions from ρ_i later. We already have discussed the idea of partial density before in one of the early classes. So, you can look up into that. Now, here when you look at this expression, we can simplify this expression a little bit further.

We can define a differential quantity called the differential optical depth d tau nu, optical depth is tau nu differential.

$$d\tau_v = -k_{v_{abs}}^{i} \rho_i dz$$

So, differential optical depth $d\tau_v$ which is unit less, which is $-k_{v_{abs}}^{i} \rho_i dz$. So in this expression, I am taking $-k_{v_{abs}}^{i} \rho_i dz$ as a common and calling it $-d\tau_v$. So, then $d\tau_v$ equal to $d\tau_v \frac{I_v}{cos\theta}$. So, this is negative of the mass absorption coefficient partial density and dz.

This optical depth is also related to the optical path length X_v along the s direction. So, optical depth is along the z direction. So, let us just the optical path length is defined along the Xs direction and is and its differential amount is given as $k_{v_{abs}}^{i} \rho_i ds$. So, this term $k_{v_{abs}}^{i} \rho_i ds$. Which is called $d\chi_v$ and the total path length is S_0 to S, S_0 is the path length up till the ground and S0 is whatever the path length you start with and it is $k_{v_{abs}}^{i} \rho_i ds'$.

So, path length from the ground to certain path length value s at a certain altitude z above the ground. So, s is smaller, s_0 is bigger, $k_{v_{abs}}^{i} \rho_i ds'$ is the dummy variable here. This gives the optical path length along a given ray length s, where s_0 is the ground and s is at a certain distance above the ground. So, s_0 is the start of the path in the direction of radiation that is top of the atmosphere in this case. So, I am mistaken S_0 is the top of the atmosphere.

So, often S_0 is taken as 0 as well. So, S_0 is the start of the path in the direction of radiation. So, you are starting from the top going downwards up to a certain path S. So, this is basically 0 to S in that sense. So, using the expression of $d\tau_v$ and d chi nu, you can express this expression.

$$dI_v = -d\chi_v I_v = -d\tau_v \frac{I_v}{cos\theta}$$

So, then we can say $\frac{dI_v}{I_v}$ the change in the radiation intensity by the radiation intensity at z equals to minus the differential change in the optical depth by $cos\theta$ equal to minus the differential change in the optical path length along the x direction. So, this is another way of expressing the same equation 80, but in terms of differential changes in the optical depth or path length. So now we can integrate this expression. The radiation is incident at the top of the atmosphere where Z is infinity, where $I_v(\text{z})$ is equals to I_v^{∞}. Thus integrating this expression from the top of the atmosphere to some altitude Z.

So we are integrating from Z equals to infinity to some altitude Z. At Z equals to infinity we have I_v^{∞} which goes into the bottom. So, this becomes and at z equals to z, the incidence, the radiation intensity is I_v at z. This is equal to exponential because it is $\frac{dI_v}{I_v}$.

So, it is d log I. So, this becomes an exponential term if you take the log out equal to $\exp\left(-\frac{\tau_v(z)}{cos\theta}\right)$, where, $\tau_v(z)$ is now the integral of this $d\tau_v$ from infinity to z. Infinity is the top of the atmosphere to whatever altitude we are thinking of z of this expression here which is basically this $d\tau_v$ expression. So, what we see here is the radiation intensity at any location z above the ground is equals to the radiation intensity at the top of the atmosphere $\exp\left(-\frac{\tau_v(z)}{cos\theta}\right)$. Whereas, where the path length optical depth up to z is from the top of the atmosphere to z minus of the mass absorption coefficient $\rho_i dz$.

That is the optical depth at z. Here ρ_i is the partial density of the absorbing component and is given as the mass mixing ratio or mass fraction yi of the component into the total density of the atmosphere at that location. So, ρ_i is also obviously a function of z, let us just write that explicitly. Similarly, the mass fraction or the mass mixing ratio of the ith component will also be a function of this. So, this is very important to remember, that both these things, the density of the atmosphere is of course dependent on the altitude. So, ρ_z is a strong function of z and the mass mixing ratio, the mass fraction of a component may also be a function of altitude z.

Especially, when you are thinking of ozone or water vapor, the mass fraction of those will change very rapidly with altitude. And the mass mixing ratio Y_i is the mass of that component by the total mass which can also be given as the molar mixing ratio or mole fraction X_i into molecular weight of the component by the molecular weight of atmosphere. So, X_i here is the molar mixing ratio, M is the average molecular weight of the atmosphere and Mi is the molecular weight of the absorber species. The plot of the molar mixing ratio with atmosphere for various gases is shown below. So, just as mass mixing ratio is a function of z, the molar mixing ratio will also be a function of z.

Here you can see especially our water vapor, the molar mixing ratio decreases very rapidly with altitude up to the troposphere, tropopause, then it reaches more or less constant in the stratosphere and then again starts to decrease beyond that. Carbon dioxide, argon, oxygen, these the mass mixing ratio or the molar mixing ratios do not change significantly for these gases. However, ozone you see a significant change. So, ozone increases very rapidly up to the troposphere and then higher lower stratosphere. Then it decreases down again rapidly till the end of the stratosphere.

Then it has again an increase decrease relationship. N2 also while its remain constant in the troposphere decreases throughout the stratosphere and continues to decrease in the later stages. CH4 again constant in the troposphere, but decreases in the stratosphere. So, you have to be a little bit careful when you are looking at the mass mixing ratios or the molar mixing ratios of these gases which have significant variation with altitudes. The density of the atmosphere at any altitude z, we have already had explicit expressions for density given below.

Here we can use the simple expression for a hydrostatic balance with a constant scale height assumption, but we have also done more complicated formulations of density based on variable scale height as well. So, we can use any of these models. For the simplest case density at a given altitude z is density at sea level exponential minus z by h where h is the scale height of the atmosphere. H value was given earlier in the class. So, you can use that and rho s is the density of the atmosphere at sea level which is also usually standard given.

This is only an approximate expression where isothermal temperature assumption was used and the scale height was considered to be the average scale height of the atmosphere. So, these expressions give all what we need to evaluate this expression 84 in terms of topical depth mass mixing ratios etcetera. Now, the optical depth value in equation 84, this term here is always a positive, $\tau_v(z)$ is always a positive and hence we see that the solar irradiance or solar radiation, not the solar irradiance, solar radiation intensity will decay exponentially as we move downwards from the top of the atmosphere. So, the solar radiation intensity I_v is exponentially decaying and is dependent on the optical depth tau nu z. The exponential coefficient itself will be given by the optical depth of the medium at z divided by the cosine of the slant angle.

For greater slant angle, the effective optical depth will be greater. Because you have tau nu z by cos theta as theta increases $cos\theta$ will decrease and hence you will have the exponential minus term will be greater and hence the decay will be sharper. Hence the expression provides the analytical expression of the attenuation effect discussed earlier. We discussed the idea of attenuation effect that at high latitudes the sun is lower in the hemisphere, so the angle of incidence is higher, hence more of the sunlight get absorbed by the atmosphere. Now here, this explicit cos theta dependence shows how much that attenuation effect is going to be and how it actually depends on the angle of incidence theta, ok.

We also see that the optical depth is a function of the mass absorption coefficient and the density of the absorbing species. So, the optical depth will be greater if the absorption coefficient is greater and or the density of the absorber in the medium is higher. So, either you have a strong absorption peak which will make the absorption coefficient high or you will have the density of the absorber medium. So, the density of the gas is higher in the system. So, the value of this mass absorption coefficient is basically given in terms of the absorption bands that we discussed earlier.

So, this kind of gives you the absorption coefficient values where the absorption bands are high the absorption coefficients are high where the absorption bands are non-existent or low the absorption coefficient is also quite low. So, here the mass absorption coefficient of CO2 and water vapor are shown in the plot. Here the values are in meter square per kg, ok. So, the units are meter square per kg for the mass based absorption coefficient and the values change quite a lot from 10^{-6} to 10^{6}.

This is the wavelength micrometers. So, this is 3.5 micrometers, so in the near infrared this is 100 micrometer. So, this is basically the terraced covering the terrestrial region and the short wave solar region here and you can see the water vapor is the black line and the CO2 is the red line. It is a function of the wavelength. The wave number is 1 by wavelength. So frequency is C by wavelength where wave number is just the inverse of the wavelength.

So, here the wavelength is given in micrometers whereas wave number is given in centimeter inverse. So, that is why the values may be a little bit different because here the wavelength is expressed in centimeter. So, you get wave number as centimeter inverse, on the top the wavelength is expressed in micrometers. So, what you see here is that CO2 has very high absorption coefficient values at certain wavelengths or wave numbers. So, at 15 micrometers corresponding to a wave number of around say 600, you have a strong peak.

You also have a strong peak for CO2 at 4.3 micrometers which is a wave number of around say 2800 or something like that. Water vapor of course between the wavelength of 10 and 100 micrometers this side will have very large absorption coefficient as you can see here. Absorption coefficients kind of peaks at around 10 to the power 4 between 25 and 100 micrometers. Here you have very strong absorption peaks of 10^2 to 10^4.

You have also another strong peak between 7.5 to 6 micrometers corresponding to wave numbers of around 1500. So, these are regions where you have large absorption coefficients, low absorption coefficients are found in the middle. So, water vapor absorption coefficients decrease to around 10^{-2}, whereas CO2 decreases even further down at certain wave number regions. So, these kind of plots help you get for a given frequency what is the absorption coefficient. Now, the absorption coefficient can also be tabulated in terms of absorption cross section which is centimeter square per molecule.

This is given as sigma I nu absorption. This sigma is not the Stephen Boltzmann's constant. It is called absorption cross section or centimeter square per molecule, and it is related to the mass absorption coefficient in this format. The absorption cross section is equal to the mass absorption coefficient in grams in centimeter square per gram. So, this unit is centimeter square per gram into the molecular weight of the absorbing species in grams per mole into the Avogadro number. So, Mi is the molecular weight in grams per mole, n is the Avogadro number which is 6.023×10^{23} molecules per mole and the mass absorption coefficient is centimeter square per gram. So, then it will get centimeter square per molecule. So, this absorption cross section is also often given in many tables and charts. So, this is the relationship to get to mass absorption coefficient from absorption cross section. For example, for ozone this is the absorption cross section that is given at various temperatures.

So, there is some temperature dependence as well and you see a large absorption cross section values at around 250 to 300 nanometers which is the UV range where ozone is getting dissociated and hence absorbing the UV radiation. You also get a large absorption cross section at around 600 nanometers which is associated with a vibrational frequency of ozone. In other places you have very low absorption cross sections as well. And specifically in this range you have more detailed experimental data that is available where the absorption cross section is basically 1.05×10^{17}. So, this is multiplied by 10^{17} and here the relationship with between with temperature is also given. So, this is 1.1×10^{17}, this is 1.15×10^{17} and the peak of ozone absorption occurs around 255 nanometers. So, these kinds of charts will help you to find out the absorption cross sections, absorption coefficients and associated optical depth and finally the net attenuation of the solar radiation as it reaches the ground.

So, we are stopping here today. We will continue the discussion in the next class and thank you for listening.