Course Name: An Introduction to Climate Dynamics, Variability and Monitoring

Professor Name: Dr. Sayak Banerjee

Department Name: Climate Change Department

Institute Name: Indian Institute of Technology Hyderabad (IITH)

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Lecture- 11

Virtual Temperature And Atmospheric Stability Concepts

Good morning class and welcome to our continuing lectures on climate dynamics, variability and monitoring. So we will continue our discussion on atmospheric humidity and conclude it in today's lecture. In the previous lecture, we looked at relative humidity, mixing ratio and specific humidities and saw how humidity is changing with both altitude and with latitudes. We also discussed qualitatively how a small change in the global air temperature can cause a large change in the amount of water vapor that air can carry. which has a very important effect on the global warming considerations because water vapor is a strong greenhouse gas. Here, in today's class, we will discuss another concept that is often used when we are dealing with moist air.This is the concept of a virtual temperature. So, The specific ideal gas constant, sorry, the ideal gas constant for air is 287 joules per kg Kelvin. This you are getting by dividing the universal gas constant by the molecular mass of air in kgs per mole. For water, the ideal gas constant is 461.5 joules per kg kelvin on a mass basis. This you are getting by dividing the ideal gas constant 8.314 by the molecular mass of water which is 18 grams per mole. So this is the gas constant for air, this is the gas constant for water. Then the ratio of these two gas constants R air by R of water, we call it this ratio as epsilon is 287 by 461.5, which is 0.622. And you can see here, that the relationship between the specific the humidity ratio or the mixing ratio and the ratio of the partial pressures of water vapor and dry air has this r air by r water ratio built into it. This is the 0.622 that is the proportionality constant.

$$\varepsilon = \frac{R_{air}}{R_w} = 0.622 \quad (30)$$

So now, let us consider a volume of moist air at a certain temperature T and the total pressure is P for this system. Usually P is in capital, so I will write this as capitals. And this volume of moist air which is at temperature T and total pressure P contains M suffix air of dry air and Mw mass of water vapor. So the total mass M is M of dry air plus M of water vapor, M air plus Mw. The density rho of this moist air is then M air plus Mw by V.which is the partial density of dry air m air by v which is rho prime of air plus partial density of water vapor which is mw by v which is rho prime w. So, the total density is the sum of the partial densities of dry air and partial density of water vapor. From ideal gas law, the partial pressure of dry air is the partial density of dry air into the gas constant of air into T. And the partial pressure of dry air is the total pressure minus the vapor pressure E. Remember the total pressure is equal to the partial pressure of dry air plus the vapor pressure. So we are replacing P air by P minus E. So, rho prime air is P air by R air into T which is equals to P minus E by R air into T. And the partial density of water vapor is the vapor pressure which is e by the gas constant Rw into T. So rho prime R is this expression and rho prime W is this expression. So we can put these two expressions in the expression for rho. So rho becomes P minus e by R air into T plus e by R W into T. Now what we do is we take P by R air into T as a common term and take it out. Then this becomes 1 the first term here p by rar into t this becomes 1 then what we get is minus e by p okay so this is minus e by p all right then the second term is e by p into Rw by Rair which is epsilon okay so it becomes e by p into epsilon So, if you take P by R air into T out within brackets then the terms that remain here can be written as 1 minus E by P into 1 minus epsilon.

$$\rho = \frac{m_{air} + m_w}{\forall} = \rho'_{air} + \rho'_w$$

By ideal gas laws $\rho'_{air} = \frac{P_{air}}{R_{air}T} = \frac{P-e}{R_{air}T}$ and $\rho'_w = \frac{e}{R_w T}$

Hence, we get

$$\rho = \frac{P-e}{R_{air}T} + \frac{e}{R_w T}$$

$$= \frac{P}{R_{air}T}\left[1 - \frac{e}{P}(1-\varepsilon)\right] \quad (32)$$

So, this is the expression we are getting for the density of the total moist air parcel. Now, what we do is that this is the like the ideal gas law.when we are taking our air into the picture. The problem here is, since the parcel of moist air has variable water content, its mass-based specific ideal gas constant changes with the water content. So, while a parcel of moist air will have the relation PV equals to RT that R term is the ideal Gas constant by the molecular weight of that moist air and the molecular weight of the moist air is molecular weight of dry air plus the molecular weight of into the mass mole fraction of dry air plus the molecular weight of water vapor into mole fraction of water vapor. This we discussed when we are looking at the composition, but the quantity of water vapor and hence the mole fraction of water vapor is changing with temperature, correct. So, that R term is not a constant. So, which makes it difficult to use the PV into RT relation directly when it comes to a parcel of moisture. Instead what we do, we use this expression. So, what we say is that this term here is some virtual temperature Tv and everything else is the same. So, we write P equal to rho into the gas constant for dry air, R air which is common which is 0.287 kilojoules per kg kelvin, correct? joules 287 joules per kg Kelvin or 0.287 kilojoules per kg Kelvin whichever you want to do into a virtual temperature Tv which is written as T by this expression 1 minus e by P into 1 minus epsilon.

$$\boldsymbol{T_v = \frac{T}{\left[1 - \frac{e}{P}(1-\varepsilon)\right]}} \quad (34)$$

So, we create an effective temperature for the moist air parcel expressed in this format so that we can write the ideal gas relationship for the moist air parcel in terms of this virtual temperature and the gas constant for dry air itself. So this is a very common way in which moist air parcels temperatures are presented which is why we are showing it here. So here this Tv will change as the e the vapor pressure of water vapor. Vapor pressure is changing with temperature within the air parcel.

So even though the temperature may remain the same as your vapor pressure increases this term decreases and you are getting the virtual temperature as greater than the actual temperature. So virtual temperature is always greater than the actual temperature based on this expression here. And we can also understand this point because moist air is less dense than dry air at the same temperature and pressure. So to balance this aspect, your virtual temperature will be greater than the actual temperature when you are using R air instead of traditional R of the system. And you can also see here that the virtual temperature will be maximum when e equals to es. Okay. So, if you do a Tv minus T, the difference between virtual temperature and the actual temperature, that difference will be maximized when the air hits its saturation point, when vapor pressure becomes equal to the saturation vapor pressure at a given pressure. All right. So how is this used? So for example, a meteorological station or an instrument can measure the temperature of air, the pressure of air, and you can condense, you can cool the air and condense the water out of it to measure the mix humidity ratio or the mixing ratio of that air. Based on that, you can evaluate the relative humidity And hence the vapor pressure of that particular amount of air at that given temperature and pressure. So, you can evaluate the e. Once you have evaluated the e, you can report the virtual temperature for that particular parcel of air. So, this is how this expression can be evaluated. So, next we will discuss a worked out example where we use these various aspects of atmospheric humidity relations that we discussed today. Suppose, we have a parcel of air with relative humidity H equals to RH equals to 0.5. So, what this tells us is that the vapor pressure is half the saturation of vapor pressure at this condition and temperature is 30 degree centigrade for this parcel of air. What is the vapor pressure? That is the first question. Then the next question is, what is the mass of water in 1 kg of this air? So, what is the amount of water present in 1 kg of this parcel of air with relative humidity 50 percent and temperature of 30 degree centigrade? And final question is what is the virtual temperature for this parcel? So, three questions. So, now we know the temperature is 303 kelvins which is equals to 30 degree centigrade ok. So, saturation vapor pressure e s is equals to 611 exponential L v by R of water.into 1 by 273 minus 1 by 303. Now, R of water is 8.314 which is the universal gas constant divided by 0.018 kgs per mole Kelvin and this is joules per mole Kelvin. So, we get joules per kg Kelvin here which is equals to 461.5 joules per kg Kelvin ok. latent heat of vaporization we value we told earlier it is 2.5 into 10 to the power 6 joules per kg. So, if we put these expressions in es we get equals to 611 exponential of 1.9646 equals to 4357.7 Pascals. So, this is the saturation vapor pressure at this temperature. But your relative humidity is 0.5, so your e is half of this. So, 2178.84 pascals, which is often also written as 21.7884 millibars. Okay. Remember, 1 millibar equals to 100 pascals. So, either you can write it in pascals or in millibars. So, this is the first part of your answer. Okay. Next, we have to find the mass of water vapor in 1 kg of air.Okay. Here, the humidity ratio omega equals to m of water by m of dry air which is equals to 0.622 e by P of dry air. But P equals to P of air plus e. Here pressure is sea level.So P is equals to 101325 Pascals. So this implies P of dry air is P minus e which is 101325 Pascals. minus 2178.84 which is equals to approximately

99146 Pascals, this is your P of dry air. So, we know the P of dry air, we know the e values here. So, this becomes equals to 0.622 into 2178.84 divided by 99146 so this becomes equals to 0.013669 okay so we get omega which is mass of water by mass of dry air equals to 0.013485 not 6 6 9 sorry hold on yeah yeah correct now specific humidity q is mass of water by the total mass which is given by omega by 1 plus omega.

In general, q is almost equals to omega. And we can show this in this context here also that q here is 0.013669 by 1 plus 0.013669 which is equals to 0.013485. So, the fraction of water vapor by mass by the total mass of air is 0.013485. So, 1 kg of moist air will contain mw equals to 1000 into cube equals to 13.48 grams of water weight. So, this is the quantity of water vapor present for this parcel of moisture 13.48 grams at 50 percent relative humidity. The last part is we have to find the virtual temperature.is T v equals to T by 1 minus E by P 1 minus 0.622. Remember epsilon is 0.622 which is the molecular weight of air by molecular weight of water. or sorry this is incorrect, this is molecular weight of water by molecular weight of air. So, this we have derived just right now. So, we can put T as 303 which is 30 degree centigrade, 1 minus e we have seen this is 2178.84 Pascals, the total pressure is 101325 Pascals and this is 0.378. So, expand calculate and you get So, at this temperature and relative humidity the virtual temperature for this moisture parcel is around 305.48 Kelvin, so 5 degrees difference. So, this kind of gives you an idea of how to use all these expressions to evaluate the amount of water vapor in a certain percent of air, how to evaluate the saturation vapor pressure and use all of these calculations. So, with this we finished our discussion of humidity. Next we will look at the idea of stability of the atmosphere which primarily is telling us how stable our atmosphere is going to be depending on its conditions. What do we mean by stability? Stability means is related to the probability of weather instabilities like storms, precipitation events, etc. happening given a certain condition of the atmosphere. So if the atmosphere is stable, what we get is fair weather conditions. That is less chance of precipitation, storms. So you will have sunny, fair days. Whereas an unstable atmosphere is likely to generate storm fronts, thunderstorms, cyclones, general precipitation events. So stability and instability criteria is very important in determining what is the weather of a certain area is going to be. And how climate change is perturbing the stability or instability criteria is an important aspect of understanding the probability of increased precipitation events or decreased precipitation events in certain parts of the world. So, first we will in this class we will just focus on the very basic generalities. The basic idea is earth surface is heated to different degrees depending on the latitude, longitude, altitude, propensity towards coastlines, etc. So, for example, near the equator the sun's rays are nearly vertical and you are getting much larger amounts of solar energy per meter square of land area compared to away from the equator. So, we will discuss this a little bit in more detail later when we look at the latitude effects on the solar radiation per unit area of land surface.

Because of this areas near the equator or land generally becomes hotter much more faster than areas away from the equator. So, we will get a warm surface and a cold surface and at temperature gradient from the warm surface to the cold surface. The air above the warm surface absorbs this heat and becomes hot. By ideal gas law, the density of a hot parcel of air will decrease as it will expand. As a result, because remember, and the pressure also will decrease as a result. So the air above a hot surface will be at a lower pressure compared to air above a colder surface so there will be a horizontal pressure gradient which will drive the movement of air mass from the high pressure area in over the cold surface to the low pressure area over the hot surface this is the surface level winds that we see on earth which are primarily driven by pressure gradients especially near the equator. So, this pressure driven winds will create a movement of air from the high pressure zone towards the low pressure zone. Furthermore, the hot and lighter air above the warm surface will have buoyancy and will tend to rise upwards. So, there will be a flow upwards creating a kind of a lower vacuum and low pressure region which will be filled by air moving from the colder regions of the world. As the air moves upwards, and we will see this, it will start to cool down. So, we will have explicit expressions of the rate of cooling that such an air parcel will undergo and how that rate of cool is affected by the amount of moisture present in that air parcel as well. So, as it cools down, what will happen? Its ability to hold water vapor will decrease. Remember, the saturation vapor pressure decreases exponentially with decreasing temperature. So, the air parcel will reach its saturation state and as it cools further, water will be condensing out of this air parcel creating clouds. And these clouds, if they accumulate long enough, will generate precipitation events above this warm surface.

The cold air, the air that has moved up here will generate a slightly higher pressure here and the air that is coming down here because of the dense air coming down will generate a low pressure zone here. So, you will have a counter wind flowing in the upper atmosphere from above the warm surface to above the cold surface. So, there is one horizontal movement from the cold to warm near the surface and a counter horizontal movement from warm to cold area in the upper atmosphere. And there is a descent of cold air over the cold surface and an ascent of warm air over the hot surface. And this ascent of warm air creates the condensation process, cloud formation process and precipitation process. Very generally, regions where this type of a convection cell has a propensity to develop will see a large amount of cloud formation and precipitation events. Whereas regions of the world where such convection or buoyancy effects are suppressed, there we will have fair weather events because there will be no vertically rising mass of air which can condense and create clouds. So, atmospheric stability and instability can be qualitatively understood as the idea that in a stable atmosphere, this buoyancy induced circulation currents are not prevalent, whereas in unstable atmosphere, this buoyancy induced circulation and the associated cloud formation is prevalent. So, we will discuss more of this in the next class. So, thank you for listening and see you later.