Finite Element Analysis and Constitutive Modelling in Geomechanics
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Lecture-33
Stress Correction Procedures in Finite Element Analysis

I hope all of you are doing well in the course; you are able to understand and also do your tutorials and other problems. And if not you just send an email to this email address and then I will try to help you to the extent possible. So, in today's class let us look at some new aspect, it is a small trick to do the stress correction after we determine the stresses as per the applied loading. We have seen that the yield function exceeds the yield limit.

It becomes greater than 0 that we have seen even both with bilinear elastic models and also with hyperbolic models. Then what do we do? Especially if you use a very coarse strain increment the F value shoots up a lot and that is what we are going to do in this today's class.

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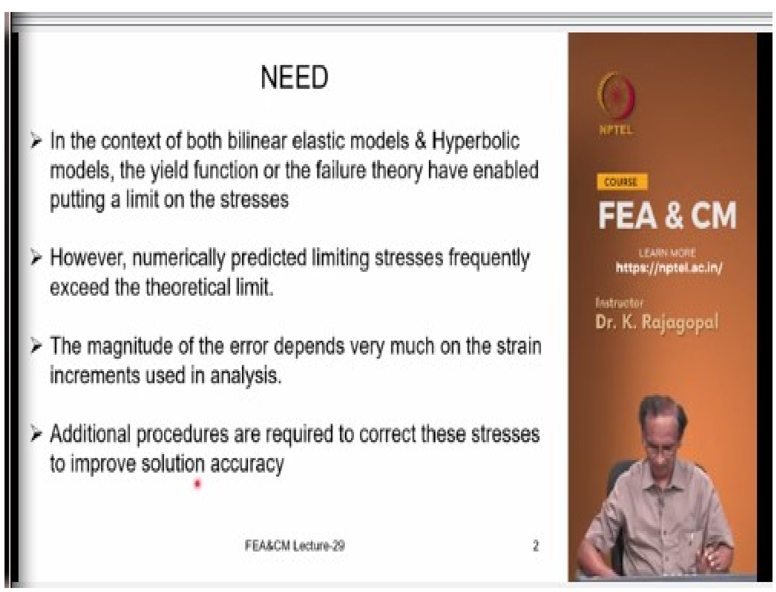

Let us see what is the need for doing the stress correction. The context of both bilinear elastic and hyperbolic models the yield function has enabled us to put a limit on the stresses. Say this sigma 1 within a triaxial compression test; we know that it should be equal to K p times sigma 3

+ 2 c square root of K p. And we were able to put a limit on this through the use of our Mohr-Coulomb relation.

But then the numerically predicted limiting stress is frequently exceed our theoretical limit. Our sigma 1 f we calculated from the Mohr circle and then the yield surface is only a theoretical limit. But then when we try to simulate the test through the finite element analysis we may or may not exactly replicate and that depends on several constraints like our strain increment. And the magnitude of error that is the difference between the theoretical limit.

And then the predicted stress limit, it depends very much on the strain increments used in the analysis. If we use a very small strain increment our result will be good very close to the theoretical limit but if you use a very coarse strain increment there could be lot of difference. So, we need to resort to some additional methods to correct or to bring the stresses back to the yield surface, so that we are always satisfying the yield limit or the yield condition.

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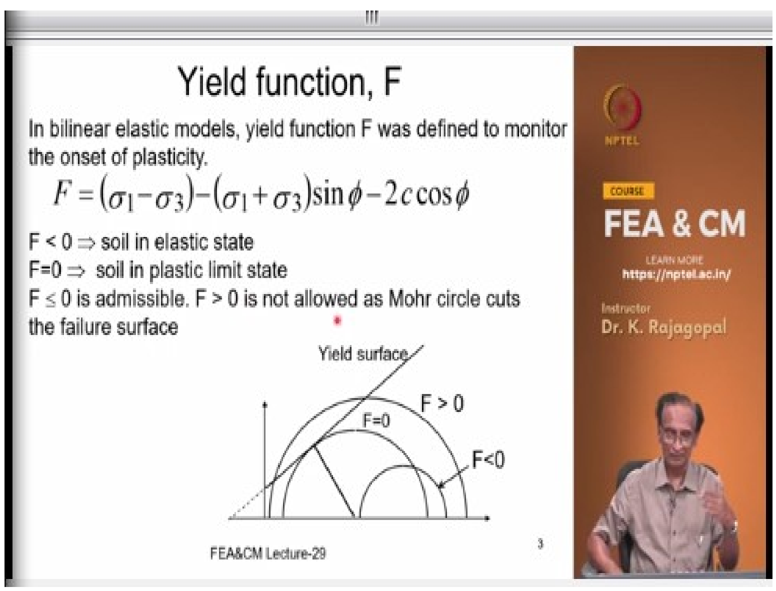

And we have seen the Mohr-Coulomb yield surface with c and phi properties and then we also know how to draw the Mohr circle. Say if the more circle is entirely within the yield surface, the yield function value will be less than 0 and we say that is an elastic state and if it is exactly tangent to the yield surface the F value will be 0 and the soil is at the limit state. And when F is greater than 0 our Mohr circle is going to intersect the yield surface and this is not permissible.

So, whenever this happens because of our numerical approaches we should somehow change the stresses such that we get back to this state where our Mohr circle is exactly tangent to this yield surface. So, we know what is the yield function F; we defined it in terms of sigma 1 and sigma 3 and then the c and phi properties like this. And if F is less than 0 we are in the elastic state and when F is exactly 0 our Mohr circle is tangent to the yield surface.

And F greater than 0 means it is exceeding the yield surface and this is not allowed, we need to do something. So, when F free is greater than 0 it is not allowed as the Mohr circle cuts the failure surface and then we do some stress correction.

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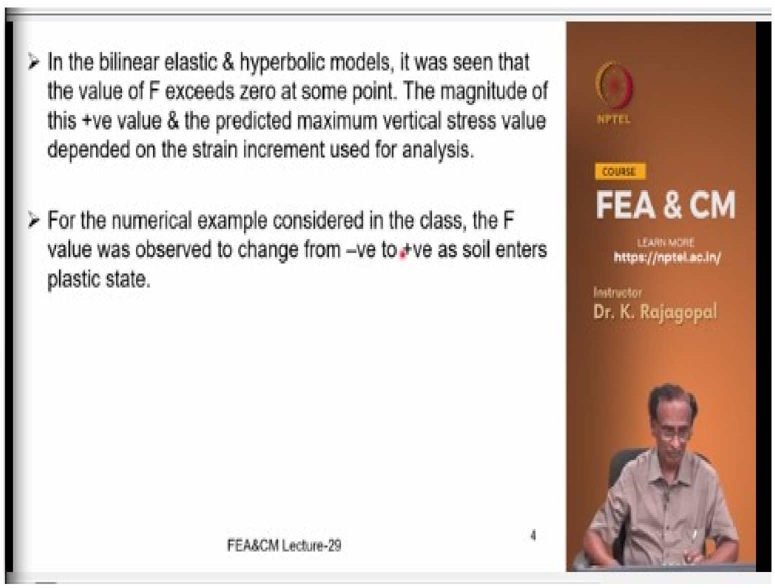

And in both the bilinear elastic and the hyperbolic models we have seen that the yield function value exceeds 0 at some point and the magnitude of this positive value and the predicted maximum vertical stress depends on the strain increment used in the analysis. And the value of F we have seen in our hand calculation that suddenly jumps from negative to positive. So, it initially the F value it has a very large negative value and as you go on increasing the deviated stress.

The F value will slowly increase from -300 to -250, -200 is something like that. Then suddenly it will become positive, at some state when the soil enters the plastic state or the yield limit state.

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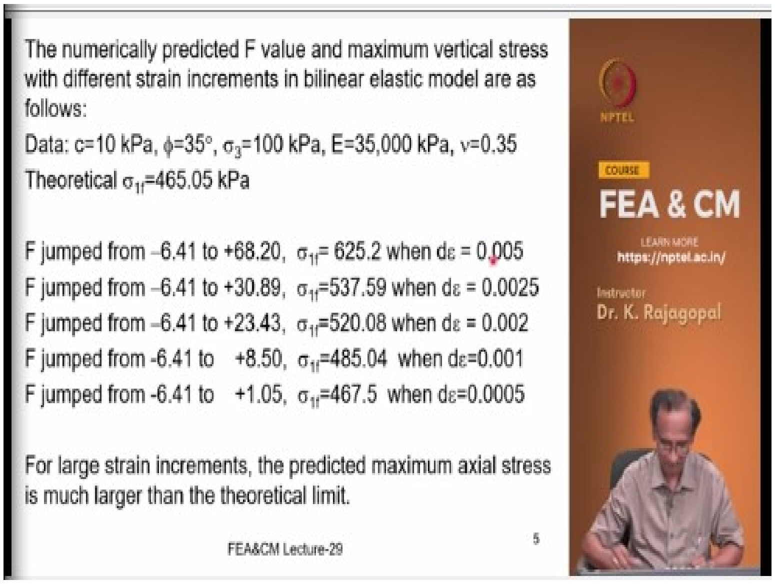

So, that is what we are going to do and before that let me just illustrate. See for the data of c of 10 kPa friction angle of 35 degrees and the confining pressure of 100 kPa our theoretical limit on the vertical stress is 465. And when we did this triaxial compression test with an axial strain increment of 0.005 the F value initially it was a very large negative value then it goes on increasing, then at some point from -6.41 it jumps to +68.20.

The numerically predicted F value and maximum vertical stress with different strain increments in bilinear elastic model are as follows:

Data: c=10 kPa, ϕ=35°, σ_3=100 kPa, E=35,000 kPa, ν=0.35

Theoretical σ_{1f}=465.05 kPa

F jumped from –6.41 to +68.20, σ_{1f}= 625.2 when $d\varepsilon$ = 0.005

F jumped from –6.41 to +30.89, σ_{1f}=537.59 when $d\varepsilon$ = 0.0025

F jumped from –6.41 to +23.43, σ_{1f}=520.08 when $d\varepsilon$ = 0.002

F jumped from -6.41 to +8.50, σ_{1f}=485.04 when $d\varepsilon$=0.001

F jumped from -6.41 to +1.05, σ_{1f}=467.5 when $d\varepsilon$=0.0005

Because as long as your yield function value is less than 0 or negative we treat the soil as in the elastic state and our Young's modulus will be very large. Like for example here is 35000 and so your stress increment also could be large and because of that your F value increase to 68.2 and that is also because our strain increment is very large. And the predicted vertical stress at the limit state is 625.2 against the theoretical limit of 465.

And if we reduce the strain increment to 0.0025, the predicted yield stress is 537 which is better but not close enough to 465. And if we further reduce the strain increment to 0.002 the sigma 1 F has become 520. And that very low strain increment of 0.0005 this sigma 1 predicted is 467.5 which is comparable to 465. So, within some limit we are able to predict the desired yield stress and then if you do any analysis with this type of increment, we may expect to get reasonably good results either for limiting bearing capacity or lateral earth pressures and so on.

That we will see later but for now I am demonstrating this only with respect to triaxial compression test because that is the simplest one that we can think of and we can also do the analysis by hand calculations. Because unless you understand hand calculations you will not appreciate what is actually happening inside the computer program. And that is the reason why I am giving you all examples with simple hand calculations.

But that is exactly what the finite element program also does but it is inside a computer program, so we will not know exactly what the program is doing. So, for very large strain increments the predicted the maximum axial stress is much larger than the theoretical limit. So, we should not expect to get good results when we use very larger strain increments.

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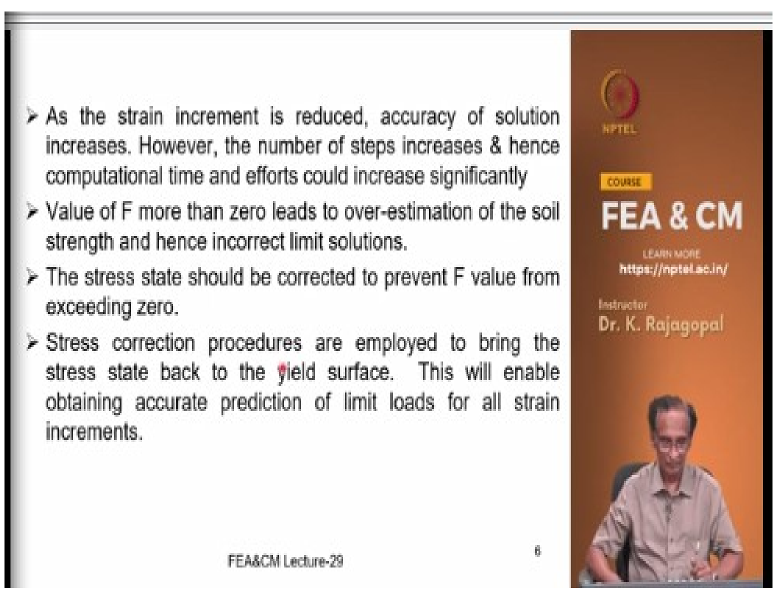

So, as the strain increment is reduced, the accuracy of solution increases but then the number of load steps of the number of times you repeat the analysis increases. And that adds to the computational time and then the computational effort. So, instead of taking 1 hour for performing a numerical analysis you may take 10 hours or even 24 hours depending on the strain increment.

And the value of F more than 0 leads to overestimation of the soil strength and hence incorrect limit solutions. So, the stress state should be corrected back to 0, so that our theoretical limits are correctly estimated by the numerical procedures. So, we should employ some stress correction procedure to bring the stress state back to the yield surface.

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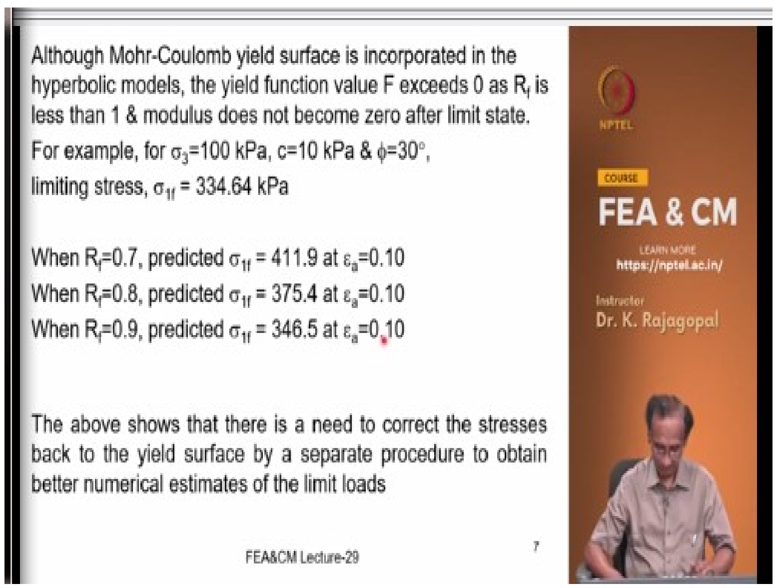

And we have also seen the same problem with hyperbolic models although the Mohr-Coulomb yield surface was implemented in the equation for the tangent Young's modulus. But because our R f value is not exactly 1, so our E tangent is never 0, it is always it has some positive value. And because of that with every strain increment your stress is going to increase and that is what we have seen.

Although Mohr-Coulomb yield surface is incorporated in the hyperbolic models, the yield function value F exceeds 0 as R_f is less than 1 & modulus does not become zero after limit state.

For example, for σ_3=100 kPa, c=10 kPa & ϕ=30°,

limiting stress, σ_{1f} = 334.64 kPa

When R_f=0.7, predicted σ_{1f} = 411.9 at ε_a=0.10

When R_f=0.8, predicted σ_{1f} = 375.4 at ε_a=0.10

When R_f=0.9, predicted σ_{1f} = 346.5 at ε_a=0.10

So, with different R f values, so for this sigma 3 of 100 c of 10, phi of 30 degrees the sigma 1f is 334. And when R f is 0.7 the predicted sigma 1f is 411 with 0.8, 375, 0.9 it is 346, all at an axial strain of 0.1 but these values could increase at a larger strain increment because our stresses will go on increasing asymptotically in a hyperbolic model. So, we need the some external correction method that will correct the stresses back to the yield surface.

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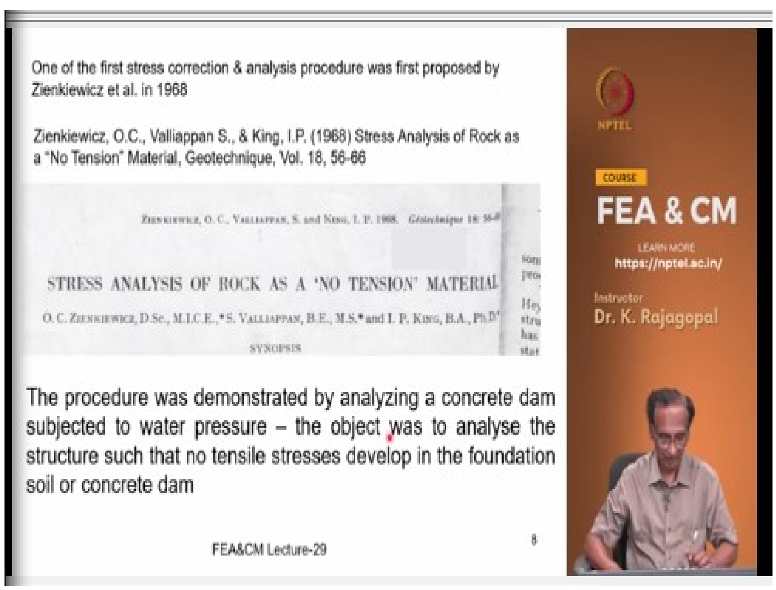

And one of the earliest methods that is other than a linear elastic method was proposed by Zienkiewicz and others in 1968. They published a paper on the stress analysis of rock as a no tension material, as a no tension material means it cannot carry any tensile stresses. So, you perform the analysis such that all the stresses within the body are compressive and this is the scan copy of the cover page of the paper.

It is one of the earliest papers and they suggested a method for achieving our stresses within the compression space and also how to satisfy the equilibrium. Because the main problem is when you change any stresses your equilibrium is not satisfied, so we have to satisfy both the equilibrium and also the yield surface in all these problems. So, they proposed a very simple method that is what we are going to see in this class.

See the procedure whatever they developed it was applied for an analysis of a concrete dam subjected to water pressure. And the object was to analyze the structure such that no tensile stresses develop either in the concrete dam or in the foundation soil.

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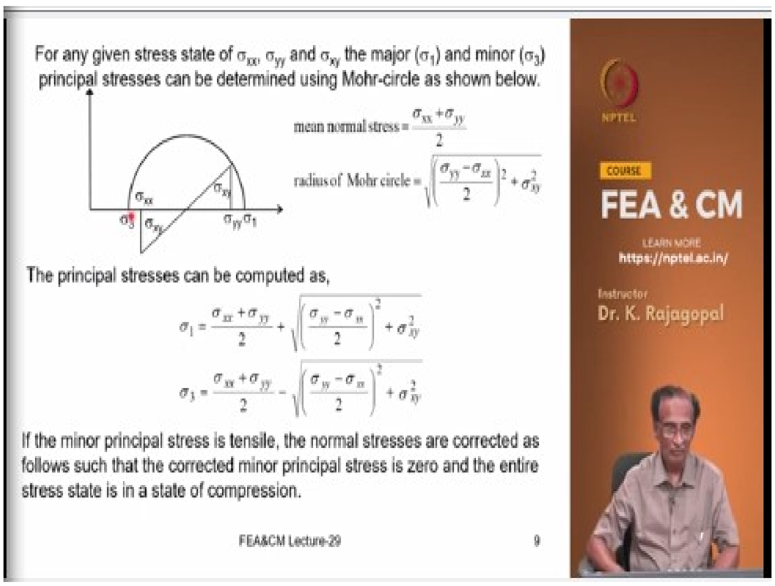
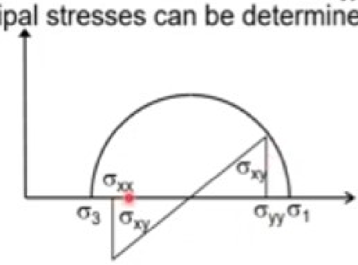


See the analysis is very simple; let us consider only two dimensional stress state, so that it is more easy to demonstrate. But whatever we are discussing for 2D you can also extend to the three dimensional problems but it becomes slightly more complicated. Let us say that we have a stress state sigma xx, sigma yy and sigma xy and we know how to calculate the principal stresses by constructing the Mohr circle.

For any given stress state of σ_{xx}, σ_{yy} and σ_{xy} the major (σ_1) and minor (σ_3) principal stresses can be determined using Mohr-circle as shown below.



$$\text{mean normal stress} = \frac{\sigma_{xx} + \sigma_{yy}}{2}$$

$$\text{radius of Mohr circle} = \sqrt{\left(\frac{\sigma_{yy} - \sigma_{xx}}{2}\right)^2 + \sigma_{xy}^2}$$

The principal stresses can be computed as,

$$\sigma_1 = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \sqrt{\left(\frac{\sigma_{yy} - \sigma_{xx}}{2}\right)^2 + \sigma_{xy}^2}$$

$$\sigma_3 = \frac{\sigma_{xx} + \sigma_{yy}}{2} - \sqrt{\left(\frac{\sigma_{yy} - \sigma_{xx}}{2}\right)^2 + \sigma_{xy}^2}$$

So, sigma y is more than sigma x, so we can plot sigma y and sigma x and then the tau xy is positive on the horizontal surface on which sigma y is acting. And then sigma xy on the vertical axis, on the vertical plane is negative, so we can plot sigma yy, sigma xx, sigma xy and then we can join these 2 lines. And then the center point is your mean normal stress or sigma xx + sigma yy by 2.

And then by taking this as the radius we can draw a circle and we get your 2 principal stresses sigma 1 and sigma 3. See this is the geometric way of interpreting, so the mean normal stress here is sigma xx + sigma yy by 2 and the radius of this Mohr circle is sigma yy - sigma xx by 2, that is this half length square plus this square sigma xy square and the square root of that will give you this radius.

And once you get the radius your sigma 1 and sigma 3 can be obtained as sigma 1 is the mean normal stress plus the radius that is sigma 1 and then sigma 3 is mean normal stress minus radius. And in all our hand calculations we are going to take compressive stresses are positive and the negative stresses are as a tensile. So, if your minor principle stress sigma 3 is tensile then we have to do some correction such that all the calculated the principal stresses are compressive and that we can illustrate like this.

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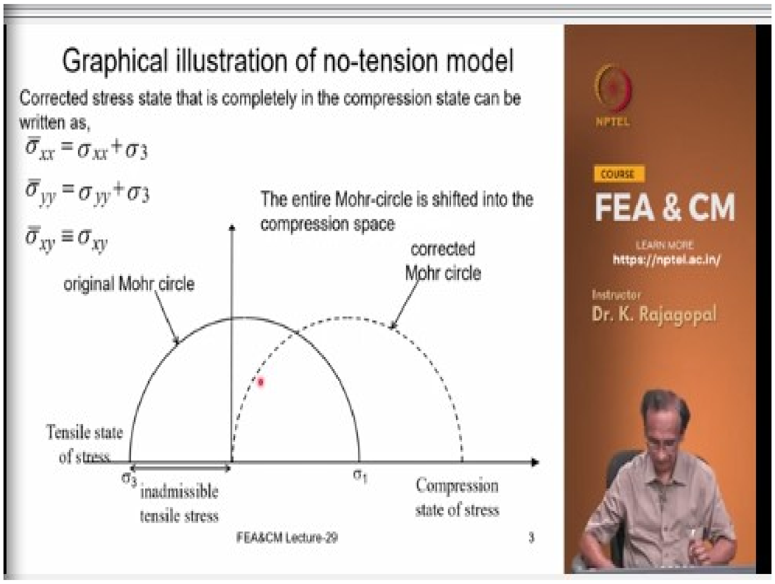
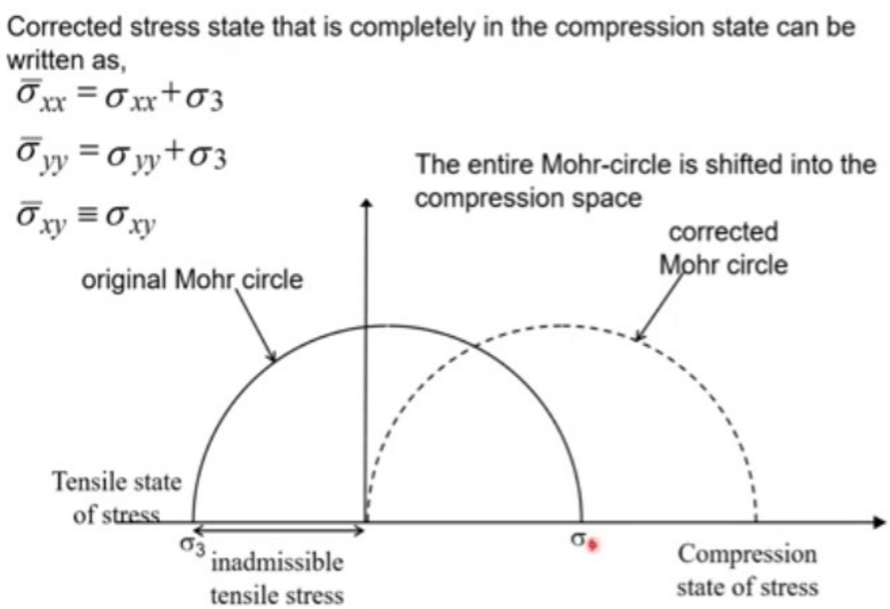


Let us say that when you calculated your principal stresses for the given stress state of sigma xx, sigma yy and sigma xy that you got a Mohr circle something like this, sigma 1 is a compressive and sigma 3 is tensile stress. And one way is that just simply push the entire Mohr circle to the compression space. And this sigma 3 is the inadmissible tensile stress because our object is to not have any tensile stresses; all the stresses should be only in compression.



And we can do this lateral translation by adding sigma 3 to sigma xx, so the modified normal stresses sigma xx bar is sigma xx + sigma 3 and sigma yy bar is sigma yy + sigma 3 and the shear stress remains the same because the radius of the Mohr circle has not changed, it is only the radial shift or the translational shift. And this is a simple method that will help us in making sure that all the stresses are within the compression space.

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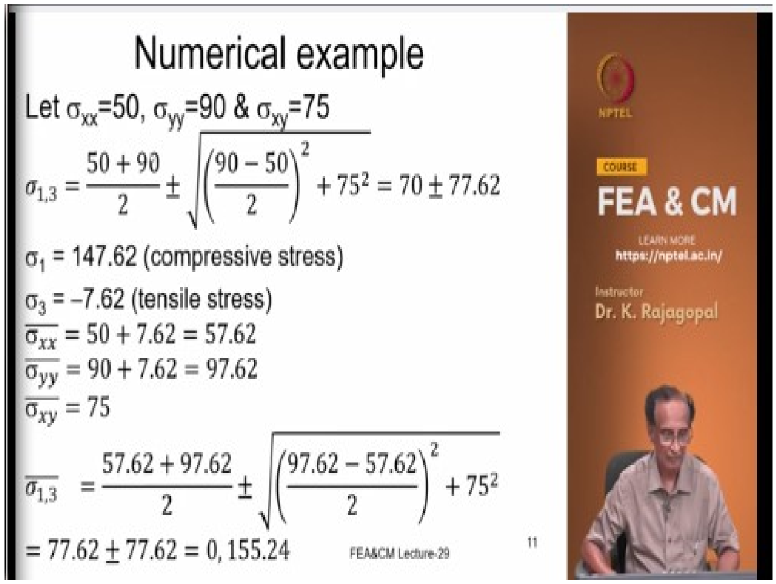

Let us look at a numerical example, our sigma x is 50, sigma y is 90 and tau xy is 75. And if you calculate sigma 1 and sigma 3, sigma 1 is 147.62 that is compressive, sigma 3 is -7.62 that is tensile. So, we should correct the stresses such that we do not end up with any tensile stresses, so our sigma xx bar is 50 + 7.62 that is 57.62. And then sigma yy is 90 + 7.62 that is 97.62, sigma xy remains the same and sigma corrected 1 and 3 are 0 and 155.24.

Numerical example

Let σ_{xx}=50, σ_{yy}=90 & σ_{xy}=75

$$\sigma_{1,3} = \frac{50+90}{2} \pm \sqrt{\left(\frac{90-50}{2}\right)^2 + 75^2} = 70 \pm 77.62$$

σ_1 = 147.62 (compressive stress)

σ_3 = −7.62 (tensile stress)

$$\overline{\sigma_{xx}} = 50 + 7.62 = 57.62$$

$$\overline{\sigma_{yy}} = 90 + 7.62 = 97.62$$

$$\overline{\sigma_{xy}} = 75$$

$$\overline{\sigma_{1,3}} = \frac{57.62+97.62}{2} \pm \sqrt{\left(\frac{97.62-57.62}{2}\right)^2 + 75^2}$$

$$= 77.62 \pm 77.62 = 0, 155.24$$

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See now with this corrected stresses if we calculate your principal stresses we see that there is no tensile stress all these stresses are in compression. Let me just make one small correction, actually the sigma 3 should be absolute sigma 3, so that is what we have seen here.

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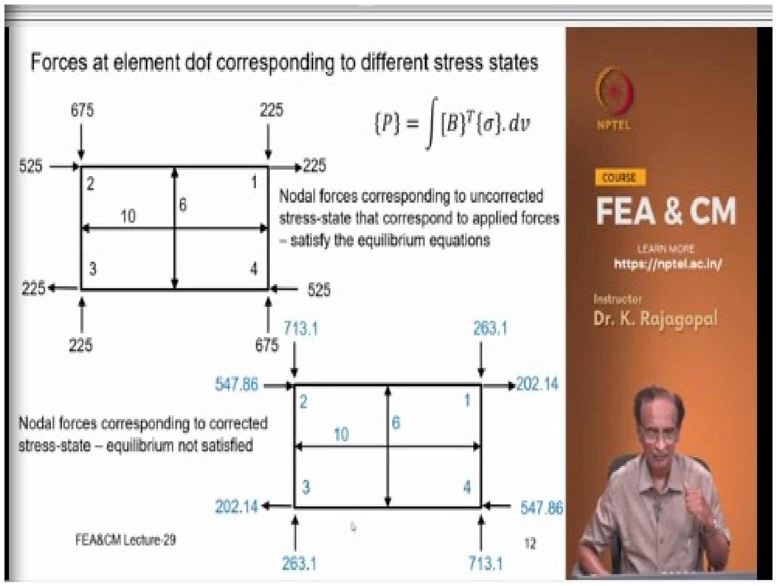
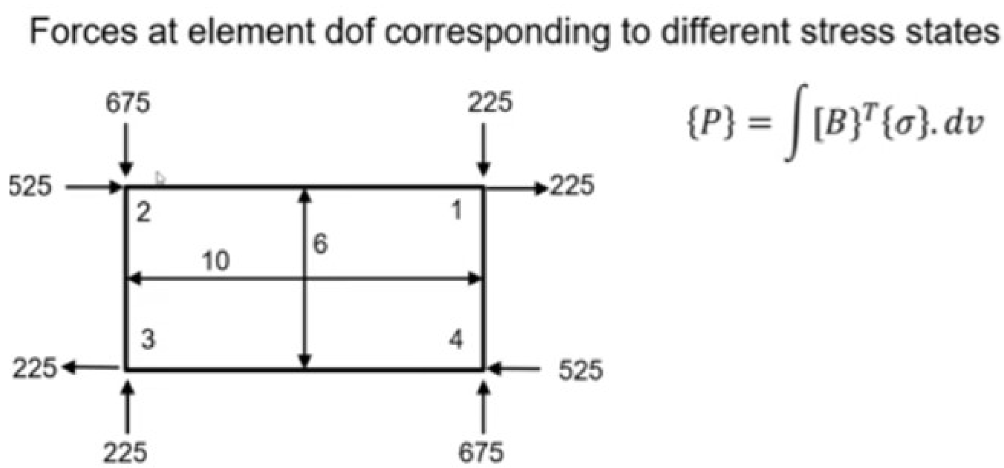
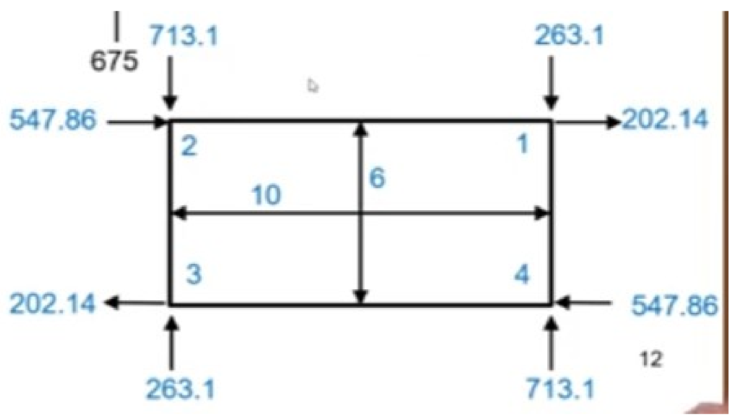


But then what happens to the equilibrium of the system? So, we can calculate the reaction forces by using our B transpose sigma dv equation. And with the original stresses these are the nodal forces corresponding to the original given stresses of sigma xx is a 50, sigma yy is 90 and tau xy is 75. So, you see here 675 + 225 is 900 divided by 10 is 90 and 525, 525 and then 225 is acting in the reverse direction.



So, it is actually it is the net compression force is the 300 divided by 6 is 50 kPa that is compression. And along this line the total shear force is 525 + 225 that is 750 divided by 10 is your shear stress of 75 acting in the positive direction. And then on the vertical plane 675 - 225 that is 450 divided by 6 that is once again 75 but that is acting in the negative direction. Because our convention is any shear force that causes positive moment about the center of the element is taken as positive.

So, on this horizontal surface our shear force is acting to the right and it is taken as positive and then on the vertical surface it is acting up, so acting in the negative direction because the moment produced around the center of the element is anticlockwise. And this system of forces is exactly equal to the applied forces, so your equilibrium is satisfied. But then our stresses are not satisfying our limit of the tensile strength and we have done the correction.



And the corrected stress state gives rise to these forces, 713 instead of 675 and 263 instead of 225 and so on. But then these forces highlighted in blue colour they will not satisfy the equilibrium because these are something else; these are obtained by doing the correction. And now this stress state is satisfying our limit of no tensile stress but then it is exceeding or it is not satisfying the equilibrium, our equilibrium gets disturbed.

And then we need to find another procedure to satisfy both the equilibrium and also the yield stress or our requirement of tensile stresses.

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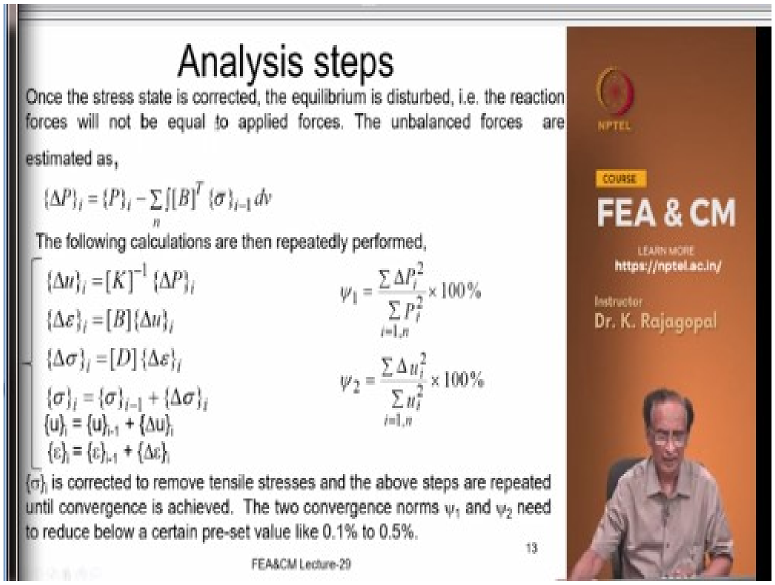

So, what Zienkiewicz and others have done is they suggested that you can calculate the reaction force dp as integral B transpose d sigma, that is d sigma is our 7.62 is our d sigma. And then corresponding to that you can calculate your reaction force and then apply it. And then later on Zienkiewicz and others in some other context that we are going to see later, they proposed more comprehensive equation that instead of writing the right hand side the force vector is delta p we can write it as the difference between the applied forces minus the reaction force.

The applied force and the reaction force and if both of them are exactly the same your equilibrium is satisfied, if not the equilibrium has to be satisfied by distributing your stresses to some other part of the body. And what they have done is in the original paper by Zienkiewicz and others that was published in 1968, the equation is not written like this, it is written as delta sigma B transpose delta sigma is dp.

So, we can calculate corresponding to that outer balance stresses we calculate the reaction force as B transpose delta sigma and then apply this on the system to get some new displacement increment delta u. Then we get the new strain increment delta epsilon and then delta sigma can be obtained as d times delta epsilon and then the total stress sigma i, the sigma i - 1 + delta sigma i, actually i is the current step and i - 1 is the previous step.

Analysis steps

Once the stress state is corrected, the equilibrium is disturbed, i.e. the reaction forces will not be equal to applied forces. The unbalanced forces are estimated as,

$$\{\Delta P\}_i = \{P\}_i - \sum_n \int [B]^T \{\sigma\}_{i-1} \, dv$$

The following calculations are then repeatedly performed,

$$\begin{cases} \{\Delta u\}_i = [K]^{-1} \{\Delta P\}_i \\ \{\Delta \varepsilon\}_i = [B] \{\Delta u\}_i \\ \{\Delta \sigma\}_i = [D] \{\Delta \varepsilon\}_i \\ \{\sigma\}_i = \{\sigma\}_{i-1} + \{\Delta \sigma\}_i \\ \{u\}_i = \{u\}_{i-1} + \{\Delta u\}_i \\ \{\varepsilon\}_i = \{\varepsilon\}_{i-1} + \{\Delta \varepsilon\}_i \end{cases}$$

$$\psi_1 = \frac{\sum \Delta P_i^2}{\sum\limits_{i=1,n} P_i^2} \times 100\%$$

$$\psi_2 = \frac{\sum \Delta u_i^2}{\sum\limits_{i=1,n} u_i^2} \times 100\%$$

$\{\sigma\}_i$ is corrected to remove tensile stresses and the above steps are repeated until convergence is achieved. The two convergence norms ψ_1 and ψ_2 need to reduce below a certain pre-set value like 0.1% to 0.5%.

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And as you are iterating or as you are applying the loads the current step is i and the previous step is i - 1, so we update our stress vector sigma i as sigma i - 1 + delta sigma i and u i that is the total displacement is u i - 1 + delta u i and then epsilon i is epsilon i - 1 + delta epsilon i. And then what we do is once again we check for the tensile stresses in this newly calculated sigma. And if that stress state is developing some tensile stress we do the correction for removing the tensile stress.

And then calculate your delta sigma and then delta p and then repeat this process and until we are happy with the convergence. And the convergence is monitored in 2 ways in terms of the norm of the outer balance forces that is the sum total of the squares of all the outer balance forces divided by the sum total of the square of the applied forces multiplied by 100 percent. Or we can also monitor in terms of the incremental displacements, the norm of the displacement psi 2 as sigma of delta u square by sigma of ui square.

And if you repeat or if you reanalyze the problem several times at some point we are bound to satisfy both equilibrium and also the yield state. And here in this case it is no tension and typically this psi 1 and psi 2 can be set to some 0.1 or 0.5 percent. Sometimes we may not be able to achieve good equilibrium even with a very large number of iterations. In that case we could increase this to maybe 1 percent or 2 percent or even 5 percent sometimes just to complete the analysis and get some results. Because we would like to see the mechanism of failure and then what is the approximate failure load.

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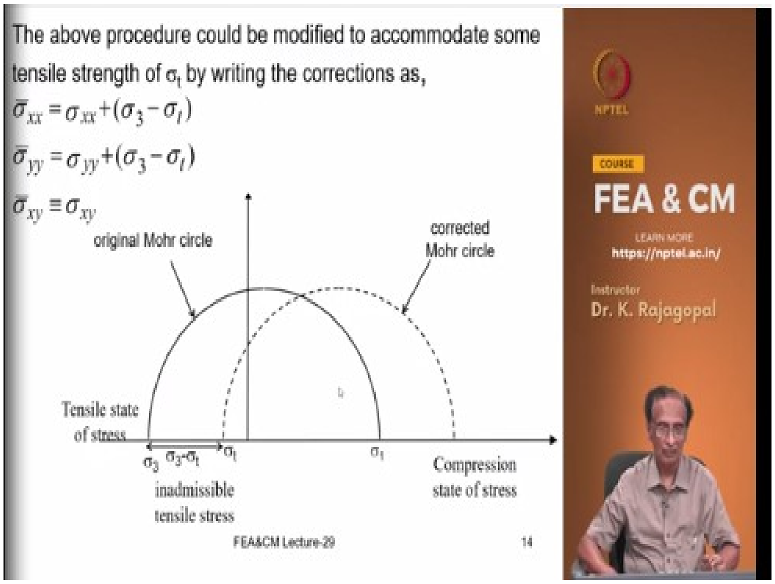
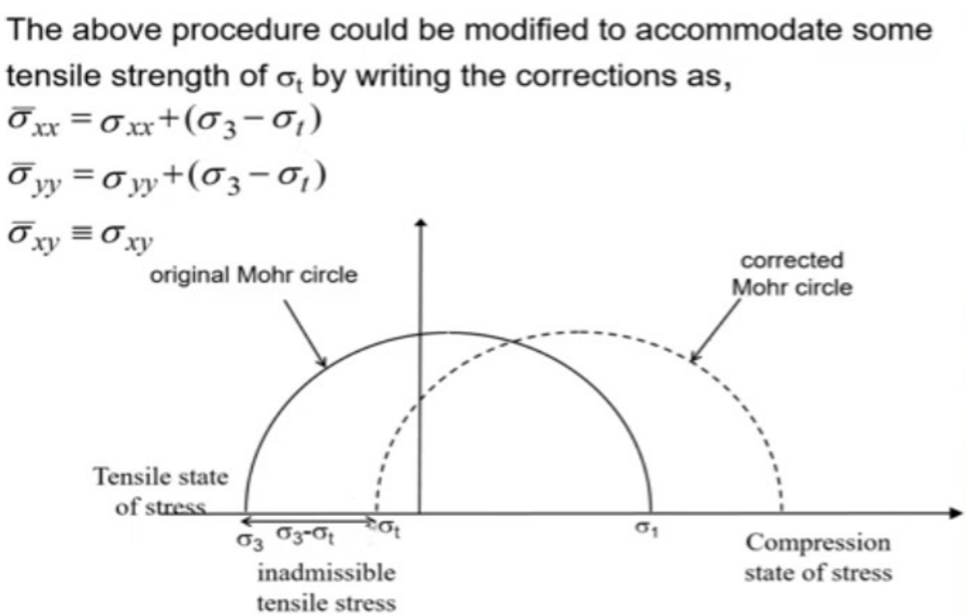


And the above procedure we can also modify it to include some tensile strength, if your body has some tensile strength of sigma t we can laterally shift the Mohr circle by magnitude of sigma 3 - sigma t like this. And the right sigma xx bar and sigma yy bar like this and this is another thing like you can set your sigma t to 0 or some value.



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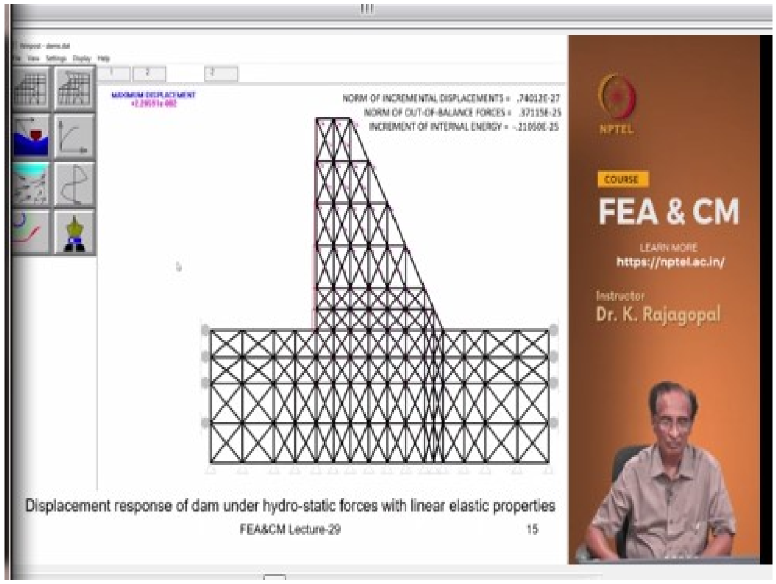

And here is the problem that Zienkiewicz and others have considered. We have a dam and the entire thing was modeled using the 3 node triangles. In those days that was 1968 the isoparametric elements have not come in and the 3 node triangular element was very popular because it is easy to program and very quick to do the computations because your the bandwidth for the stiffness matrix is not very large.

So, this is the dam and then there is some hydrostatic pressure indicated by this red line and the first analysis that we are seeing is with the linear elastic properties. The maximum displacement that is shown by the largest vector I do not know exactly where but we can find out by looking at the results. The maximum displacement is 2.28 times 10 to power of -2 and in all the finite element analysis the program does not know the units.

So, we should use some consistent units, so if we define the length in meters your displacements also will be in meters and then your modulus should be in kilo Newtons per meter square or Newtons per meter square. And then your unit weights are should be in the same units as your modulus and everything should be consistent. And the program as such it does not know the units, it will only give you the value.

And here because we are using linear elastic properties the outer balance forces are very, very small 10 to power of -25. Because at every stage where exactly satisfying the equilibrium whatever stresses are applied on the body the material is able to take, so it will produce equal and opposite reaction force. It is the norm of the outer balance force is a 10 to power of -25 and even the norm of incremental displacements is very, very small because the equilibrium is satisfied.

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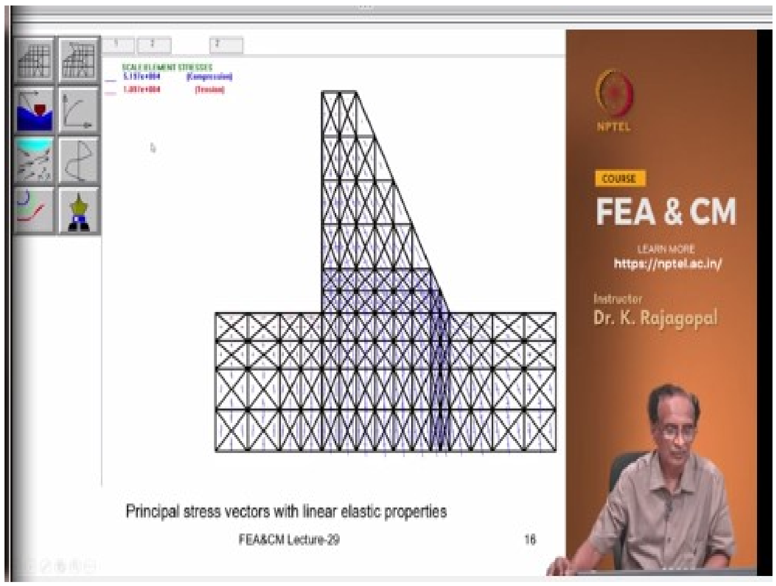

But then if we look at the stresses, what you are looking at the principal stress vectors the direction and then the magnitude. Magnitude is drawn to some scale, the blue lines are compression and the red lines are tensile stresses. And the blue line of this length represents a stress of 5.197 times 10 to the power of 4 and the red line of this length represents a tensile stress of 1.097 times 10 to the power of 4.

And you see lot of tensile stresses are developed here along this line and then somewhere inside the soil and then within the dam body some tensile stresses are developed. And we like to perform the no tension analysis; so that we can get rid of the tensile stresses.

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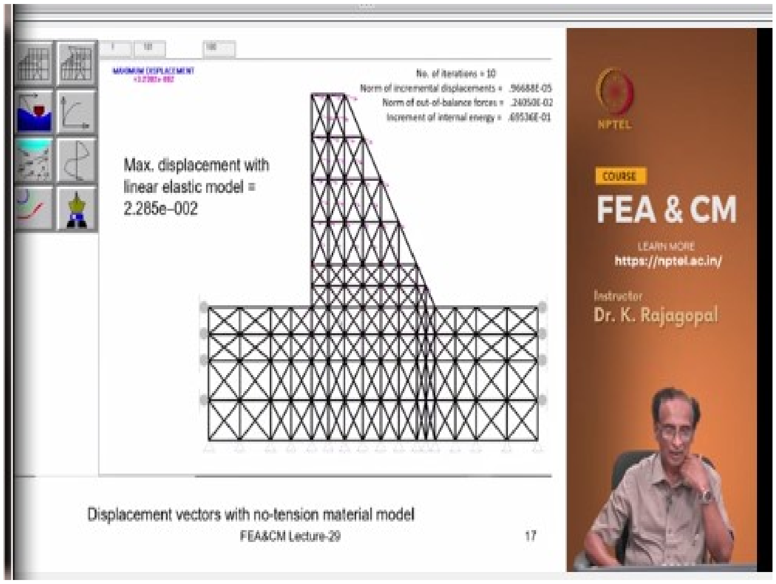

And here the same dam was analyzed by using the no tension model. And now our displacement is slightly increased, now it is 3.23 times 10 to the -2 and with the linear elastic model, these were the displacements. And if you look at the norm of outer balance forces it is a 0.002 that is 0.2 percent this is actually it is good, it is able to achieve the convergence after about 10 iterations at every step.

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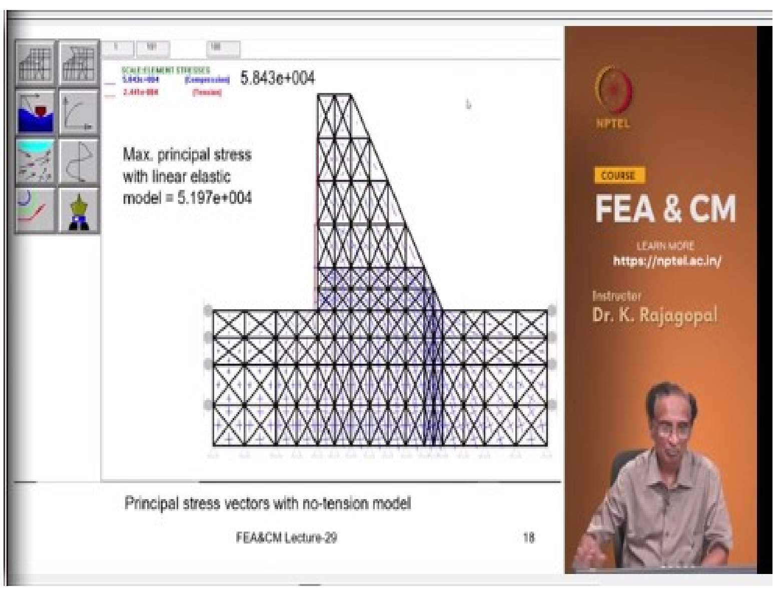

And then if you look at the stresses there are no compression stresses, the maximum compression stress is 10 to the power of -4 which is negligible. And our maximum compression stress is 5.843 times 10 to the power of 4 and when we had the linear elastic model where we allowed some tensile stresses the maximum compression stress was only 5.197 times 10 to the power of 4. But now here you see because of redistribution of stresses there is a higher stress and the higher stress is happening somewhere here.

And you see here all the stresses have become very small if you look at the stress vectors. So, previously when we had the linear elastic material all these were red coloured lines indicating tensile stress. But now there are no red colour lines because the most of the tensile stresses are dissipated and we end up with only compression stresses.

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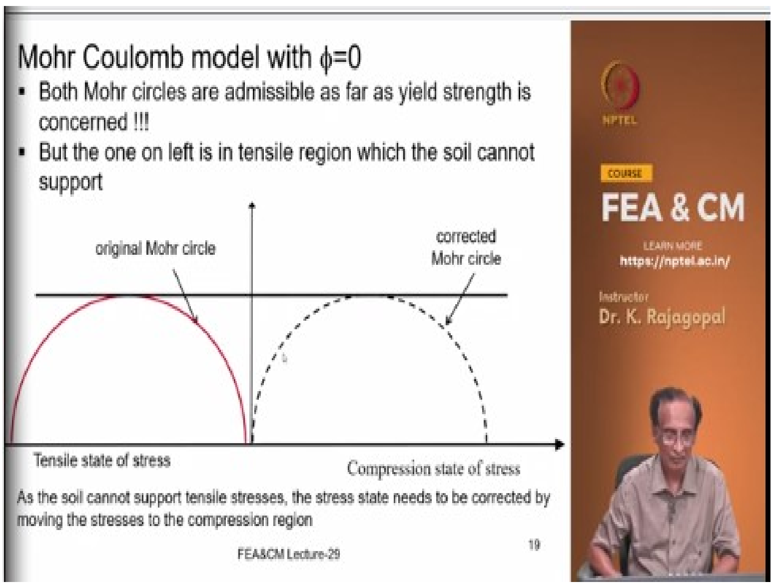
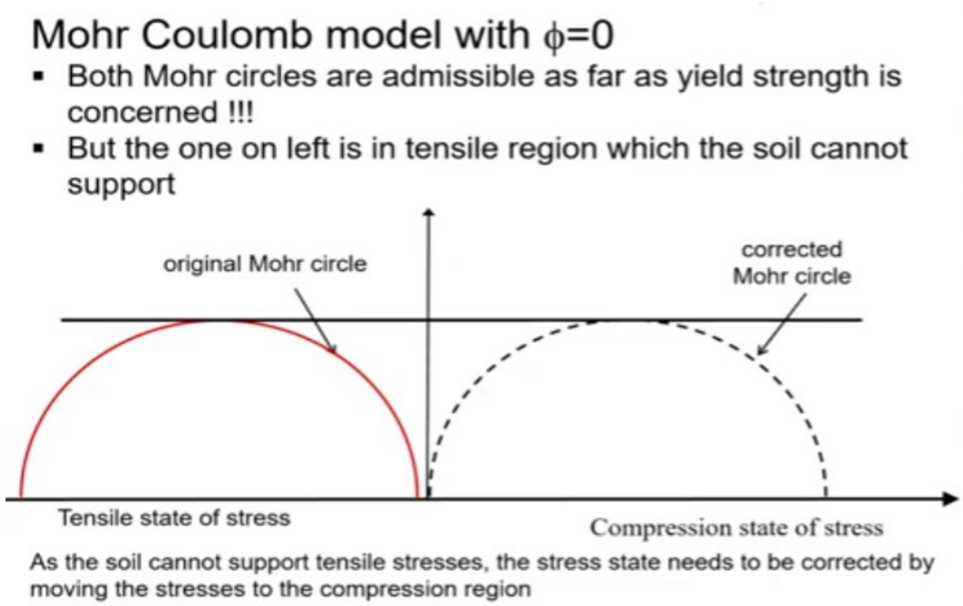


And this same problem of no tension case might arise when we use the Mohr-Coulomb model with phi = 0. See with phi = 0 our yield surface is horizontal, so that means that wherever you draw the Mohr circle whether in the compression space or the tension space where F value is going to be exactly 0. So, if you look at only the F value then you might think okay the soil is satisfying the yield condition, so there is no problem.



But then when you look at the stress this entire Mohr circle is in the tensile space and our soil may not be able to carry the tensile stresses. So, actually this Mohr-Coulomb model with phi = 0 requires some additional check for the tensile stresses. And then in most of the computer programs they give you an option to check for tensile stresses because especially when we are doing any analysis of the soil structures we should get rid of all the tensile stresses.

Because by default the soil does not have any tensile capacity and if you are dealing with any rock medium then you can define some tensile strength and make sure that your tensile stresses do not exceed the tensile stress. So, in this case also we can laterally shift the Mohr circle into the compression space and then get rid of our tensile stresses.

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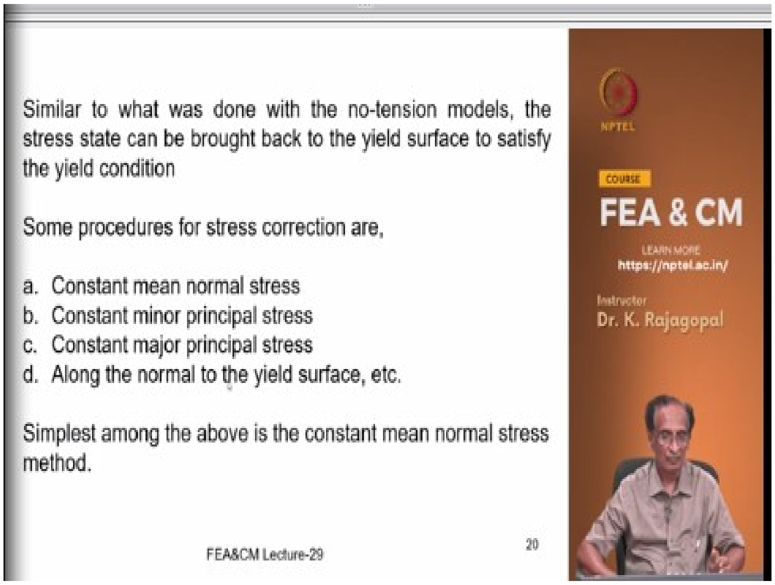

So, similar to what was done with the no tension model we can think of some procedures for correcting the stresses and there are different methods. The simplest one is the constant mean normal stress or the constant minor principle stress or constant major principal stress or along the normal to the yield surface is actually we are going to see only one method that is the constant mean normal stress in this class.

Then the fourth one along the normal to the yield surface is anywhere that we are going to do when we deal with plasticity analysis of the soils. We are going to correct the stresses normal to the yield surface and that will give us the dilation also.

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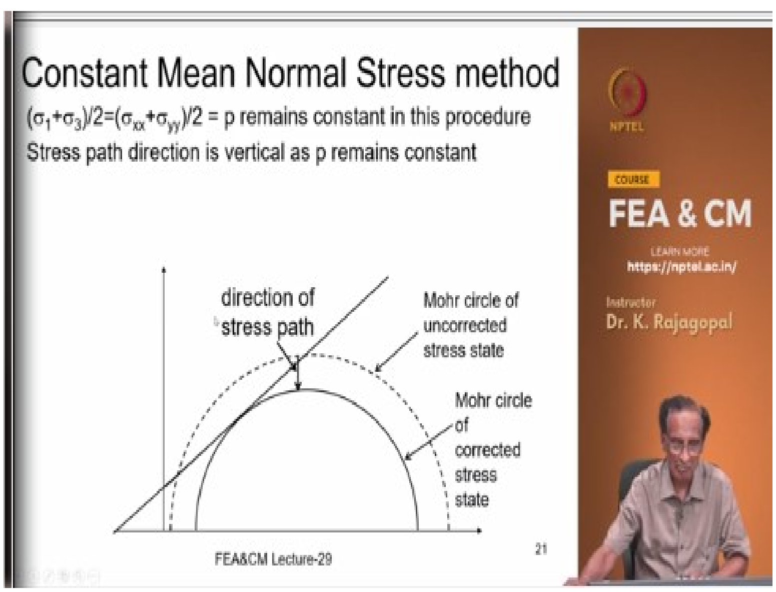
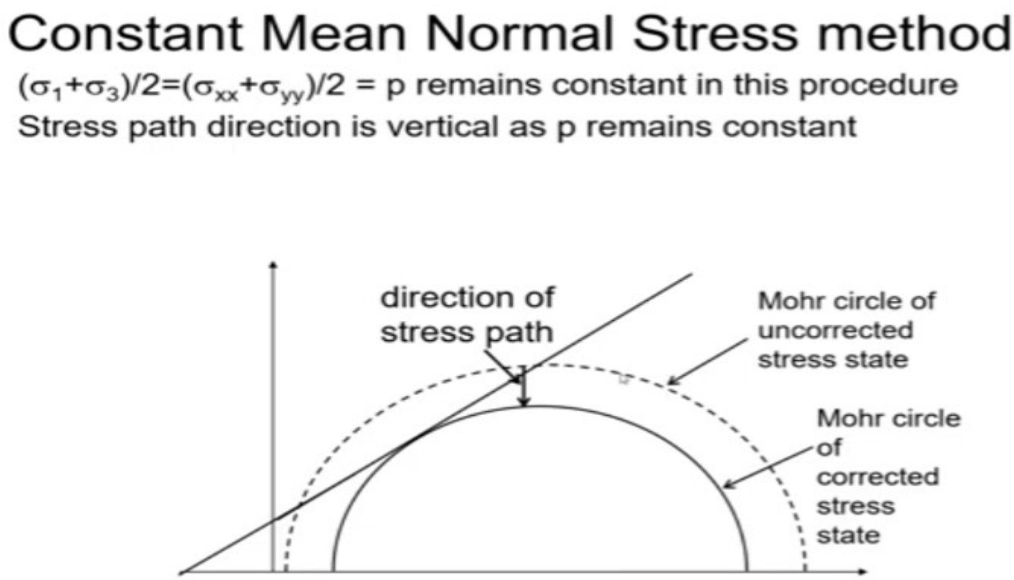


So, the constant mean normal stress method is very simple, so this dotted line is the original Mohr circle or the uncorrected stress state and this solid line is the corrected Mohr circular or the corrected stress state and the mean normal stress is kept constant during. So, the stress path that is followed by the stresses is this P-Q diagram is like this, so here we are only changing q but not the p. Such that our Mohr circle is exactly tangent to the yield surface and the mean normal stress is sigma 1 + sigma 3 by 2 or sigma xx + sigma yy by 2 and that remains constant in this method.



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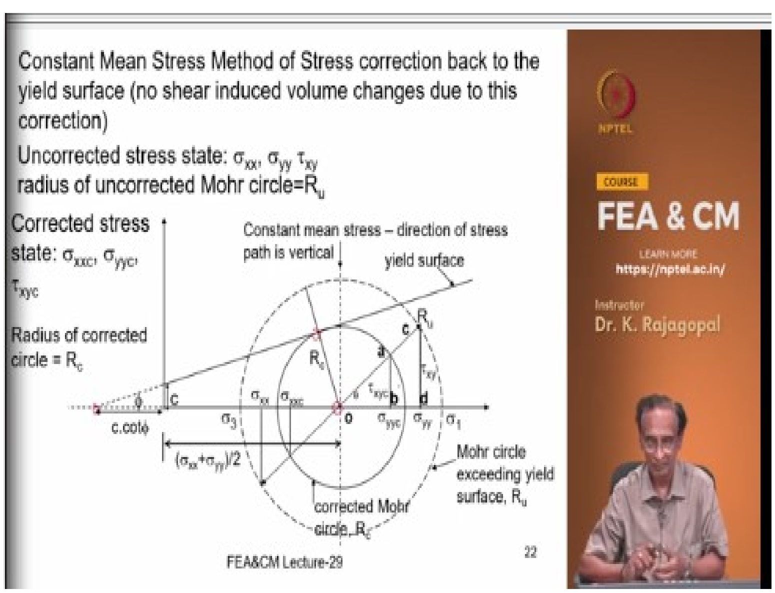
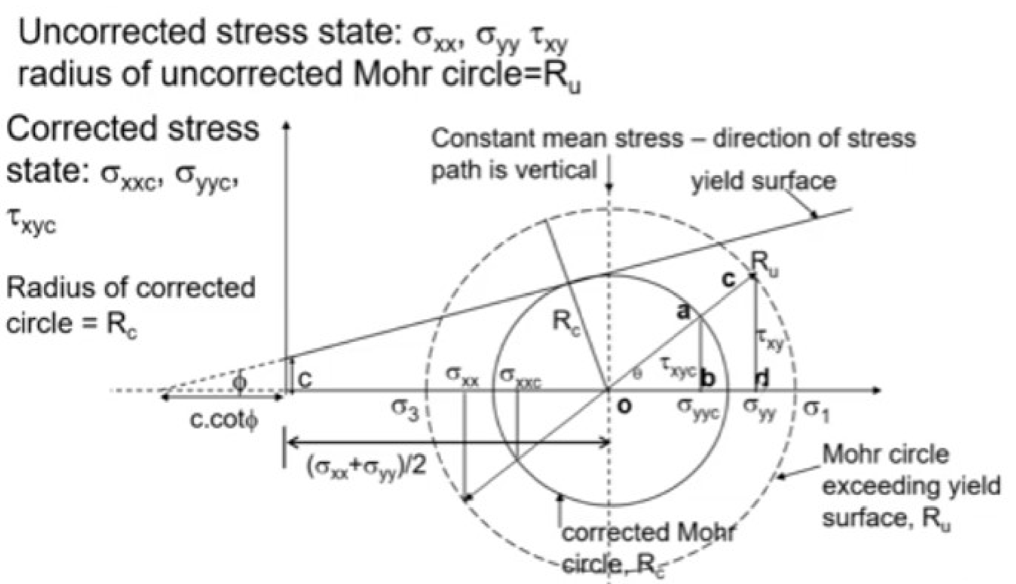


So, let us look at the complete stress state sigma yy, sigma xx and tau xy they were the original stresses as per the equilibrium and then if you plot the Mohr circle you get this dotted line and that is exceeding the yield surface. And that sigma yy corrected sigma yy c, sigma xx c and tau of xy c are the corrected stresses where the Mohr circle is exactly tangent to the yield surface.



And that R u is the radius of the uncorrected Mohr circle and the R c is the radius of the corrected Mohr circle. So, because our mean normal stress sigma xx + sigma **yy** by 2 is not changing, we can calculate the radius of the corrected Mohr circle R c from this triangle let me just show you with. So, if you look at this triangle, I think later I use it as some other symbols I will not use the symbols.

But if you look at this triangle highlighted by this red circles R c is sine phi multiplied by this length that is c cotangent phi + sigma xx + sigma yy by 2, whereas R u is calculated based on the stresses that we have.

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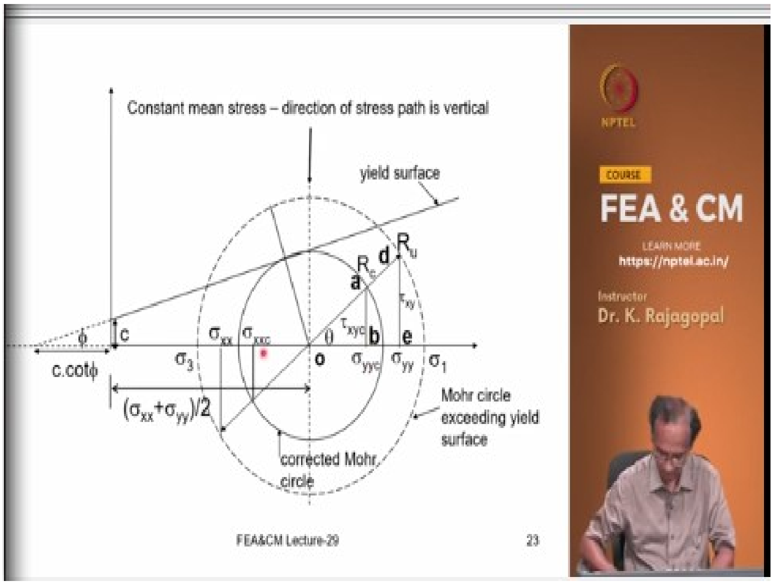

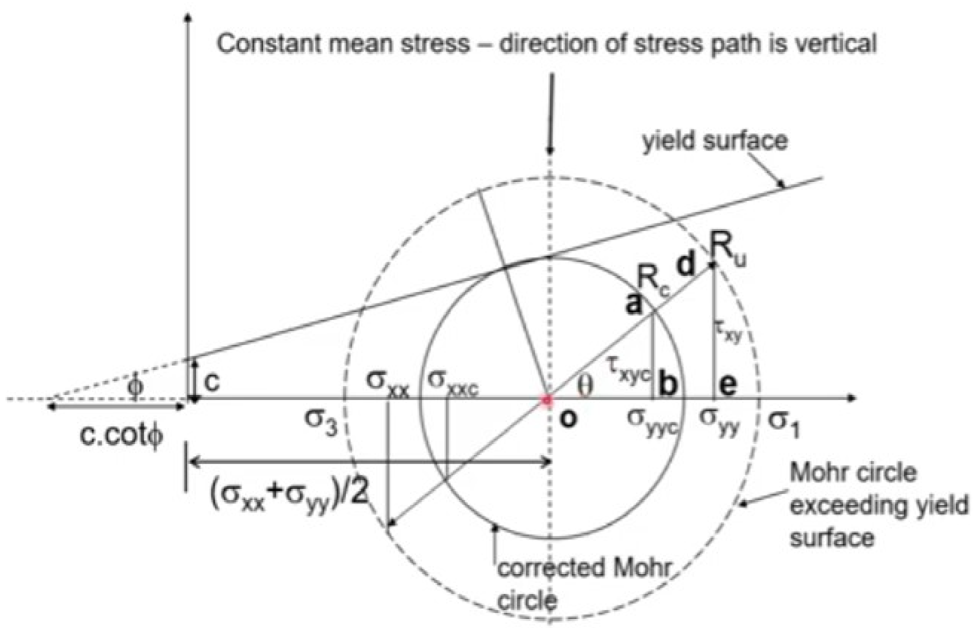

So, let us look at the 2 Mohr circles closely and this bigger circle is the original stress state and the smaller circle is the corrected stress state. See if we look at this 2 triangles oab and ode we see that they are similar triangles because they have the same angle theta. So, the sine theta or tan theta we can calculate and then get some equations for tau xy and then corrected and the sigma yy corrected, sigma xx corrected like this.

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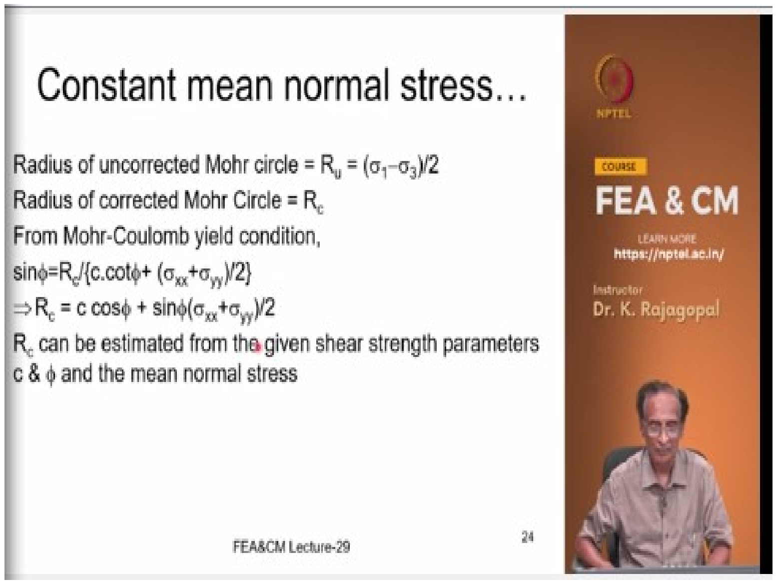

So, our radius of the uncorrected Mohr circle is R u is sigma 1 - sigma 3 by 2 because we can calculate sigma 1 and sigma 3 based on the given stress state. And then R u is that and the radius of the corrected Mohr circle R c is by looking at this triangle with this highlighted circles sine phi is R c divided by this hypotenuse, that is c cotangent phi + sigma xx + sigma yy by 2. So, our R c we can calculate like this and the R c once you know the shear strength properties and then the mean normal stress we can calculate R c.

Radius of uncorrected Mohr circle = $R_u = (\sigma_1 - \sigma_3)/2$

Radius of corrected Mohr Circle = R_c

From Mohr-Coulomb yield condition,

$\sin\phi = R_c/\{c.\cot\phi + (\sigma_{xx}+\sigma_{yy})/2\}$

$\Rightarrow R_c = c\cos\phi + \sin\phi(\sigma_{xx}+\sigma_{yy})/2$

R_c can be estimated from the given shear strength parameters c & ϕ and the mean normal stress

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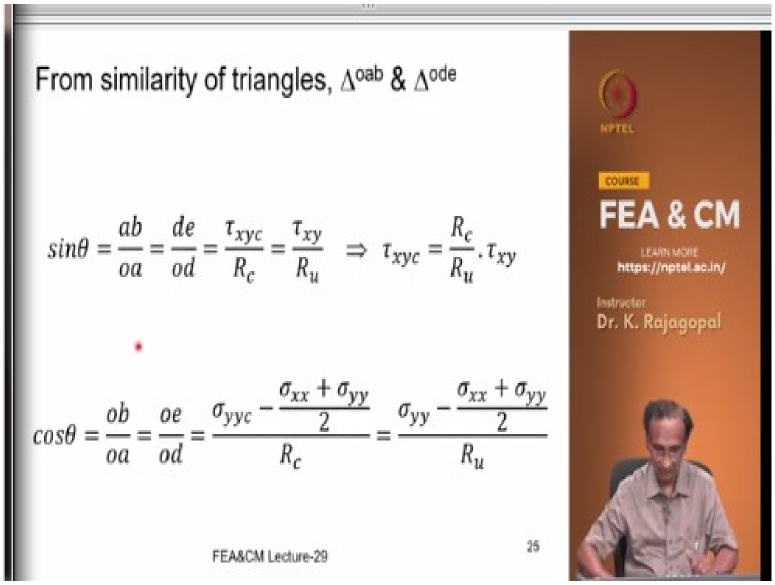

Then if you look at these 2 similar triangles, oab and ode sine theta is ab by R c or de by R u, right. And the oa is R c and od is R u and so our corrected shear stress tau xy c is R c by R u times tau xy, R c is the radius of the corrected Mohr circle and R u is the radius of the uncorrected Mohr circle and tau xy is your shear stress. And then if you look at this cosine theta it is ob by oa and oe by od and oa is R c and od is R u.

From similarity of triangles, Δ^{oab} & Δ^{ode}

$$sin\theta = \frac{ab}{oa} = \frac{de}{od} = \frac{\tau_{xyc}}{R_c} = \frac{\tau_{xy}}{R_u} \Rightarrow \tau_{xyc} = \frac{R_c}{R_u}.\tau_{xy}$$

$$cos\theta = \frac{ob}{oa} = \frac{oe}{od} = \frac{\sigma_{yyc} - \frac{\sigma_{xx}+\sigma_{yy}}{2}}{R_c} = \frac{\sigma_{yy} - \frac{\sigma_{xx}+\sigma_{yy}}{2}}{R_u}$$

And ob we can ok just let me and ob is the sigma yy corrected minus the mean normal stress. See this ob is sigma yy corrected minus this length and that is ob and oe is a sigma yy minus this mean normal stress. And then the corresponding radius values R c and R u.

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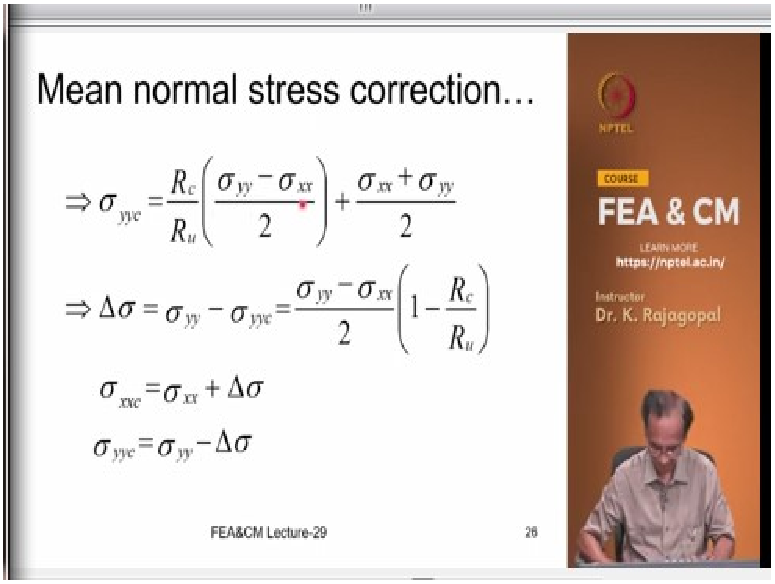

And sigma yy corrected is this like once you simplify the previous equation you get this. And the delta sigma is sigma yy - sigma xx corrected is this and sigma xx corrected is sigma xx + delta sigma and sigma yy corrected is sigma yy - delta sigma. It is actually this calculation assume that sigma yy is greater than sigma xx but it is a generic process and this method works even with the K naught greater than 1.

Mean normal stress correction…

$$\Rightarrow \sigma_{yyc} = \frac{R_c}{R_u}\left(\frac{\sigma_{yy}-\sigma_{xx}}{2}\right)+\frac{\sigma_{xx}+\sigma_{yy}}{2}$$

$$\Rightarrow \Delta\sigma = \sigma_{yy}-\sigma_{yyc} = \frac{\sigma_{yy}-\sigma_{xx}}{2}\left(1-\frac{R_c}{R_u}\right)$$

$$\sigma_{xxc} = \sigma_{xx}+\Delta\sigma$$

$$\sigma_{yyc} = \sigma_{yy}-\Delta\sigma$$

If K naught is greater than 1 your sigma x maybe more than sigma y and the signs will reverse and one of the stress components will decrease and the other will increase, so that your yield stress is exactly satisfied.

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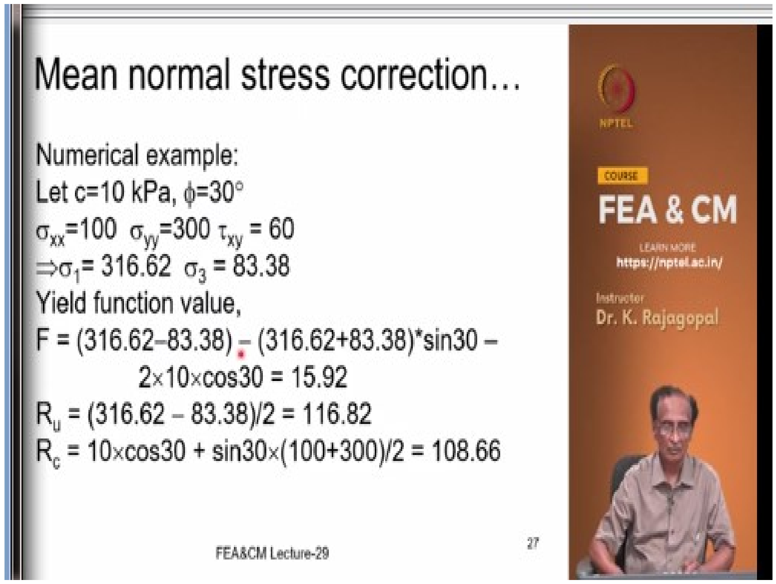

So, let us look at an example, let us take a material with a c of 10 and a phi of 30 degrees and sigma xx is 100 sigma yy is 300, tau xy 60 and sigma 1 and sigma 3 are this. And your yield function value F is 15.92 and that is positive and we like to correct the stresses, so that the yield function value is exactly 0. So, R radius of the uncorrected Mohr circle is sigma 1 - sigma 3 by 2 that is 116.82 and the radius of the corrected Mohr circular R c is c times cosine phi + sine phi times mean normal stress and that is 108.66.

Numerical example:

Let c=10 kPa, $\phi=30^\circ$

$\sigma_{xx}=100 \quad \sigma_{yy}=300 \quad \tau_{xy} = 60$

$\Rightarrow \sigma_1 = 316.62 \quad \sigma_3 = 83.38$

Yield function value,

$$F = (316.62-83.38) - (316.62+83.38)*\sin 30 - 2\times 10\times \cos 30 = 15.92$$

$$R_u = (316.62 - 83.38)/2 = 116.82$$

$$R_c = 10\times \cos 30 + \sin 30\times(100+300)/2 = 108.66$$

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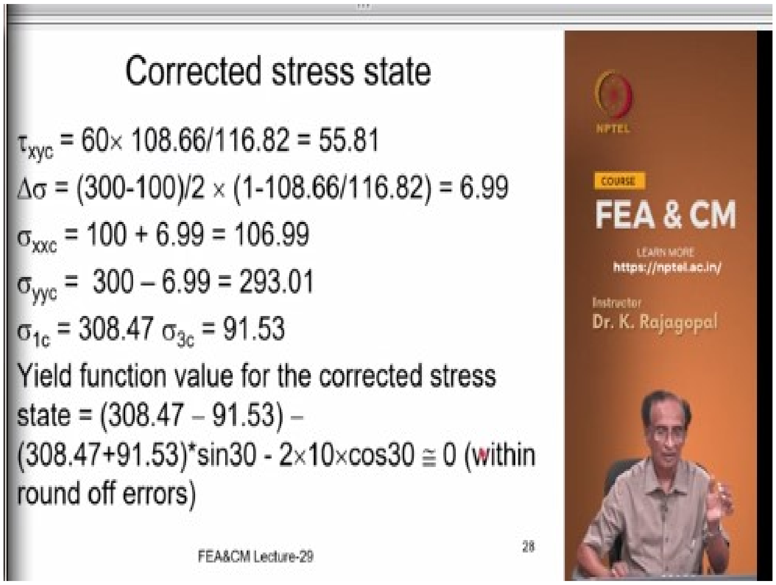

And the tau xy corrected is a tau xy multiplied by R c by R u and delta sigma is sigma 1 - sigma 3 by 2 times 1 - R c by R u that comes to 6.99, so our sigma xx corrected is 100 + 6.99 that is 106.99, sigma yy corrected is 293.01 and then tau xy is 55.81. And if you calculate sigma 1 for this corrected stress state sigma 1c is 308.47, sigma 3 for the corrected stress state is 91.53.

Corrected stress state

τ_{xyc} = 60× 108.66/116.82 = 55.81

$\Delta\sigma$ = (300-100)/2 × (1-108.66/116.82) = 6.99

σ_{xxc} = 100 + 6.99 = 106.99

σ_{yyc} = 300 – 6.99 = 293.01

σ_{1c} = 308.47 σ_{3c} = 91.53

Yield function value for the corrected stress state = (308.47 – 91.53) – (308.47+91.53)*sin30 - 2×10×cos30 ≅ 0 (within round off errors)

So, if you substitute them back in the yield function equation it comes to almost 0, see within the round of errors like because I am showing only 2 decimal places, so this value will not be exactly 0 but it is close enough to 0 but definitely not like the previous value of 15.92.

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So, this is one method for correcting the stresses and the only problem is with this corrected stresses will not be able to satisfy the equilibrium, we have to do something to satisfy the equilibrium by repeatedly doing the analysis. So, we imply some iterative procedures, similar to those that we discussed some classes back when we discuss the non-linear analysis methods either in terms of the initial stress method tangent stiffness or secant stiffness method.

And in all these problems if we are able to update the stiffness matrix you get a faster convergence and if you do not update you may need the more number of iterations. So, in the initial stress method we are not going to update the stiffness matrix and then we repeatedly apply the outer balance forces and then calculate the new stress state and so on. Whereas in the tangent stiffness and secant stiffness we are going to update our stiffness matrix, so it is a slightly it requires lesser number of iterations.

But then they might take a much longer time because you need to reformulate your stiffness matrix assemble it and then makes it as a upper triangular matrix and so on. Whereas in the case of initial stress method very fast because it is only back substitution.

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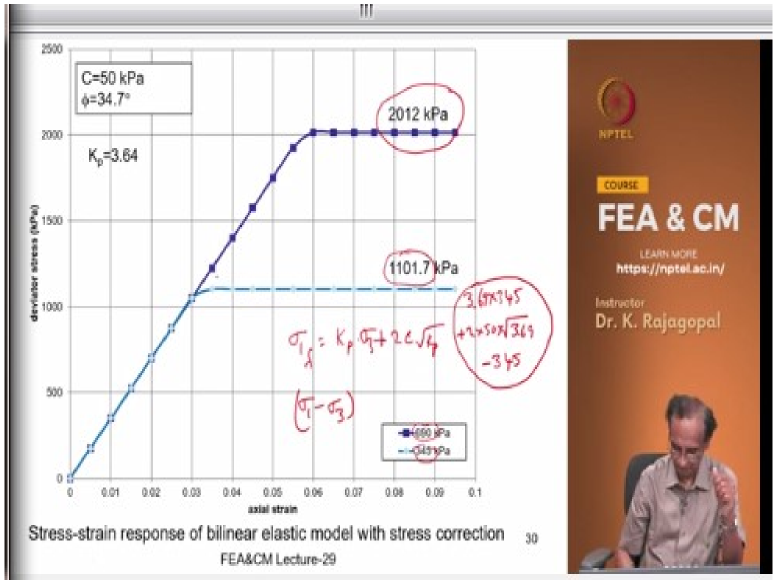
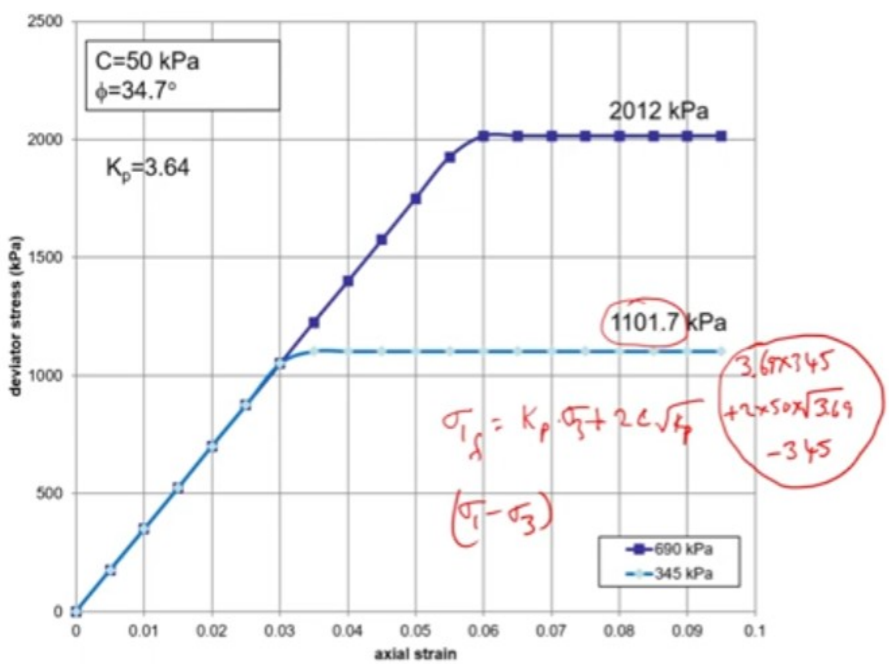


So, here I am showing you 2 results using the bilinear elastic model for C of 50 and phi of 34.7 and then at 2 different confining pressures of 345 and 690, these are the sigma 1 maximum 1101.7 and 2012. See the sigma 1f. So, our sigma 1 maximum is K p sigma 3 + 2C square root K p and what is plotted on the y axis is deviated stress sigma 1 - sigma 3.And this 1101.7 is 3.69 times 345 + 2 times 50 times square root 3.69 - 345. And if you do this calculation you should get this value, this is what is predicted by our numerical method. And similarly for a confining pressures of 690 this is the pressure that you compute.



Stress-strain response of bilinear elastic model with stress correction 30

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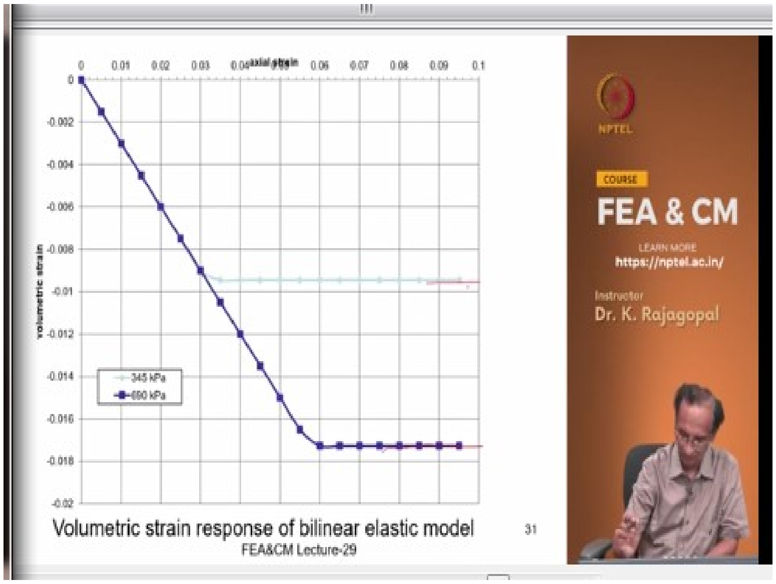

And these are the volumetric strains that are predicted beyond your yield limit, your Poisson's ratio approach is 0.5 and you get the constant volume state. And this being a bilinear elastic model, you see that the slope of this stress strain curve is the same whatever may be the confining pressure. See even whether it is a confining pressure of 345 or 690, the slope is the same because our Young's modulus is not a function of the confining pressure.

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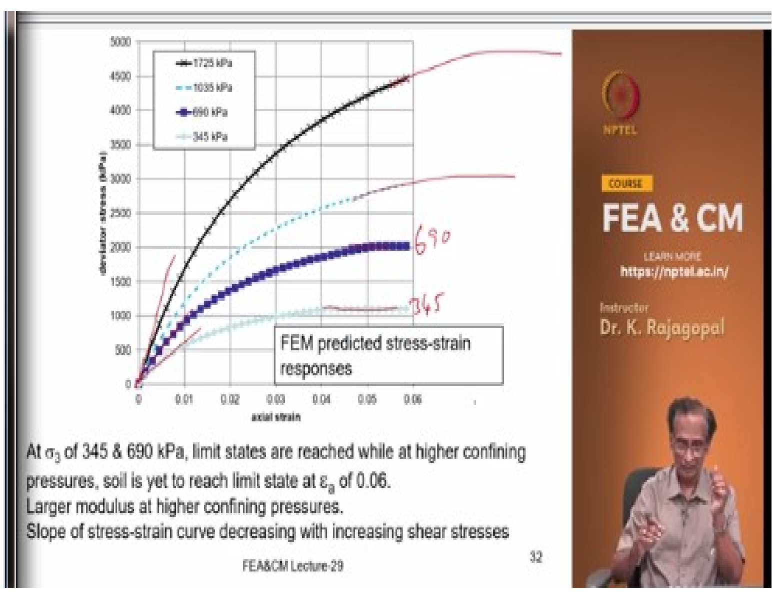

But if you look at our hyperbolic model, it is a different confining pressures the strength is different and then the initial slope. You see here the slope is higher for higher confining pressure and then even the strength is higher. And if you look at the stress strain response are 2 confining pressures of 345 and 690 within an axial strain of 6 percent both of them have reached the ultimate stress states.

But at a higher confining pressure of 1035 and 1735 you see that there is still increase like maybe they will read some asymptote at a much larger strain level. So, that is the difference between hyperbolic model and then other models like our bilinear elastic model. The hyperbolic model is representing, the influence of the confining pressure on the modulus and also the influence of the shear strength properties C and phi and the ultimate stress.

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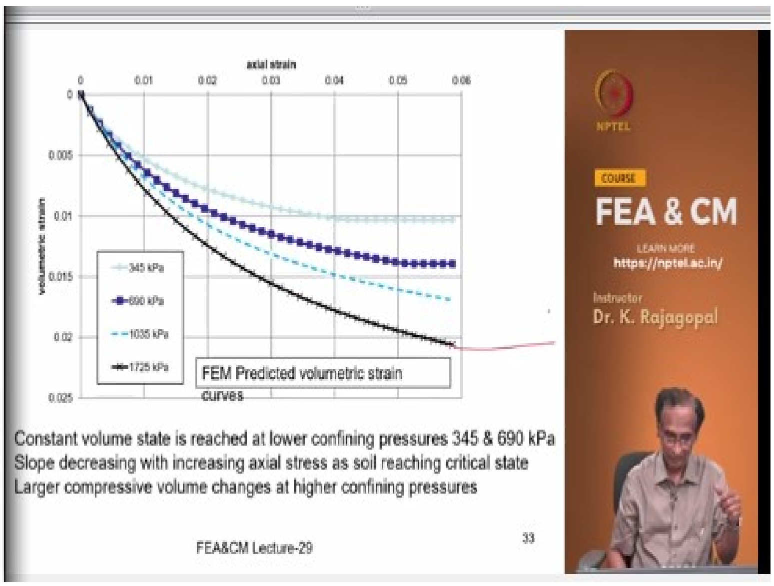

And these are the predicted volumetric strains set a higher confining pressure soil will undergo more volumetric compression before it reaches the constant volume state because it is strength is higher.

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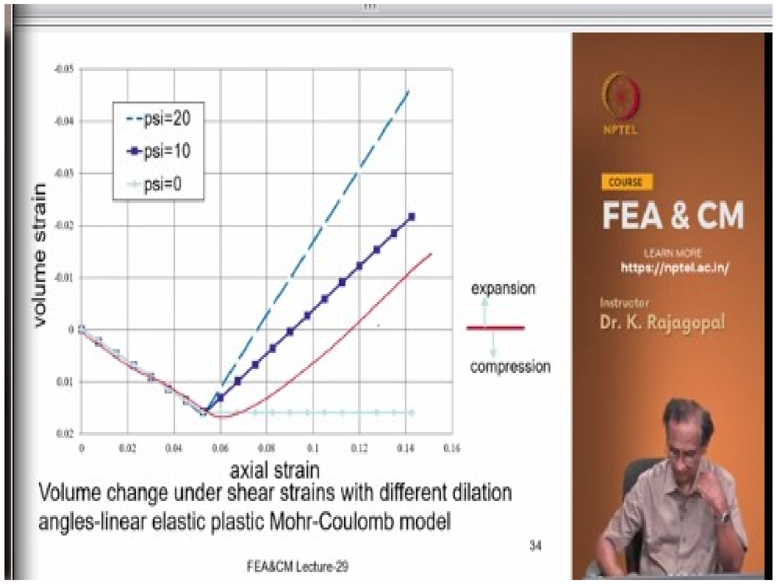

So, **so** this hyperbolic model being a non-linear elastic model, it can only predict volumetric compression under shear. But then if you look at the actual volumetric response our soil will initially compress and then it will undergo some volume expansion that is called as the dilation. And this dilation we can simulate by some other procedures that we will do next because till now we were looking at only elastic and non-linear elastic models.

And we can look at more advanced models like our elastic-plastic models that can simulate the shear induced dilation that we will do later.

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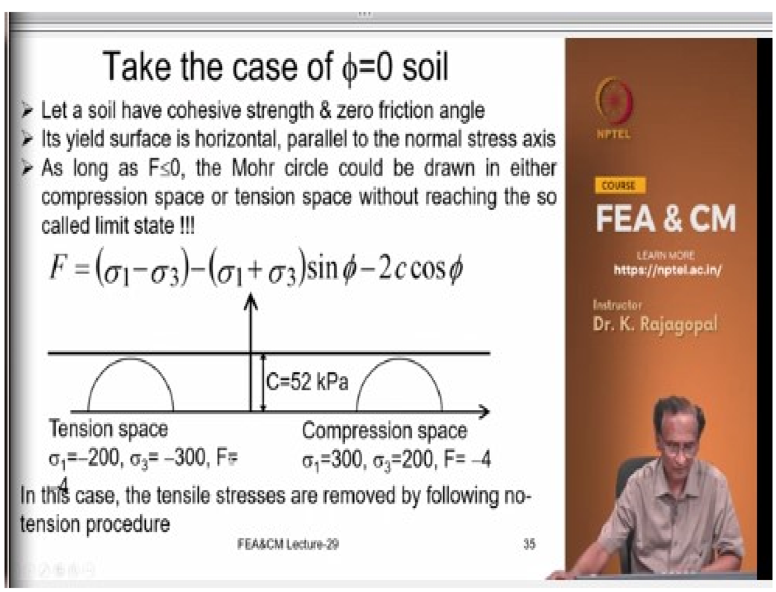
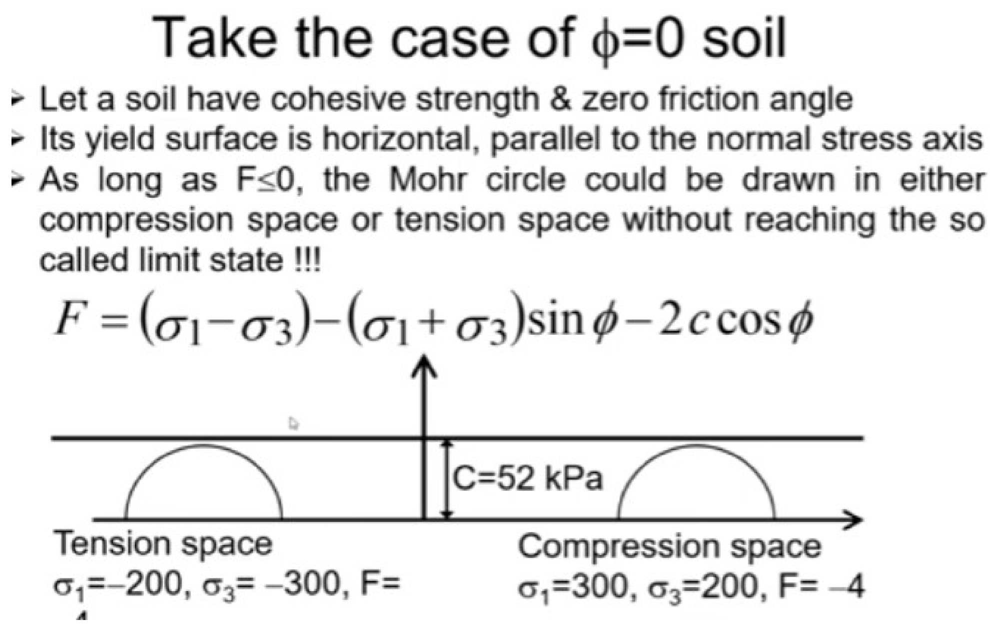


So, actually this is a numerical example of our case with phi = 0, see the tension space or compression space our F value is -4 and both these Mohr circles they are within this yield surface these cohesion is 52 kPa and both of them have an F of less than 0.



So, that means that they both represent elastic state but then this Mohr circle is in the tension space, so the soil we know that it cannot support any tensile stresses, so we need to move the Mohr circle to the right.

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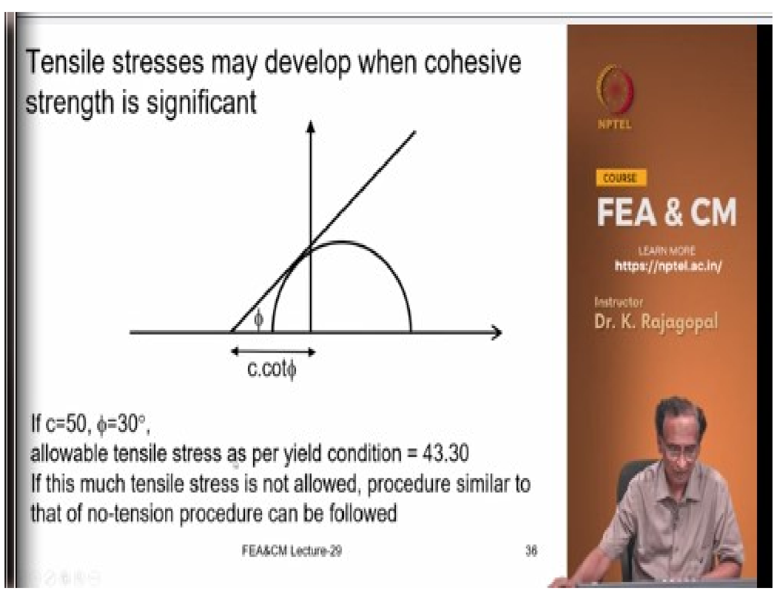
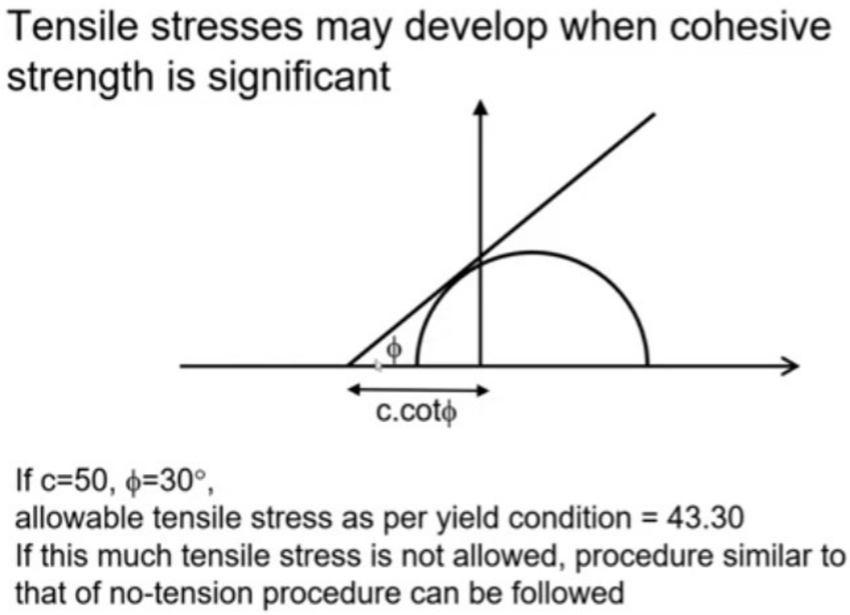


So, even this might happen even in c, phi soils where you have significant c and reasonably smaller phi, you might end up with a lot of tension and we can do something, so that we shift the

Mohr circle into the compression space. That is actually for this properties of C of 50 and phi of 30 degrees, the allowable tensile stress that is C times cotangent phi is 43.3, so that means that you can get a Mohr circle with the tensile stress of 43.3.



And it will not be detected, it will not be flagged by the yield condition because our F value is less than 0 but then we have to check separately for whether you have very large tensile stress within the material, if that is so we need to do some correction.

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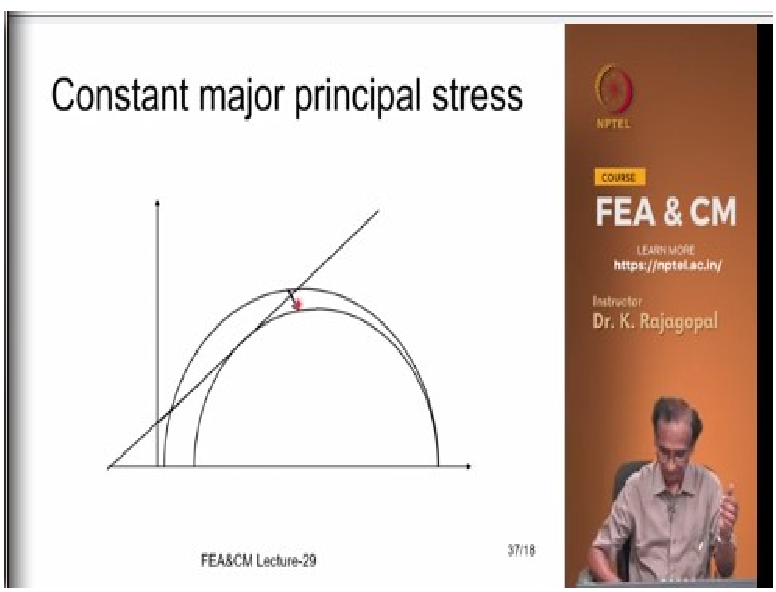

And there are other methods, see we are going to look at the methods that change the stresses along the normal to the yield surface. This is what we are going to see, that will give us the ability to model the dilation.

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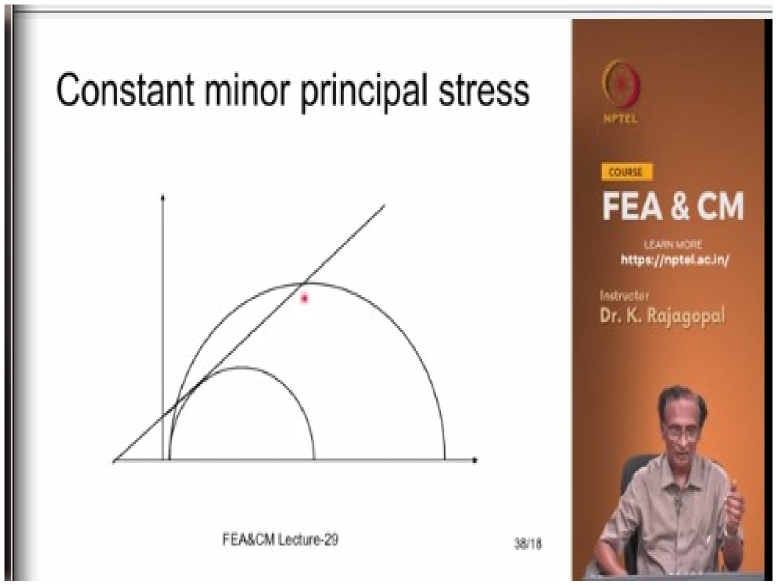

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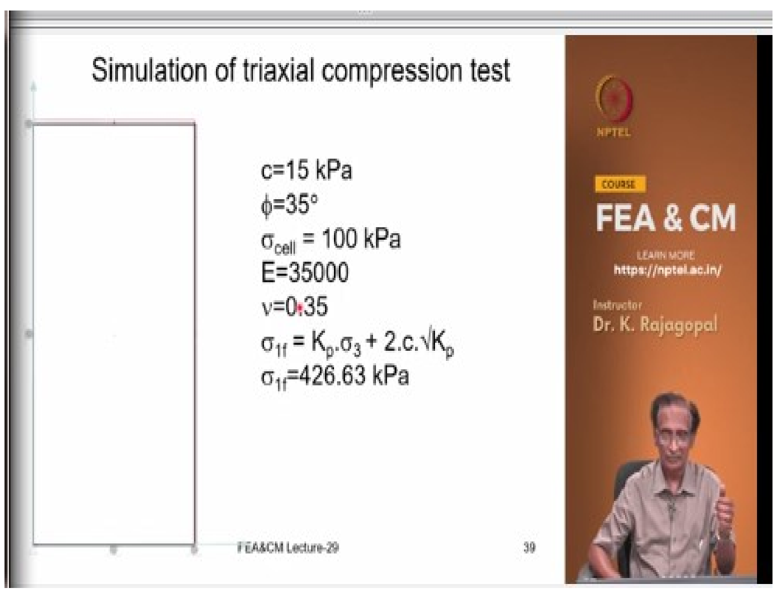

So, I think that we will see later. And I think this simulation of triaxial compression test that we will do in the next class, I will explain how we can work with hyperbolic and then the bilinear elastic models for simulating the triaxial compression test.

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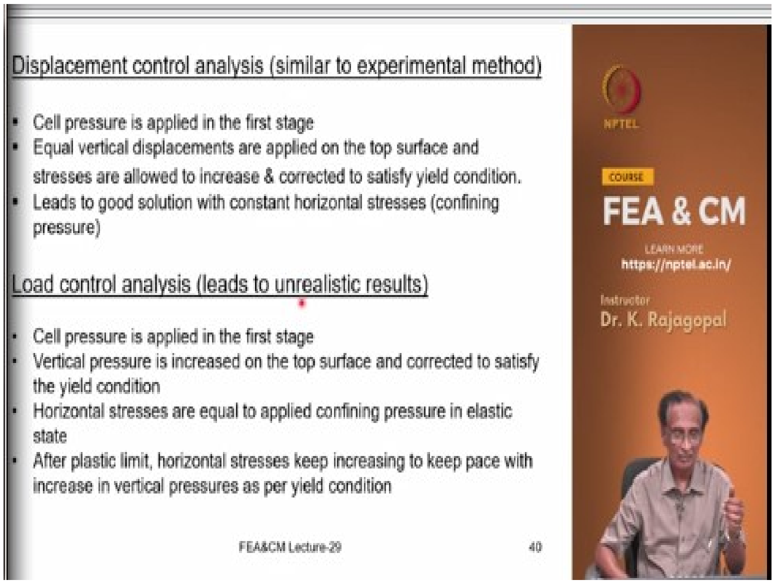

So, I think that is the end of today's lecture and if you have any questions please send an email to this address profkrz@gmail.com, so thank you very much.