FEM and Constitutive Modelling in Geomechanics
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Lecture: 11
2-Dimensional Approximations of Continuum

Hello students in the previous class we had looked at three dimensional continuum we have seen the stresses and the strains and then the definitions. And in today's class let us look at some special cases where we can approximate a three dimensional stress State as an equivalent to two dimensional states so, that we can gain some computational efficiency.

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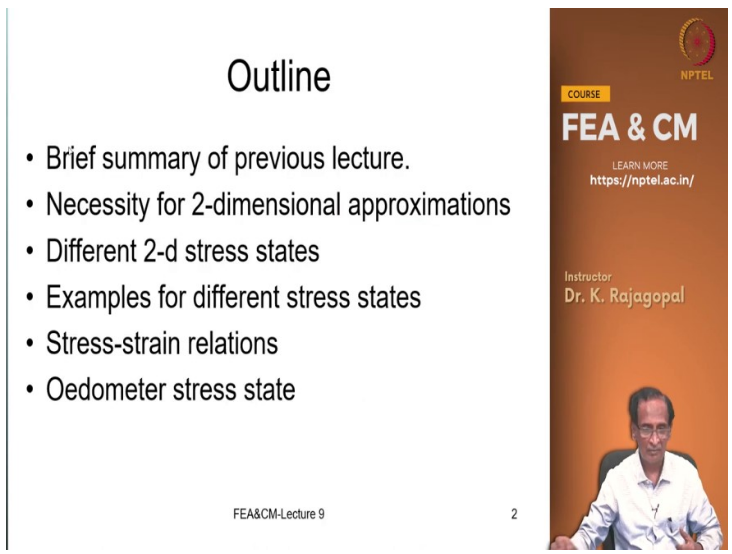

So, let us look at the brief outline say I will briefly summarize the previous lecture so, that we can have the continuity and then the necessity for two dimensional approximations different 2D stress states what are the examples and then the stress strain relations. And then I will also consider one special case that is unique to to geotechnical engineering that is the odometer stress state.

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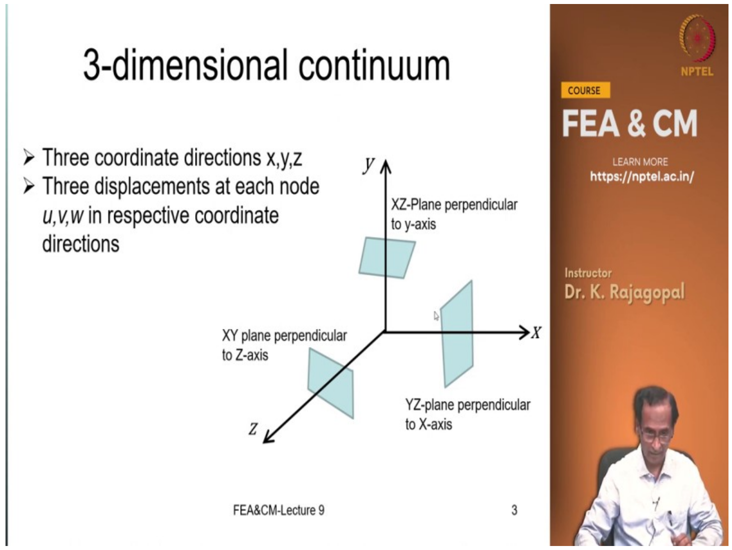

So, we have seen the three dimensional continuum with three axis x, y and z then we have defined the three planes perpendicular to these axis perpendicular to x axis is our yz plane perpendicular to z axis is our xy plane perpendicular to to y axis is our xz plane and so on. And then the stresses acting normal to these planes are called as the normal stresses and then the strains are the stresses with parallel to these planes they are called as the shear strains or the shear stresses and so on.

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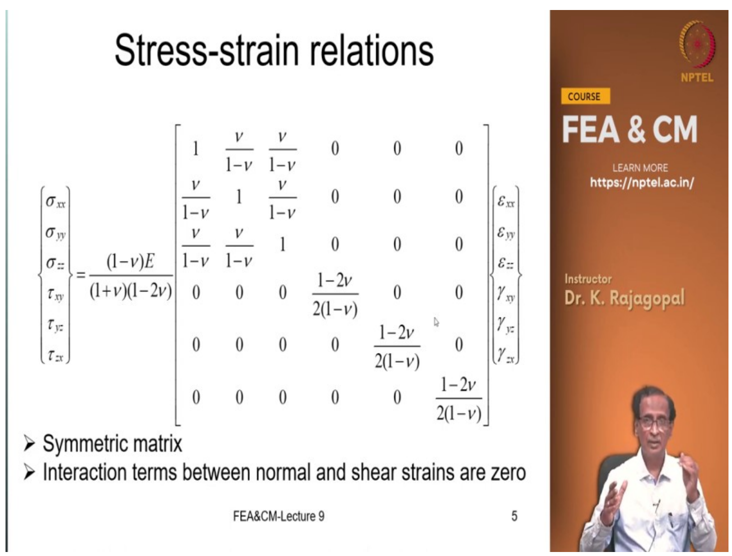

And in general our constituent equation is also more complicated it is a six by six Matrix because we have six stresses and six strains.

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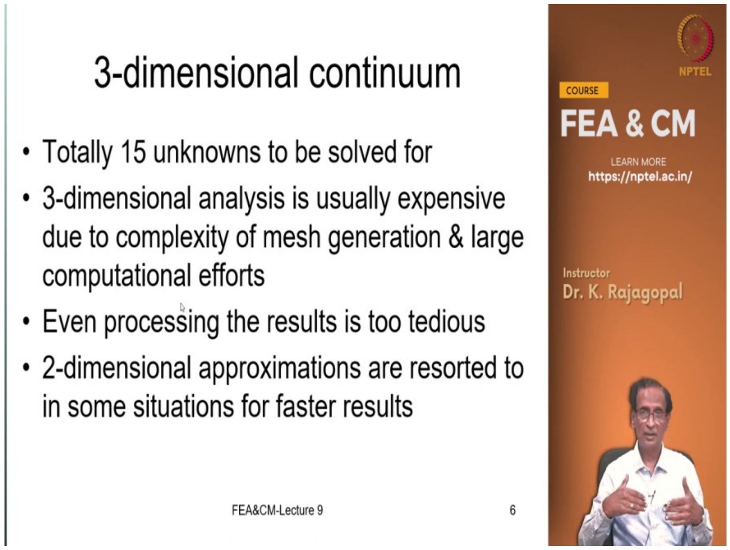

And in the three dimensional problems we have 15 unknowns and the 3D analysis is very cumbersome because first thing is even generating the mesh for our analysis is very complicated because we have to imagine the third dimension. And so, it becomes very expensive for doing these analyses not only for mesh generation but also the computational effort you will be dealing with very very huge stiffness matrices and even after running the program processing the results could take a long time.

So, it is most ideal if we can come out with the two dimensional approximations. So, that we can reduce our computational effort and do everything faster. And as a corollary so, all our geotechnical analysis are two dimensional like when we design a retaining wall or when we do a slip circle analysis for slopes these are all two dimensional and even the bearing capacity analysis it is basically a two dimensional like either a circular footing or a slip footing or a square footing and with correction factors and so on.

So, it makes sense even with the finite element analysis if we can come out with a simpler situation where we can do more efficient analysis.

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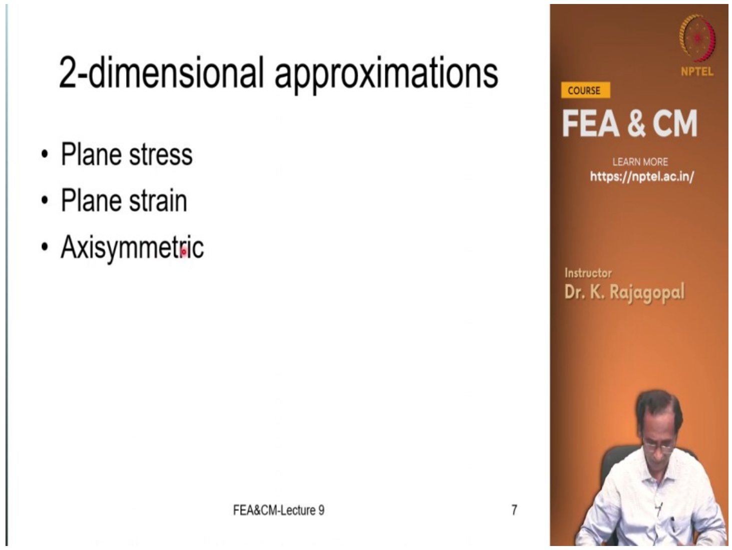

So, the two dimensional approximations there are three types one is the plane stress let me just get my laser pointer. So, we could have a plane stress or a plane strain or axis symmetric. So, let us see what they are.

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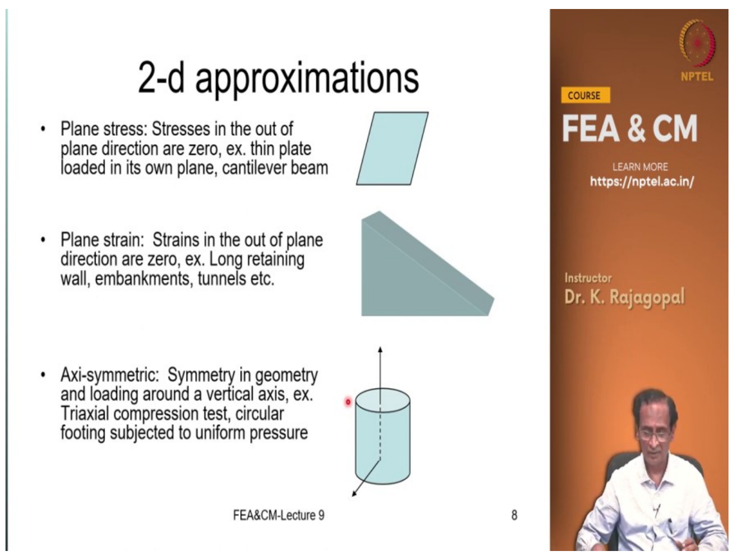

So, the plane stress state is actually the stresses in the outer plane direction are 0. It is actuallyyou can imagine the plane stress the plane stress means basically all the stresses are acting in one plane and the outer plane stresses are 0. And this could happen in the case of very thin plates loaded in their own plane like our cantilever beams. And we can have plain straight case where all the significant strains are in one plane and the outer out of plane strains are 0.

Like for example in the case of a very long retaining wall or a long embankment and so on. Then axis symmetric when we have radial symmetry in both the loading and also in the geometry like for example you take a cylindrical soil sample and subject it to all round pressure in our triaxial compression test. And there is a radial symmetry and we do not need to consider a three-dimensional case for this and this is a classical case of axis Symmetry and we will see what how we can simulate them.

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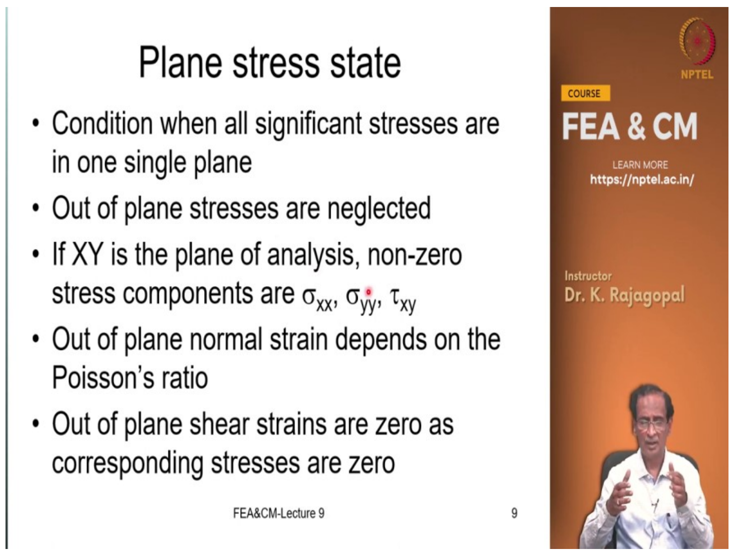

So, the plane stress state is a condition when all the significant stresses are acting in one plane. So, that we can neglect the outer plane stresses see they need not be 0 like Engineers we make lot of approximations and if we can neglect the stresses that are acting in the outer plane direction we can consider that as a plane stress case and do the analysis in two dimensions rather than in 3D.

And if our xy is the plane of analysis our non-0 stress components are Sigma xx Sigma yy these are the two normal stresses and then one shear stress Tau xy.

$$\sigma_{xx},\ \sigma_{yy},\ \tau_{xy}$$

See we have only three stress components instead of six and the outer plane a normal strain need not be 0 it depends on the Poisson's ratio. See we are saying out of plane stress is 0 but the outer plane strain may be non-0.

And the outer plane shear stress shear strains are 0 anyway because the outer plane shear stresses are 0 the shear stress and shear strain are directly related to each other through the shear modulus. So, actually in all these approximations say the plane stress see even if you do not calculate the outer plane strain nothing is going to happen because when we do the virtual work calculation as the sigma times strain.

Since our outer plane stress is 0 the virtual work done because of this outer out of plane stress and strain is 0. So, we do not really need to consider other than these three stresses and these three strains.

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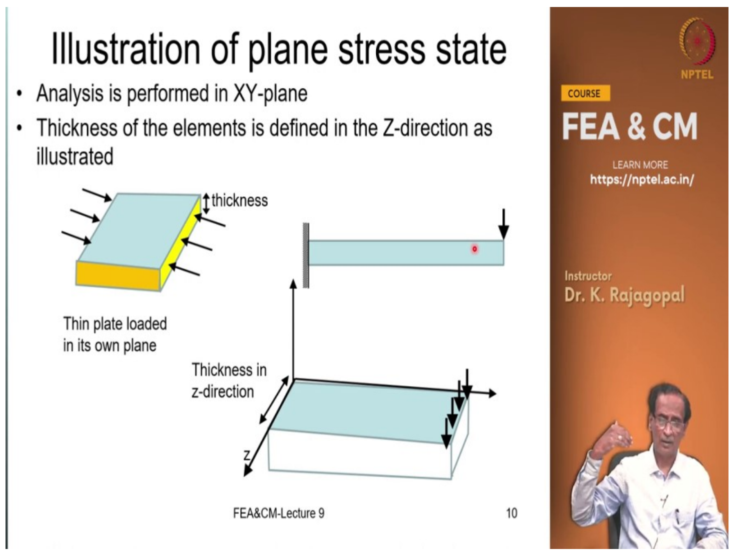
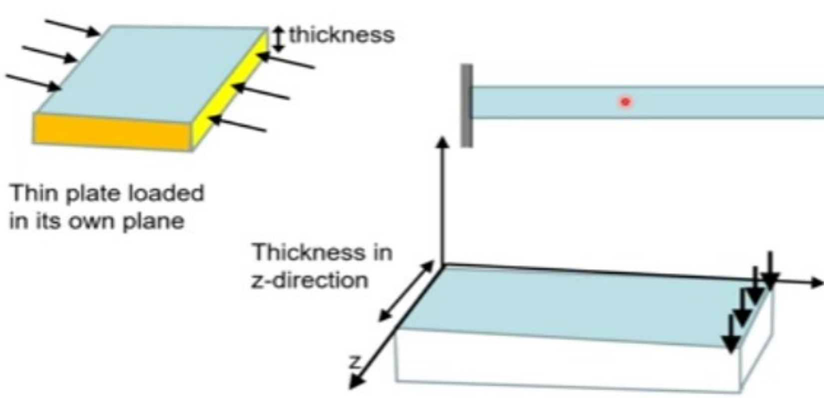


So, some illustrations for the plane stress state. So, you can consider a thin plate and lower it in its own plane and that can be analyzed as a as a plane stress case. And this is our thickness it could be anything it could be a unit thickness or some thickness and we could have cantilever beam loaded in its own plane either with a tip load or with the moment in cross section it is like this and if you look at the perspective view it could be like this it has some thickness in the outer plane direction.



So, just imagine our computer screen as the plane of analysis our xy plane is our computer screen and anything going out of the computer screen is our z direction. And our thickness is in the z Direction and our loading actually in the 2D when we draw we only show one load but it is actually it is a line load. It is a line load continuously along the perpendicular direction to the plane of analysis.

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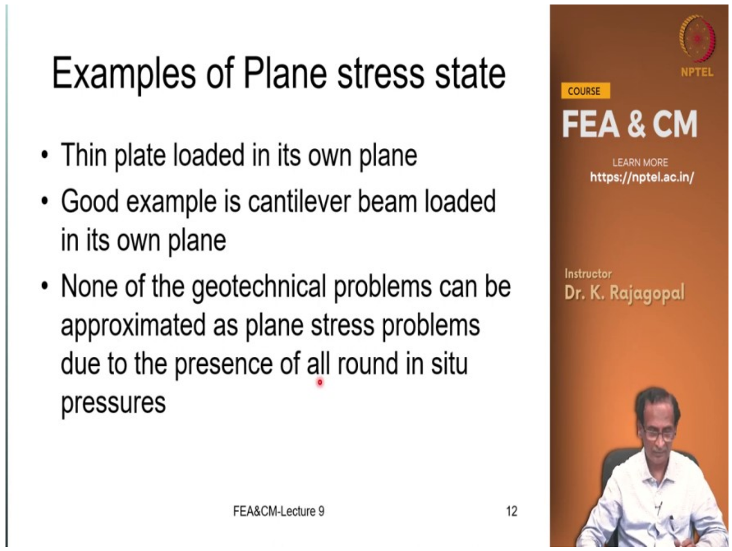

And in fact in all these whether it is a plane stress or plane strain when we consider point load it is not really point load it is a line load ok and our uniform pressure we can think of either applying line loads or uniform pressure. And but then what happens if you have a concentrated load a point load that becomes a fully three-dimensional we cannot have any plane stress idealization or a plane strain idealization.

Examples for the plane stress problems thin plate loaded in its own plane and another good example is the cantilever beam loaded in its own plane and the plane stress state is more applicable for structural problems. But when it comes to soil mechanics problems the geotechnical engineering we have the self weight and because of that we have the stresses in all the directions whether it is x, y or z we have the stresses.

So, technically speaking we will not be able to come across or we will not be able to solve any geotechnical problem reasonably with a plane stress approximation we need to go in for something else because of the presence of all round in-situ pressures.

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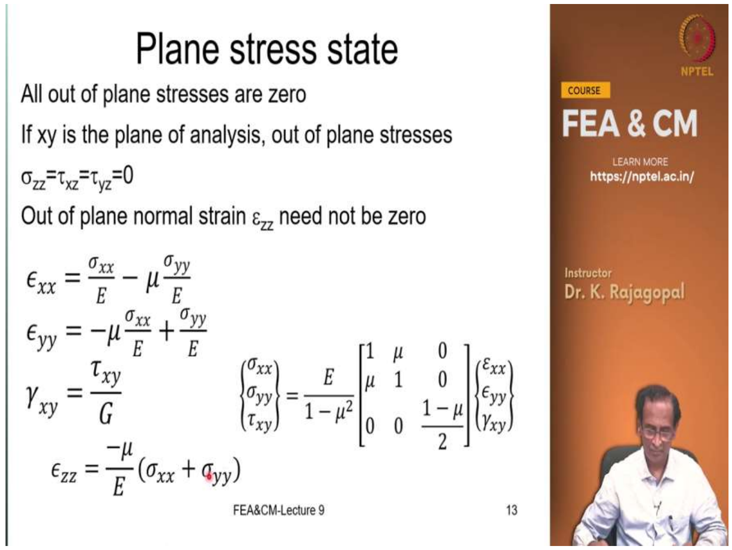

If xy is the plane of analysis, out of plane stresses

$\sigma_{zz}=\tau_{xz}=\tau_{yz}=0$

Out of plane normal strain ε_{zz} need not be zero

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \mu\frac{\sigma_{yy}}{E}$$
$$\epsilon_{yy} = -\mu\frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E}$$
$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

And the constituted matrices for constitute matrix for the plane stress state is very simple. Say we have only three significant strains Epsilon xx Epsilon yy and Gamma xy and we can invert this and the right this strain the stress in terms of strains like this Sigma xx Sigma yy Tau xy is E by 1 minus mu square multiplied by this some Matrix 1 mu 0 and so on. And then Epsilon xx Epsilon yy gamma xy these are the three strains in the in the plane strain case.

$$\begin{Bmatrix}\sigma_{xx}\\\sigma_{yy}\\\tau_{xy}\end{Bmatrix} = \frac{E}{1-\mu^2}\begin{bmatrix}1 & \mu & 0\\ \mu & 1 & 0\\ 0 & 0 & \frac{1-\mu}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\\epsilon_{yy}\\\gamma_{xy}\end{Bmatrix}$$

And after you perform the analysis we can calculate the strain in the outer plane direction Epsilon zz is minus mu by E times Sigma x Plus Sigma y right.

$$\epsilon_{zz} = \frac{-\mu}{E}(\sigma_{xx} + \sigma_{yy})$$

And the Poisson's ratio here can be anything it can be any value within the reasonable limit like -1 to 1 to 0.5.

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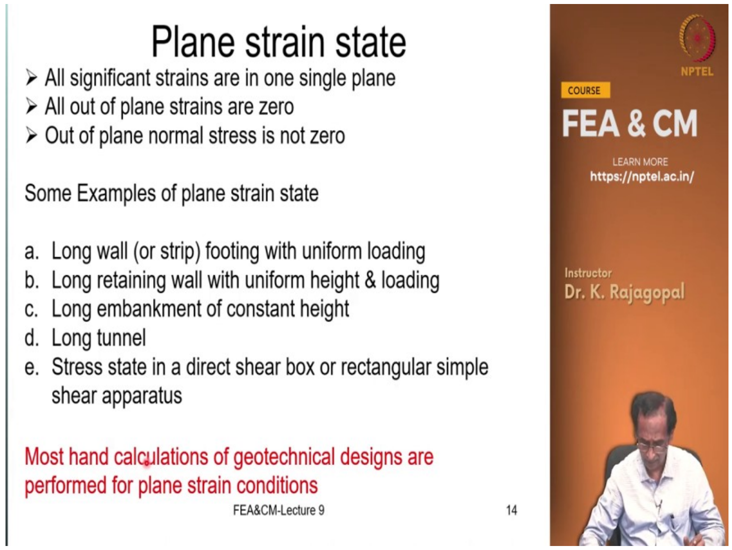

Then similar to plane stress case we have a plane strain case where all the significant strains are acting within one plane say for xy is the plane of analysis will have Epsilon xx Epsilon yy and then within the same plane our gamma xy and all the outer plane strains are 0. So, in the Epsilon z is 0 gamma yz gamma xz are 0. And but then the outer plane strain is 0 but outer plane stress need not be 0 it depends on the Poisson's ratio.

Some examples for the plane strain state if you have a long retaining wall or a sorry long wall footing or a strip footing with a uniform loading along the length that we can consider as a plane strain state by considering unit length or we could have a long retaining wall or of uniform height and loading. So, if you have let us say along the length your height of the wall is increasing gradually that cannot be simulated as a as a plane strain case we should go in for full three dimensional.

But then as engineers we can approximate it with an average height of the wall and then take some some unit length along the length unit width along the length and then do the analysis. We could have a long tunnel or stress state within a direct shear box is also a plane strain state because say along the Shear direction will have the stresses and then perpendicular to that your strain is 0 because of the rigid shear box.

Or it could also be the same thing with the simple shear operators or rectangular with a rectangular shape. See most hand calculations in our geotechnical designs are done in the plane state no plane strain condition like either the design of retaining walls or embankments or with the design of sheet pile walls everything is done for a unit length in the outer plane direction assuming the plane strain situation.

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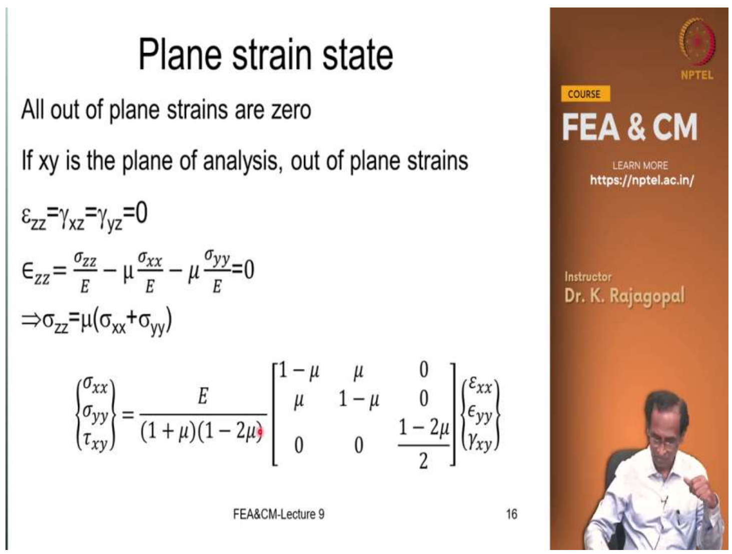

If XY is the plane of analysis, strains in z-direction are zero, i.e. $\varepsilon_{zz}=\gamma_{xz}=\gamma_{yz}=0$

$$\sigma_{zz}=\mu(\sigma_{xx}+\sigma_{yy})$$

So, the plane strain state is if our xy is the plane of analysis or the 0 strains are Epsilon zz gamma xz gamma yz dot 0 and the normal stress in the z direction is Sigma z is Mu times Sigma x Plus Sigma y right and we can invert our constitute of matrices and write the stress in terms of strain like this Sigma xx Sigma yy Tau xy is E by L plus mu times 1 minus 2 mu multiplied by this Matrix and the strain vector.

$$\varepsilon_{zz}=\gamma_{xz}=\gamma_{yz}=0$$
$$\epsilon_{zz}=\frac{\sigma_{zz}}{E}-\mu\frac{\sigma_{xx}}{E}-\mu\frac{\sigma_{yy}}{E}=0$$
$$\Rightarrow\sigma_{zz}=\mu(\sigma_{xx}+\sigma_{yy})$$

This is the relation between the stress and strain in the case of plane strain and if you look at this equation we have 1 minus 2 mu in the denominator and what happens if you substitute a

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{Bmatrix} = \frac{E}{(1+\mu)(1-2\mu)}\begin{bmatrix}1-\mu & \mu & 0\\ \mu & 1-\mu & 0\\ 0 & 0 & \frac{1-2\mu}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \epsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}$$

Poisson's ratio 0.5 the whole thing will blow up because we have 0 in the denominator and the 1 by 0 is it is infinite and our computer program will just simply crash. So, if you are Poisson's ratio is a 0.5 what we do is we can play 0.49 or 0.499 or something like that and then get around this problem of the 0 being in the in the denominator.

See this problem we did not have with the plane stress case because in the denominator we have 1 minus mu Square and even if mu is 0.5 is not a problem we can will have an finite value.

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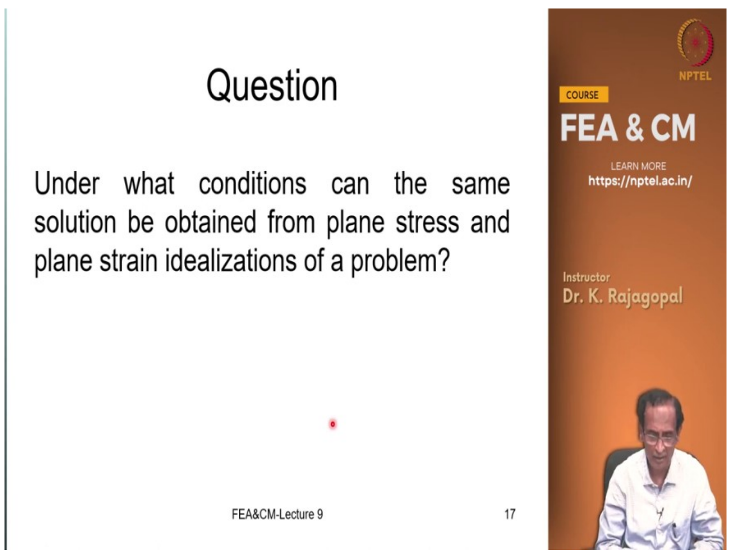

So, now that we have seen both the plane stress and plane strain a question can arise. So, you have a problem and you can idealize that as a plane stress case or idealize that as a plane strain case separately and is it possible that both the solutions are the same whether you approximate it as a plane stress case or a plane strain case can we get the same result. If we can get the same result what are the requirements what are the boundary conditions under which we can get the same result you think about and you can send me the answers.

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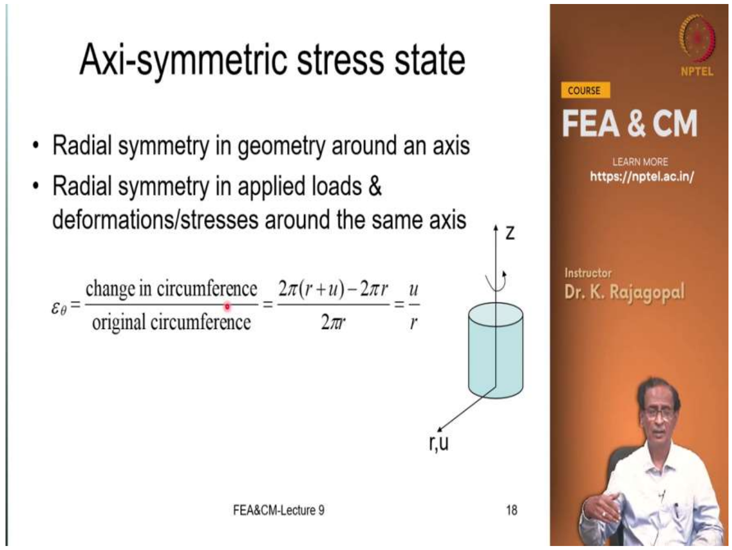

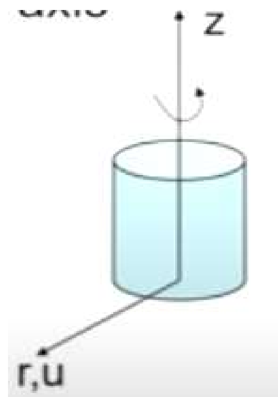

Now there is one more simplification that is the axi-symmetric case this radial symmetry in both the geometry and also in the loading so, that we can consider a section of this and then do the analysis in the two dimensional two dimensional framework. So, in the case of axis symmetric analysis will have one more strain called as the is the circumferential strain or the hoop strain.

So, just imagine a cylinder and as you go on compressing its diameter will go on increasing. So, that means that the circumference is going to increase. So, based on this we can define one circumferential strain epsilon theta as the change in the circumference divided by the original circumference. So, if r is the radius and u is the radial deformation the change in the circumference is 2 pi times r plus u minus 2 pi r.

$$\varepsilon_\theta = \frac{\text{change in circumference}}{\text{original circumference}} = \frac{2\pi(r+u)-2\pi r}{2\pi r} = \frac{u}{r}$$

And the original circumference is a 2 pi r and that Epsilon theta becomes u by r, u is the radial displacement at the radius of r. So, Epsilon theta is the additional quantity that we need to consider in the axis symmetric case.

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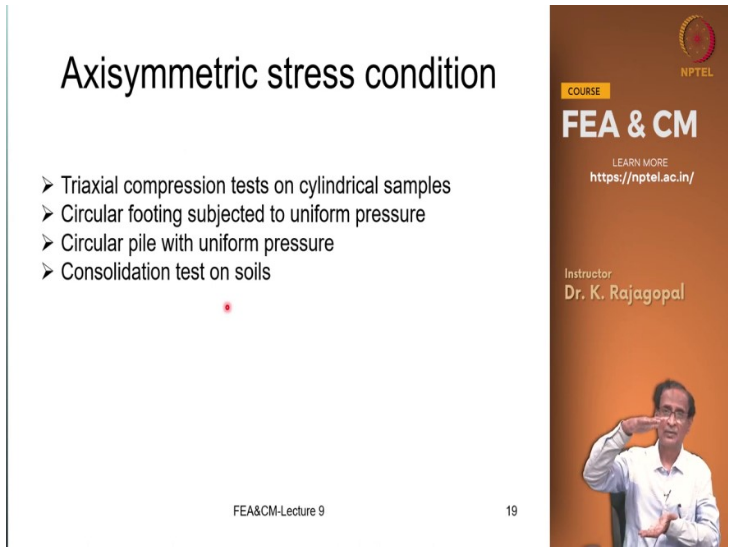

So, one good example for the axis symmetric case is our triaxial compression test performed on cylindrical soil samples and our circular footing subject at uniform pressure or a circular pile with the uniform pressure and then the typical consolidation test on soils. So, the consolidation test is done in a on a cylindrical sample it is a round sample 60 millimeter diameter and the height is 20 millimeters and then we apply uniform pressure and the entire surface of the soil. And that is also a typical axis symmetric case.

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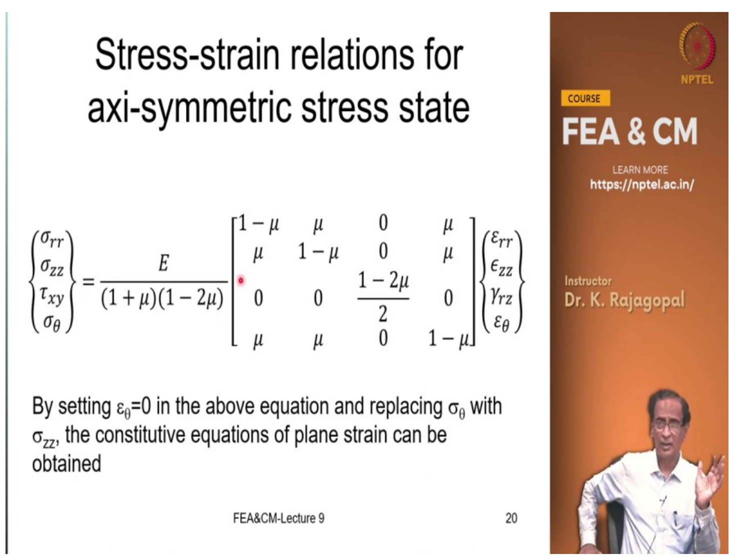

And we can derive the stress strain relations like this in fact this stress strain relation is very close to the tough the plane strain case except that we have the fourth row corresponding to circumferential stress and circumferential strain. And actually this has a four rows and four columns because we have four stress and four strain components Sigma R Sigma z Tau rz and then Sigma theta.

$$\begin{Bmatrix}\sigma_{rr}\\ \sigma_{zz}\\ \tau_{xy}\\ \sigma_{\theta}\end{Bmatrix} = \frac{E}{(1+\mu)(1-2\mu)}\begin{bmatrix}1-\mu & \mu & 0 & \mu\\ \mu & 1-\mu & 0 & \mu\\ 0 & 0 & \frac{1-2\mu}{2} & 0\\ \mu & \mu & 0 & 1-\mu\end{bmatrix}\begin{Bmatrix}\varepsilon_{rr}\\ \epsilon_{zz}\\ \gamma_{rz}\\ \varepsilon_{\theta}\end{Bmatrix}$$

So, actually this should be xy and in this equation by setting Epsilon theta to 0 we can make this constitute matrix applicable even for the plane strain case. Because in the plane strained case you have Sigma theta that is Sigma zz but your outer plane strain is 0. So, we can set this to 0 and use exactly the same constitute Matrix even for the plane strain case.

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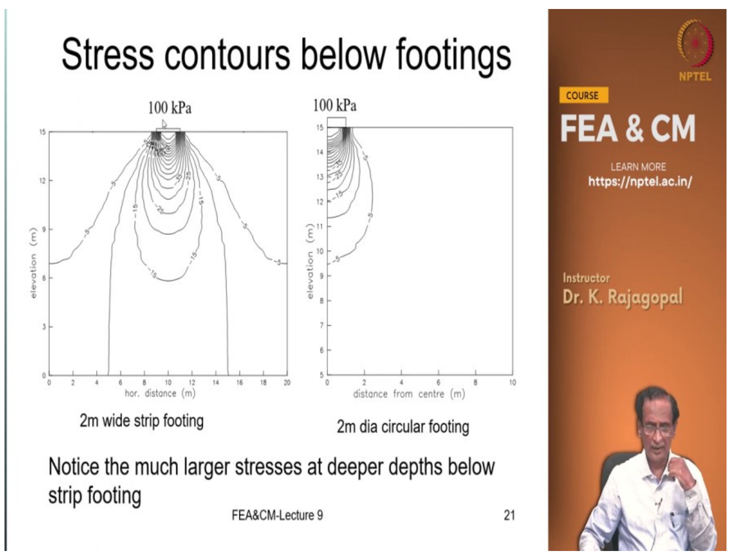
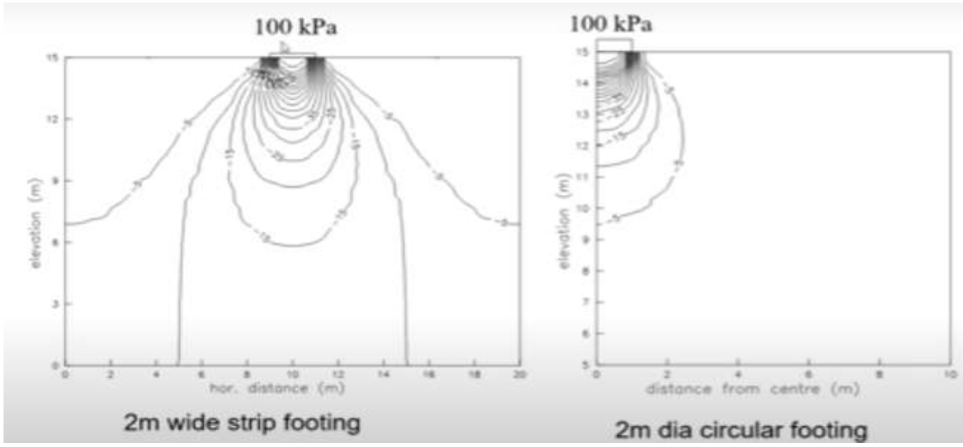


So, just give you one example see we have a two meter wide strip footing and then a two meter diameter circular footing and both are loaded with the same pressure of 100 kPa. And the strip footing is a typical plane strain case whereas circular footing with the uniform pressure you can consider as a as an axis symmetric one. So, our access of symmetry accesses along the central axis.



So, just imagine you rotate it then you will get back your cylindrical shape and if you plot your stress contours this contouring was done in a separate program not in the geofem what we do is we get the result in the geofem and give this data to some other contouring packets that can draw the contours and these are the stress contours below a strip footing. So, applied pressure is 100 then -35, -25, -15 and so on.

And these are the stress contours below the circular footing and what you can notice is that within a short depth most of the stress is dissipated. Because in the case of circular footing or square footings we have load dispersion in all the directions whereas in the case of strip footing you have load the dispersion only in one direction. So, the load dispersion is very very slow in the case of strip footing.

So, that means that even if you go very deep there will still be some significant pressure. So, for example let us take let us say 15 kPa pressure because I can see that here and also there and the 10 kPa contour line is maybe this is the 10 kPa contour line which may be occurring at a large depth. Say 15 kilopascal Contour is acting at a depth of almost 9 meters below the footing.

Whereas in the below the strip fitting whereas in the case of circular footing this is the 15 kPa contour and it is acting at a depth of three meters. So, at a height of 15 meters 100 kPa pressure was applied and within 3 meters a distance significant amount of the applied pressure is dissipated. So, you see that the stresses are dissipated faster in the case of circular footing. So, actually remember one question that we normally ask in the ground improvement whether the depth of borehole can be the same for design of a strip footing.

Or for a circular footing assuming that both are having similar dimensions like the same width of the footing or the diameter. And the answer expected answer is the borehole should be deeper below the below the strip footing and the reason for that answer is this. So, the pressure is dispersed over a much larger depth below the strip fitting compared to the compared to the square or circular footing.

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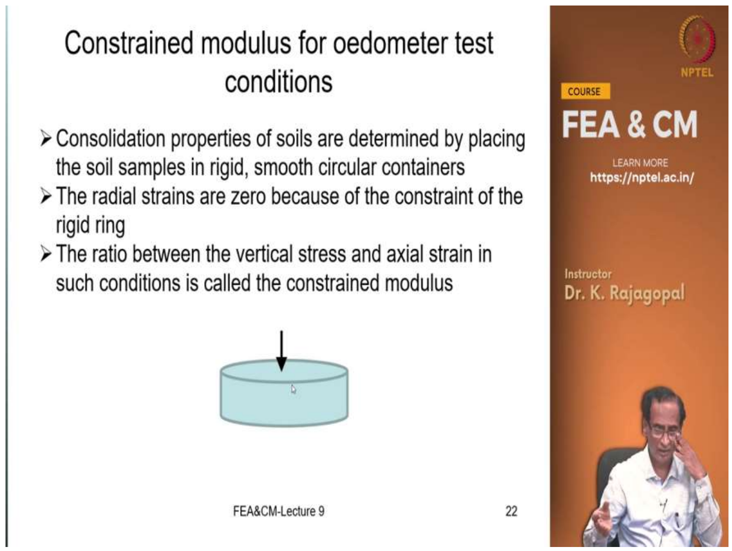
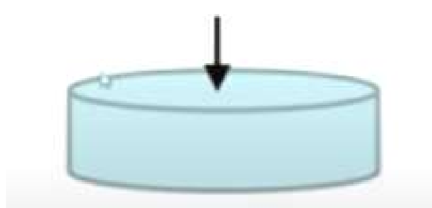


So, let us look at constrained modulus in the odometer test. Odometer test is the consolidation test that we perform in a consolidation ring that is cylindrical in shape it is a 60 millimeter diameter the height is only 20 millimeters and we determine the consolidation properties and the radial strains are 0 because of the rigid ring that we have around the soil sample. And the ratio between the vertical stress and the vertical strain is given as the in the constrained modulus and this constrained modulus could be higher than the Young's modulus that we have.



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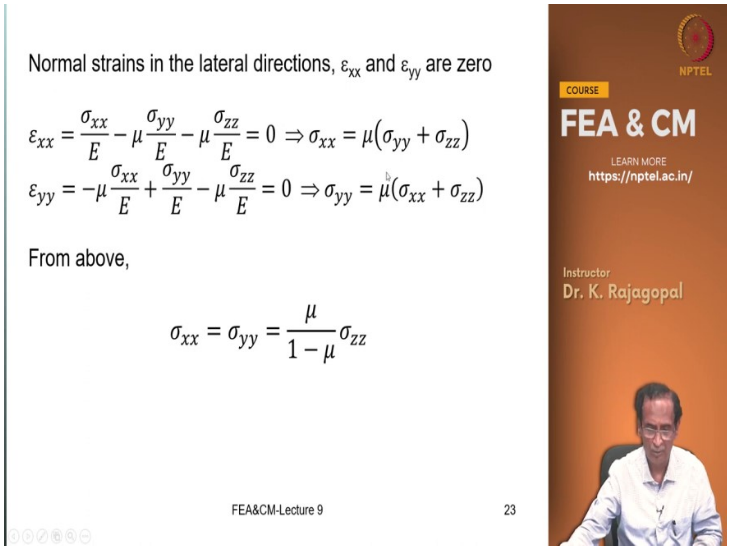

And so, we can actually apply lateral strains at 0 because our Sigma x and sigma y they are in the lateral directions. So, we can set them to 0 and by setting Epsilon xx to 0 we get Sigma xx is Mu times Sigma y plus Sigma z and Epsilon y is 0. So, from here we get Sigma y is a mu times Sigma x Plus Sigma z. And by solving these two simultaneous equations we get Sigma x and sigma y as Mu by 1 minus mu times Sigma z. The lateral stress is a mu by 1 minus mu times Sigma z.

Normal strains in the lateral directions, ε_{xx} and ε_{yy} are zero

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \mu\frac{\sigma_{yy}}{E} - \mu\frac{\sigma_{zz}}{E} = 0 \Rightarrow \sigma_{xx} = \mu(\sigma_{yy} + \sigma_{zz})$$

$$\varepsilon_{yy} = -\mu\frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E} - \mu\frac{\sigma_{zz}}{E} = 0 \Rightarrow \sigma_{yy} = \mu(\sigma_{xx} + \sigma_{zz})$$

From above,

$$\sigma_{xx} = \sigma_{yy} = \frac{\mu}{1-\mu}\sigma_{zz}$$

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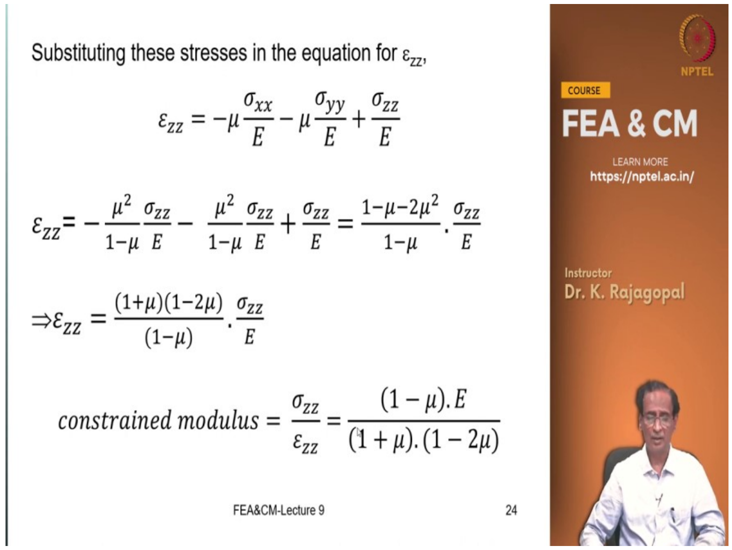

And so, by substituting these two equations in the equation for Epsilon z, Epsilon z is minus mu times Sigma x by E minus mu Sigma y by E plus Sigma z by E and sigma x and sigma y they are determined as Mu by 1 minus mu times Sigma z. So, that becomes mu Square by 1 minus mu and so on. and we can simplify this whole thing at the right hand side as 1 plus mu times 1 minus 2 mu by 1 minus mu times Sigma z by E.

Substituting these stresses in the equation for ε_{zz},

$$\varepsilon_{zz} = -\mu\frac{\sigma_{xx}}{E} - \mu\frac{\sigma_{yy}}{E} + \frac{\sigma_{zz}}{E}$$

$$\varepsilon_{zz} = -\frac{\mu^2}{1-\mu}\frac{\sigma_{zz}}{E} - \frac{\mu^2}{1-\mu}\frac{\sigma_{zz}}{E} + \frac{\sigma_{zz}}{E} = \frac{1-\mu-2\mu^2}{1-\mu}.\frac{\sigma_{zz}}{E}$$

$$\Rightarrow \varepsilon_{zz} = \frac{(1+\mu)(1-2\mu)}{(1-\mu)}.\frac{\sigma_{zz}}{E}$$

$$constrained\ modulus = \frac{\sigma_{zz}}{\varepsilon_{zz}} = \frac{(1-\mu).E}{(1+\mu).(1-2\mu)}$$

Now our constrain modulus can be written as Sigma z by Epsilon z as E times 1 minus mu by 1 plus mu times 1 minus 2 mu. So, depending on the Poisson’s ratio that you have the constraint modulus could be very high.

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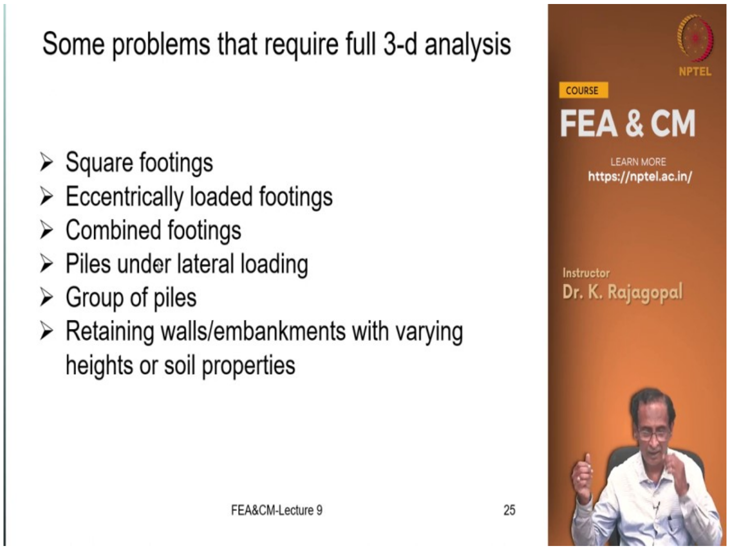

Let us look at some other cases that require full three dimensional analysis. So, if you have a square footing you cannot simulate any symmetry like you cannot you do not have a radial symmetry unlike in a circular footing the square footing has to be analyzed as a three dimensional problem. Or let us say you have a circular footing but with some eccentric loading then the geometries axis symmetric because it is circular but then because of the eccentricity of loading you are you will not have any radial symmetry in the in the response.

So, we should ideally analyze that as a three dimensional problem and we may have some combined footings they also require full 3d analysis are the piles and the lateral loading. Let us say you have a circular pile but apply some lateral loading. The soil stresses are not symmetric with respect to any radial line that you have. So, we need to go in for full three dimensional analyses for those cases the group of piles.

Or if you have any retain wall or embankment their height is changing or the soil properties are changing then we cannot ideally simulate them as a plane strain case or axis symmetric case. So, we have to go in for full 3d.

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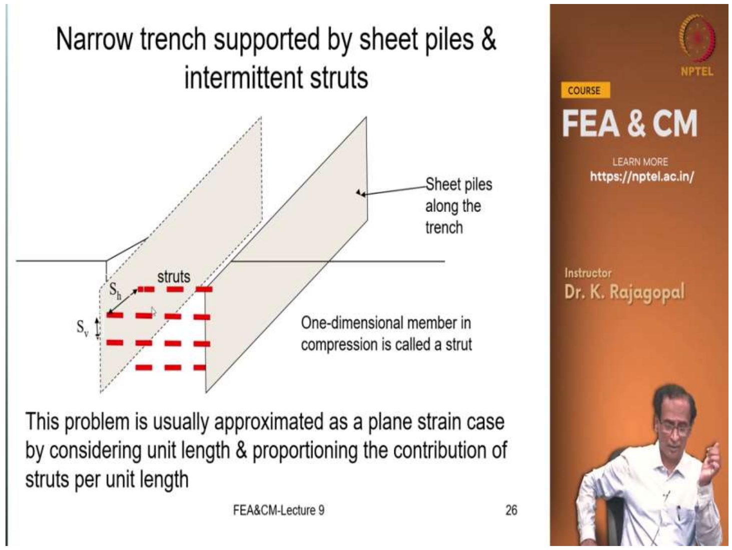
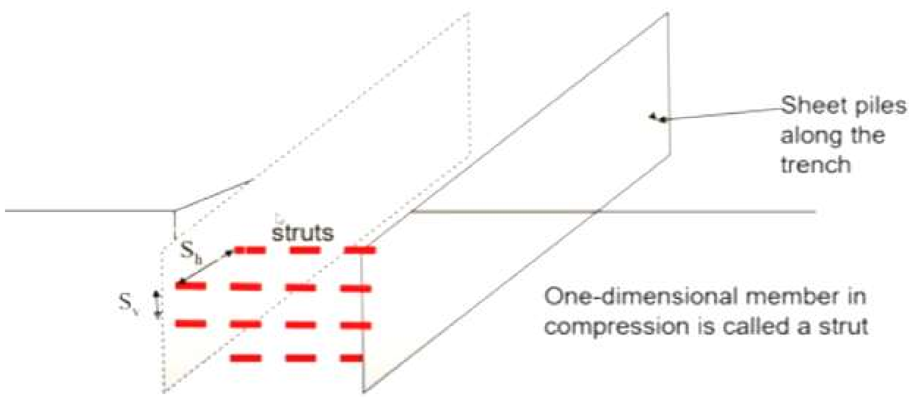


Or sometimes we may have this type of problems. So, along the length we have a sheet pile wall and then we placed some struct elements along the length and the height or we may have some anchor elements say if this is the situation can we do some simplification or should we analyze this only as a three dimensional problem. So, this can be I idealized it as a as a plane strain Case by using some engineering approximations.



So, we take unit length and the perpendicular direction and the prorate the contribution of our rankers are the struts over that the unit length one meter length and then do the analysis using a plane strain approximation.

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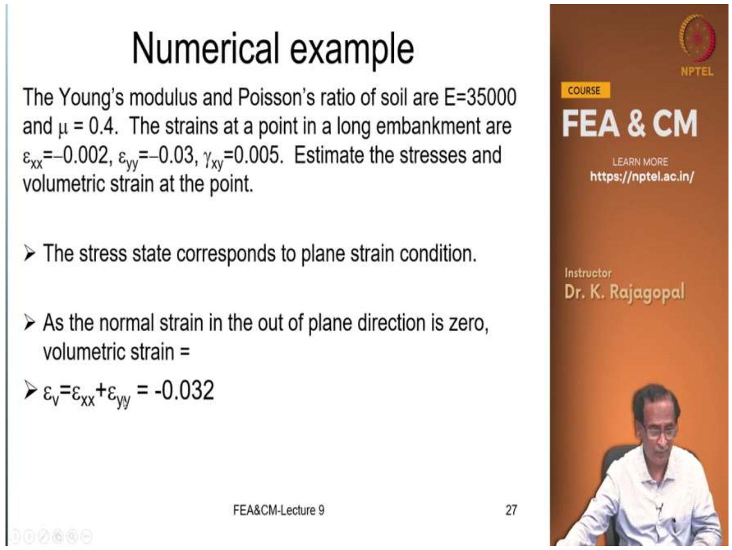

And let us look at a small numerical example to illustrate our calculations. Let us say that our soil sample has a Young's modulus of 35 000. and the Poisson's ratio of 0.4 and the strains at a point in a long embankment or like this Epsilon xx Epsilon yy and Gamma xy and we are asked to estimate the stress state corresponding to these strains and then the volumetric strain. See we are referring to a long embankment and nothing else is given.

The Young's modulus and Poisson's ratio of soil are E=35000 and μ = 0.4. The strains at a point in a long embankment are $\varepsilon_{xx}=-0.002$, $\varepsilon_{yy}=-0.03$, $\gamma_{xy}=0.005$. Estimate the stresses and volumetric strain at the point.

So, we can assume that the embankment is of uniform height and then the soil properties are uniform. So, we can consider this stress State as a plane strain condition and has the normal strain in the outer plane direction is 0 Epsilon z is 0. So, our volumetric constraint Epsilon V is just simply Epsilon xx Plus Epsilon yy and the sum total of all the normal strains is called as the volumetric strain so, Epsilon x Plus Epsilon y that comes to -0.032.

$$\varepsilon_v=\varepsilon_{xx}+\varepsilon_{yy} = -0.032$$

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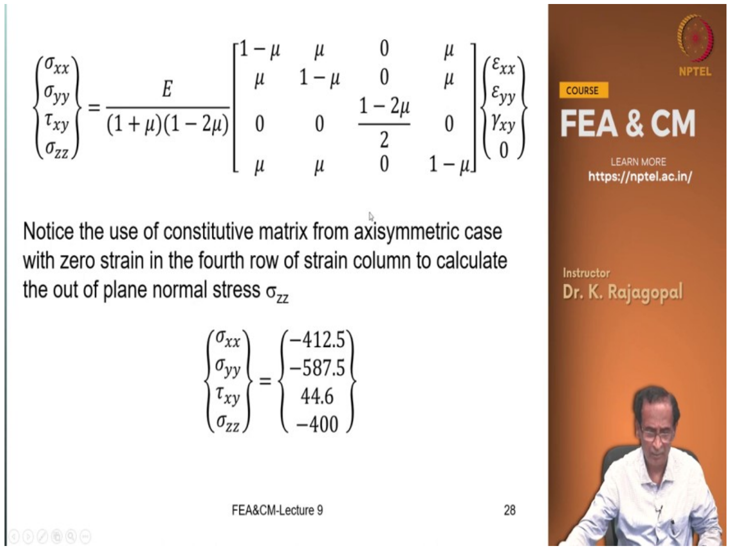

And our constitute equation for the plane strain case can be written with the same equation that we had with the axis symmetric problem except in place of Epsilon z we put a 0 and the advantage that we get here is directly by doing this calculation we get the sigma z also but that is only like an ornament it is not going to add any value it is because the virtual work done in terms of Sigma z and Epsilon z is 0.

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \sigma_{zz} \end{Bmatrix} = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1-\mu & \mu & 0 & \mu \\ \mu & 1-\mu & 0 & \mu \\ 0 & 0 & \dfrac{1-2\mu}{2} & 0 \\ \mu & \mu & 0 & 1-\mu \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ 0 \end{Bmatrix}$$

So, this information is only additional information but in terms of the computation it has no significance. So, if you compute your Sigma xx Sigma yy Tau xy and sigma z they are computed using this constitutive relation between the stress and the strain. So, in this class we have seen the two dimensional approximations of the full 3d problem and in general we prefer using a two dimensional analysis because it is much faster.

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \sigma_{zz} \end{Bmatrix} = \begin{Bmatrix} -412.5 \\ -587.5 \\ 44.6 \\ -400 \end{Bmatrix}$$

The first thing is that for solving the equations and even the mesh generation is very simple and we can imagine the shape and everything because it is two dimensional we can draw a sketch on the on a piece of paper and then see how it looks. And there are some cases that you cannot simplify either as a plane stress or plane strain or axis symmetric we have to go in for full 3d that I have mentioned I have given some examples.

So, if you have any questions please write to me at profkrg@gmail.com then I will reply back to your questions. So, thank you very much we will meet next time.