FEM and Constitutive Modelling in Geomechanics
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Lecture: 10
Stresses and Strains in continuum

So, hello students let us continue from our previous discussions on Prismatic elements and from today onwards we will be looking at the continuum like soil or plate or something. And before we go into the other aspects let us first look at what are the stresses and strains within a continuum how do we define them how do we develop our different equations of equilibrium and then how do we calculate the strains and stresses within these elements and let us see.

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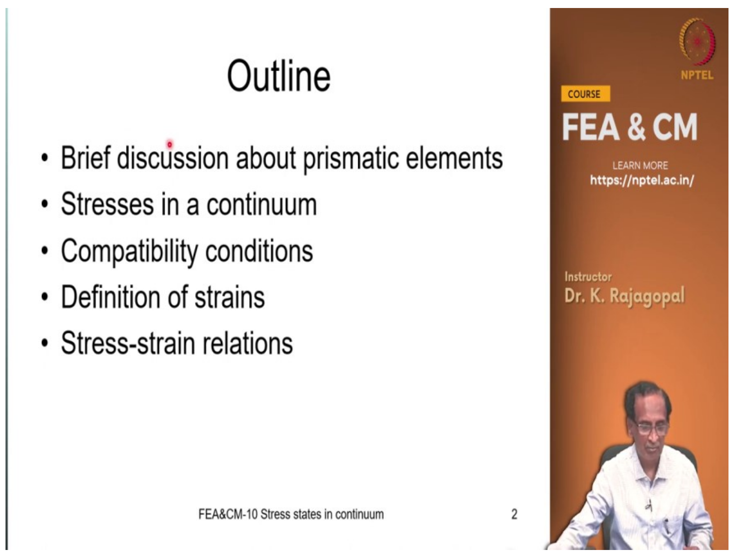

And before that let us briefly discuss about the Prismatic elements so, that we have been discussing of the past few classes then after that we will move into the stresses in a continuum. And then see the compatibility conditions and then the definition of different strains and then the stress strain relations.

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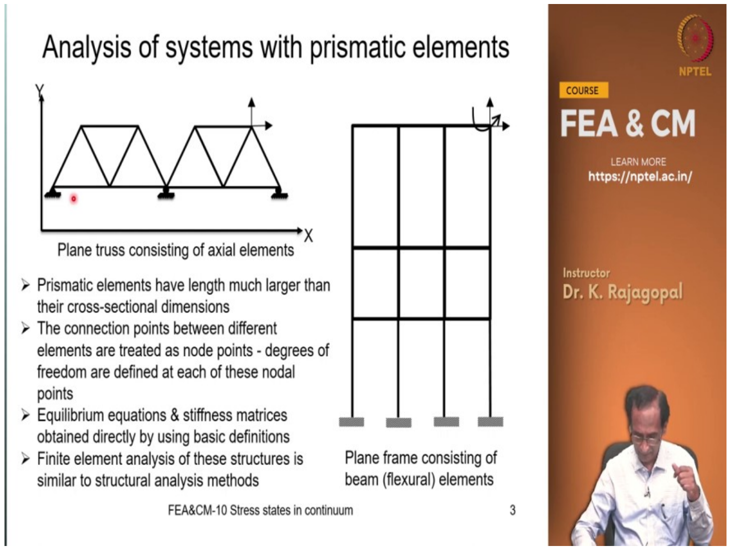

So, till now we were looking at only the prismatic elements which are long compared much longer compared to their cross-sectional area and we have seen different type of structures like the the plane trusses and the plane frames and so on. And the advantage that we had is that we can readily identify all these connection points where the different members are joining and we can treat them as nodes.

And define our degrees of freedom like x and y direction displacements for bar elements or for be beam elements we have three degrees of freedom two displacements and one in plane rotation and once we define the degrees of freedom then we can form the equilibrium equations for each element and then assemble them. And so, these equilibrium equations for these Prismatic elements we were able to obtain by using the fundamental definitions like for bar elements it was AE by L. And then we had rotated that in different directions for getting our global matrices.

Similarly for the beam elements also these are very simple slightly more complicated compared to Bar elements but we can directly write the stiffness coefficients that is called V i by L Cube 4E by L and so on. And the finite element analysis of this structures is similar to structural analysis methods and till now we have seen how to determine the stiffness matrix or equilibrium equation of individual elements and then how to assemble them to get the global matrix, matrix for the entire structure.

Then how to apply our boundary conditions like how do we tell whether support is a hinge or a roller and that we have seen and then we have also seen some methods for applying non-0 displacements. And so, that was a brief introduction to finite element analysis through these Prismatic elements.

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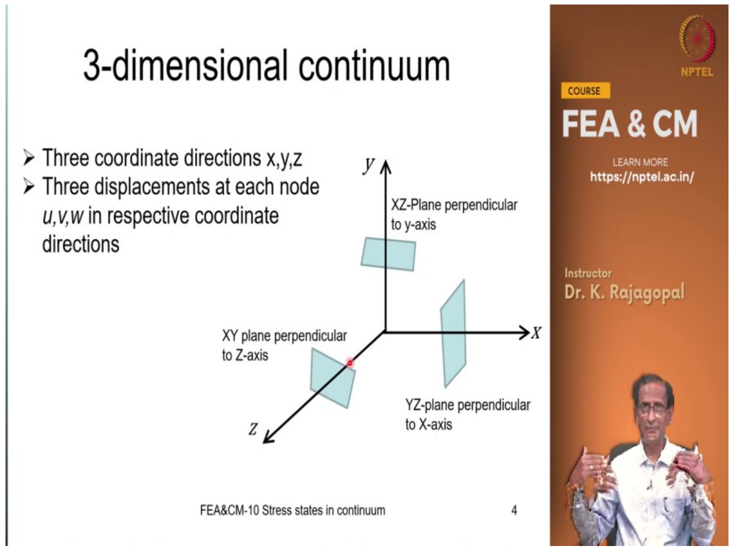
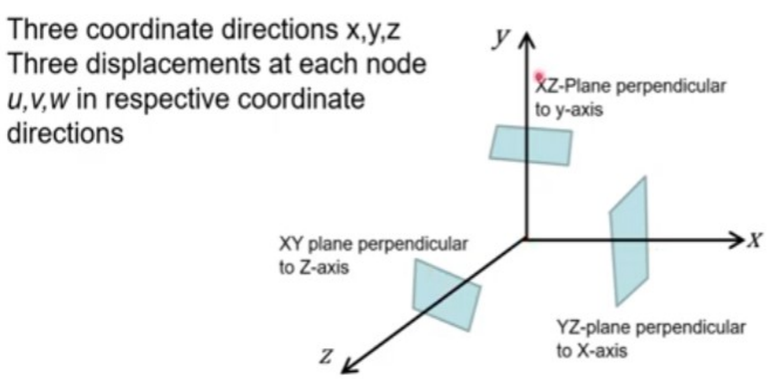


And now let us move on to more complicated materials like our structures like our continuum and in general a continuum could be in three dimensions. So, we define three coordinate axis x, y and z and then we have three displacements u along the x axis V along the y axis and then w along the z axis. And we can also as we draw these three axis we can also imagine three different planes say y z x z and then xy planes like this.



Like perpendicular to x axis we have the y z plane and perpendicular to y axis we have this xz plane and then perpendicular to z axis we have our xy plane and because we need these planes to be able to imagine our stresses and the strains.

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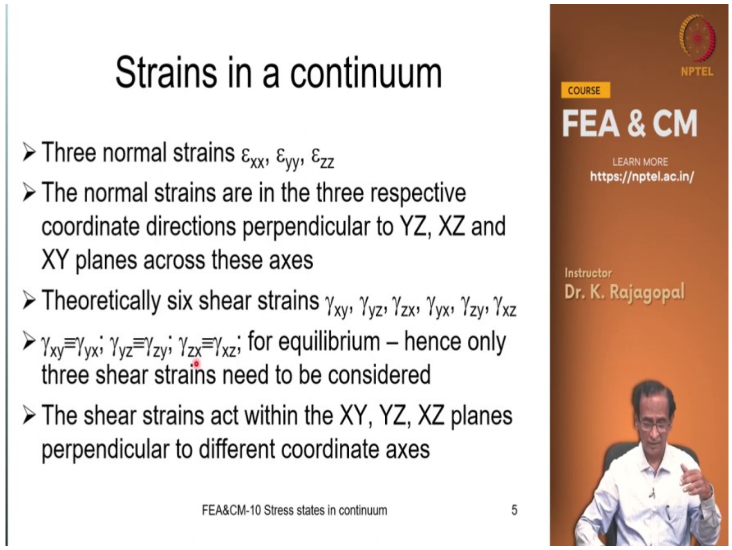

And the strains within a continuum we could have a different type of strains. So, we can have three normal strains Epsilon xx Epsilon yy Epsilon zz.

Three normal strains ε_{xx}, ε_{yy}, ε_{zz}

I will explain in a few minutes why we have a double subscript xx, yy, zz and so on. And these are three normal strains they are in the three respective coordinates x axis y axis and z axis and the strain along the x axis is actually acting perpendicular to the to the plane of yz that is what we had indicated here.

See this any strain in the x axis is acting normal to this plane and normal strain in y is acting perpendicular to this xz plane and then Epsilon zz is acting perpendicular to this xy plane. And we can also have theoretically six Shear strains gamma xy gamma yz gamma zx gamma yx gamma zy gamma xz and for rotational equilibrium these gamma xy should be equal to gamma yx and so on.

Theoretically six shear strains γ_{xy}, γ_{yz}, γ_{zx}, γ_{yx}, γ_{zy}, γ_{xz}

And so, although the theoretically we can have six independent shear strains we will end up with only three of them because of the Symmetry and then the equilibrium rotational equilibrium considerations. So, we end up with only three normal strains and then the three Shear strains and when we say gamma xy it is a Shear strain acting in the xy plane gamma yz means it is a Shear strain acting in the yz plane.

Actually shear these are also called as planar stresses the shear stresses like the name itself indicates it is because of relative deformation between two surfaces and this gamma xy gamma yz gamma zx they act on individual planes xy yz and then xz planes.

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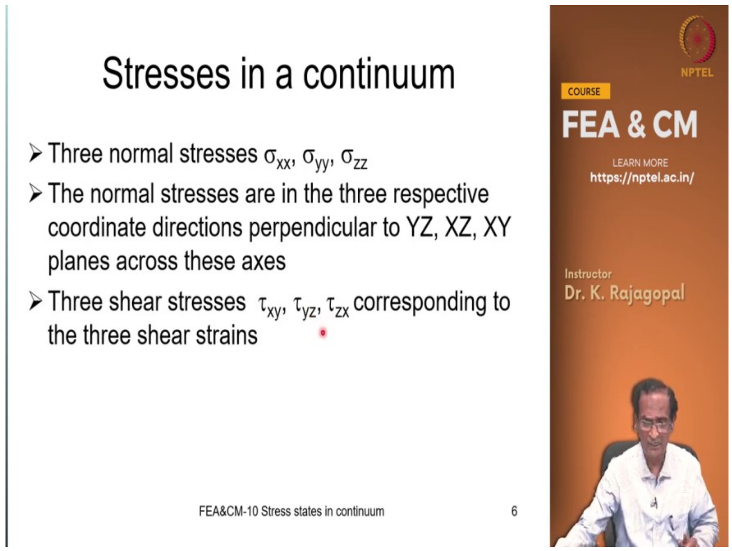

And then corresponding to these strains we will have stresses we have three normal stresses Sigma xx Sigma yy Sigma zz and these are along the three coordinate axis x, y and z normal to some planes as indicated earlier and then we will also have three Shear stresses Tau xy Tau yz Tau zx and the normal stresses that normal to some planes whereas the shear stresses that act along the plane like xy yz and zx and so on.

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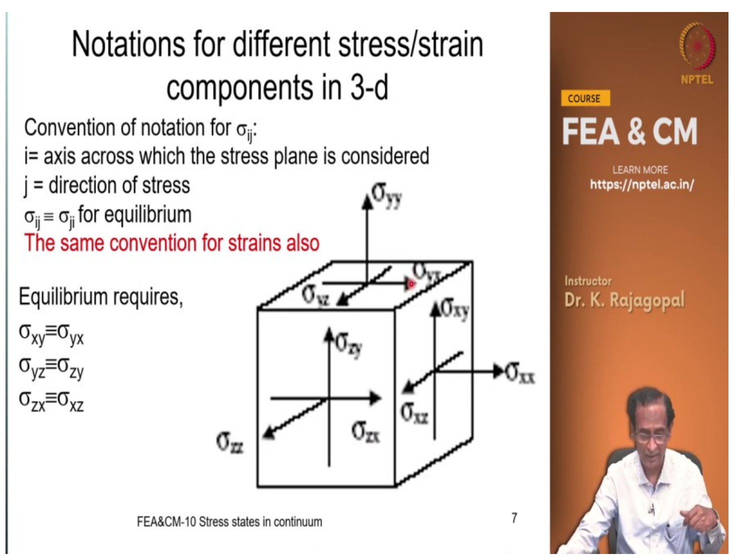
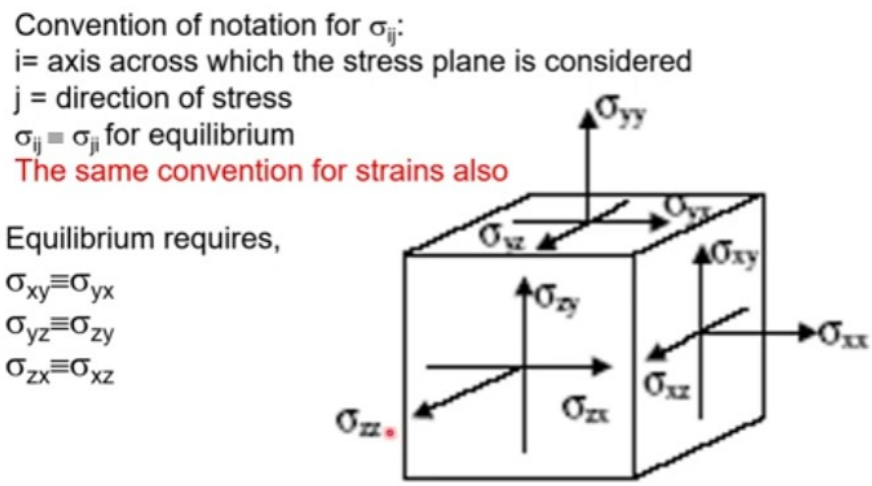


So, pictorially this is what we can see we have three coordinate axis x, y and z and these are the positive directions and we can imagine the right hand side the screw for all our sign conventions. And the convention in elasticity is to denote all these quantities with two subscripts and sigma ij like our Sigma xx Sigma xy and so on. And i and j they refer to two different things i is the axis across which the stress plane is considered.



Like for example if you are looking at along the x axis we have the yz plane. And so, whatever stress that you are considering is acting a normal stress is acting across that particular plane and the j the second subscript j refers to the direction of stress and this Sigma ij is identical equal to Sigma ji for our equilibrium purpose. Because so, Sigma xy is equal to Sigma yx and is actually sometimes I use the Tau for shear stress sometimes Sigma xy but just bear with me.

But in most cases I think I'm using only Tau for the shear stress Sigma for the normal stresses. So, here if you look at this pictorially this is the yz plane right and this Sigma xx is actually this Sigma x is is the on a plane perpendicular to yz plane and then along the x axis. So, we have Sigma xx and sigma xy because this is the stress acting on a on a plane perpendicular to this x axis and in the direction of y.

So, Sigma xy and then Sigma xz this Sigma this shear stress is acting on the plane perpendicular to x axis but in the direction of the z direction. So, it is a sigma xz then if you look at the complementary shear stress on the other phase it is Sigma zx and here we have Sigma xy. And the horizontal surface we have Sigma yx right and same thing Sigma yz and sigma zy.

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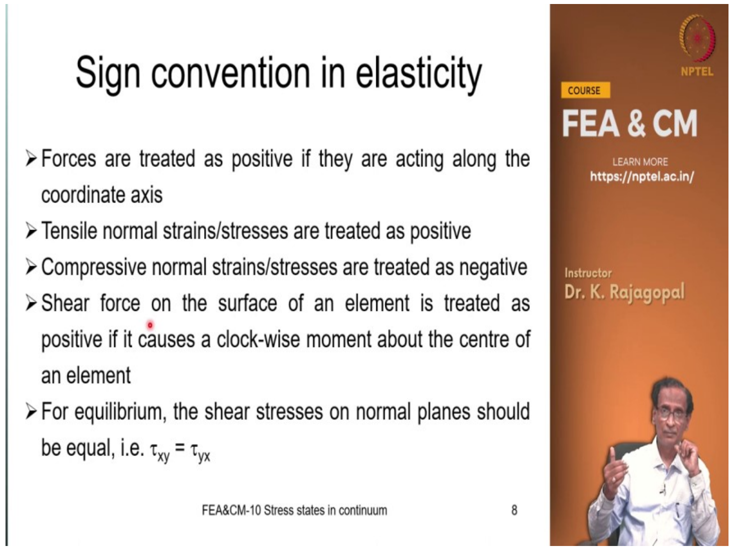

So, what is the sign convention? The forces and displacements that treated as positive if they are acting along the coordinate axis and that also depends on whether you are in the positive quadrant or the negative quadrant. So, if it is in the positive quadrant the positive displacement means it is going towards the positive x axis let us say we are in the considering along x axis. But then if you are in the negative quadrant any force acting along the negative direction that is along the x axis in the direction it is taken as a positive quantity.

And the tensile normal stresses and the strains are considered as positive. So, it is like tensile is pulling just imagine that along the x axis you have a positive force on one face and then another side also it is a positive force but its acting on the in the other direction the net result is to apply tensile force within your body and the tensile forces are treated as positive and the compressive forces are the compressive normal strains under stresses they are treated as negative.

Then the shear stresses it is a bit complicated because there is nothing like tension and compression when it comes to Shear force on a surface we treat it as a positive force if it causes a clockwise moment about a point in the interior of the element but the center of the element. And for equilibrium as I mentioned Tau xy should be exactly equal to Tau yx and this is the sign convention for shear force we have to carefully consider but the rule is any Shear force acting on a surface of an element if it is going to cause a clockwise moment.

If you take a moment of that Force about any point interior that we call as a positive shear force that I will illustrate.

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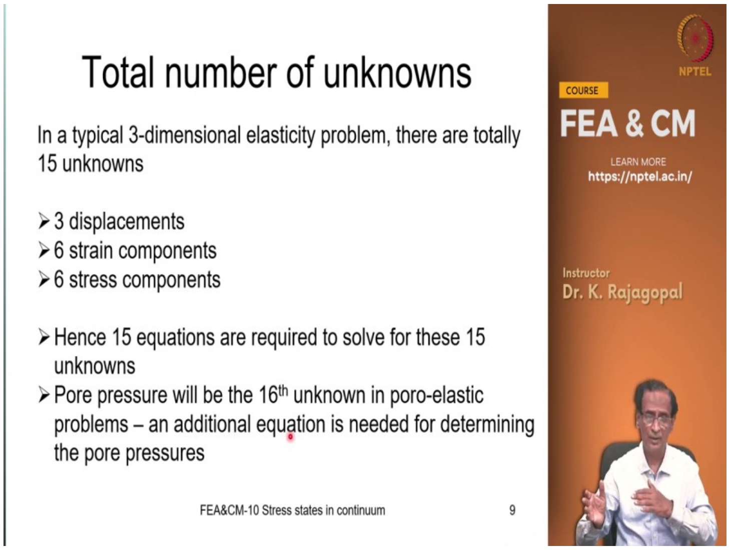

And before that let us look at how many unknowns we have to solve for. See we have totally 15 unknowns unknowns in any three dimensional problem we have three displacements we have six strain components then we have six stress components. So, we have totally 15 unknowns. So, we need at least 15 equations to solve for these 15 unknowns but then we have one more extra unknown because we are dealing with the geotechnical problem. So, we should not forget about our pore pressures.

So, the pore pressure will be the 16th unknown for all the poor elastic problems it is not just simply elasticity but we call it as a pore elasticity because it is including at the water. So, we require some additional equation for determining the pore pressures that we will see later.

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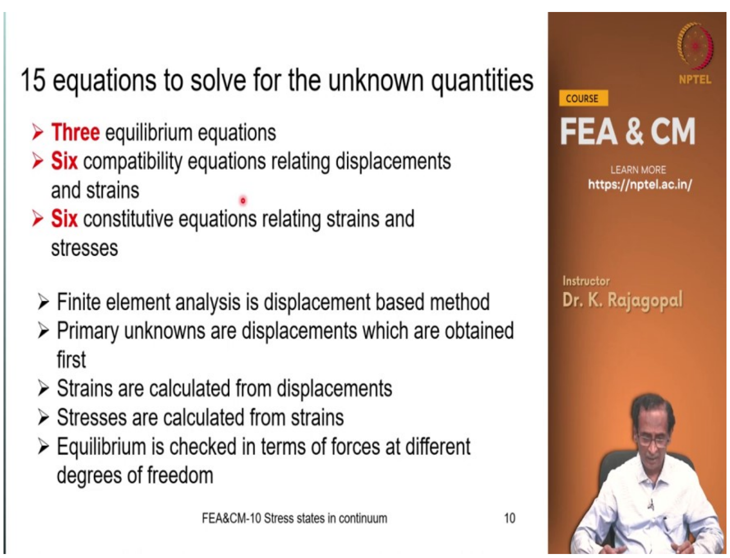

And what are the 15 equations that we have to solve for the 15 unknowns? See we have three equilibrium equations the sum of forces in x direction is 0, y direction is 0 the sum total of z direction force is 0. So, we call them as equilibrium equations and then we have six compatibility equations these compatibility equations they relate the displacements and strains and so, that when you have a body it should not deform into some unreasonable shape.

So, we put some constraint through these compatibility equations or basically they are the definitions of the strains and we also need six constituent equations to relate the stresses to the strains. But in our finite element analysis we are dealing only with the displacement based finite element analysis. So, that means our displacements are the primary unknowns and we determine the displacements first and then later we calculate the strains from the displacements and then we calculate the stresses from the strains.

So, in fact when we check for equilibrium we are only going to check for equilibrium in terms of forces when we do the finite element calculations but when you go into the pure elasticity all the equilibrium equations are written in terms of stresses but that I will explain a bit later. So, in the finite element analysis we are going to check for equilibrium only in terms of forces are different not the different degrees of freedom.

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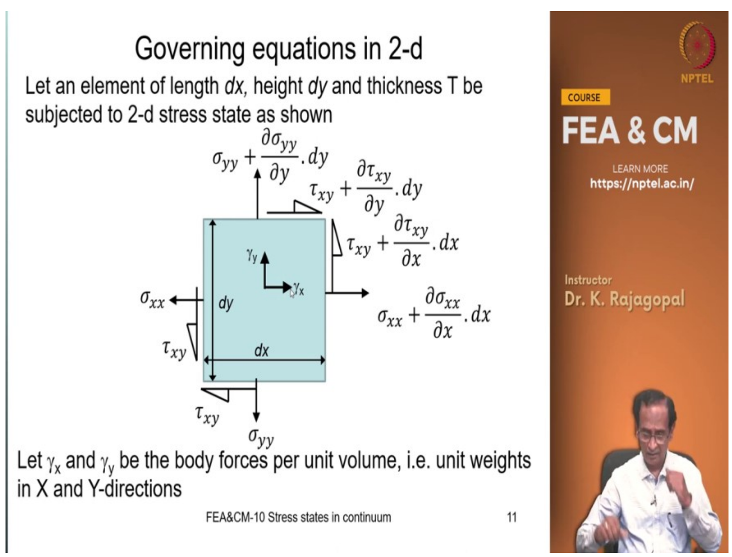
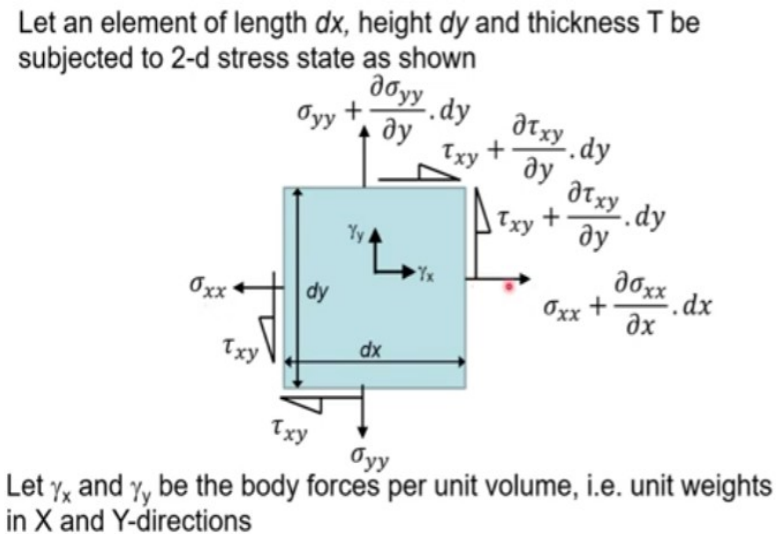


So, let us look at a two dimensional stress state because more easy to imagine and let us consider an element having a length of dx and the height of dy and in the outer plate direction let it have a thickness of t and it is subjected to some a two dimensional stress State Sigma x and then on the left hand side and the sigma xx plus some other quantity on the right hand side. And these are the positive quantities because they are acting along the Positive directions of the coordinates.



On the positive quadrant it is acting in the along the positive direction in the negative quadrant it is acting in the negative direction similarly you have The Shear sorry the normal force in the y direction Sigma yy then Sigma yy plus something that I will explain. And then we have one shear stress Tau xy because we are dealing with only with one plane xy plane. So, we will have only one shear stress Tau xy.

And this is positive shear force because it is going to cause at clockwise moment if we take a moment about the center of this element. Then similarly the shear force on the bottom surface bottom horizontal surface is also positive quantity whereas the shear acting and the vertical lines are vertical surfaces they are treated as negative because if we take moment it will be an anticlockwise moment.

And so, we take that as negative and Tau xy is equal to Tau yx. So, that we do not end up with infinite rotations. And let us consider a stress state which can vary because we do not need to have the uniform stress state within the body let us say along the x axis at this place this surface we have Sigma xx and then the right hand side we have Sigma xx Plus dou Sigma by dou x times dx a dou sigma x by dou x is the general variation or the change in x direction stress with x multiplied by dx that will give you your the net change.

Then similarly on the along the y axis we have Sigma yy and then Sigma yy plus d Sigma y that is the rate of change of Sigma y multiplied by this length dy and then same thing with Tau xy Tau xy sorry this should be this should be dx should be sorry and then this is along the y axis Tau xy Tau xy plus dou xy by dou y dy. And we can consider the equilibrium along the x axis and y axis.

Say along the x axis our this this Sigma xx on the left hand side phase is acting in the negative direction. So, we can write minus Sigma xx multiplied by dy that is the the length over which this stress is acting multiplied by thickness in the outer plane direction t minus Sigma xx dy times T is your is your force in the horizontal direction on this phase. Then on the right hand side face we have Sigma xx plus dou Sigma by dou x dx multiplied by dy times t.

And then we have this the shear stress Tau of xy multiplied by dx multiplied by by the thickness and then Tau xy here Tau xy plus dou Tau by dou y dy multiplied by this length dx the thickness T and then apart from this we could have some body forces these body forces are the force per unit volume I am indicating with the gamma x and Gamma y gamma x is the unit body Force acting along the x axis.

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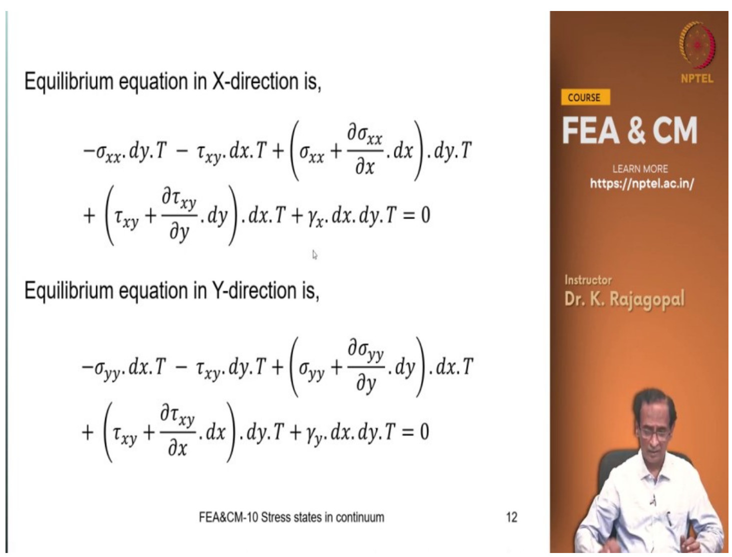

And so, we can write all these quantities and this unit Force sorry I think it should be gamma x but it got mixed up. So, gamma x and Gamma y these are basically our unit weights in x direction y direction these are the body forces per unit volume.

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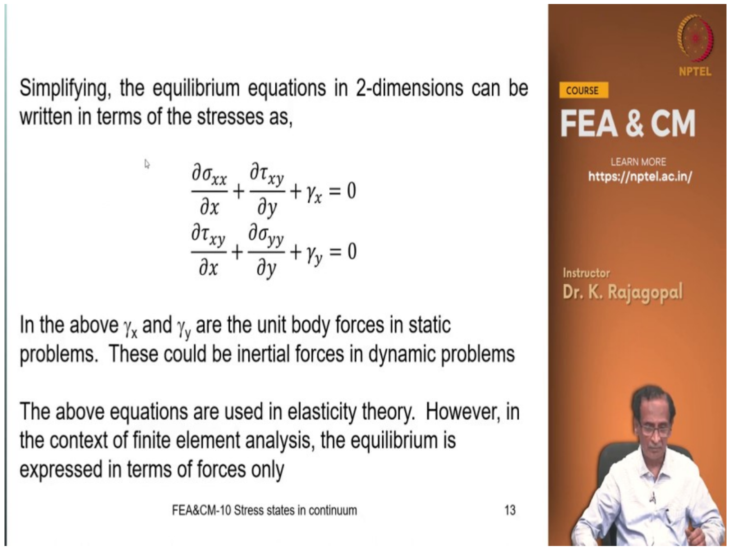

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \gamma_x = 0$$
$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \gamma_y = 0$$

And by simplifying we get two equations two equilibrium equations in terms of stresses dou Sigma xx by dou x plus dou Tau xy by dou y plus gamma x is 0 and similarly in the y direction dou tab xy by dou x plus dou Sigma yy by dou y plus gamma y. And these gamma x and gamma y in static problems we can treat them as unit weights but in dynamic problems we could have some inertial forces Mass multiplied by acceleration is gamma x and gamma y are the generic forces that we consider as additional forces in our equilibrium equations.

So, actually in our finite element analysis we are not going to directly operate on these on these equations because we are actually considering this type of these equations because the stress multiplied by the area that is the force. So, we are considering only the forces for our equilibrium equations in the finite element analysis.

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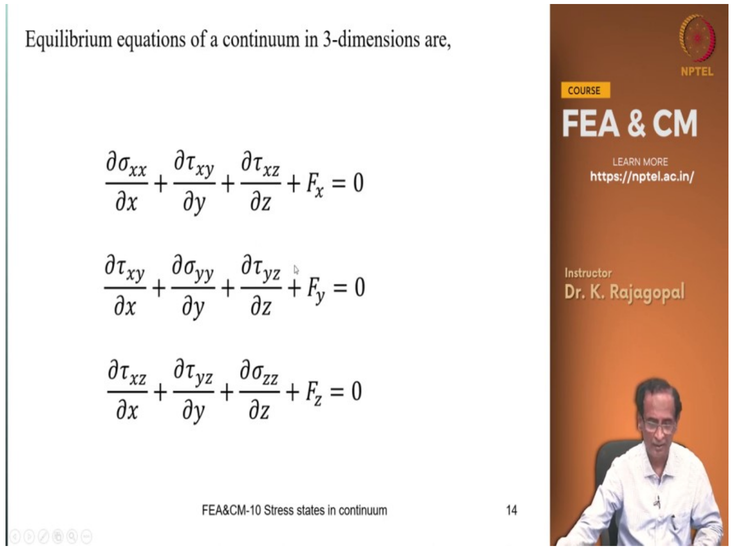

So, in general for three dimensional case we can write like this along the x axis.

$$\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xz}}{\partial z}+F_x=0$$

$$\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}+\frac{\partial \tau_{yz}}{\partial z}+F_y=0$$

$$\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \sigma_{zz}}{\partial z}+F_z=0$$

So, you have dou x dou Sigma xx by dou x plus dou Tau xy by dou y plus dou Tau xz by dou z Plus the unit Force f x is 0 and so on. Like this is the x direction this is in the y direction this is in the z direction.

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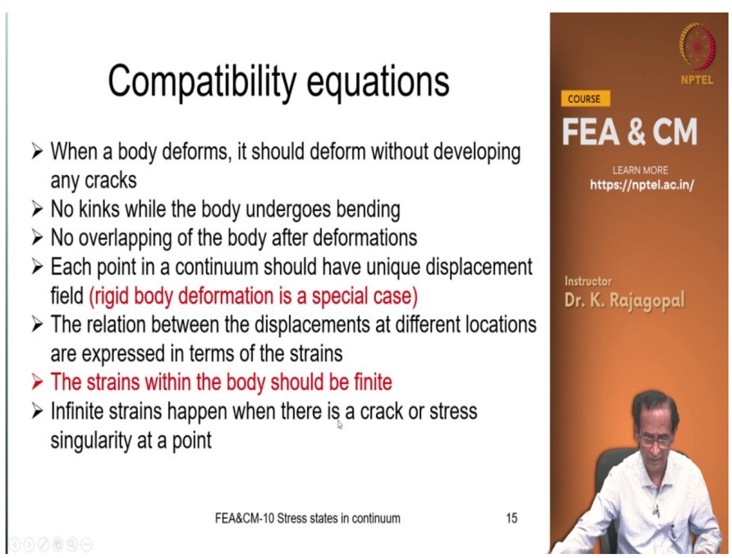

Then apart from this we need to have some conditions so, that our body even after deformation will have its shape. And so, this when our body is deforming it should deform without developing any cracks and there should not be any kinks developed within the body when it is bending that is a kink means there is a discontinuity in our shape of the element. And there should not be any overlapping of the body after deformation because every point should have some unique displacement so, that there is no over overlapping.

And each point in a continuum should have unique displacement field say only exception is in the case of rigid body deformation rigid body deformation is when we move the entire body by the same displacements. So, all the points within this continuum have the same displacements but in general each point will have some set of deformation so, that there is some strain. And the relation between the displacements, at different locations are expressed in terms of the strains.

See we have two points and each can deform in different directions and then in different magnitudes but we need to have some check on how the entire body is deforming so, that even after our deformations the shape of the body is not unduly changed or there should not be any cracks ok and these are called as the compatibility relations. And then most of all the strains within the body should be finite.

But then when we have a crack we will have some discontinuity and the strain across this discontinuity will be infinite and obviously that we cannot handle and the this point with infinite stress we call as a singularity point stress singularity and the stress is infinite resulting in the in the cracking.

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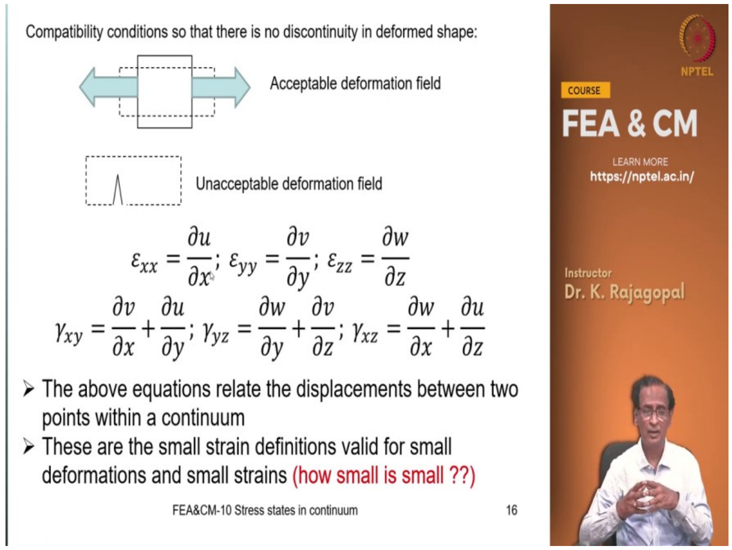
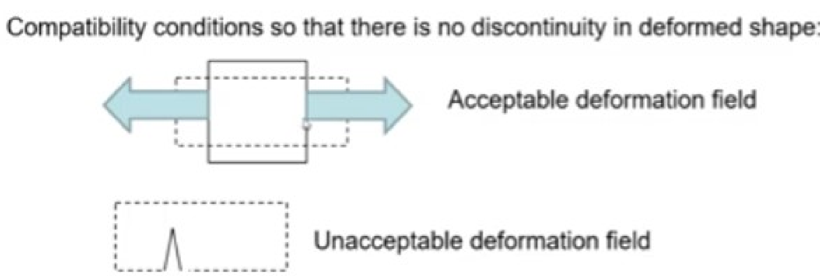


So, the acceptable deformation is like this let us say you have a square then you apply some tensile force or the tensile stress at the most it could stretch into a rectangle like this like this dotted line. So, one direction it is elongating whereas in the other direction it is compressing. So, we cannot have say this is the particular for the uniaxial loading but if you apply some other forces in the other direction you may have elongations. But in general so, you have a square and if you elongate in one direction its length has increased but the height has decreased.



But we should not end up with a crack like this and this we cannot we will not be able to simulate because our stress is infinite. And for simulating this type of problems we have to use some other methodology with the crack tip elements or the some other elements for fracture mechanics analysis. But in this course we're not going to consider any fracture or anything and the relation between the different displacements and then the strains can be written like this.

$$\varepsilon_{xx} = \frac{\partial u}{\partial x};\ \varepsilon_{yy} = \frac{\partial v}{\partial y};\ \varepsilon_{zz} = \frac{\partial w}{\partial z}$$

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y};\ \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z};\ \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

Epsilon xx is dou u by dou x the rate of change of displacement along the x axis we can define as the normal strain in the x axis Epsilon xx. Similarly Epsilon yy is dou V by dou y that is the rate of change of displacement along the y axis and then Epsilon zz is the rate of change of displacement in the in the z direction though w by dou z and gamma xy is the shear strain in the xy plane dou v by dou x Plus dou u by dou y gamma y z in the shear strain in the yz plane dou w by dou y plus dou y by dou z.

And similarly gamma xz is dou w by dou x plus dou u by dou z section of the shear strains they are considering only the two displacements that are acting within the plane. So, if you consider xy plane along the x axis we have the u and along the y axis we have v. So, gamma xy is written in terms of u and v variations as dou u by dou y plus dou v by dou x that is what we have written here.

And they put a bind or some type of relation between two different points. So, that after deformation we have some continuity and whatever definitions that have written here they are meant only for small strains and small deformations but for higher order strains we could write epsilon xx is dou u by dou x Plus dou Square u by dou x square times dx and so on. Like we can define higher order terms and but these are meant only for small strains.

Then the question comes how small is small is it point one percent is a small strain or one percent is a small strain. So, actually that is where the engineering judgment comes it is up to the user. Sometimes we call even five percent as a small strains and then do the analysis just for simplicity because if you go for large strain formulation the computational effort will significantly increase that we will see later.

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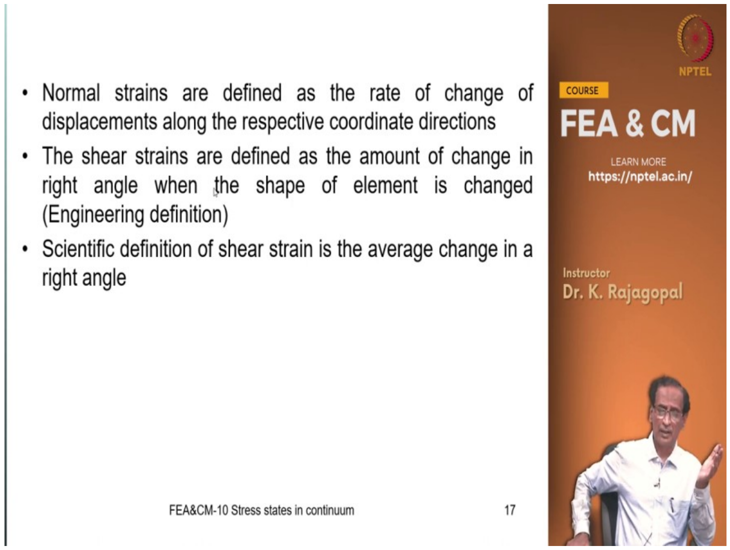

So, the normal strains are defined as the rate of change of displacements along the respective coordinate directions. So, Epsilon xx is a dou u by dou x and Epsilon y y is a dou v by dou y Epsilon zz is dou w by dou z the rate of change of displacement along the three respective coordinates. On the other hand our shear strain is defined as the amount of change in the right angle. So, when we twist some element out of shape the change in the right angle.

Like let us say we have a square element initially and then we are we applied some shear strain and then the shape has changed so, that 90 degrees is normal at 90 degrees it could be only 89 degrees or 88 degrees and so on. And the change in that right angle is defined as the shear strain and there is another definition scientific definition of the shear strain it is the average strain. So, here we have written gamma xy as dou v by dou x Plus dou u by dou y and this is the engineering definition.

And the scientific definition will be E xy is one half of dou v by dou x Plus dou u by dou y. And so, we and in all the finite element calculations we use only the engineering definitions not the scientific definition.

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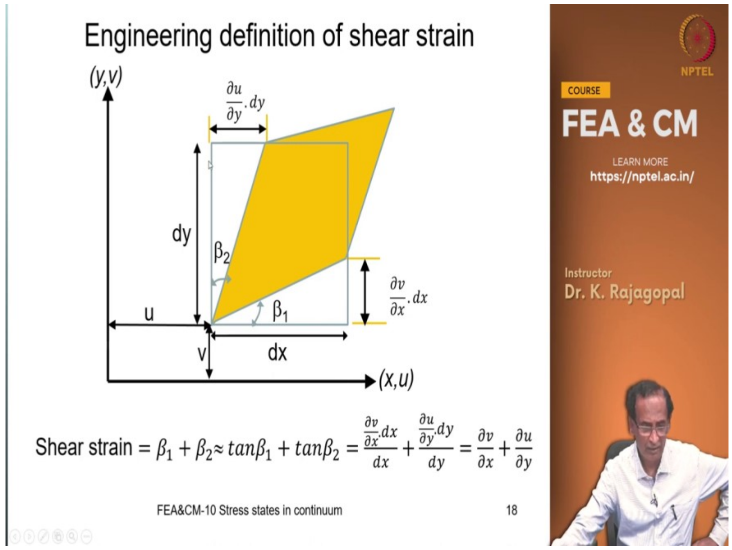
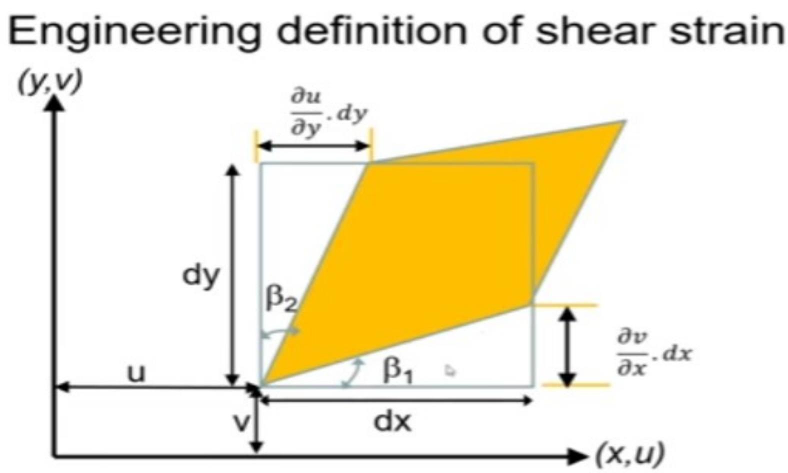


Let us look at what is this shear strain let us take a rectangular element like this and we twist it out of shape. So, that there are some angles beta1 and beta2 and our Shear strain we can write as beta 1 plus beta2.



And since we are that is our basic definition or the definition of Shear strain is the change in the right angle. And since we are dealing with the small deformations beta1 can be approximated as tan beta1 and beta2 can be approximated as tan beta2.

$$\text{Shear strain} = \beta_1 + \beta_2 \approx tan\beta_1 + tan\beta_2 = \frac{\frac{\partial v}{\partial x}.dx}{dx} + \frac{\frac{\partial u}{\partial y}.dy}{dy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

And the tan beta1 is this relative change divided by dx. And so, this along x axis we have u and along y axis we have the v and so, let us say that this point it has undergone a displacement of u and v. And if you consider this point the the height has changed by as a doe v by dou dou x the rate of change of V along the x axis multiplied by dx will give you this change in the height here.

Similarly dou u by dou y that is the rate of change of u with respect to y direction multiplied by dy is your change in the mind the length here and our Shear strain is beta 1 plus beta2 and that is approximately tan beta 1 plus tan beta 2. So, by using these equations for Tan beta 1 is the height divided by this length tan beta 2 is this height divided by this length. So, that comes to dou v by dou x Plus dou u by dou y. Similarly we can derive the other two components gamma yz and Gamma zx.

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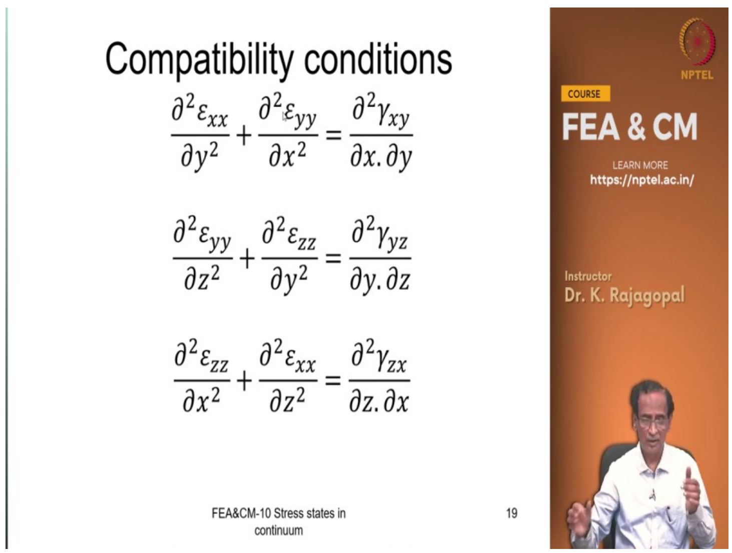

We can also write this compatibility equations in terms of the strains Epsilon xx and Epsilon yy and Gamma xy it is actually remember that our gamma xy is defined in xy plane and xy plane means we have two displacement components u and v right and along the x axis we have the Epsilon x x and along the y axis we have Epsilon yy.

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x . \partial y}$$

$$\frac{\partial^2 \varepsilon_{yy}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial y^2} = \frac{\partial^2 \gamma_{yz}}{\partial y . \partial z}$$

$$\frac{\partial^2 \varepsilon_{zz}}{\partial x^2} + \frac{\partial^2 \varepsilon_{xx}}{\partial z^2} = \frac{\partial^2 \gamma_{zx}}{\partial z . \partial x}$$

So, to relate the normal strains to the shear strains we can express our compatibility equations like this also this puts a relation between the normal strains and then the shear strains so, same thing with the other direction gamma yz and Gamma zx.

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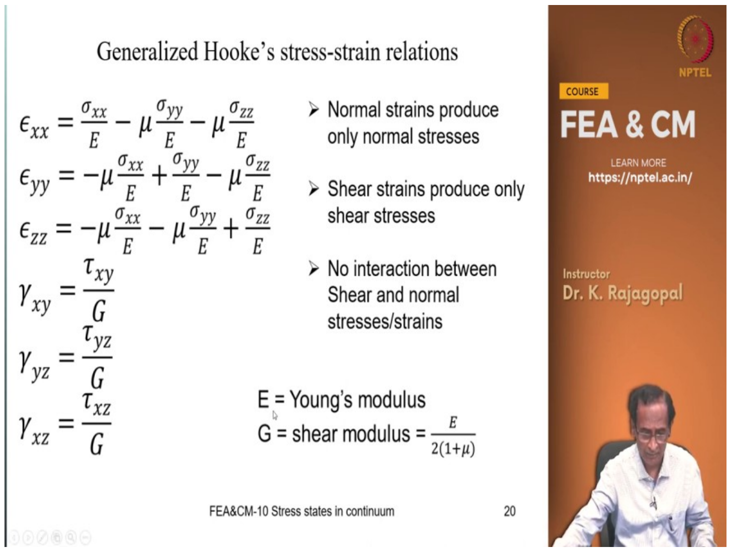

And we need some relation between the stress and strain. So, that if you know the strain we can calculate stress or if you know the stress we can calculate the strain and for this we go back to generalized hooks loss that were given to us in I think in the 1700's 1772 or something like that our Epsilon xx is Sigma xx by E minus mu times Sigma yy by E minus mu times Sigma zz by e.

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \mu\frac{\sigma_{yy}}{E} - \mu\frac{\sigma_{zz}}{E}$$
$$\epsilon_{yy} = -\mu\frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E} - \mu\frac{\sigma_{zz}}{E}$$
$$\epsilon_{zz} = -\mu\frac{\sigma_{xx}}{E} - \mu\frac{\sigma_{yy}}{E} + \frac{\sigma_{zz}}{E}$$

So, these are the three normal strains and our shear strain gamma xy is a Tau xy by z gamma y z is Tau y z by G and Gamma xz is Tau xz by G. And so, we have three normal strains and three normal three Shear strains and if you see in these normal strain equations we do not have any shear stress. And these three shear strain equations we do not have any normal stresses sigma x and sigma y and so on.

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$
$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$
$$\gamma_{xz} = \frac{\tau_{xz}}{G}$$

Gamma xy is Tau xy by G and the shear strains will produce only the shear stresses they will not be able to produce any pure normal stresses and if you apply any normal stress or normal strain you will develop only the normal component in that respective direction and if you apply any shear strain we will produce. So, only the shear stress the two only in that direction whereas x and y the normal strains they may be related to each other because of her poisons poisons ratio and in general our G is the Young's modulus sorry E is the Young’s modulus and the G is our shear modulus E divided by 2 times 1 plus mu.

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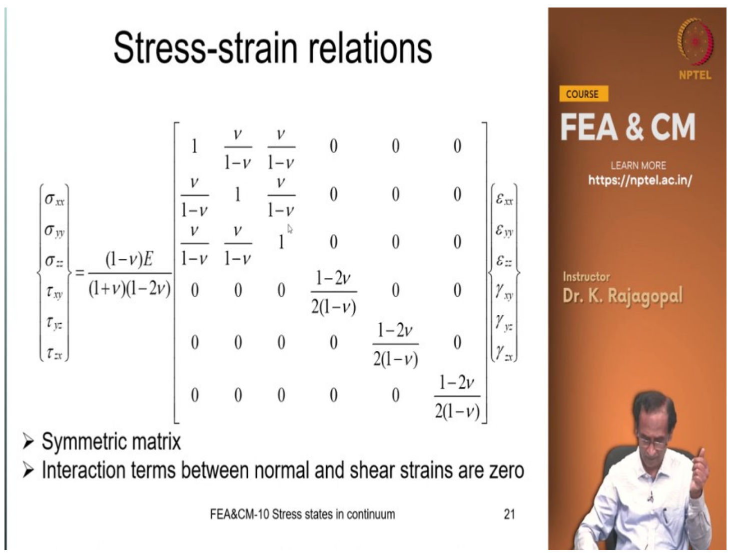

And by inverting these relations we can get our sigma in terms of the strain like this Sigma xx Sigma yy Sigma z Tau xy Tau y z Tau zx is is this product multiplied by this Matrix and then we have the six strain components Epsilon xx Epsilon yy Epsilon zz gamma xy gamma yz gamma zx and if you notice we have a symmetric constitute to matrix. See the same thing we have seen even with this the constituted Matrix for bar element and beam element actually.

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & 1 & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}$$

There we did not have a constitute to Matrix but we had stiffness Matrix directly because we were directly deriving them. So, the consequence of this the Symmetry and the constitutive Matrix means even in our stiffness matrices will have the symmetry.

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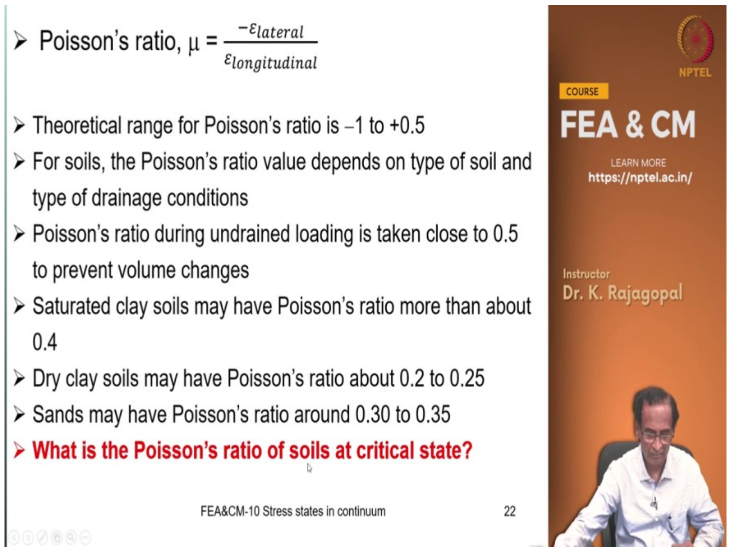

And how do we define the Poisson's ratio this is how we define minus of Epsilon lateral divided by Epsilon longitudinal the minus sign is to indicate or to take care of the the change in the sign. So, if you apply tension in one direction the other direction we have compression or if you apply compression in one direction in the other direction there is elongation. And our Poisson's ratio is a positive quantity cannot be negative.

$$\text{Poisson's ratio, } \mu = \frac{-\varepsilon_{lateral}}{\varepsilon_{longitudinal}}$$

So, we write minus Epsilon lateral divided by Epsilon longitudinal knowing that they do not have the same sign. If you have tension in one direction you will have compression in the other direction. And the theoretical range for these Poisson's ratio can be animated from -1 to plus 0.5. And for soils the Poisson's ratio depends on the type of soil and then the type of drainage conditions and so on.

So, if you are dealing with extremely loose sand's that will collapse under small Shear strain you may have a negative Poisson's ratio because you may have collapse in all the directions. So, that means that in the x-axis you have compression y axis also there is a compression but that is an extreme case and in fact you cannot handle the negative Poissons ratio in our analysis and the Poisson's ratio during the undrained loading we take it very close to 0.5. Because we know undrained loading means there will not be any volume changes especially if you consider saturated soil.

So, actually when we say undrained loading we only mean saturated soils and saturated soils means all the pores are filled with incompressible pore water. So, if you apply any volumetric compression or retention there will not be any change in the volume. So, we take the Poisson's ratio is close to 0.5 during these calculations. So, in general the saturated clays could have Poisson's ratio more than about 0.4 maybe 0.45, 0.46 and so on.

And the dry clay soils may have poisons ratio of 0.2 to 0.25 or maximum about 0.3 depending on the nature of the soil whether it is severely over consolidated or normally consolidated clay and so on. And the sands the Sandy soils may have poisons ratio in the range of 0.3 to 0.35 and in general the Poisson's ratio of the sands will not change much in the presence of water.

So, this Poisson's ratio for the sands could be about 0.3 and 0.35 and when we deal with sands we assume that the sand has very high permeability. So, the pore pressures are not really considered like we can consider effective stresses. And now the question comes what is the Poisson's ratio of soils at critical state. A critical state is a state where you achieve this at a very large shear strain the application of any Shear strain will not be associated with any volume changes the critical state that is our limit state.

So, what is the Poisson's ratio of soils at critical state you think about and send me an answer by email.

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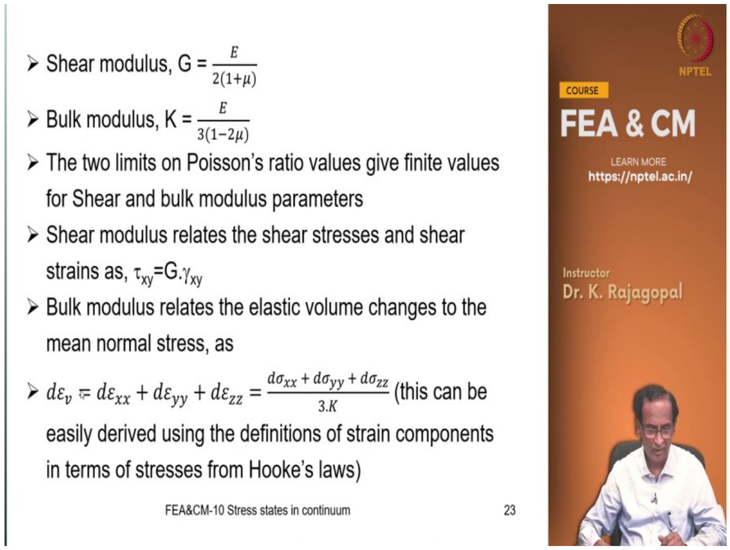

We can define two modulus parameters one is the shear modulus G as E by two times one plus mu and bulk modulus K is E by 3 times 1 minus 2 mu. And the two limits are the Poisson's ratio we can imagine that is minus one to plus 0.5 if you look at these two relations the shear modulus and the bulk modulus. So, if your mu is exactly equal to minus 1 or more than minus one you will have a problem because you will your shear modulus is negative.

$$\text{Shear modulus, } G = \frac{E}{2(1+\mu)}$$

$$\text{Bulk modulus, } K = \frac{E}{3(1-2\mu)}$$

So, that means that even if you apply some positive Shear strain you will get a negative stress and our shear stresses under Shear strains they are related through the shear modulus as a Tau is G times gamma and our bulk modulus it relates the one the volumetric changes to the mean normal stress. Say d Epsilon v is the volumetric constraint that we can write as Epsilon xx Plus Epsilon yy plus Epsilon zz.

And this is actually we write everything in an incremental form in our geotechnical engineering because even with a small change in the load your stiffness may be different. So, our d Epsilon v is d Epsilon xx plus d Epsilon yy plus d Epsilon zz that is the mean normal stress that is d Sigma x plus d Sigma y plus d Sigma z that whole thing divided by 3 divided by bulk modulus K will be your volumetric strain.

$$d\varepsilon_v = d\varepsilon_{xx} + d\varepsilon_{yy} + d\varepsilon_{zz} = \frac{d\sigma_{xx} + d\sigma_{yy} + d\sigma_{zz}}{3.K}$$

And in fact the equations for G and K we can derive from the fundamentals that have not done here but you can do it. So, I think this is my last slide. So, in this lecture we have looked at the stress states within a three dimensional continuum and the equilibrium equations in terms of in terms of the stresses the constituted equations relating the stress on the left hand side Sigma and to the Strain on the right hand side through some constituent equation that is what we have seen here.

This entire thing and this matrix written in terms of the Young's modulus and poisons ratio it relates the strains to the stresses. So, thank you very much I think that is the end of this lecture and if you have any questions please send an email to this email address profkrg@ gmail.com. So, thank you very much and we will meet in the next class.