Course Name: An Introduction to Climate Dynamics, Variability and Monitoring

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Lecture 23

DERIVATION OF BEAM SPREADING EFFECT, DERIVATION OF THE GREENHOUSE EFFECT

Good morning class and welcome to our continuing coverage on the climate dynamics, climate variability and climate monitoring lecture. Today we will discuss a few worked out examples and a few derivations to clarify some of the concepts that we have discussed in this week. One of the concepts that we discussed was the beam spreading effect. So, let us understand very simply how the beam spreading effect works. So, suppose, so let us just start this discussion with beam spreading effect. So let us consider a ground and this dotted line is the normal to the ground.

Now the Sun is at a certain angle theta with respect to the ground. So the Sun's rays if you remember they come in parallel lines. The Sun's rays are coming in parallel lines at an angle theta with respect to the normal to the ground. radiative flux that is normal to the sun's rays will be significantly larger than the radiative flux which is normal to the ground and let us see how we can show this.

Let us draw a line here that is connecting this point with this point here in a perpendicular direction. So, this is 90 degrees. So, this angle is 90 degree, this angle is also 90 degrees. So, we shall consider this length to be L1 and this length here to be L2. Let us name this thing.

So, suppose this point is O, this point is This point is B. So we have a right triangle OAB where angle OBA is 90 degrees. So angle OBA is equals to 90 degrees. Notice this angle is theta. So this angle is 90 minus theta.

Correct. So this angle must be theta as well. This angle here. So, angle BAO is theta, where theta is the angle of incidence. Let us assume that the solar flux is S watt per meter square.

The solar flux here normal to this direction BA is S watt per meter square. S watt per meter square. So, suppose we take a square area of length a rectangular area of length L1 and width W into the plane of the paper. So, the total energy incident on an area L 1 into w which is oriented normal to the direction of incidence is equal to S which is the flux normal to the plane along which the sun is incident into L1 which is the length of this line BA into some width w perpendicular to the plane, this is the watts. So, this flux is finally incident on the ground on this area OA. this flux is finally incident on the ground where the area is OA into width W. The width W is not changing because it's perpendicular to the plane of paper, but this flux, this length OA is changing which is equals to L2 into W meter square. So, this same energy is spreading over a length L2 into width W meter square. Hence, flux on the ground, let us call this Sg is equals to the total energy coming into the ground which is this area here which is equals to E dot, total incident energy, incident power. because it is in watts, is equals to E dot by this area on the ground which is L2 into W, watt per meter square.

Now, what is this E dot? We have, this is S into L1 into W, this is equals to S into L1 into by L2 into W equals to S into L1 by L2, which basically means it is S into, what is L1? This length AB and what is L2? This length OA. So, AB by OA. Now, in this triangle, let us draw this triangle now. This is the vertex B. This is the vertex A.

This is the vertex O. This is 90 degrees. OAB is theta. So, OAB, this is theta.

Okay. And what we are trying to find is OA by AB. or AB by OA. Clearly, because this is right triangle, we must have AB by OA equals to cos theta. So, this AB, this OA is equals to cos theta, basic geometry. So, here this becomes S So, due to beam spreading effect if the angle of incidence is theta then flux normal to the ground is S cos theta where S is the solar flux along the direction of the sun, fine.

$$S_G = \frac{\dot{E}}{L \times w} \, \mathrm{W/m^2}$$

$$= \frac{S \times L_1 \times w}{L \times w} = S \times \frac{L_1}{L}$$

$$= S \times \frac{AB}{OA} = S \cos\theta$$

So, clearly as theta tends to 90 cos theta tends to 0, so at higher and higher angles of incidence you will get lower and lower ground flux and when the sun is directly overhead it will be the maximum. So this is the basic idea of the beam spreading effect. This happens every day of course as the sun rises and sun sets you have very high angles of incidence so by beam spreading effect you have much lower ground flux which is one of the reasons why mornings and evenings are much cooler than in the afternoons where the sun is much higher up in the sky so the angle of incidence is lower. However, in high latitudes even at noon times the angle of incidence is much higher than near the equator where the sun is far more overhead the sky. Hence overall at higher latitudes you have due to beam spreading effect much lower ground flux throughout the day than a corresponding day in the low latitudes in the same meridian for example.

So this kind of gives an example of how the beam spreading effect works. The next example that we will give is the greenhouse effect for planets with extremely thick absorbing atmospheres. So greenhouse effect for planets with extremely thick absorbing atmospheres. So if you consider the example of Venus, Venus is a planet which has extremely thick atmosphere, much thicker significantly, order of magnitudes thicker than that of Earth. What happens in such cases is that modeling the greenhouse effect by a single isothermal layer of the atmosphere is no longer sufficient because the entire surface heat flux gets absorbed much lower down in the atmosphere and when that atmosphere re-emits towards the sky, there is sufficient layer of the atmosphere above it to again reabsorb it and then again re-emit it to the next layer of atmosphere and so on.

So, in this case, we have the model is multiple isothermal layers of absorbing atmosphere. Let us understand how this system works. So again we assume that the atmosphere is not absorbing any solar energy but can absorb all the thermal energy associated with emissions from the ground or emissions from other layers of atmosphere. So firstly the solar energy is passing through the atmosphere without getting absorbed, but it may get reflected. So, you will get again the solar flux per unit area is S0 by 4 into 1 minus alpha, where remember alpha is the albedo.

Now, here we will be assuming multiple perfectly absorbing layers of the atmosphere. These are all individual isothermal layers of the atmosphere. Which fully absorb any thermal radiation that is incident on it. They emit thermal radiation at their own temperature. Assuming they are black body atmospheres.

These layers are perfect black bodies. They are transparent to short wave or solar radiation. So here we have. Layer 1 with temperature TA1, layer 2 with atmosphere temperature TA2, layer 3 with temperature TA3, this is say layer n minus 1, this is layer n and then this is the surface which is the ground. What are the approximations? atmospheric layers are individually isothermal.

They are perfect black bodies for thermal or long wave radiation, they are transparent to solar or short wave radiation. So, then we will treat each of these layers as we have treated before. Each of these layers will emit upwards and downwards at a rate of sigma into Ta1 watt per meter square. The surface also is a perfect black body. So, surface is a black body for thermal radiation.

So, these are the assumptions. So, this is assumption 1, this is assumption 2, this is assumption 3, this is assumption 4. Fine. Let us now do the energy balance for each of these cases and continue. So, first is the blackbody emission temperature of the planet as a whole.

What is the blackbody emission temperature of the planet as a whole? The net solar flux this planet is absorbing which is S0 by 4, 1 minus alpha. S0 is the solar flux per unit area of the projected circular disk for that planet, remember. And S0 by 4 is the solar flux per unit area of the spherical surface area of the planet. Alpha is its albedo and this is equals to sigma into Te to the power 4 where Te is the emission temperature.

So, this is equation 1. Then energy balance at the top of the atmosphere. So, if you go top of the atmosphere is only emitting thermal radiation. from is only seeing thermal radiation from the first atmospheric layer Ta1. Remember these atmospheric layers are absorbing all ground radiations as soon as they hit. So, the space is not seeing any layer other than the topmost layer which is at temperature Ta1.

Hence S0 by 4 1 minus alpha which is the net solar flux entering the planet equals to sigma So, equation 1 and 2 implies that the blackbody emission temperature is equal to the temperature of the topmost layer of absorbing atmosphere. So, this is clear. So, this is expression 3, alright. Now, we do the balance of the first atmospheric layer.

$$\frac{S_0}{4}(1-\alpha) = \sigma T_e^4 \qquad \text{(i)}$$

$$\frac{S_0}{4}(1-\alpha) = \sigma T_{A_1}^4 \qquad \text{(ii)}$$

$$T_e = T_{A_1} \qquad \text{(iii)}$$

So, first atmospheric layer. The first atmospheric layer is emitting both upwards and downwards at temperature TA1 and receiving energy from the second atmospheric layer emitting upwards at temperature TA2. TA1 to the power 4 net emission equals to sigma TA2 to the power 4 is the net incoming emission into this layer and this is the net outgoing emission from this layer which implies TA2 is 2 to the power 1 fourth TA1 equals to 2 to the power 1 fourth Te. So, this is expression 4. Now, we will go to the next atmospheric layer.

For the second atmospheric layer. this layer here, what you are getting? Emissions from the top atmospheric layer and the third atmospheric layer going into and twice sigma Ta to the power 4 going outwards. So, you get the second atmospheric layer 2 sigma TA2 to the power 4 equals to sigma TA1 to the power 4 plus sigma So, if you just look at it, this is TA2, this is TA1, this is TA3, emissions are coming from the top and the bottom and emissions are going out on two sides from the atmospheric layer TA2. Similarly, here first atmospheric layer. Emissions are going up and down and emission is only coming from the bottom. So, in this case, so if you eliminate sigma, this implies TA3 to the power 4, this one here because it twice TA2 to the power 4 minus TA1 to the power 4.

Now, what is T A 2 to the power 4? 2 into Te, right. So, this is 2 into twice Te just from this expression here, alright. So, T A 2 to the power 4 is 2 into Te to the power 4. into Te to the power 4.

$$2\sigma T_{A_1}^4 = \sigma T_{A_2}^4 \Rightarrow T_{A_2} = 2^{\frac{1}{4}} T_{A_1} = 2^{\frac{1}{4}} T_e \qquad \text{(iv)}$$

And what is Ta1? Ta1 is Te. So, this is Te to the power 4. So, this becomes thrice Te to the power 4, implies Ta3 is 3 to the power 1 fourth Te. Similarly, if you do the third atmospheric layer, Similarly, for atmospheric layer 3 energy balance, we get TA4 equals to 4 to the power 1 4 Te. It is always going to go by one value upwards. So, this was TA1 is 1, basically you can think of it this way, TA1 is 1 to the power 1 fourth Te, TA2 is 2 to the power 1 fourth Te, TA3 is 3 to the power 1 fourth Te, TA4 is 4 to the power 1 fourth Te.

$$2\sigma T_{A_2}^4 = \sigma T_{A_1}^4 + \sigma T_{A_3}^4$$

$$\Rightarrow T_{A_3}^4 = 2T_{A_2}^4 - T_{A_1}^4$$

$$\Rightarrow T_{A_3}^4 = 2 \times (2T_e^4) - T_e^4 = 3T_e^4$$

$$\Rightarrow T_{A_3} = 3^{\frac{1}{4}} T_e$$

Similarly, for the third atmospheric layer:

$$T_{A_4} = 4^{\frac{1}{4}} T_e$$

So, as you go down and down to finally the last atmospheric layer, you get So, for the last it is nth atmospheric layer above ground T A n equals to n to the power one-fourth T e.

$$T_{A_N} = N^{\frac{1}{4}} T_e$$

Now that we know this, let us do the ground surface balance. energy balance of ground. This is tan, we have S0 by 4 1 minus alpha, energy is coming here sigma tan to the power one-fourth and energy is going out sigma Ts to the power one-fourth. So, net outgoing is sigma Ts to the power one-fourth, Ts is the surface temperature. Net incoming is S0 by 4, 1 minus alpha plus sigma TAn to the power one-fourth. Now, what is S0 by 4, 1 minus alpha? Sigma TA1 to the power 4. So, we can replace that here. equals to sigma and what is TA1? TA1 is Te. So, this is just sigma te to the power 4 and what is sigma tan to the power 4? sigma n te to the power 4 equals to sigma n plus 1 te to the power 4 which implies that the ground surface temperature is n plus 1 to the power one-fourth into the blackbody emission temperature Te.

$$\sigma T_s^4 = \frac{S_0}{4}(1-\alpha) + \sigma T_{A_N}^4$$

$$= \frac{S_0}{4}(1-\alpha) = \sigma T_e^4 + \sigma N T_e^4$$

$$\sigma T_s^4 = \sigma(N+1)T_e^4$$

$$T_s = (N+1)^{\frac{1}{4}} T_e$$

This is what we wanted to find. So if you have N perfectly absorbing atmospheric layers modeling a thick atmosphere of a planet, then you will get this expression here where Ts, the surface temperature, will be N plus 1. However, atmospheric layers are there, plus 1 to the power 1 fourth into the blackbody emission temperature of the planet. This kind of modeling is very useful for planets with extremely thick atmospheres where the surface temperature is much much larger than the blackboard emission temperature because of the thickness of the atmosphere. So in the next class we will do an example using the planet Venus where this kind of situation is common. Thank you for listening and see you in the next class.