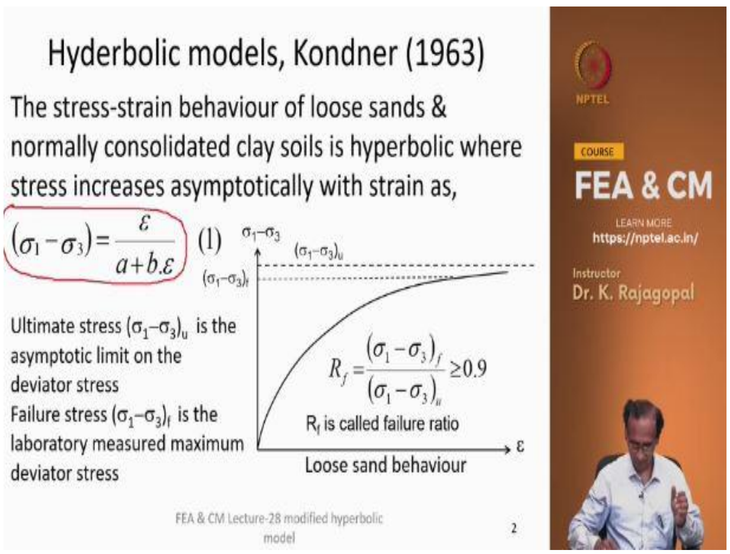
Finite Element Analysis and Constitutive Modelling in Geomechanics
Prof. K. Rajagopal
Department of Civil Engineering
Indian Institute of Technology-Madras

Lecture-32
Modified Hyperbolic Model and Determination of Material Parameters

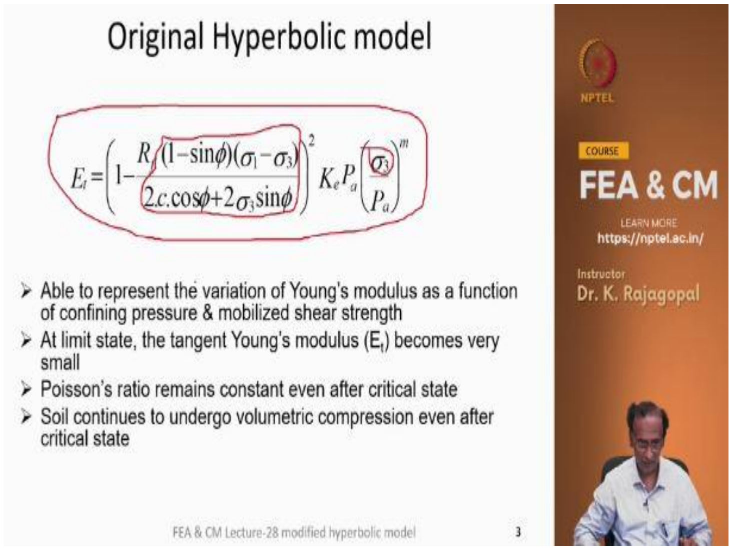
Let us continue our discussion on the hyperbolic model. Let us try to modify so that we can incorporate the effect of constant Poisson's ratio or after the critical state, the poisons ratio should approach 0.5 and then we will also see how to determine the material properties in this lecture.

(Refer Slide Time: 00:44)



So, in the previous lecture, this is what we had seen**.** This is called as the hyperbolic equation given by Kondner. And this equation was slightly modified for adapting it to finite element implementation. By differentiating this with respect to strain, we can get our tangent modulus and d sigma by d epsilon is defined as the tangent modulus.

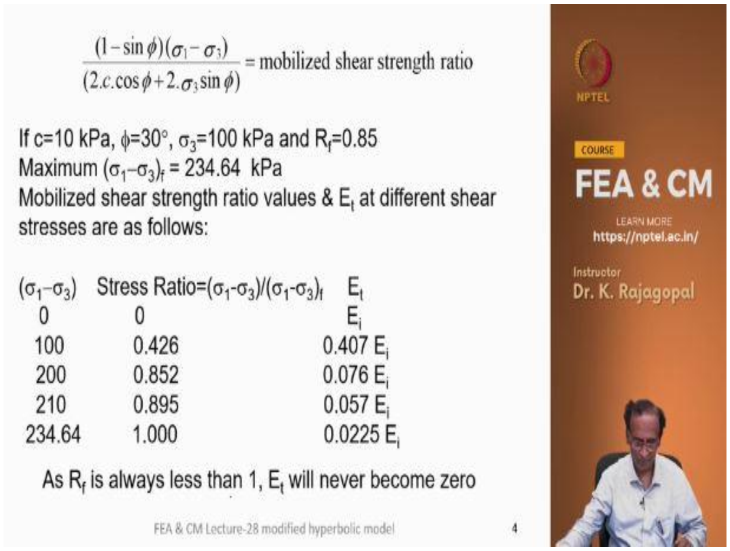
(Refer Slide Time: 01:25)



And then we have seen this equation; this is another beautiful equation. And this is our original hyperbolic model and here we are able to represent the effect of sigma 3, that is the confining pressure on the initial modulus and then this quantity that we had seen, 1 - sine phi times the shear stress divided by 2 c cosine phi + 2 sigma 3 sine phi is actually the mobilized shear strength. This 2 c cosine phi + 2 sigma 3 sine phi by 1 - sine phi is our sigma 1 - sigma 3 failure.

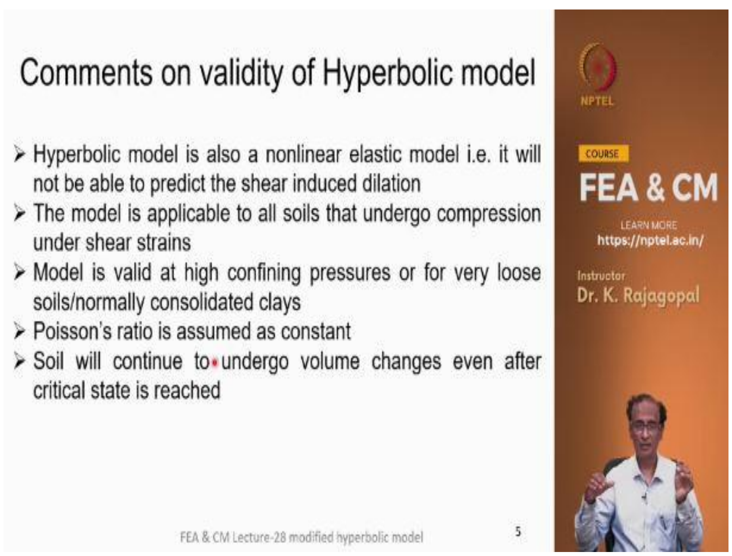
So, this is our mobilized shear strength. And in one equation, Duncan and Chang they are able to take care of the reduction in the young's modulus with increase in the shear stresses and then increase in the shear modulus because of confining pressure. So, and the limit state of stresses through the Mohr–Coulomb relation is also incorporated through this column relation. And the only limitation here is the soil continues to undergo volumetric compression even after the limit state is reached because we assume the Poisson’s ratio to remain constant.

(Refer Slide Time: 03:08)



So, that let us try to see how we can incorporate. And this is what we had seen in the previous class as the shear stress is increasing, your shear modulus will go on reducing. So, initially we have the Young's modulus as E i and then as the shear stress is increasing, your modulus is reducing. And ultimately, it has come down to 0.0 to 25 times E i and because our R f is always less than 1, it is about 0.85 to 0.9 or sometimes even 0.6**,** because R f is less than 1, our tangent modulus will never become 0.

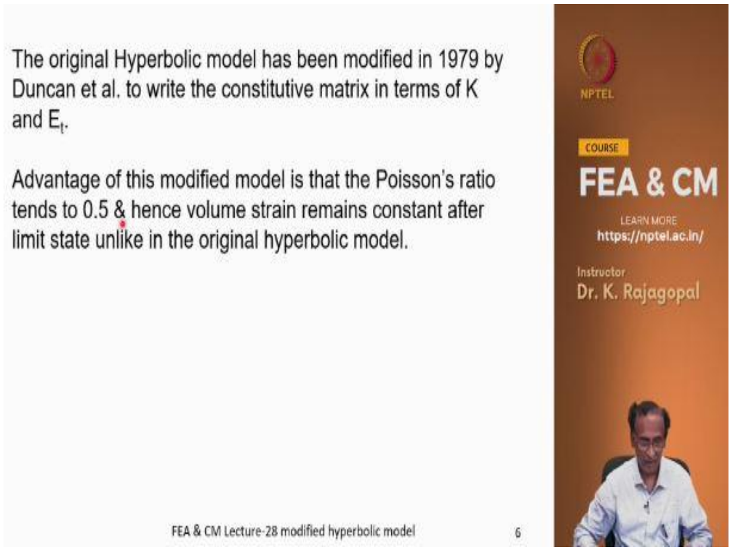
(Refer Slide Time: 03:59)



And these are all what we had seen in the previous class that the hyperbolic model; let me just get back to laser pointer**.** So, our hyperbolic model is also a non-linear elastic model, that means that the stress and strain are non-linearly related and then the unloading will recover all the strain that is elastic and it will not be able to simulate the shear induced dilation. And this model is applicable to all the soils that undergo volumetric compression during the loading.

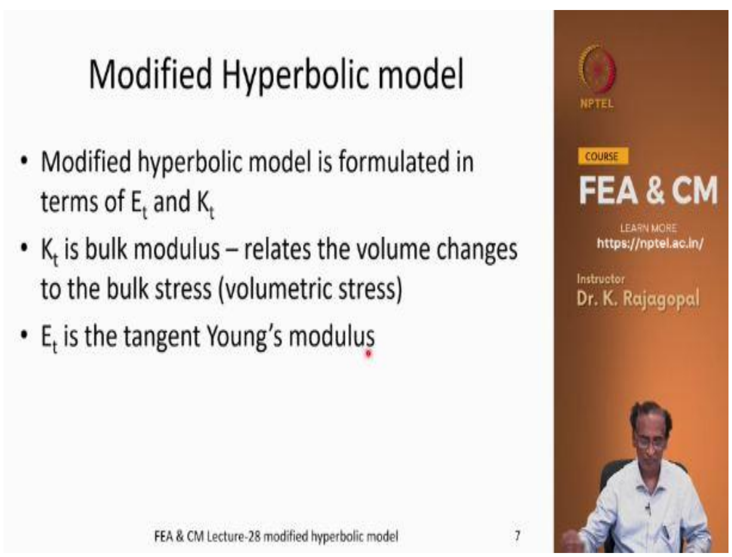
And then the Poisson's ratio is assumed constant. So, the soil continues to undergo volumetric compression even during the critical state.

(Refer Slide Time: 04:58)



So, we need to modify this. In 1979 Duncan and others they have modified the original hyperbolic model by Poisson's ratio in terms of bulk modulus K and then Young's modulus E t. And the advantage of this is that the Poisson's ratio tends towards 0.5 as our tangent modulus reduces to near zero and our bulk modulus K remains constant during the analysis that we will see. And the tangent Poisson's ratio tends towards 0.5.

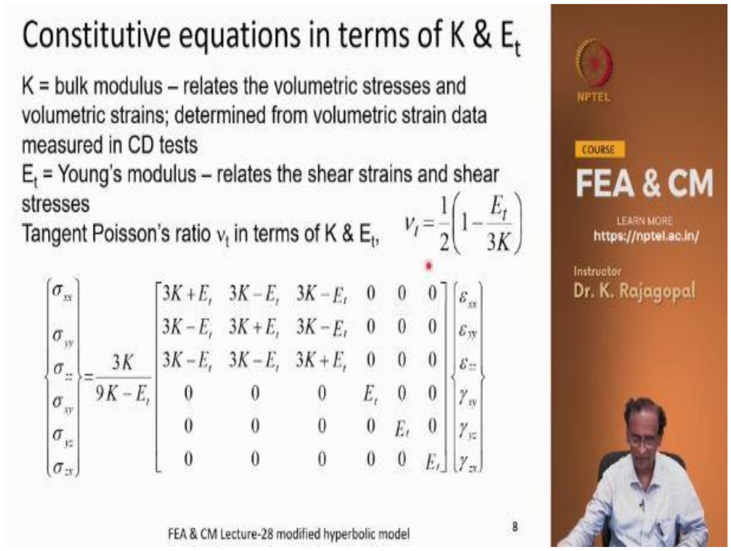
(Refer Slide Time: 05:49)



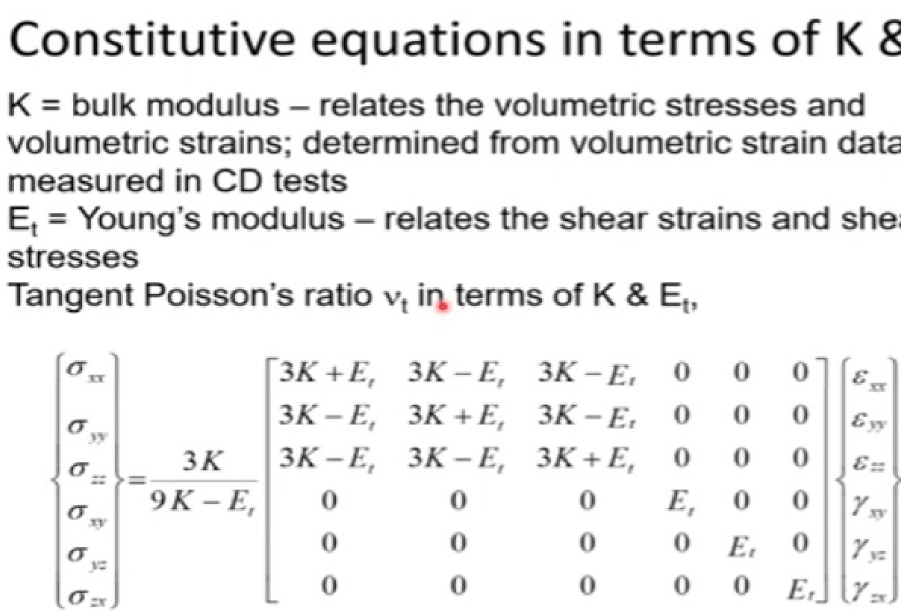
And because of that we will be able to represent the constant volume after the critical state is reached. And this particular modified hyperbolic model is written in terms of tangents

Young’s modulus E t and then tangent bulk modulus K t. And K t is the bulk modulus that relates the volume changes to the bulk stress and E t is the tangent Young's modulus.

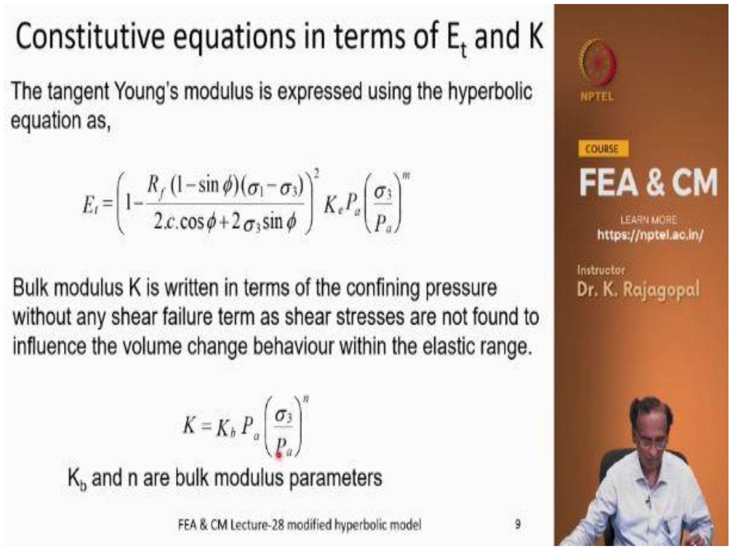
(Refer Slide Time: 06:21)



And the K is the bulk modulus; it relates the volumetric stresses and volumetric strains. And this particular parameter we can determine from the volume strain data from CD test. And our Young's modulus E t relates the shear strains and shear stresses. And then the tangent Poisson’s ratio nu t is written in terms of our K and E t as one half of 1 - E by 3 K. And our constitute matrix the relation between the stress and strain can be written in terms of bulk modulus K and the tangents Young's modulus E t like this. And our nu t is one half of 1 - E by 3 K as E tends towards 0, our Poisson’s ratio tends towards 0.5.



(Refer Slide Time: 07:30)



And the tangent Young's modulus is represented by the same equation that was derived earlier. And then the bulk modulus K is expressed in an equation similar to our initial modulus as K b times P a sigma 3 by P a to the power n and there is no shear failure term here because the shear stresses do not affect our bulk modulus. The shear stresses and shear modulus are related and then the bulk modulus is only related to the bulk stresses or the volumetric strains.

The tangent Young's modulus is expressed using the hyperbolic equation as,

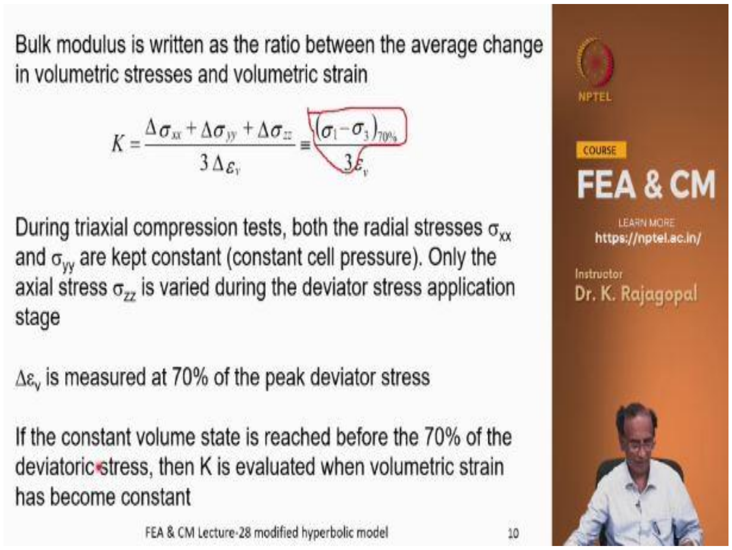
$$E_t = \left(1 - \frac{R_f (1-\sin\phi)(\sigma_1 - \sigma_3)}{2.c.\cos\phi + 2\sigma_3 \sin\phi}\right)^2 K_e P_a \left(\frac{\sigma_3}{P_a}\right)^m$$

So, the Mohr–Coulomb failure relation is not incorporated in this equation for K. So, if your sigma 3 is remaining constant, your K will remain constant during the analysis and our Young's modulus will go on decreasing with increasing shear stresses. And in this equation, we have number of parameters, the c and phi are the strength parameters and R f, K e and m these are related to the Young's modulus parameters. And then the K b and n are our bulk modulus parameters.

$$K = K_b P_a \left(\frac{\sigma_3}{P_a}\right)^n$$

K_b and n are bulk modulus parameters

(Refer Slide Time: 09:04)

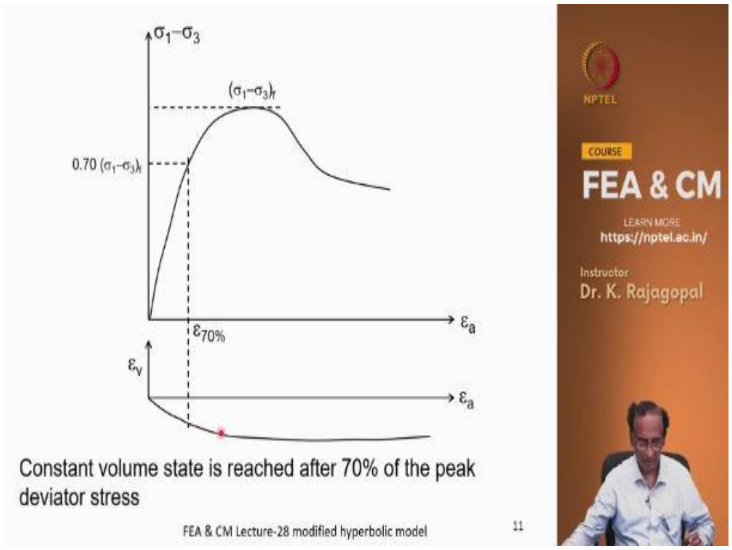


And where bulk modulus K can be represented as the change in the average change in the bulk stress is divided by volumetric strain or if you relate to the triaxial compression test, we can express the K as sigma 1 - sigma 3 at 70 percent stress level divided by 3 epsilon v. This is our mean normal stress because, in the triaxial compression test, during the application of the deviator stress, the delta sigma x and delta sigma y are 0 and we have only the axial stress that is the deviated stress.

$$K = \frac{\Delta\sigma_{xx} + \Delta\sigma_{yy} + \Delta\sigma_{zz}}{3\,\Delta\varepsilon_v} \equiv \frac{(\sigma_1 - \sigma_3)_{70\%}}{3\varepsilon_v}$$

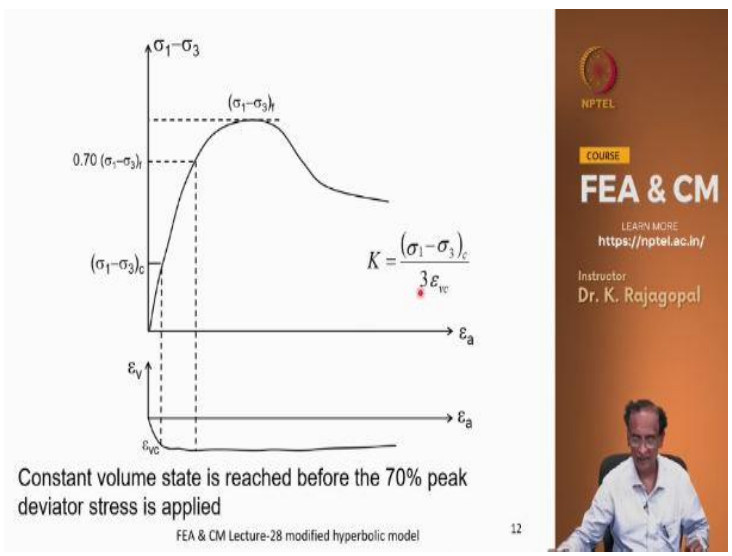
And the bulk modulus is related to the deviator stress at 70 percent strength level divided by 3 epsilon v. This is our mean normal stress, because our sigma x and sigma y are 0 and the only normal stress that we have is the axial stress. So, this is our mean normal stress divided by our volumetric strain, epsilon v is our bulk modulus. And in case we reach the constant volume state before the 70 percent of the deviator stress is reached, then the K is evaluated when the volumetric strain has become constant like this.

(Refer Slide Time: 11:05)



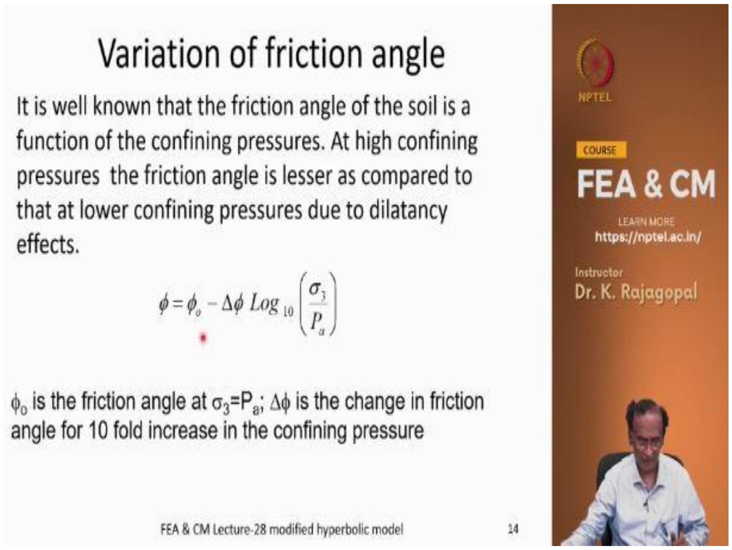
Say in general our volumetric the bulk modulus is represented as the ratio of this divided by 3 2 volumetric strain at this 70 percent of the peak stress.

(Refer Slide Time: 11:32)



If the constant volume state is reached at a very large strain level, but sometimes at very low axial strains itself, we might reach the constant volume state in that case, we use this epsilon v constant and then sigma 1 - sigma 3 c that is when the constant volumetric strain is reached. In this case, our K is sigma 1 - sigma 3 at the constant volume strain divided by 3 epsilon v.

(Refer Slide Time: 12:02)



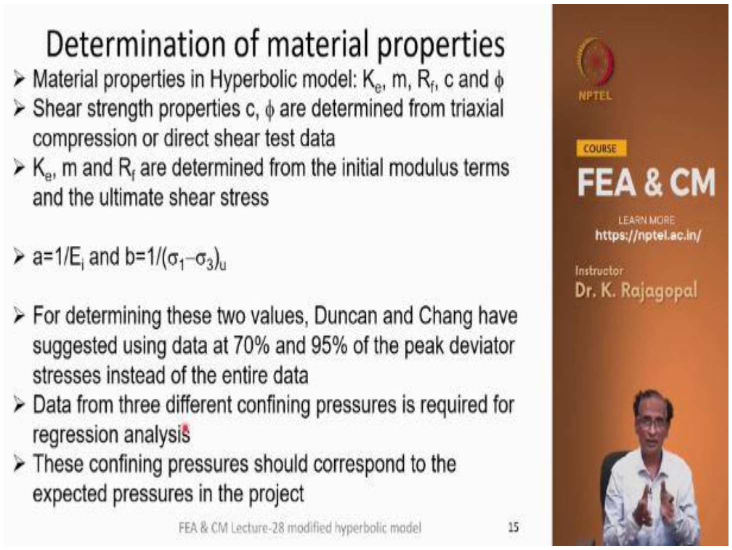
And we can actually relate the variation of the friction angle with respect to the confining pressure through this relation phi is phi naught - delta phi log sigma 3 by P a to the base 10. And basically at higher confining pressures our shear strength will reduce and that is expressed by this equation as your sigma 3 is increasing your phi will reduce. The phi naught could be your shear strength at an average confining pressure equal to atmospheric pressure P a.

$$\phi = \phi_o - \Delta\phi \; Log_{10}\left(\frac{\sigma_3}{P_a}\right)$$

ϕ_o is the friction angle at σ_3=P_a; $\Delta\phi$ is the change in friction angle for 10 fold increase in the confining pressure

And that lower confining pressures, so at a sigma 3 less than P a, this quantity becomes positive because your log of sigma 3 by P a becomes negative, your friction angle will increase; at higher confining pressures your friction angle will reduce.

(Refer Slide Time: 13:15)



So, our material parameters for the hyperbolic model are K e, m, R f, c, phi and then our K b and n and the c and phi can be determined from triaxial compression tests or direct shear test data. And K e, m and R f are determined from the initial modulus terms and the ultimate shear stress that I will explain. And basically, we have seen a graph between epsilon and epsilon by sigma 1 - sigma 3 and we said that for a truly hyperbolic behaviour that graph is a straight line.

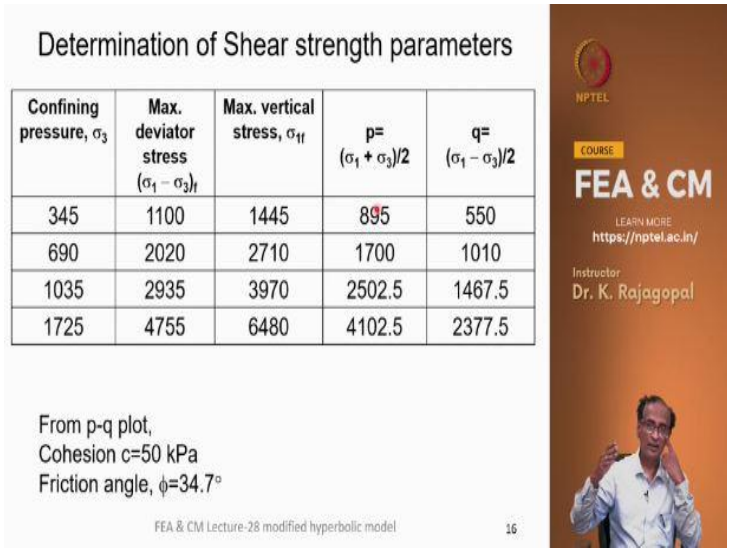
$$a=1/E_i \text{ and } b=1/(\sigma_1-\sigma_3)_u$$

And what Duncan and Chang have done is that you do not need to plot that line based on all the data points; but you select two data points corresponding to 70 percent of the peak stress and 95 percent of the peak stress and plot this graph of epsilon versus epsilon by sigma 1 - sigma 3. And then the intercept will give you this a or the reciprocal of the initial modulus and then the slope is your reciprocal of the ultimate deviator stress.

So, basically, it is a very simple one because now instead of using all the data points we use only two data points, one corresponding to 70 percent of the peak stress and the other corresponding to 95 percent of the peak stress and for determining the model parameters we need to perform the triaxial compression tests or different confining pressures and the range of these confining pressures should correspond to what we expect in the field.

So, if you are dealing with a dam of some 20 meters height, we can calculate what are the operating range of pressures like 20 meters height multiplied by an average gamma of 20 means that 400 kPa and if your K naught is about 0.5 then you are confining pressure could be of the order of 200 kPa. So, based on some empirical calculations like this we can decide what should be the range of your confining pressures in the laboratory test.

(Refer Slide Time: 16:12)



And let me illustrate this procedure with an example and here we have data from four different triaxial compression tests performed at four different confining pressures of 345, 690, 1035 and 1725 and then these are the corresponding maximum deviator stresses 1100, 2020, 2935, 4755 and then the maximum vertical stress is basically sigma 1 - sigma 3 + sigma 3, 1445, 2710, 3970 and 6480.

Determination of Shear strength parameters

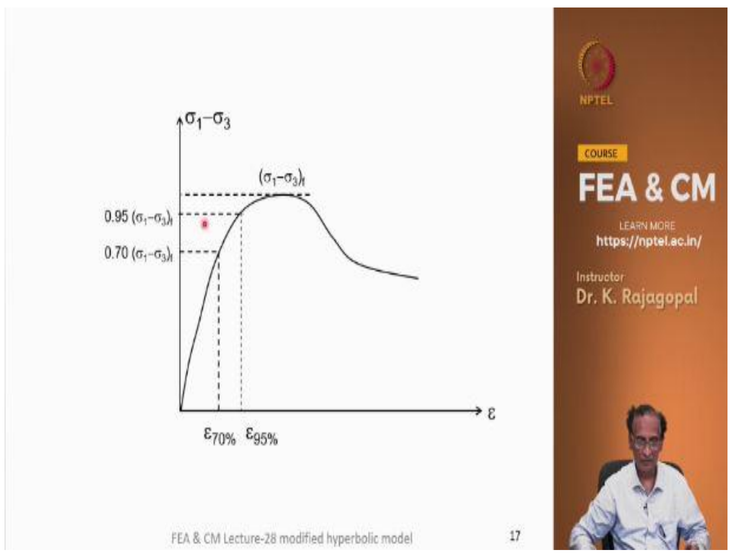
Confining pressure, σ_3	Max. deviator stress $(\sigma_1 - \sigma_3)_f$	Max. vertical stress, σ_{1f}	p= $(\sigma_1 + \sigma_3)/2$	q= $(\sigma_1 - \sigma_3)/2$
345	1100	1445	895	550
690	2020	2710	1700	1010
1035	2935	3970	2502.5	1467.5
1725	4755	6480	4102.5	2377.5

From p-q plot,
Cohesion c=50 kPa
Friction angle, ϕ=34.7°

And then we can determine our p and q values, p is a sigma 1 + sigma 3 by 2, q is sigma 1 - sigma 3 by 2. Then we can plot a graph between the p and q, p on the x axis and the q and the y axis and then get the cohesion as 50 kPa and friction angle phi of 34.7 degrees and this is a very simple thing that we learned in the advanced soil mechanics courses on the shear strength of the soils.

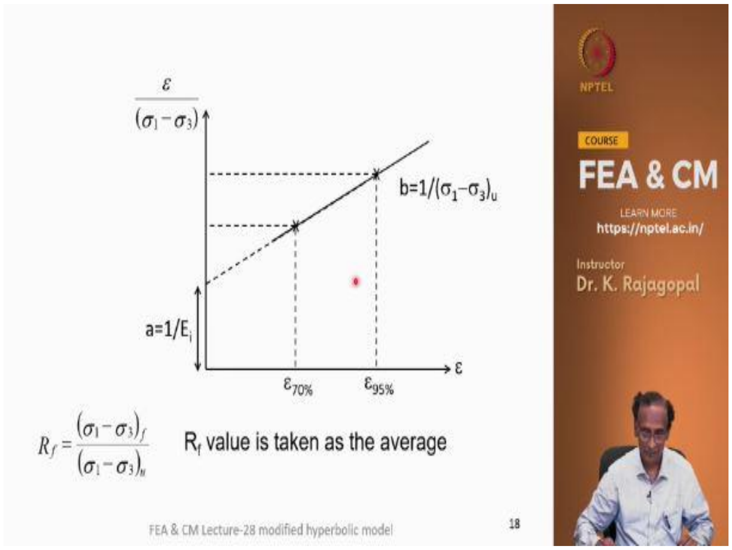
If you do not want to draw the p-q diagram, we can draw four more circles and then draw a common tangent and then we can get these values. The advantage of the p-q diagram is that we can do the regression analysis. We can give this data to any calculator or Excel spreadsheet program and then fit a straight line and get our intercept on the y axis and then the slope and from that the phi is sine inverse of that slope and then the c is the intercept divided by cosine phi. And in this case, the c is 50 kPa and phi is 34.7.

(Refer Slide Time: 18:38)



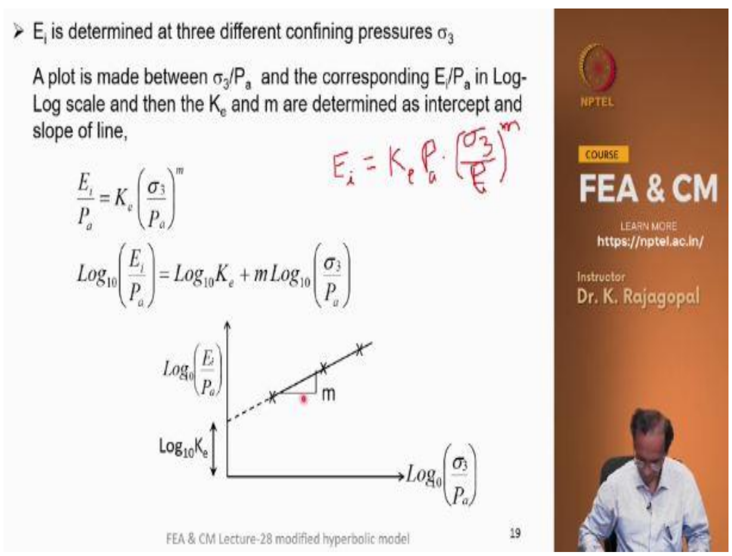
And then for determining our Young's modulus parameters, our K e, m and R f we need two data points corresponding to 70 percent of the peak stress and 95 percent of the peak stress and then here I have illustrated that this is your peak stress 70 percent of this, 95 percent of this and then the corresponding constraint we indicated as epsilon 70 percent that is the strain at a stress equal to 70 percent of the peak stress and then epsilon 95 is the strain corresponding to the 95 percent of the peak stress.

(Refer Slide Time: 19:29)



And then we can plot a graph between epsilon and epsilon by sigma 1 - sigma 3 and then get the intercept and the slope and the intercept is 1 by E i and b is the reciprocal of the ultimate stress and R f is sigma 1 - sigma 3 f by sigma 1 - sigma 3 ultimate, that is this b.

(Refer Slide Time: 20:00)



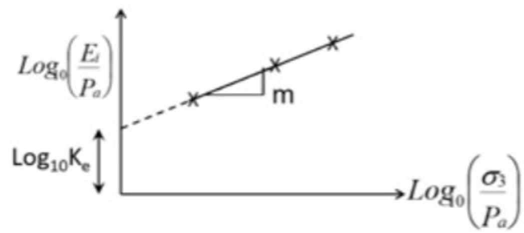
So, our E i we need to determine at least at 3 different confining pressures and then we can plot a graph between log sigma 3 by P a and log E i by P a. So, our equation for Young's modulus but anyway this is our equation for the Young's modulus and this we can rewrite as E i by P a = K e times sigma 3 by P a to the power m and by taking the log on both the left hand side and the right hand side, we get this.

E_i is determined at three different confining pressures σ_3

A plot is made between σ_3/P_a and the corresponding E_i/P_a in Log-Log scale and then the K_e and m are determined as intercept and slope of line,

$$\frac{E_i}{P_a} = K_e\left(\frac{\sigma_3}{P_a}\right)^m$$

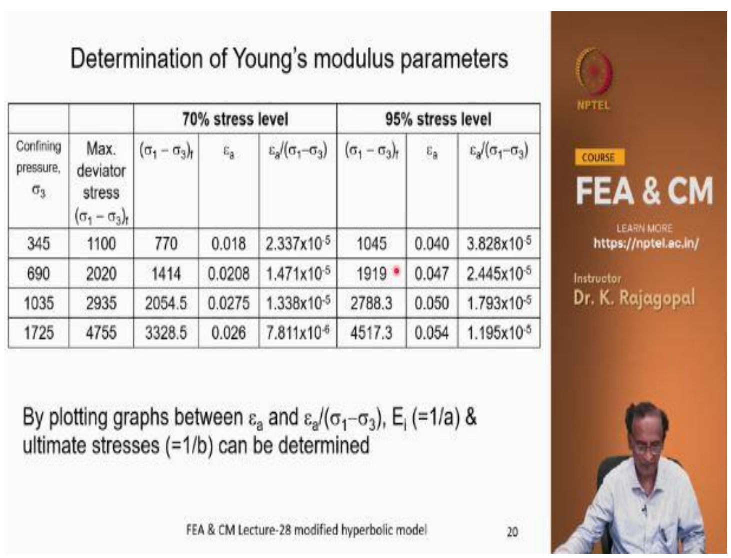
$$Log_{10}\left(\frac{E_i}{P_a}\right) = Log_{10}K_e + m\,Log_{10}\left(\frac{\sigma_3}{P_a}\right)$$



So, if you plot a graph between the log of E i by P a and log of sigma 3 by P a, this quantity log K e is the intercept on the y axis and then the slope is your m.

$$E_i = K_e P_a \cdot \left(\frac{\sigma_3}{P_a}\right)^m$$

(Refer Slide Time: 21:52)



Determination of Young's modulus parameters

Confining pressure, σ_3	Max. deviator stress $(\sigma_1-\sigma_3)_f$	70% stress level $(\sigma_1-\sigma_3)_f$	ε_a	$\varepsilon_a/(\sigma_1-\sigma_3)$	95% stress level $(\sigma_1-\sigma_3)_f$	ε_a	$\varepsilon_a/(\sigma_1-\sigma_3)$
345	1100	770	0.018	2.337×10^{-5}	1045	0.040	3.828×10^{-5}
690	2020	1414	0.0208	1.471×10^{-5}	1919	0.047	2.445×10^{-5}
1035	2935	2054.5	0.0275	1.338×10^{-5}	2788.3	0.050	1.793×10^{-5}
1725	4755	3328.5	0.026	7.811×10^{-6}	4517.3	0.054	1.195×10^{-5}

By plotting graphs between ε_a and $\varepsilon_a/(\sigma_1-\sigma_3)$, E_i (=1/a) & ultimate stresses (=1/b) can be determined

So, we have the basic data from four different confining pressures. And then we can take the 70 percent stress level and 95 percent stress level at each of the confining pressures. So, at a confining pressure of 345, the maximum deviator stress is 1100. So, 70 percent of this is 770 and 95 percent of this is 1045 and then the corresponding axial strain is epsilon a and this is epsilon a by sigma 1 - sigma 3 and epsilon a is of 0.04 at 95 percent, this is your this thing.

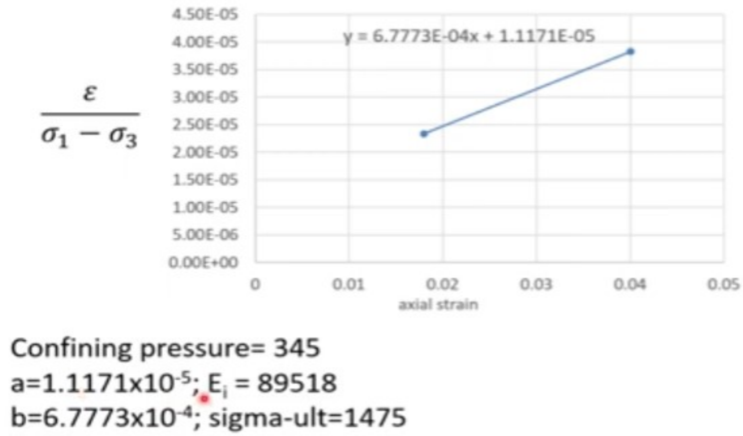
Determination of Young's modulus parameters

		70% stress level			95% stress level		
Confining pressure, σ_3	Max. deviator stress $(\sigma_1 - \sigma_3)_f$	$(\sigma_1 - \sigma_3)_f$	ε_a	$\varepsilon_a/(\sigma_1-\sigma_3)$	$(\sigma_1 - \sigma_3)_f$	ε_a	$\varepsilon_a/(\sigma_1-\sigma_3)$
345	1100	770	0.018	2.337×10^{-5}	1045	0.040	3.828×10^{-5}
690	2020	1414	0.0208	1.471×10^{-5}	1919	0.047	2.445×10^{-5}
1035	2935	2054.5	0.0275	1.338×10^{-5}	2788.3	0.050	1.793×10^{-5}
1725	4755	3328.5	0.026	7.811×10^{-6}	4517.3	0.054	1.195×10^{-5}

By plotting graphs between ε_a and $\varepsilon_a/(\sigma_1-\sigma_3)$, E_i (=1/a) & ultimate stresses (=1/b) can be determined

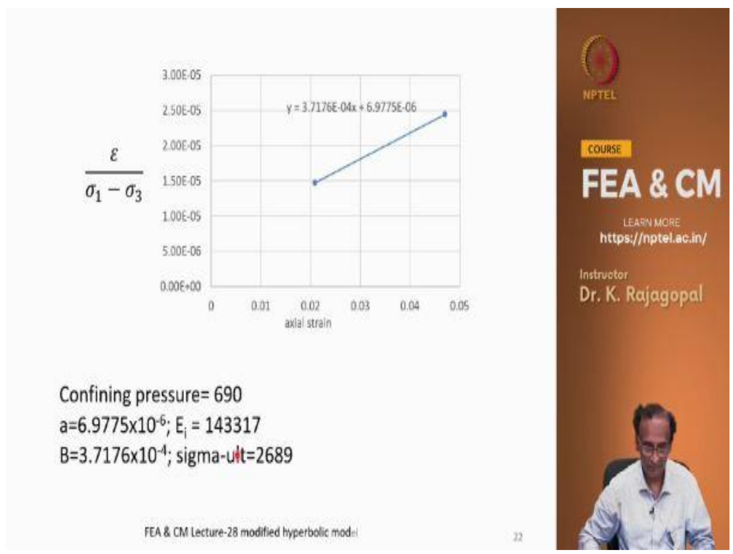
(Refer Slide Time: 22:49)

Then we can plot graph between epsilon and epsilon by sigma 1 - sigma 3 at different confining pressures. So, this is the graph for confining pressure of 345. I plotted a graph between epsilon and epsilon by sigma 1 - sigma 3 and the intercept and the y axis a is this and the initial Young's modulus is reciprocal of this that is 89518 and the b is this slope of this line that is the inverse of the ultimate stress.

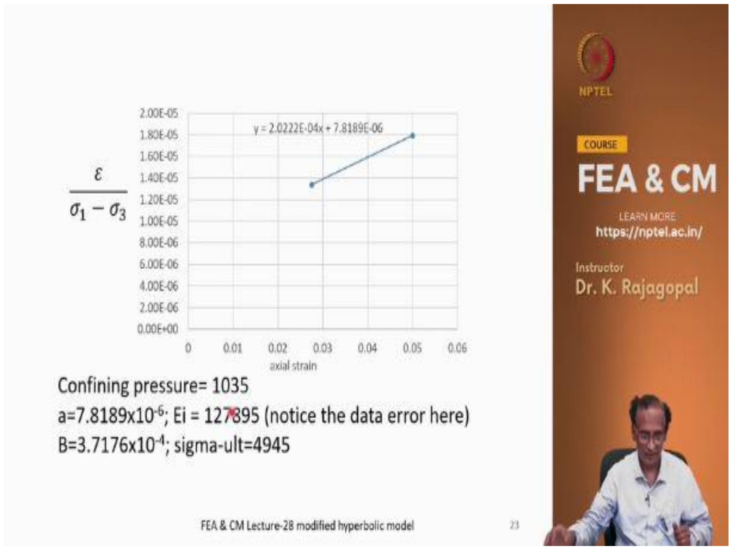


So, our sigma 1 ultimate is 1475 and this is the test, but 345 confining pressure and the sigma 1 - sigma 3 failure is 1100 and sigma 1 - sigma 3 ultimate is a 1475. So, our R f can be determined as 1100 by 1475.

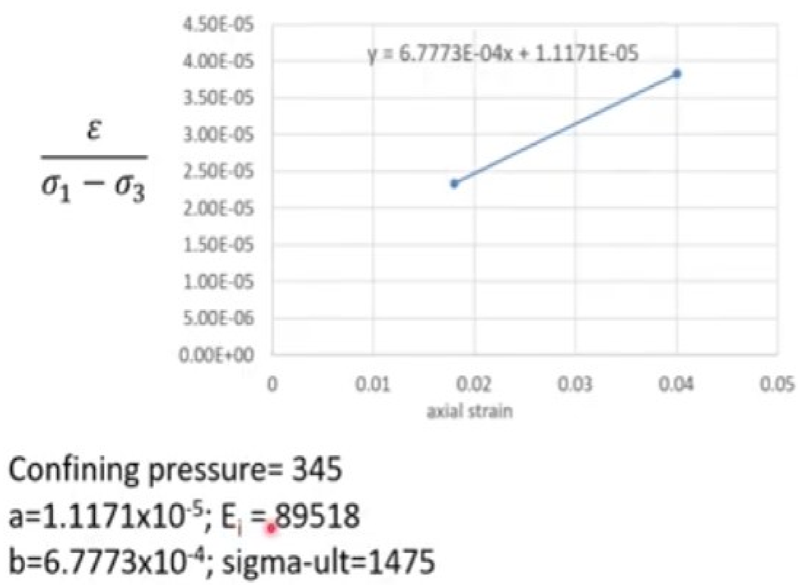
(Refer Slide Time: 24:02)

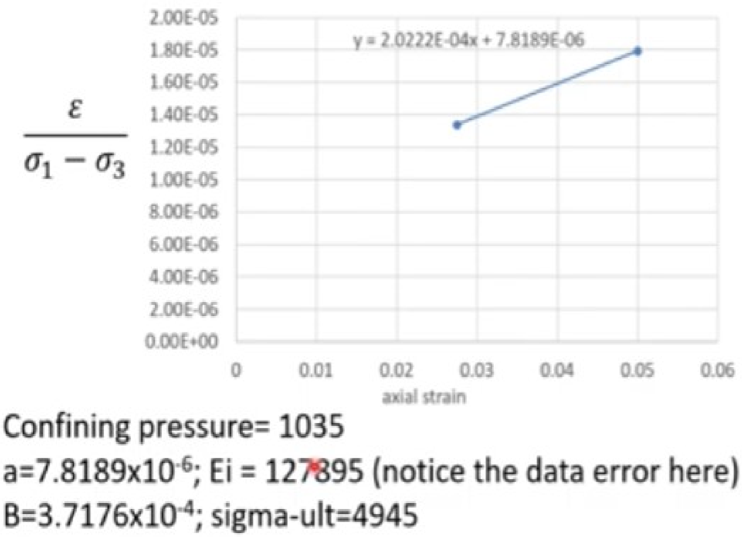


(Refer Slide Time: 24:10)



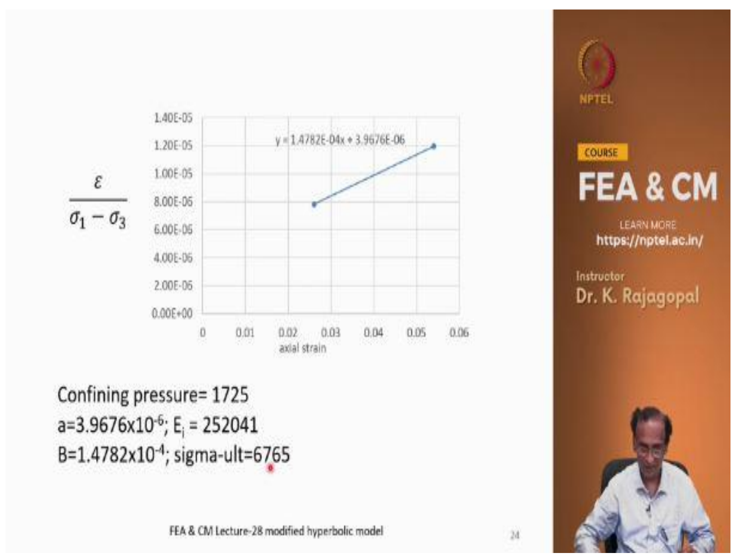
And similarly at a confining pressure of 690, our a and B are this and confining pressure of 1035. And fact as we see at confining pressure of 345 our initial modulus is 89518 and at a confining pressure of 690, our initial modulus has increased to 143317, but then at a confining pressure of 1035, the initial modulus is reduced. That means that there is something wrong with the data and that is not consistent with what we expect but, sometimes this is what happens.



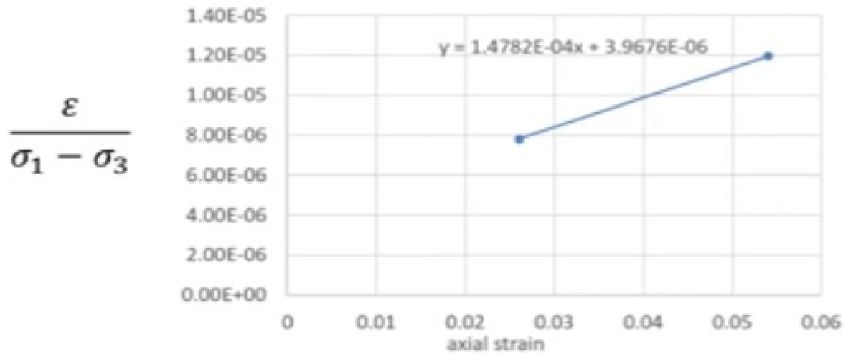


When we do a laboratory test, we may get some value that is not consistent that could be either because of the sample disturbances or because of not following the standard procedures for doing the laboratory test. So, our sigma 1 ultimate is 4945.

(Refer Slide Time: 25:26)

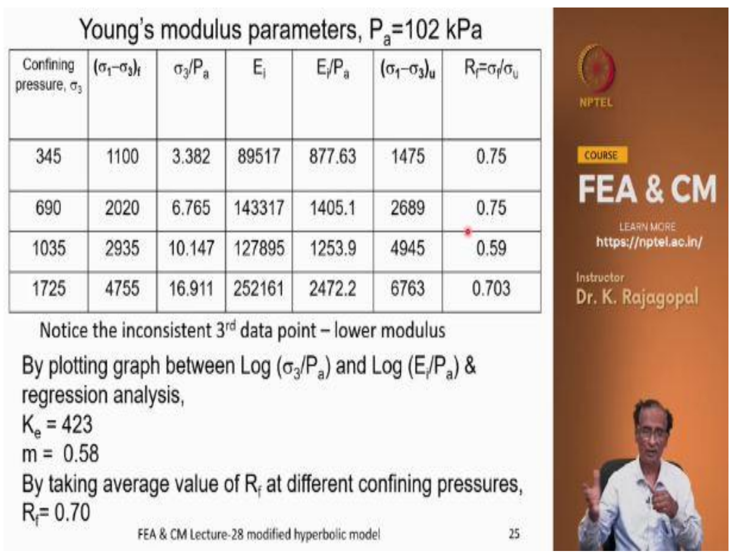


Similarly at a confining pressure of 1725 the E i is this and sigma 1 ultimate is 6765.



Confining pressure= 1725
a=3.9676x10^{-6}; E_i = 252041
B=1.4782x10^{-4}; sigma-ult=6765

(Refer Slide Time: 25:34)



So, if you look at our data at 345, our initial modulus is 89517, 690, 143317 and our 1035 is 127895, 1725, 2502161 and our sigma 1 - sigma 3 ultimate values are like this: Our R f is sigma 1 - sigma 3 f divided by this ultimate value. So, at a confining pressure of 345, the R f is 1100 by 1475, so approximately 0.75. Once again at 690 it is about 0.75 and at 1035 it is about 0.6, 1725 0.703.

Young's modulus parameters, P_a=102 kPa

Confining pressure, σ_3	$(\sigma_1-\sigma_3)_f$	σ_3/P_a	E_i	E_i/P_a	$(\sigma_1-\sigma_3)_u$	$R_f=\sigma_f/\sigma_u$
345	1100	3.382	89517	877.63	1475	0.75
690	2020	6.765	143317	1405.1	2689	0.75
1035	2935	10.147	127895	1253.9	4945	0.59
1725	4755	16.911	252161	2472.2	6763	0.703

Notice the inconsistent 3rd data point – lower modulus

By plotting graph between Log (σ_3/P_a) and Log (E_i/P_a) & regression analysis,

K_e = 423

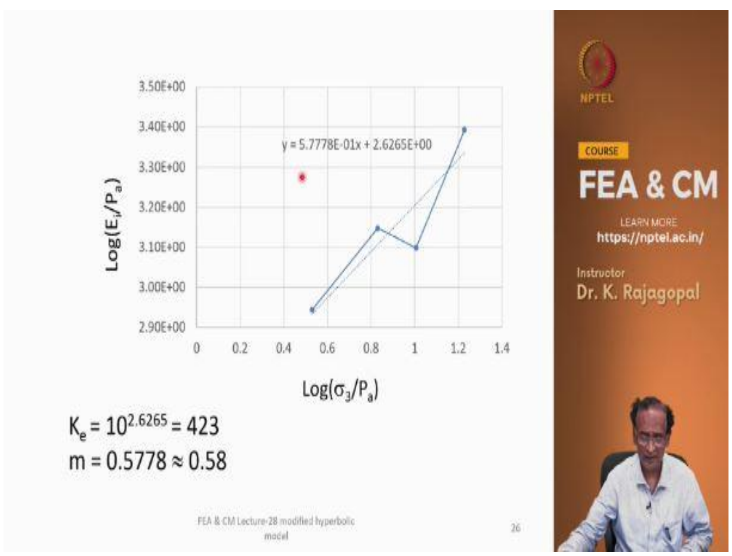
m = 0.58

By taking average value of R_f at different confining pressures,

R_f= 0.70

And what we do is we just simply take an average value of R f because we do not have a different R f values a different confining pressures in that equation. So, we take an average value of R f and use that same R f at all the confining pressures.

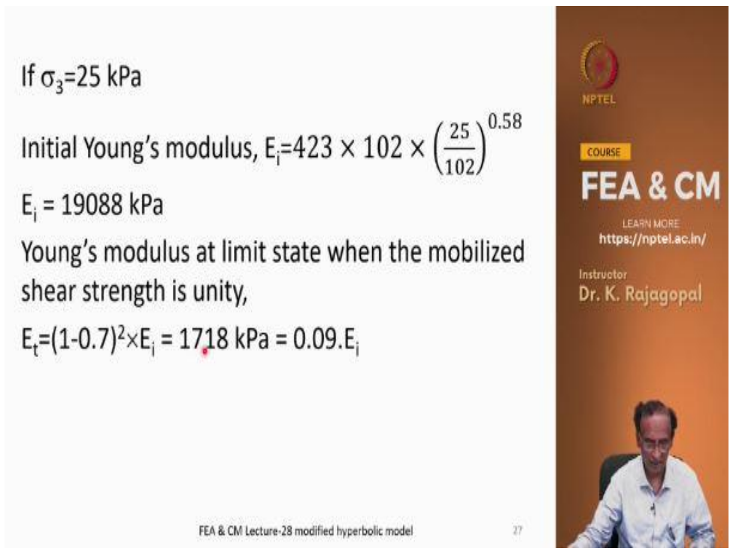
(Refer Slide Time: 27:01)



And if you plot a graph between so here, this we have at a sigma 3 of 345; the initial modulus is 89517 and this is sigma 3 by P a and E i by P a and then at 690, 1035, 1725, we get different sigma 3 by P a and E i by P a. And then we take the log of sigma 3 by P a and the x axis and the log of E i by P a and the y axis and you see that at this confining pressure your Young's modulus is reduced instead of increasing.

So, we might have as well omitted this data point, but anywhere just to illustrate I have included this in the regression analysis. And if you do a regression analysis, you will get an equation like this. So, our intercept in the y axis is 2.6265; our K e is 10 to the power of 2.6265 that is approximately 423 and our exponent is the slope of this line, there is approximately 0.58.

(Refer Slide Time: 28:30)



If σ_3=25 kPa

Initial Young's modulus, $E_i = 423 \times 102 \times \left(\frac{25}{102}\right)^{0.58}$

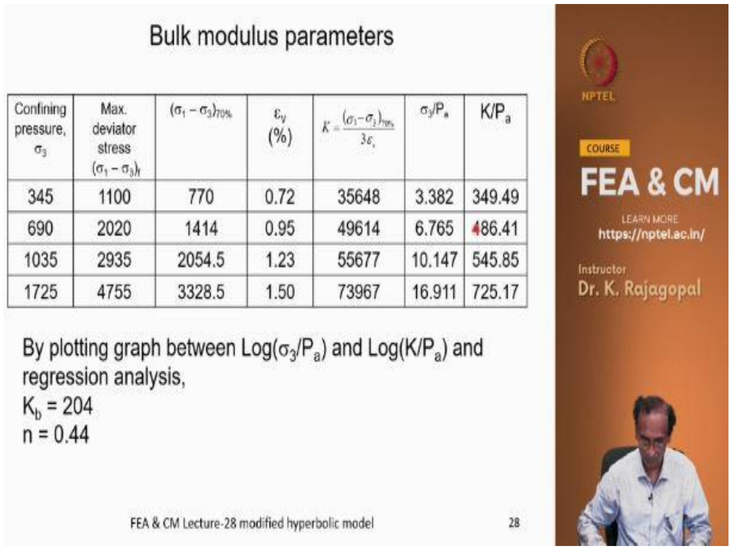
E_i = 19088 kPa

Young's modulus at limit state when the mobilized shear strength is unity,

$E_t = (1-0.7)^2 \times E_i$ = 1718 kPa = $0.09.E_i$

So, if you do this calculation at different confining pressure at sigma 3 of 25 our initial modulus is 19088. So, for this particular case our R f is 0.7 and so if the limit state is reached in that bracket 1 - sin phi times sigma 1 - sigma 3 by 2c cosine 5 + 2 sigma 3 sine phi, that becomes 1. So, that E tangent becomes 1 – R f times 1 square multiplied by E i, that comes to about 1718 and that is 9 percent of the initial modulus.

(Refer Slide Time: 29:25)



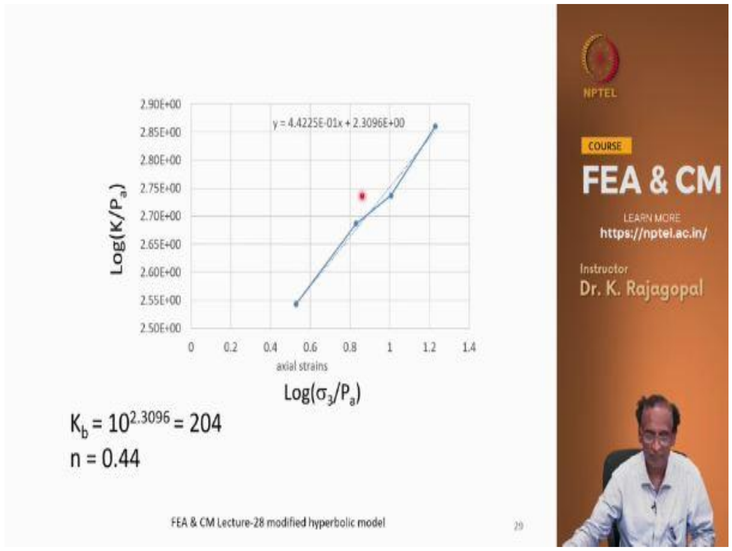
And we can do similar calculations for the bulk modulus and different confining pressures and at 70 percent of the maximum deviator stress; the volumetric strains from different tests are like this. And our bulk modulus is the mean normal stress that is sigma 1 - sigma 3 at 70 percent divided by 3 divided by epsilon v is your bulk modulus and then our sigma 3 by P a is this K by P a.

Bulk modulus parameters

Confining pressure, σ_3	Max. deviator stress $(\sigma_1 - \sigma_3)_f$	$(\sigma_1 - \sigma_3)_{70\%}$	ε_v (%)	$K = \frac{(\sigma_1 - \sigma_3)_{70\%}}{3\varepsilon_v}$	σ_3/P_a	K/P_a
345	1100	770	0.72	35648	3.382	349.49
690	2020	1414	0.95	49614	6.765	486.41
1035	2935	2054.5	1.23	55677	10.147	545.85
1725	4755	3328.5	1.50	73967	16.911	725.17

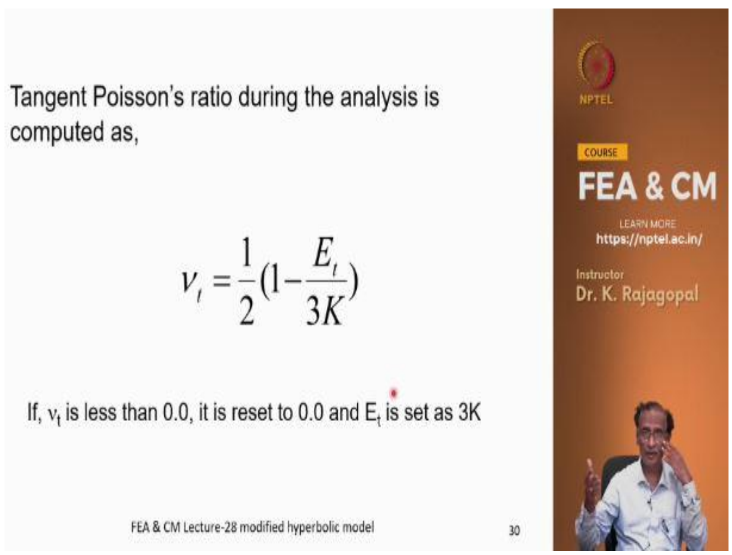
By plotting graph between Log(σ_3/P_a) and Log(K/P_a) and regression analysis,
$K_b = 204$
$n = 0.44$

(Refer Slide Time: 30:04)



And we can plot a graph between log of sigma 3 by P a and log of K by P a and then we get a regression equation like this. And our K b is a 10 to the power of this intercept that is 204 and n is the slope of this line that is 0.44.

(Refer Slide Time: 30:26)



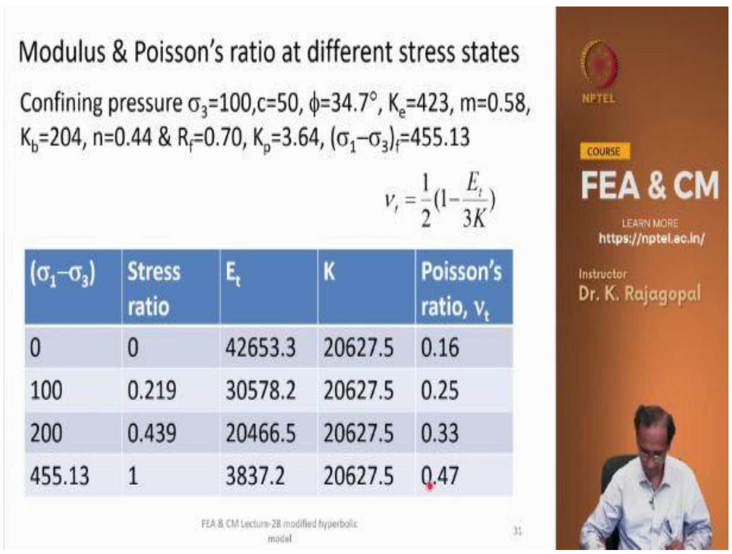
So, our tangent Poisson's ratio is now written as one half of 1 - E t by 3 K. In fact, we are not going to use any tangent Poisson's ratio and finite element calculations, because we are going to use only the bulk modulus K and then the tangent Young's modulus E t that we get from that long equation. So, sometimes depending on the initial values, if your K is very small your Poisson's ratio might become negative.

Tangent Poisson's ratio during the analysis is computed as,

$$\nu_t = \frac{1}{2}\left(1 - \frac{E_t}{3K}\right)$$

In that case we reset the Poisson's ratio to 0 and then our E t is set as 3 times the K. In case, E is more than 3 times the K, it is reset to 3K, because we do not want any negative Poisson's relation our analysis, our reasonable values for the Poisson's ratio about maybe 0.25 to 0.45, 0.5.

(Refer Slide Time: 31:42)



And just to illustrate at different shear stresses we see that our tangent modulus at the start is 42653 and our bulk modulus K remains constant, because it is not a function of the shear stress is only a function of sigma 3. And our bulk modulus remains constant. And then our tangent modulus will go on decreasing at higher shear stresses. So, to start with your Poisson's ratio is one half of 1 - E by 3K, E divided by 3 times this.

Modulus & Poisson's ratio at different stress states

Confining pressure σ_3=100,c=50, ϕ=34.7°, K_e=423, m=0.58, K_b=204, n=0.44 & R_f=0.70, K_p=3.64, $(\sigma_1-\sigma_3)_f$=455.13

$$\nu_t = \frac{1}{2}(1-\frac{E_t}{3K})$$

$(\sigma_1-\sigma_3)$	Stress ratio	E_t	K	Poisson's ratio, ν_t
0	0	42653.3	20627.5	0.16
100	0.219	30578.2	20627.5	0.25
200	0.439	20466.5	20627.5	0.33
455.13	1	3837.2	20627.5	0.47

Now, you will get about 0.16. And as E is reducing, your Poisson's ratio will increase. This is what happens? At stress of 100 our Poisson's ratio is increased to 0.25, at the stress of 200 0.33 and deviator stress of 455 our stress ratio is 1, that is the sigma 1 - sigma 3 by sigma 1 - sigma 3 failure that is 1, the mobilized shear strength ratio is 1 and our Poisson's ratio is 0.47 which is close to 0.5.

(Refer Slide Time: 33:19)

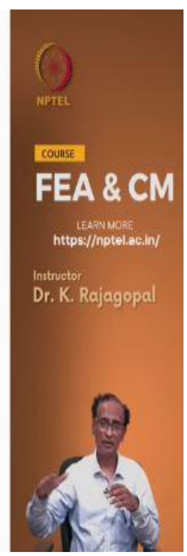
Modulus & Poisson's ratio at different stress states

Confining pressure σ_3=200,c=50, ϕ=34.7°, K_e=423, m=0.58, K_b=204, n=0.44 & R_f=0.70, K_p=3.64, $(\sigma_1-\sigma_3)_f$=818.78

$$\nu_t = \frac{1}{2}(1-\frac{E_t}{3K})$$

$(\sigma_1-\sigma_3)$	Stress ratio	E_t	K	Poisson's ratio, ν_t
0	0	63760	27983	0.12
200	0.244	43840	27983	0.24
400	0.488	27639	27983	0.335
818.78	1	5738	27983	0.466

FEA & CM Lecture-28 modified hyperbolic model 32



Modulus & Poisson's ratio at different stress states

Confining pressure σ_3=200, c=50, ϕ=34.7°, K_e=423, m=0.58, K_b=204, n=0.44 & R_f=0.70, K_p=3.64, $(\sigma_1-\sigma_3)_f$=818.78

$$\nu_t = \frac{1}{2}\left(1-\frac{E_t}{3K}\right)$$

$(\sigma_1-\sigma_3)$	Stress ratio	E_t	K	Poisson's ratio, ν_t
0	0	63760	27983	0.12
200	0.244	43840	27983	0.24
400	0.488	27639	27983	0.335
818.78	1	5738	27983	0.466

So, actually let us just let me show you the Excel spreadsheet program for the modified hyperbolic model. **(Video Start: 33:33)** In this modified hyperbolic model, we have more number of parameters. The c and phi are the shear strength parameters, sigma 3 let me just do it at 100 and K e and P a are 423 and 102 and the exponent for the Young's modulus term is 0.48.

And then our E initial is 42653, our incremental strain is 0.002, for doing our triaxial compression test R f is 0.85, K b and n or 204 and 0.44**.** And then the bulk modulus is 20727 that will remain constant all through. And we can perform the triaxial compression test in the same manner as what we had done in the case of bilinear elastic model and t states we calculate the tangent Young's modulus.

And then, the incremental stresses is incremental strain 0.002 multiplied by the tangent modulus. So, basically if you see this equation c 16 times b 8, b8 is a 0.002 and c 16 is 42653 and incremental stress. And then the total axial stress is previous stress is 100 + 85 185 and then as you go along your tangent Young's modulus goes on reducing and your Poisson's ratio initially it is about 0.166 then it has increased to 0.298364 and so on.

So, our Poisson's ratio will go on increasing and our maximum are the limiting axial stresses 407 and let us see so here, around this 407 we have reached the limit state and our Poisson's ratio is reached about 0.491. And beyond that also it is changing, because our Young's modulus will go on reducing. And you see at the limit state the initial the tangent Poisson's ratio has increased from an initial value of 0.155 to about 0.49.

So, that is what when we look at the volumetric strain graph it will initially compress at a very fast phase, but then towards the limit state as the Poisson's ratio approaches 0.5 your further volumetric strains will not increase much. So, your volume strain graph will look something like this, which is almost like our constant volume state. And then the stress strain curve is similar to this, initial sigma 1 is 100, because our sigma 3 is 100.

Then with increasing axial strain, we get increasing stresses. And in this particular case, I have plotted only up to 5 percent to axial strain. So, at 5 percent axial strain the sigma 1 is 412. And as we go on increasing the axial strain, the axial stress will go on increasing without any stop. And let me just illustrate, at a confining pressure of 100, our limiting Poisson's ratio is about 0.49 actually it is it is gone going almost to 0.5.

And the initial Poisson's ratio is about 0.155 and let us see what happens at a confining pressure of 500 kPa. And we see our initial Poisson's ratios very small 0.06. And it becomes more and more brittle, because the ideal brittle material has a Poisson's ratio of close to 0. So, at a higher confining pressure our soil is becoming more and more brittle. So, this is what we see 0.068 for the initial value.

And then as the shear stress has gone increasing, your Poisson's ratio tends towards 0.5. So, if your stress strain graph is something like. This initially it will increase very fast and then after reaching the limit state it will increase very slowly. And the volumetric strain graph also is very beautiful, because after limit state your further increase in the axial strains is not much. Let me just change the limit on the x axis how do I change the limits.

The none I think I not able to get the change in the x value, but if you plot it the volumetric strain, you see it is continuing to increase. Then after the limit state is reached, it is more or less remaining constant. As you can see from the values here 1208, 1209, 121 and so on it is basically it has reached constant volume state. So, because we are representing the constitutive matrix in terms of a tangent Young's modulus that is a function of the shear stresses and then a bulk modulus that is not a function of the shear stresses.

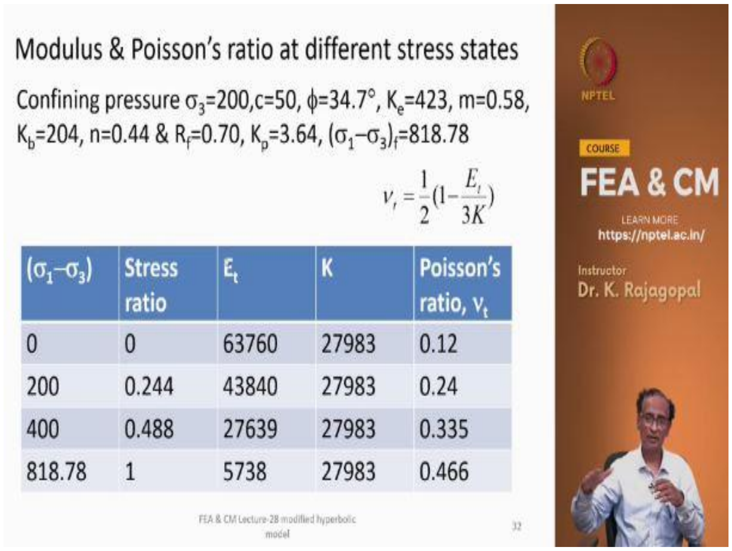
There is no failure term in the bulk modulus. So, because of that, as the E reduces your Poisson's ratio will go on increasing towards 0.5. So, that is what we see here. So, the initial Poisson's ratio was 0.068 and then quickly it has increased to 0.175, 0.246, 0.295 and so on. So, the other thing that we need to see is our sigma 1 failure should be 1883 as per hour Mohr–Coulomb relation.

But then if we see this predicted sigma 1 it is much more than 1883, it is increase into to more than 2000**,** that is what we see here. So, if you look at this it is you actually have to plot beyond. I will not change this value, but you can do it yourself like you can increase the range of the x values and the plot up to larger axial strain. Then we will see that your volume will remain more or less constant.

So, let us just do for one more confining pressure. Let us say 200 confining pressure. And here at a confining pressure of 200 our Poisson's ratio is 0.12, at a confining pressure of 100 the initial Poisson's ratio was about 0.15. Now, it is 0.12 and if I change this confining pressure to 50. So, our initial Poisson's ratio will be higher 0.18. So, we see that this modified hyperbolic model is able to also incorporate the influence of tangent Poisson's ratio on our predicted volume changes and towards the limit state our volume more or less remains constant.

And the only thing that we notice here is that both the original hyperbolic model and also in the modified hyperbolic model the predicted deviator stress is much higher than the theoretical limit. Say for example, for this case of 50 confining pressure, the maximum sigma 1 is 222, but your stress is predicted are much higher than 222. So, that is one thing that we need to fix and that we can fix by decreasing the incremental strain ratio we can work with smaller strain increments to increase our accuracy.

That is what we had seen with the bi-linear elastic model. The same principle works even with the hyperbolic models. And here also we see our incremental stresses are much lower, because our incremental strain is only 0.001 and because of that our stress predictions are slightly better and they are more accurate. So, let us go back to our Power point file. **(Video End: 46:50)**
(Refer Slide Time: 46:53)



So, the modified hyperbolic model is able to represent the stress strain behaviour and also the constant volume state after the limit state.

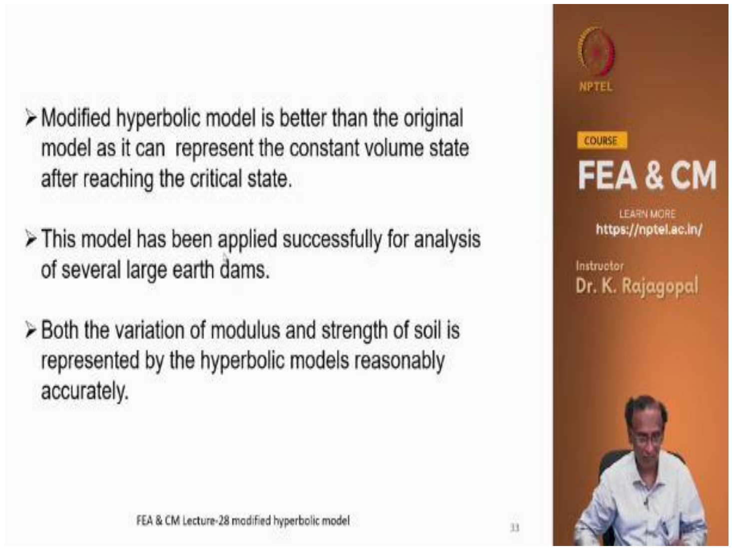
Modulus & Poisson's ratio at different stress states

Confining pressure σ_3=200,c=50, ϕ=34.7°, K_e=423, m=0.58, K_b=204, n=0.44 & R_f=0.70, K_p=3.64, $(\sigma_1-\sigma_3)_f$=818.78

$$\nu_t = \frac{1}{2}\left(1-\frac{E_t}{3K}\right)$$

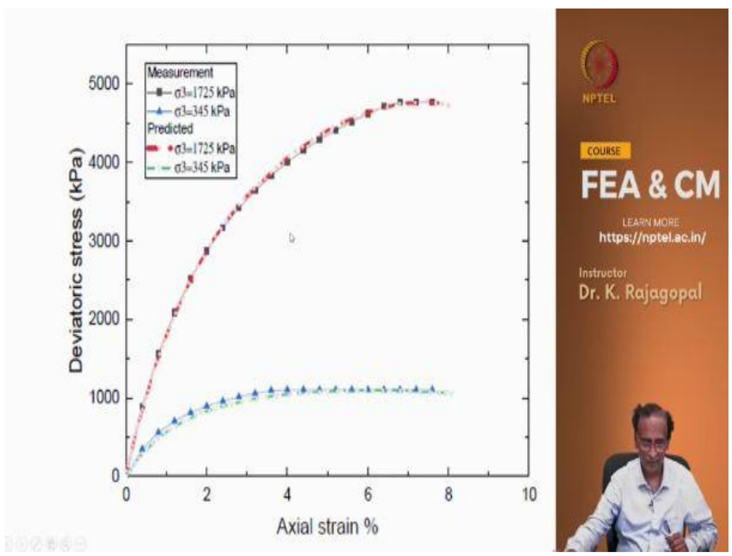
$(\sigma_1-\sigma_3)$	Stress ratio	E_t	K	Poisson's ratio, ν_t
0	0	63760	27983	0.12
200	0.244	43840	27983	0.24
400	0.488	27639	27983	0.335
818.78	1	5738	27983	0.466

(Refer Slide Time: 47:13)



And it is slightly better than the original hyperbolic model, but then it is also elastic. So, you will only predict volumetric compressions, you will not be able to predict the dilation.

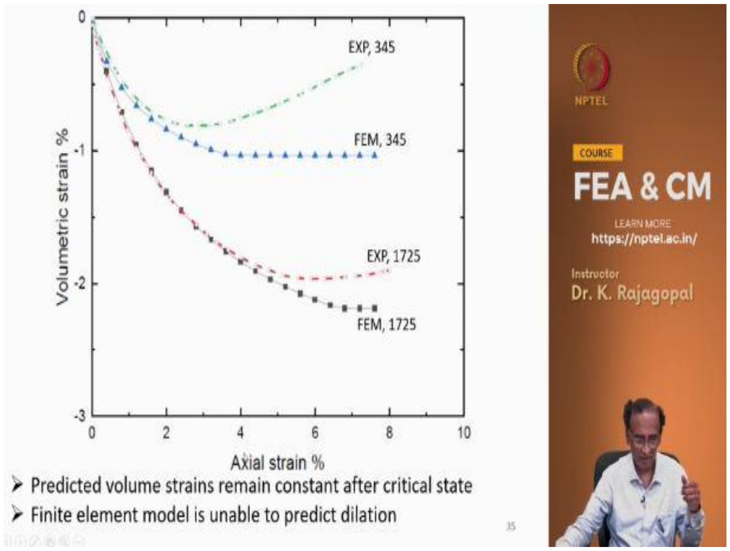
(Refer Slide Time: 47:32)



So, this is the comparison between the experimental data and then the finite element predicted the value in stress strain graphs, at confining pressure of 345 and then 1725. These are the two extreme pressures. And our hyperbolic parameters they were estimated based on only two values, one at 70 percent stress level and the other at 95 percent stress level, but still we are able to get a good prediction at all the stress levels.

So, that is the feature of this model. The Duncan and others they have analyzed a lot of data and then based on that they selected the only two data points, one at 70 percent and 95 percent. And then they said that you will be able to represent the entire stress strain curve. So, you get a good match at both the confining pressures using the same K and m parameters.

(Refer Slide Time: 48:54)



And then the volumetric strain graphs also are like this. And this is the experimental data. There is lot of dilation, after reaching the limit state whereas, in the finite element model there is only volumetric compression. So, initially there is volumetric compression and then after some time after reaching the limit state, the volumes remain constant in the finite element analysis, because we have not modeled the dilation.

So, at a higher confining pressure, because you reach a limit state at a larger axial strain we see that up to a larger axial strain, we have not reached the plastic limit state. So, we were compression even in the experimental data. Then after this the volume strains have slightly started increasing, but then if you look at the slope of this line and this line, there are two different values.

The slope at a lower confining pressure is higher, that means you get more dilation; whereas, at a higher confining pressure you do not get too much of dilation, because the confining pressure is so much that the soil cannot expand much. And this particular one is the finite element predicted volume change data. After the critical state the finite element predicted the volume strains they have remained constant, but then the finite element model is not able to predict the dilation.

So, that is a slight improvement in the original hyperbolic model that we are able to predict the constant volume state, after the limit state, but we are not able to represent the failure. And at higher strain rates like if you apply the strain increments at very coarse increments our predicted axial stresses are much higher than the theoretical limit. So, that we need to somehow correct, that we will see in the next class.

How to do that and before that if you have any questions please send an email to this address profkrg@gmail.com and then I will respond back to you. So, thank you very much.