Course Name: An Introduction to Climate Dynamics, Variability and Monitoring

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TEMPERATURE VARIATION WITH CHANGES IN VERTICAL PRESSURE

Good morning and welcome to our continuing online lectures on climate dynamics, climate variability and climate monitoring. In the previous class, we had started the derivation of the expression for the saturation adiabatic lapse rate. Which is the rate at which the temperature of a parcel of saturated air, that is a parcel of air which has reached the saturation vapor pressure condition, changes as it moves up and down in altitude and changes its temperature through an adiabatic process. So, here we consider a parcel of air, which is initially situated at an altitude z, where the surrounding temperature as well as the parcel temperature is T_z and it moves adiabatically to a new altitude z+dz, so, a small change in altitude through an adiabatic process. Initially at z, the humidity ratio or the saturation humidity ratio of this parcel of air was ω_s at z, whatever the saturation humidity ratio is that, and then it changes to a new saturation humidity ratio at z+dz. The change in the saturation humidity ratio $d\omega_s$ is the difference between the saturation humidity value of this parcel of air at z and the saturation humidity value of the same parcel of air at z+dz.

Now, what is this change? So, remember ω_s is the mass of basically water vapor under saturation conditions by mass of dry air, correct? So, ω_s at z is equals to mass of water under saturation condition for this parcel of air at z by the mass of dry air. Now, this same parcel has moved upwards, so the mass of dry air is not changing, okay. What is changing is the mass of saturated vapour that is still present in that parcel of air, okay. So, $\omega_s(z+dz)$ is the mass of water under saturation conditions at its new altitude z+dz by the same mass of air.

So, $d\omega_s$ is the difference between these two. So, $\frac{m_w^{sat}(z+dz)-m_w^{sat}(z)}{m_{air}}$. So, basically the change in the mass of water vapor under saturation condition between z+dz and z by the mass of air, clear? Now, what will this change be? Note this change is going to be negative if the altitude is increasing. As the altitude increases, the temperature falls. In an adiabatic process, the temperature must fall as you move up in altitude as we have looked before.

So, clearly if the temperature is falling, then the saturation vapor pressure is also falling, ok. So, now remember that ω_s is basically $0.622\frac{e_s}{\rho_{air}}$, where e_s is the saturation vapor pressure. This is obtained from the Clausius Clapeyron relation that we discussed a few lectures before. So, since temperature of the parcel will decrease with increasing altitude.

This implies ω_s (z+dz) will be less than ωs(z). Clear? What this means is $d\omega_s$ which is $\frac{\Delta m_w^{sat}}{m_{air}}$ is less than 0 for dz greater than 0. So, if the altitude change is positive, if you are moving up in altitude, you must have a decrease in the saturation humidity ratio for this parcel of air. So, $d\omega_s$ will be less than 0. What does this practically mean? So, if you think in the physical condition, because the temperature is falling, the partial saturation vapor pressure is also falling.

So, the air will be able to hold less water. So, a mass of water vapor of the amount Δm_w^{sat} will condense out of this partial of air as it moves from z to z+dz. So, this mass of water vapor which is the difference between the mass of water vapor that this partial at z could have contained and the mass of water vapor at its new temperature z+dz could contain, that amount of water vapor will condense out. So, what does this mean? If a mass of water vapor is condensing out of the system, it is releasing the latent heat of condensation to the air around it. So, as the water vapor condenses into liquid water, it releases the latent heat of condensation, which is absorbed by the dry air which is contained within it.

So, what happens is, as altitude increases, a mass of water vapor Δm_w^{sat} will condense into liquid water. As it condenses, it will release latent heat of vaporization or condensation, it is the same thing, the reverse process. So, the amount of heat absorbed during the vaporization process is the same as the amount of heat released during the condensation process. So, we can call it the latent heat of vaporization also without any problems, which will be absorbed by the dry air in the parcel. So, what happens therefore? The dry air is absorbing this heat released by this water vapor.

So, this is a net source of heat. So, the condensation process acts like a heat, internal heat source, just like an internal heat source within this parcel. So, heat absorbed by the parcel due to condensation is $\delta_{qin} = -L_v d\omega_s$. L_v is the latent heat of vaporization, since $d\omega_s$ is less than 0 for dz greater than 0. So, this term is negative.

So, this negative term balances out. So, δ_{qin} becomes positive. So, as the humidity ratio is decreasing with increasing altitude, this $d\omega_s$ term will be a negative term which will be multiplied by L_v and the negative term will be balanced to the net heat input into the system will be an effective positive term. So, if the parcel of air is rising, saturated air is rising, then the heat is getting absorbed by this system due to the release of the condensation process. This is the condensation process.

So, now we can do a first law analysis for our system. First law analysis for the partial of air. So, we have looked at the first law expression earlier. Here, we have a heat source. So, it will be heat input into the system plus work done due to change in the specific volume of the system, αdp.

$$\delta_{qin} + \alpha dp = c_p^{air} dT$$

This is heat input, this is the differential work output by that parcel of air due to change in specific volume equals to $c_p^{air} dT$, ok. So, this expression we have derived earlier as a generic expression for this parcel of air. If you remember, we will just go over it after this derivation is complete just to show you where the derivation was done. This is the net heat input into the system due to the condensation process. This is the work transfer term due to the change in the specific volume.

This is basically the change in energy within the system due to change in temperature. Now, from hydrostatic balance relation, we have, $\alpha dp = -gdz$. Ok. So, we can write $\delta_{qin} = c_p^{air} dT + gdz$ and here we have δ_{qin} term shown here. So, we get putting the expression for δ_{qin}, we have $c_p^{air} dT + gdz + L_v d\omega_s = 0$.

Now, we will try to simplify this expression. ω_s is $0.622 \frac{e_s}{\rho_{air}}$, clear. So, $\ln(\omega_s) = \ln(0.622) + \ln(e_s) - \ln(\rho_{air})$.

So, taking differential, $\mathrm{d}(\ln(\omega_s)) = \mathrm{d}(\ln(e_s)) - d(\ln(\rho_{air}))$ d of log omega s equals to, this term is constant, so differential will make it 0. So, $\mathrm{d}(\ln(e_s)) - d(\ln(\rho_{air}))$. Now, the Clausius-Clapeyron equation gives this $\mathrm{d}(\ln(e_s))$ that we expressed earlier. By Clausius-Clapeyron equation $\mathrm{d}(\ln(e_s))$. e_s is the saturation vapor pressure, right.

This equals to latent heat of vaporization, molecular mass of water by ideal gas constant. This becomes R of water basically equal to $\frac{dT}{T^2}$ which becomes, latency of vaporization by the molecular weight of water, sorry by the ideal gas constant of water, mass based gas constant of water R of water $\frac{dT}{T^2}$.

$$\mathrm{d}(\ln(e_s)) = \frac{L_v M_w}{\hat{R}} \frac{dT}{T^2} = \frac{L_v}{R_w} \frac{dT}{T^2}$$

So, this is the expression of $\mathrm{d}(\ln(e_s))$ that we will put here. $d(\ln(\rho_{air}))$, what is this? By hydrostatic balance relation, $d(\ln(\rho_{air})) = -\frac{gdz}{R_{air}T}$. So, this is another expression for the hydrostatic balance relation. So, basically here we have replaced the density using the ideal gas law.

This is basically, just to show the original hydrostatic balance relation is $g = -\alpha \frac{dp}{dz}$ and $g = -\frac{R_{air}T}{\rho} \frac{dp}{dz}$. So, here $\frac{dp}{p}$ is $d(\ln(\rho))$ and the rest is put in on this side. So, we get $-\frac{gdz}{R_{air}}$. So, we know, we have seen this expression before.

So, this is $d(\ln(\rho))$. This we will put here. Now, one point, since partial pressure of water vapor Es is much, much less than the total pressure P, this implies $d(\ln(\rho))$ is almost equals to P of log of the partial pressure of dry air. So, the dry air partial pressure is much, much greater than the saturation vapor pressure of water. So, one approximation that we are making is this $d(\ln(\rho\,))$ is almost equals to $d(\ln(\rho_{air}))$ which is the expression here. So, if we do that, this expression we can now, sorry, this expression we can now expand.

So, $\mathrm{d}(\ln(\omega_s)) = \frac{L_v}{R_w}\frac{dT}{T^2} + \frac{gdz}{R_{air}T}$. What is $\mathrm{d}(\ln(\omega_s))$? It's basically, $\frac{\mathrm{d}\omega_s}{\omega_s} = \frac{L_v}{R_w}\frac{dT}{T^2} + \frac{gdz}{R_{air}T}$. So, this kind of gives the rate of change of the humidity ratio as a function of change in temperature and change in altitude. Where we will use this? We will be using this expression here. So, we want to replace this d omega s term with this term.

Alright. By first law, $C_p^{air}dT + gdz + L_v d\omega_s = 0$. This implies, $C_p^{air}dT + gdz$. So, instead of $d\omega_s$, we put this expression. So, $C_p^{air}dT + gdz + L_v\omega_s\left[\frac{L_v}{R_w}\frac{dT}{T^2} + \frac{gdz}{R_{air}T}\right] = 0$

This is the relationship. So, what we have done here is we replace the d omega term in terms of dt term and dz term. So, now we will take all the dt terms together and all the dz terms together. So, we get or, $[C_p^{air} + \frac{{L_v}^2\omega_s}{R_wT^2}]dT + g[1 + \frac{L_v\,\omega_s}{R_{air}T}]$dz=0. Now, this entire expression was evaluated based on the saturation parcel of air expression. Saturated parcel of air is moving from z to (z+dz) under adiabatic condition.

So, here then the saturation adiabatic lapse rate Γ_s is $-\frac{dT}{dz}$ under saturation conditions and this $\frac{dT}{dz}$ if we take that will be this expression here, $\frac{g}{c_p^{air}}$. So, we are just taking this term on this side then dividing by dz and putting this term on the denominator on this side, $\frac{L_v\,\omega_s}{R_{air}T}$ divided by $1 + \frac{{L_v}^2\omega_s}{c_p^{air}R_wT^2}$.

$$\Gamma_s = -\frac{dT}{dz}|_{sat} = \frac{g}{c_p^{air}}\frac{1 + \frac{L_v\,\omega_s}{R_{air}T}}{1 + \frac{{L_v}^2\omega_s}{c_p^{air}R_wT^2}}$$

This is the expression we are getting. Remember, dry adiabatic lapse rate $\Gamma_d = \frac{g}{c_p^{air}}$. Final expression becomes,

$$\Gamma_s = \Gamma_d\frac{1+\frac{L_v\,\omega_s}{R_{air}T}}{1+\frac{{L_v}^2\omega_s}{c_p^{air}R_wT^2}}.$$

This is the expression for saturation adiabatic lapse rate in terms of dry adiabatic lapse rate and obviously, Γ_s is less than Γ_d. Remember, $\Gamma_d = \frac{g}{c_p^{air}} = 9.76\,K/km$. The other terms are $L_v \cong 2.5\times10^6\frac{J}{k}$, $c_p^{air} = 1.005\times10^6\,J/kgK$. $R_{air} = 287\,J/kgK$ is equal to 287 joules per kg Kelvin, $R_w = 461.5\,J/kgK$ and $g = 9.81 m/s^2$. So, this kind of gives us the saturation adiabatic lapse rate, the rate of change of temperature with altitude. One final derivation that we will do. In many meteorological and climatological applications, pressure, because pressure is changing consistently with altitude, we often use the, instead of z, we replace with pressure on the y axis. That is how the pressure isobars are changing with altitude, that becomes our measure of how the altitude is changing. So we can write, $\frac{dT}{dp} = \frac{dT}{dz} \times \frac{dz}{dp}$.

Now, from hydrostatic balance, $gdz = -R_{air}T\frac{dp}{p}$. Basically, this is d log P, the same expression just writing it this way. So, $\frac{dz}{dp} = \frac{-R_{air}T}{gp}$. This implies, $\frac{dT}{dp} = \frac{dT}{dz} \times \frac{-R_{air}T}{gp}$. So, the expression of change in altitude can be rewritten as an expression of change of temperature with pressure.

So, here this is the expression. So, $\frac{dT}{dp}|_{dry} = -\frac{dT}{dz}|_{dry}\frac{-R_{air}T}{gp}$. And this $-\frac{dT}{dz}|_{dry}$ is Γ_d. So, $\frac{dT}{dp}|_{dry} = \Gamma_d\frac{-R_{air}T}{gp}$. Similarly, $\frac{dT}{dp}|_{sat} = \Gamma_d\frac{R_{air}T}{gp}$. So, we can express the rate of change of temperature with pressure in terms of gamma d and gamma s also. And this term is often used in graphical plots in climatological and meteorological applications. So, how temperature is changing with the vertical variation of pressure.

So, I will stop here today. We will wrap up this part on stability in the next class and start in the next section. Thank you for listening and have a great day.