Course Name: An Introduction to Climate Dynamics, Variability and Monitoring

Professor Name: Dr. Sayak Banerjee

Department Name: Climate Change Department

Institute Name: Indian Institute of Technology Hyderabad (IITH)

Week- 03

Lecture- 14

DERIVATION OF POTENTIAL TEMPERATURE, DERIVING THE EXPRESSION FOR SATURATED ADIABATIC LAPSE RATE

Good morning class and welcome to our continuing course on climate dynamics, climate monitoring and climate variability. In the previous few lectures, we had been discussing about atmospheric stability, what is adiabatic lapse rate for dry conditions, what is adiabatic lapse rate for moist conditions And we looked at the various conditions which can create either unconditional stability, unconditional instability or conditional stability situations. So, today we will begin with a few extra derivations specifically regarding helping you to, helping to show how these things are arrived at. That is why is it that atmosphere becomes unstable when you have a specific type of lapse rate condition valid. We will also do an explicit derivation of the moist adiabatic lapse rate during the course of this lecture. So, let us begin our discussion today.

So, here we will look at a generic case. Suppose you have two levels, ok. So, this is an altitude Z1. This is at altitude Z2 here.

This is altitude Z. This is your ground level. Suppose we have a parcel of air. that is initially at altitude Z1 and due to certain dynamical conditions in the atmosphere this parcel of air moves to a new altitude Z2 through an adiabatic process that is there is no heat transfer between the air parcel and the surrounding during the course of this upward movement. Now initially at altitude Z1 this parcel of air was at equilibrium with its surroundings.

So at this altitude Z1 the corresponding temperature and pressure was T1 and P1. So the temperature of air at this altitude was T1, the pressure of air at this altitude was P1. At altitude z2, the corresponding temperature and pressure at this location is T2 and T2. However, because this parcel of air has moved swiftly to this new altitude Z2, it has not yet thermally equilibrated with its surroundings. So, this has a different temperature than P2.

So, let us say that the initial temperature and pressure of this parcel was also P1 and P1 and the final temperature and pressure of this new parcel here T prime and P prime okay and in general because this movement was quick and there is no heat transfer between the parcel and the surrounding air this T prime will not be equal to T2 okay so because the air parcel is had risen adiabatically, it is not in thermal equilibrium with its surroundings. and hence T prime is not equal to T2. In general, pressure equilibration happens much faster than thermal equilibration. Because this parcel of air is kind of just parcel of air within the surrounding, as it moves up, it is always matching the pressure of the surroundings. If it is not matching the pressure of the surrounding, it will just expand or contract to match that pressure.

So, since Pressure equilibration is much faster than thermal heat transfer. The parcel quickly attains the pressure of air at its new surroundings and hence P prime is equal So now, since we have an adiabatic process by the adiabatic relations derived earlier we have T prime P2 minus R by Cp, P2 to the power minus R by Cp equals to T1 P1 to the power minus R by Cp.

Hence $$\boxed{T' = T_1 \left(\frac{P_1}{P_2}\right)^{-R/C_p}}$$

See T P to the power minus R by C P is a constant in an adiabatic process. T prime and P2 are the final temperature and pressure of the parcel at Z2. So, these are put here and T1 and P1 are the final initial temperature and pressure of the parcel at Z1.

So, this is put here and so this relation must hold. Hence, T prime which is the temperature of the parcel at Z 2 will be equals to T 1 into T 1 by P 2 to the power minus R by C. So, this will be the temperature of the parcel at Z 2, temperature of parcel at Now, this is a dry parcel of air. So, this is a dry parcel of air. And suppose, see for every altitude we can define the potential temperature theta, correct? So, suppose we know that the potential temperature at Z1 is theta 1.

So, here the potential temperature is theta 1 and here the potential temperature is theta 2. This is for unsaturated conditions. So, potential temperature at unsaturated conditions. Now suppose we know the potential temperature of dry air at these two altitudes. as theta 1 is the potential temperature at Z1 and theta 2 is the potential temperature at Z2.

$$\Theta = T\left(\frac{p_0}{p_z}\right)^{R/C_p}$$

$$\Theta_1 = T_1\left(\frac{p_0}{p_1}\right)^{R/C_p} \quad \& \quad \Theta_2 = T_2\left(\frac{p_0}{p_2}\right)^{R/C_p}$$

Now, what is the definition of potential temperature of dry air? Theta is equals to T P0 by P at the corresponding altitude Z by R by C, right. So, this implies theta 1 is P1, P0 by P1 to the power R by Cp and theta 2 is T2 P0 by P2 to the power R by Cp, where T1 and the T2 are the temperatures at these two stations of the surrounding air, okay. Now, suppose we know that theta 2 greater than theta 1. That is the potential temperature at Z2 which is at the higher altitude is greater than the potential temperature of dry air at Z1 which is at the lower altitude. So, this means this expression is greater than this expression.

So, we get T2 P0 by P2 to the power R by Cp is greater than P1 P0 by P1 to the power R by Cp, which means T2 by T1 is greater than P2 by P1 to the power R by Cp. So, this is the expression we get if theta 2 is greater than theta 1. However, based on this expression here we can write T prime by T1 is equal to T2 by P1 to the power R by Cp. So, this two expressions together implies, hence T2 is greater than T prime if eta 2 is greater than eta1, clear? So, this expression, if the potential temperature at station 2 is greater than the potential temperature at station 1 for dry air, then the temperature of the surrounding air at station 2 is greater than the temperature that the parcel of air will have when it reaches that station 2. Hence, if theta 2 is greater than theta 1, then the temperature of the adiabatically rising parcel of air will be lesser than the surrounding air at Z2.

$$T_2\left(\frac{p_0}{p_2}\right)^{R/C_p} > T_1\left(\frac{p_0}{p_1}\right)^{R/C_p}$$

$$\Rightarrow \boxed{\frac{T_2}{T_1} > \left(\frac{p_2}{p_1}\right)^{R/C_p}} \quad \text{if } \Theta_2 > \Theta_1$$

Hence, it will be colder and denser than the surrounding air at Z2 and will tend to sink back down. Thus for dry air if theta 2 is greater than theta 1 for Z2 greater than Z1, then atmosphere is stable. So this is something that we asserted without demonstrating it in the previous class and now we are showing the demonstration. We have shown now that if the potential temperature gradient is such that for dry air it is increasing with altitude then we have an atmospheric stability condition field that is any adiabatically rising parcel of dry air will be colder than its surroundings and hence will tend to sink back down hence large scale convection currents cannot form. One important point, we have said that a colder parcel of air will be denser than its surroundings.

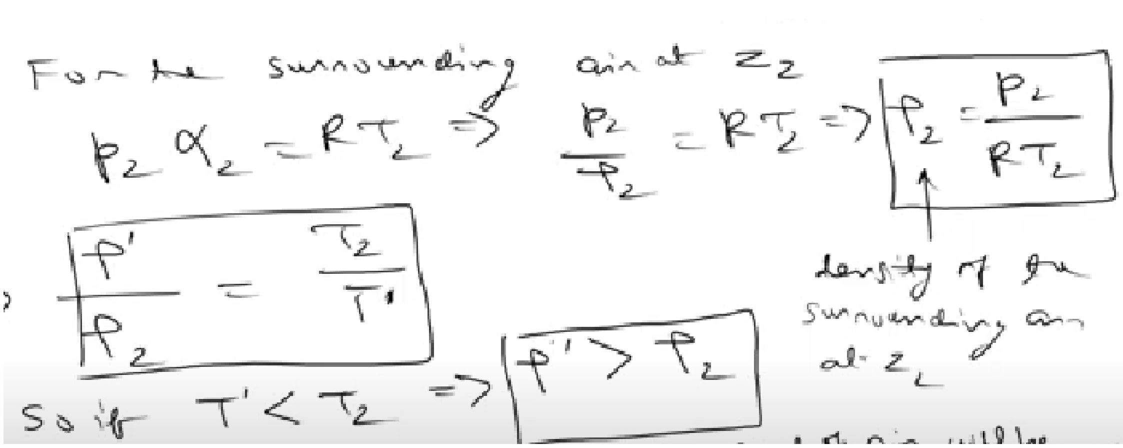
This can also be easily proved though it is intuitive. So, just for completeness, I will just prove this because the air is an ideal gas. So, at station 2 for the parcel of air we have P2 alpha 2, alpha 2 is a specific volume if you remember is equals to RT prime, implies P2 by rho 2. Remember, density is the inverse of specific volume alpha. So, we are writing P2 by rho 2 equals to R theta.

So, this implies that rho 2, if you go back, you take rho 2 here, it is P2 by RT prime equals to P2 by RT prime. Sorry, let us go back. I made a mistake. This will be alpha prime, the specific volume of that parcel of air. And this will be rho prime, the density of that parcel of air as it has moved adiabatically to the station Z2.

This need not be equal to the specific volume or density of the surrounding air. In fact, it will not be. That is why we are keeping alpha prime, alpha as alpha prime and rho as rho prime. The pressure is the same between the surrounding and the parcel of air, but temperature and density values will be different. So, we are keeping the prime symbol here.

So, this expression also becomes rho prime equals to P2 by RT prime. So, this is the density that the parcel of air will have at z2. Now, for the surrounding air at Z2, it is P2 alpha 2 equals to RT2 which implies P2 by rho 2 equals to RT2 which implies rho 2 equals to P2 by RT. So, this is the density of the surrounding air at Z2. So, this implies rho prime rho 2.

So, rho prime by rho 2, P2 P2 cancels out, R R cancels out, gas constant for air. So, this will be equal to T2 by T prime, correct. So, if T prime is less than T2 as has been shown, this implies rho sorry rho prime is greater than rho 2. So, parcel of air will be denser than surrounding. So, we have derived this point as well just using ideal gas law.



So, here the main consideration is the pressure is the same between the parcel and the surroundings. The temperature and density therefore tracks the relationship of the adiabatic processes. And based on that we can derive all these relations of what will be the stability condition, what will not be the stability condition etc. So, next up we will start the derivation today, we will continue in the next class and this is about we have evaluated the lapse rate for dry air. We have not derived the lapse rate for the saturated parcel of air and we will do that as well.

So, we will start the derivation today and we will continue in the next recording. So, next set is deriving the expression for saturation adiabatic lapse rate. So, here we are looking at a small change. So, we have a saturated parcel of air at a location z. This parcel of air rises adiabatically to another location just a little bit above this which is at z plus dz.

Very simple z plus dz, let us call it del z for now. No, let us just do dz ok, alright. Now, this is a saturated parcel of air ok. The amount of water vapor present in the saturated parcel of air at z is given by omega s, ok. The saturated specific humidity, saturated specific humidity, ok.

So, just remember, just for recall. omega s is equals to mass of water vapor at saturation condition by mass of dry air ok. This is also called humidity ratio or humidity ratio. Better to write this as humidity ratio only for now.

Saturation humidity ratio. basically, m of water under saturation conditions by m of air, m of dry air. So, next because of this increment there are changes in temperature and pressure. So, what you will get is that the omega s at this station will be different from omega s at z. So, change in humidity ratio due to rise of this saturated parcel is d omega s equals to omega s at z plus dz minus omega s at z. So, because the parcel has risen a small amount, saturation humidity ratio has changed ok it has changed from the original value which was omega s value at the altitude z to the omega s value at altitude z plus dz now remember here because the temperature is changing here ok so here this is Tz Because it has moved upwards, there is a change in temperature.

So, that is why your parcel of air's temperature has changed and hence its saturation specific humidity value itself has also changed. So, we will explore those things and that will be based on what we will discuss on saturation adiabatic lapse rate. So, we will continue this work in the next class, how the explicit derivation happens. So, stay tuned for the next recording. Thank you.