**Analysis and Design of Bituminous Pavements**

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**Lecture - 07**

**Numerical Problems in Two-Layer Theory**

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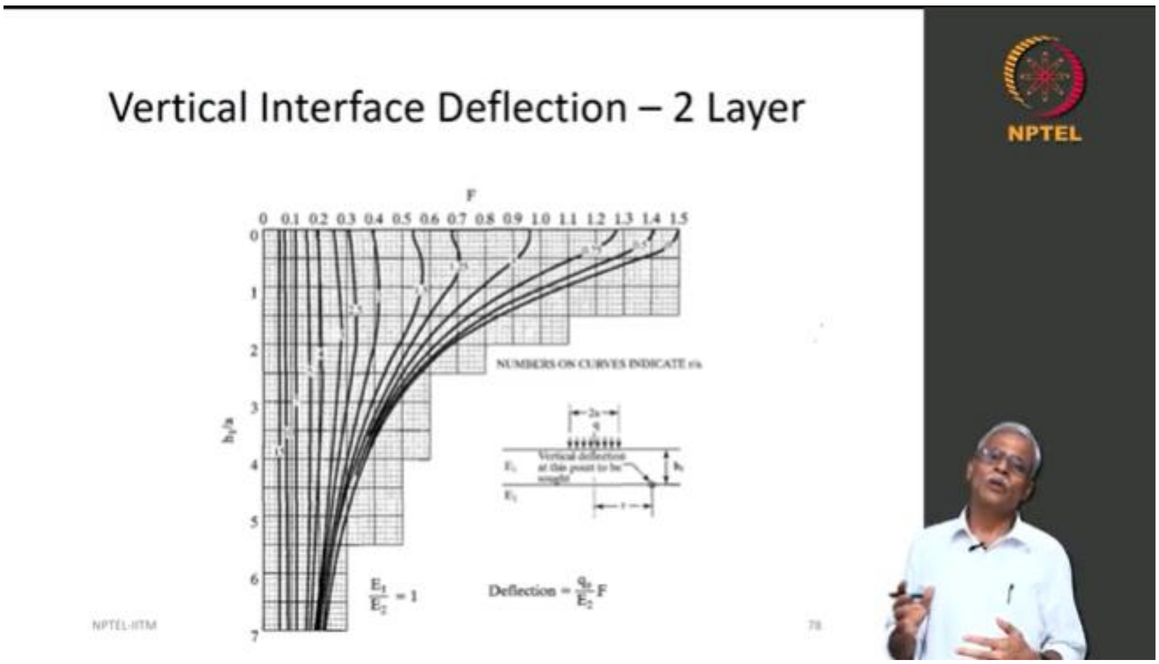


So, hello everyone let us continue our discussion on how to compute various components of stresses and strains using this chance. In fact, what I am going to do is I am going to spend some more time on explaining some of these things on how to compute the interface deflection. But later what we will do is to introduce KENPAVE and using the KENPAVE how to go about computing all these things and in Chapter 2 of the textbook that you have there are a lot of examples at the end which you should be able to do using the chart as well as the software and try and see whether there are any differences. I will also be sharing with you some example problems to be worked out from some other textbook. So that will be for you to find out how to get comfortable with using the chart. So let us continue our discussion so we are going to talk about what is really called as the vertical interface deflection.

So what exactly is this vertical interface deflection please take a look at this particular picture. So this is two layers so this is layer 1 this is layer 2 the E value is $E_1$ here $E_2$ and $h_1$ and of course you are talking about h infinity. Now this is 2a this is your contact diameter and this is the load per unit area q. Now what we are interested to find out is we need to find out at some point here, which is at a distance r from the reference line what is the deflection we need to find that out. So how do we find that out we use the formula below,

$$Deflection = \frac{qa}{E_2}F$$

Now since this is two-layer theory you are going to have now separate charts for separate ratios of $E_1/E_2$. So this is going to be for $E_1/E_2 = 1$ of course $E_1/E_2 = 1$ means what this basically reduced to the one layer Boussinesq problem. So this is $h_1/a$ and whatever is the lines that you the graphs that you see corresponds to various r/a. So use $h_1/a$ and use r/a find out what is F substitute the F here you know q you know a you know $E_2$ you can find out what is the deflection. Now this particular chart that you see is for E1/E2 is equal to 1.

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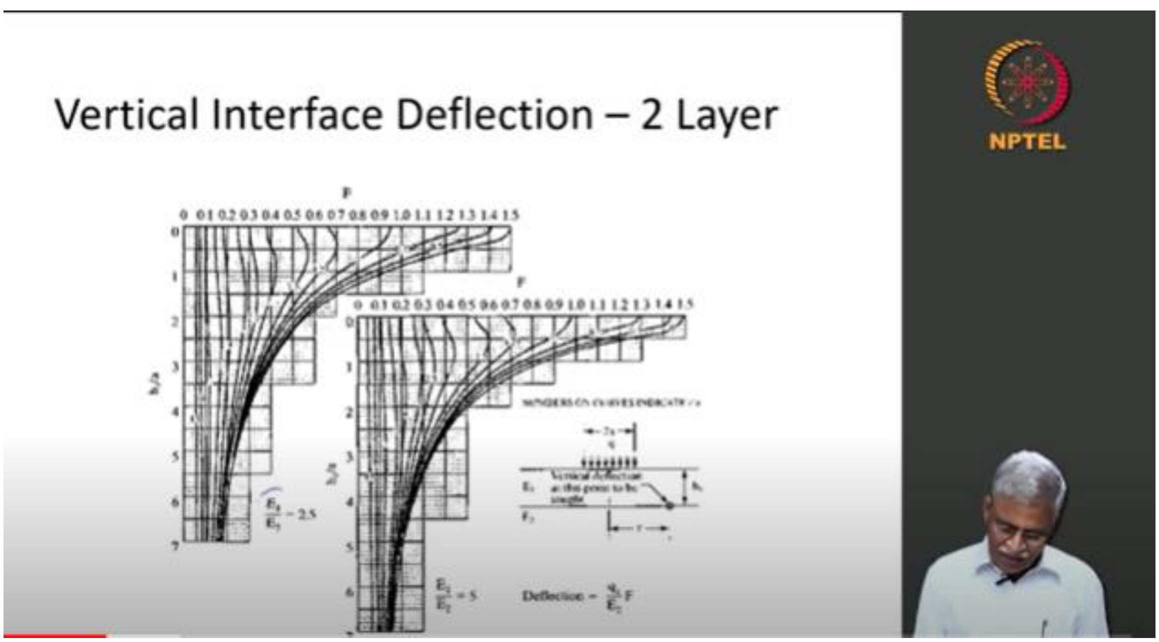

Now you are given charts like this for $E_1/E_2$ =2.5 and $E_1/E_2$ = 5

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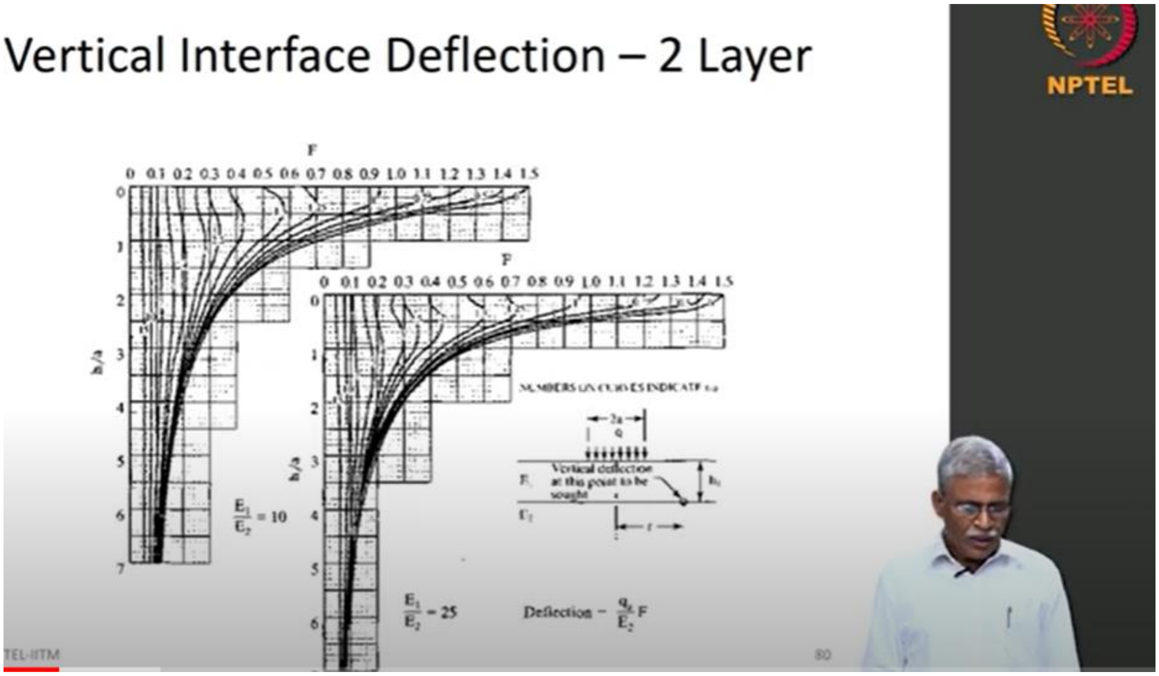


Here $E_1/E_2$ = 10 as well as $E_1/E_2$ = 25

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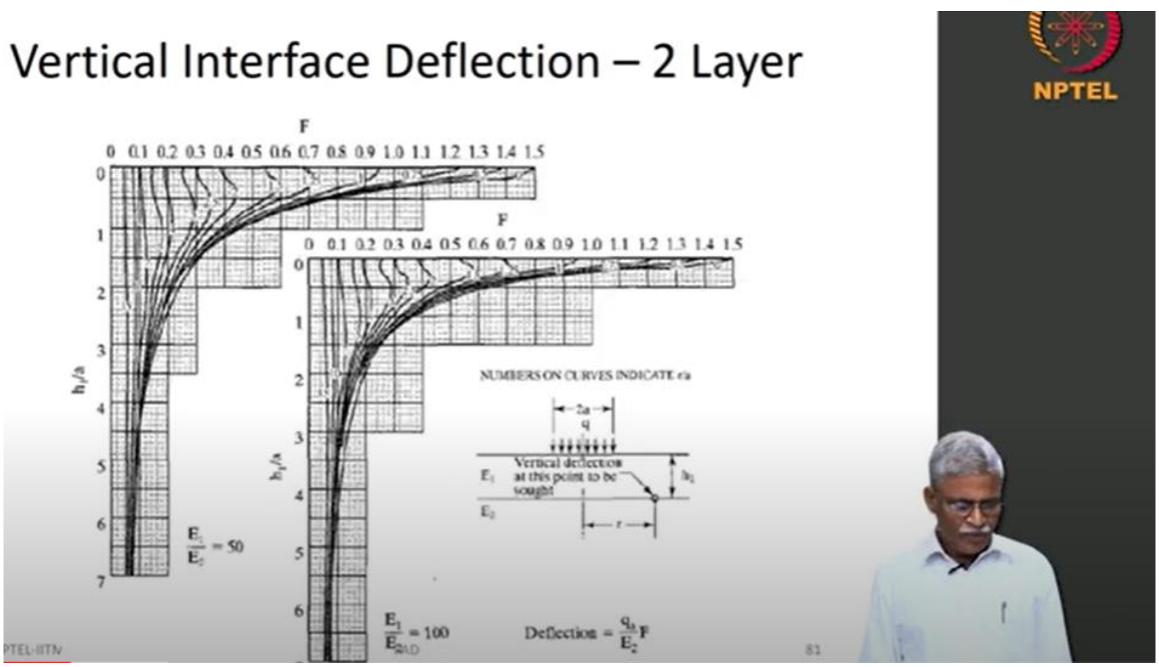

Similarly, $E_1/E_2$ = 50 and $E_1/E_2$ = 100. So there are a variety of charts that are given here in fact in those days when I started taking the pavement analysis and design course we used to go to the exam with so much of design charts in our hand okay and you basically read from the chart and then compute the stresses and strain and means many times my whoever taught me basically used to mask this formula here and give us the chart only this particular chart. These days when I teach pavement analysis and design and I have been teaching at IIT Madras for close to 18 years this particular course I do not ask in the final semester examination anything related to computing the interface deflection or any of these things which uses the chart. I introduce the student to the chart because it gives them the big picture of how this actually evolved because you know you are when you see this you are able to feel it get a feel of how the deflections will vary. See because what it means is q is constant and a is constant for any given problem and the only thing that is variable is F. F depends on r/a and h/ a and you will be able to see how as r/a varies and h/ a varies, the deflections keeps reducing. So in a sense whatever you see here is the actual variation of the deflection for various combinations of h and r.

So that is very clear to you. So we use the chart and then do design problems. See for instance you would like to proof check IRC 37 design we train the students to use the software which is what we are going to do later now. But nevertheless let us get used to the idea of the chart here.

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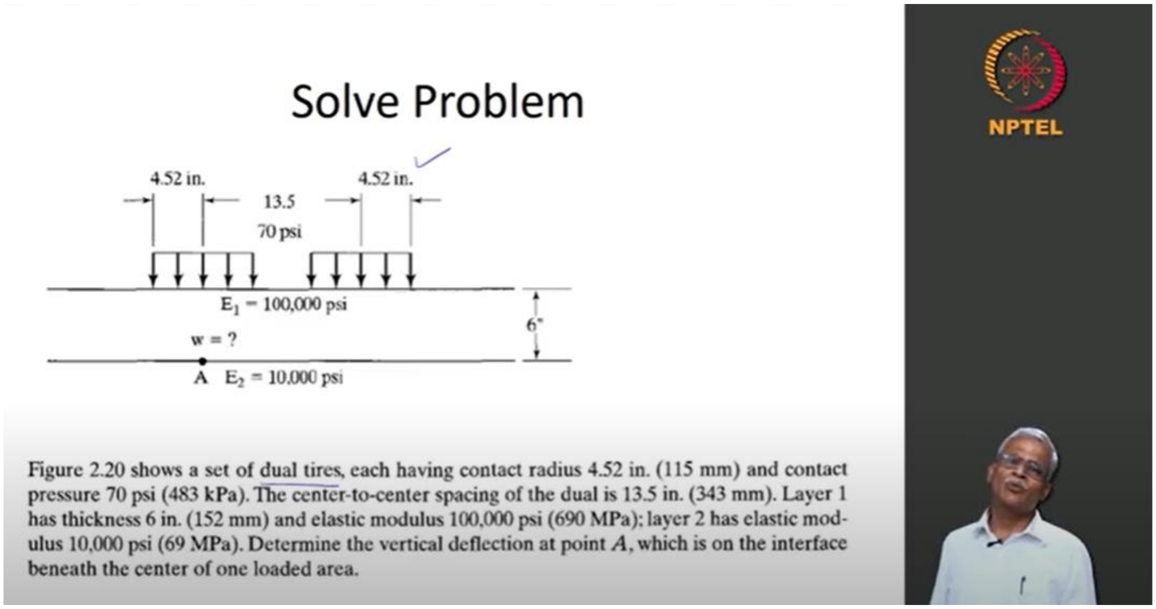


So now we will solve one simple problem. What is this problem? It says there is a dual tire each having a contact radius of 4.52 inches. I know that writing in inches and $lb/in^2$ and all those things might be uncomfortable to most of you. But since the textbook the only standard textbook that is available for this is given in this unit I do not want to confuse you so I just want to use the same thing. But you should be able to convert it into the system that you are familiar with. See for instance even here also in this book they have written in the brackets whatever is written in inch they have written it in mm but that is okay. So we let us learn the concepts. So this is 70 psi the center to center distance spacing of the dual is 13.5 inches. Now this is a very important quantity and this is normally written as S suffix d. Layer 1 has thickness of 6 inches and elastic modulus of 100,000. Layer 2 has elastic modulus of 10,000 psi.

Determine the vertical deflection at this point which is on the interface beneath the center of one loaded area. Now what is that you need, basically what are all the things that you need? You need $E_1/E_2$. What is $E_1/E_2$? 100,000/10,000. So your $E_1/E_2 = 10$. So immediately you know that we have a chart for 10. Then you have need $h_1/a$ and $h_1/a = 6/4.52$. So how did we get to say 6/4.52. So this is 6 h, this is a = 4.52 and that is how you got this value. So that comes to be 1.33. Now what you really need to do is if you want to compute the deflection at A you need to compute the deflection at this point due to this load I will call it as the left load and the right load. So for the left load you need to look at E1/E2 of 10.

What is that you need? You need r/a and h/ a that is what you really need here. So for r/a = 0, h is the same thing in both the cases. Only in the right you are going to have a different r/a. So how much it is going to be there? That is going to be 13.5/4.52. So for left hand side r/a is 0 and for right hand side r/a is 13.5/4.52. So let us write for left r/a = 0 for right r/a = 2.99. So now what will really happen? The deflection factor for this case. So you know what is the h by a here right? So you use the values you know take $h_1/a$ here. You know how much is $h_1/a$. You know what is r/a. So what you really need to do is to find out the interfaces deflection factor. So this is going to be 0.56, this is going to be 0.28. So the total deflection factor is going to be 0.56 + 0.28 and that is going to be 0.84. So now how much is the total the interface deflection? It is $qa/E_2 \times F$. So what is q here? q in your case is 70 that is 70 here and a is 4.52 and what is $E_2$? $E_2$ is 10,000 psi times 0.84 and this is what you get.

Now we need to understand few things. So immediately your question that should come to you is where will be the maximum deflection? So this is something worth talking about. Will the maximum deflection be at A? Because this is symmetry so whatever you see here at the center of left hand wheel is what you are going to see for at the right hand wheel. So what normally we should do is let us say this is distance and this is the deflection. So let us mark the position of A and B. Let us also mark the position of the outer and the inner and let us mark the position of the center. So if you try to find out the deflection here so right now you have found out the deflection at this point and it will be the same as this particular point. So it might increase and come something like this. So that basically depends on the interface distance that you heave or the center to center distance. Normally we see that this portion where a tire edge is there it is actually subjected to maximum deflection.

So what I think you should do is take this point and three points. So one is the center point another is this one and the third one is either the difference means exact midpoint between these two point or the tire edge and then compute the deflection you might see something like this. So you may want to work it out on your own and find out where exactly is the maximum deflection coming in. So that is the most important thing.

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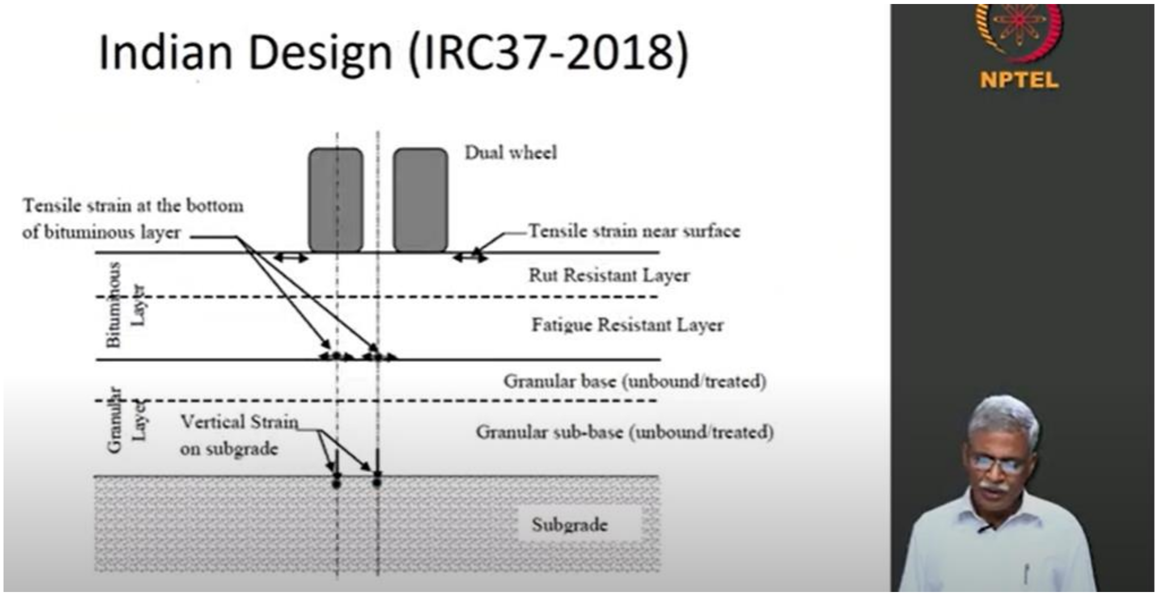


Now let us relate it to what we discussed earlier about the pavement cross section. Because we have been talking about computing the stresses and the strains and interface deflection. So now let us revisit our IRC cross section and you are going to see that there is some tensile strain at the bottom of the bituminous layer that is mentioned here. Now how do we really compute this tensile strain?

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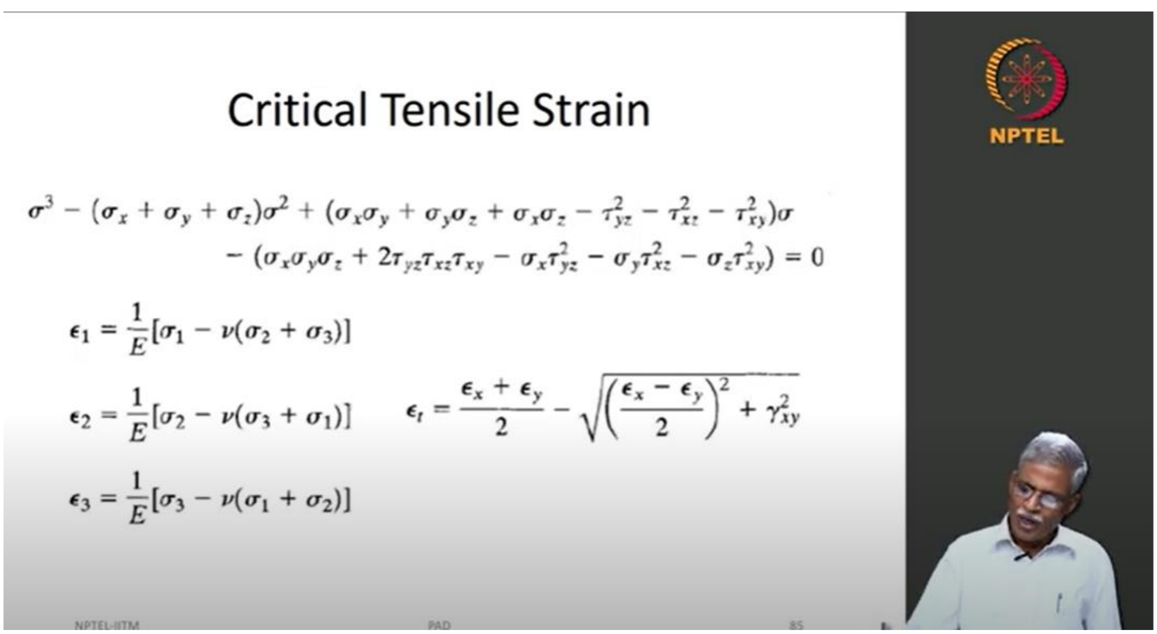

So for that I just need to tell you something about the critical tensile strain. So I assume you know what is this? This is your characteristic equation of the stress.

$$\sigma^3 - (\sigma_x + \sigma_y + \sigma_z)\sigma^2 + (\sigma_x\sigma_y + \sigma_z\sigma_y + \sigma_x\sigma_z - \tau_{yz}^2 - \tau_{xz}^2 - \tau_{xy}^2)\sigma - (\sigma_x\sigma_y\sigma_z + 2\tau_{yz}\tau_{xz}\tau_{xy} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{xz}^2 - \sigma_z\tau_{xy}^2) = 0$$

So we will revisit this in the next class when we talk about how KENPAVE will implement these things. So this is your characteristic equation of the stress. So when I solve this what will be the solution? What will be the solution of this equation? I am sure you know it. You are going to get your principle stresses. Now knowing the principle stresses you can compute your principle strains $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$.

$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu(\sigma_2 + \sigma_3)\right]$$

$$\varepsilon_2 = \frac{1}{E}\left[\sigma_2 - \nu(\sigma_1 + \sigma_3)\right]$$

$$\varepsilon_3 = \frac{1}{E}\left[\sigma_3 - \nu(\sigma_1 + \sigma_2)\right]$$

Now what is the stress that we need to do use here? Because it is written as tensile strain. So ideally we should be using the principle strains. So it should be the major principle strain or the minor principle strain. But what normally pavement engineers do is to use this formula to compute the tensile strain.

$$\varepsilon_t = \frac{\varepsilon_x + \varepsilon_y}{2} - \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \tau_{xy}^2}$$

Now this tensile strain that is computed here for this particular load is substituted in your distress equation to find out what is the expected number of load repetitions that a pavement cross section can actually take. So that is the whole idea.

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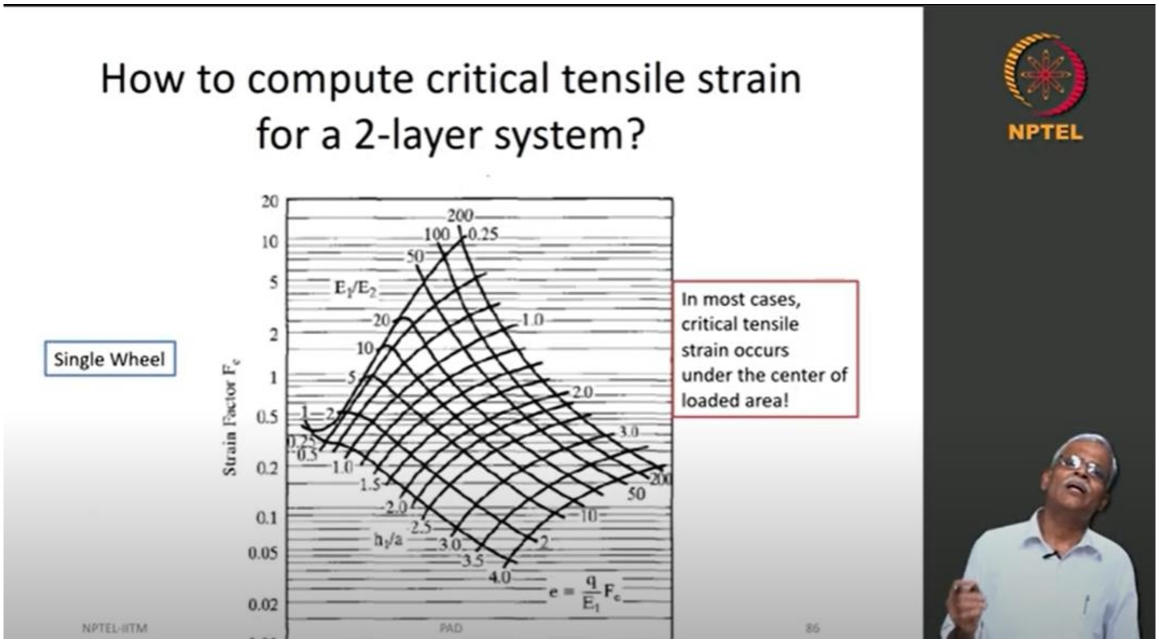


Now let us try to see how we can compute this critical tensile strain for a two-layer system. I hope I made the connection very clearly. If not, I will repeat it once more again. So what we have been doing we have been using this charts to find out vertical stress, interface deflection and many other things and all we have been computing here. In fact, if you recollect we had these many charts here. So let us just go take a look at all these charts. So we know how to compute it. So we know how to compute $\sigma_z$, we know how to compute $\sigma_r$, we know how to compute $\sigma_t$, we know how to compute $\tau_{rz}$, we also know how to compute the deflection vertical deflection for one layer. And now knowing these things we also know how to compute the strain. Then we differentiate it between the flexible plate and the rigid plate because typically when you are going to do the plate load test you are talking in terms of the rigid plate and there is going to be a non-uniform pressure distribution. We discussed all those things and we worked out a lot of interesting problem and then we went into two-layer theory how to do because all these things that were discussed were based on the one-layer theory. So we went into the two-layer theory and then we talked about the interface stress here and then we talked about interface vertical surface deflection for a two-layer system and so on and so forth and we also talked about the vertical interface deflection. So the one that we talked about was the surface deflection $w_0$ at the surface.

Now we want to know what is the interface deflection at the interface. Now the next logical thing will be to find out what is the strain at that particular interface because basically we believe that this pavement that you see here when it is subjected to load if you can assume it in that way it is not really a correct way of visualizing it, but to motivate a discussion you can say that this is going to bend something like this. So let me do it do not take all these things in a more literal sense I am just trying to motivate a discussion here so you are going to have critical strains here. Now we need to compute this critical strain. Now comes an interesting thing computing the deflection is fairly straightforward. Computing the strains is going to be little bit more involved. So you are trying to solve the characteristic equation of the stress get the roots of this equation nothing but which are principal stresses. Now compute the principal strain and knowing the principal strain try and compute your tensile strain. Now it could be a major principal strain or a minor principal strain, but this is the form of the strain that we are going to use. So now what we are planning to do is how do we compute the critical tensile strain for a two-layer system that is what we are going to do.

Now this particular case that you see here is for single wheel. In typically the most of the cases the critical tensile strain will occur under the center of the loaded area. Now the calculations now start becoming more involved. It is not the computation of the vertical deflection which is straightforward. When it comes to computation of the critical tensile strain now we need to make the distinction between whether it is going to be under the single wheel or within dual wheel. Within dual wheel it becomes little more involved and that is where I am going to say that you are going to use a software to compute the whole thing otherwise you will be repeating this exercise again and again.

And one more thing before I just forget see we discussed here $E_1/ E_2$ = 50, 100, 10, 25, 2.5, 5 as well as 1. So you might ask so what if my $E_1/E_2$ is 4. You can do linear interpolation. So that means you will find out the deflection factor for 2.5, you will find out the deflection factor for 5 and then you do a linear interpolation for $E_1/E_2$ = 4. Is that correct? That is okay. There will not be that much difference. But again these charts are provided for the ease of you to understand some of these things.

So let us just get back here and you can essentially see this is the critical tensile strain. So it is going to be $q/E_1 \times$ Fe. So you can actually see this is $E_1/E_2$ is given here. This is the strain factor which is given here and these are all $h_1/a$ that is very clearly given here.

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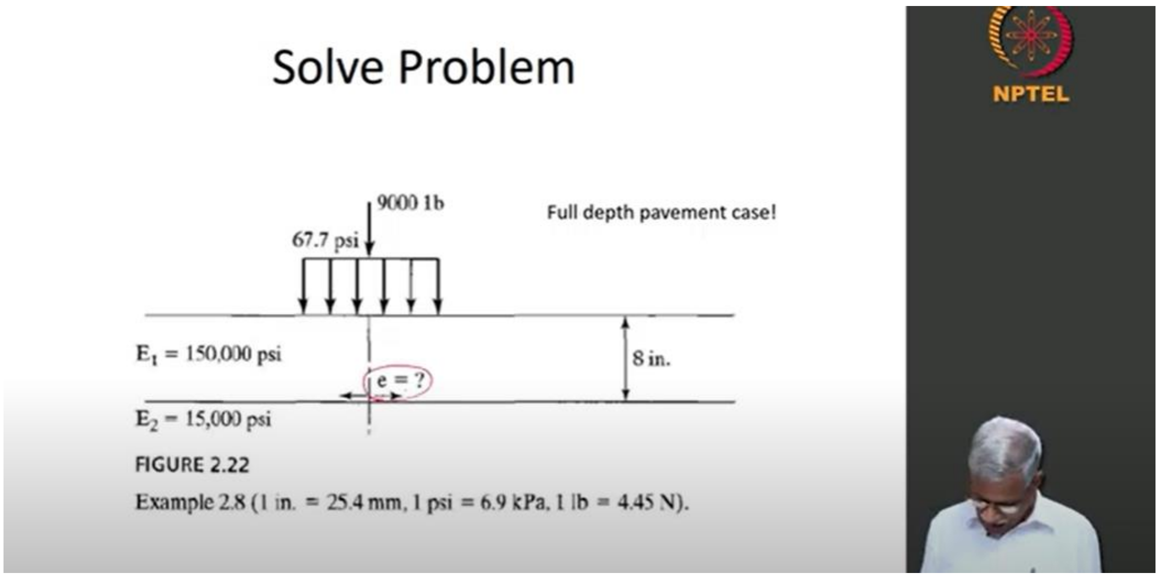


So now let us try and see how to solve this problem. So what we really want to do is to find out the critical strain. And in fact you will notice here that there is no r/a or anything that is given here right. So you have an $h_1/a$ is given here and $E_1/E_2$ is given here because this is basically for a single wheel and the critical tensile strain is going to exactly happen under the center of a loaded area. So if I have a pavement cross section like this with 9000lb and then this is the area. So we did not we do not have a so we need to compute a, $E_1$ is given, $E_2$ is given. What is really needed is how to compute this E. So now what you are going to do is a = $\sqrt{9000 \times \pi \times 67.7}$. So that is going to give you 6.5 inches. Now $h_1/a$ =8/6.5. So 8, a is that is what we have computed now, it is going to be 1.23 and $E_1/E_2$ is equal to 150000/15000 that is going to be 10. So now what you need to do is to take the 10 here that is given here and the $h_1/a$ that is found out by you as 1.23 somewhere here and somewhere here and see where is the factor that is coming here. So you are going to get the factor as close to 0.72 and so E is equal to which is what I have written here, you can say q / $E_1$ × Fe, so q = 67.7, $E_1$ = 150000, Fe = 0.72 and your critical tensile strength is $3.25 \times 10^{-4}$.

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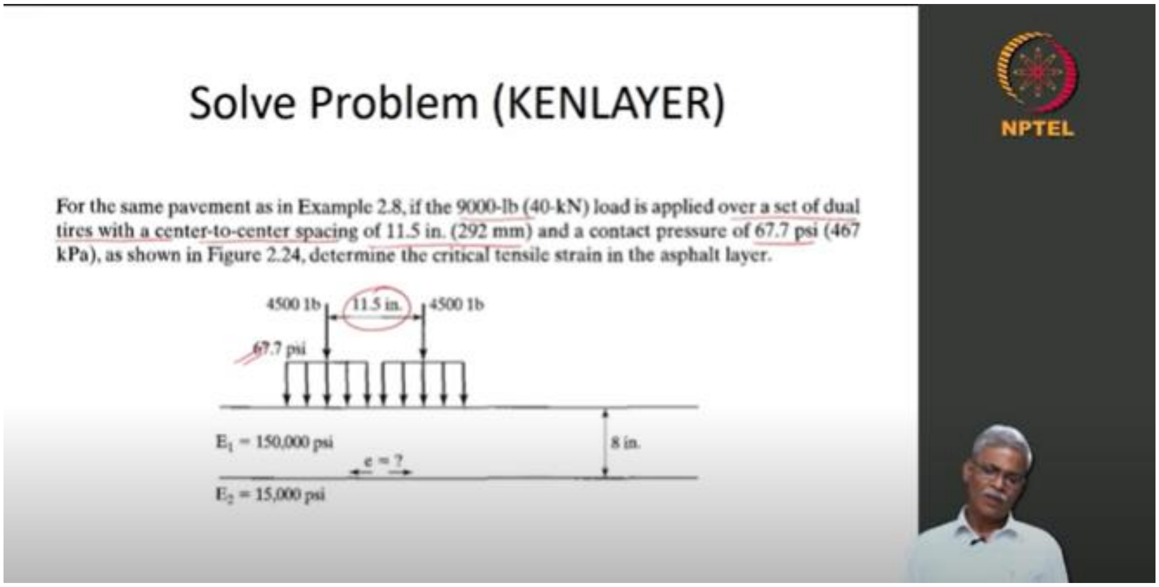


So this is what we are going to do. Now what will really happen here? So let us discuss this for the same pavement as shown in Example 2.8, if this 9000lb load is applied over a set of dual tires with a center to center spacing of which is this is the critical one and a contact pressure of the same as shown in Figure 2 point. So critical tensile strain in the asphalt layer. Now here comes the crux, so how do we really determine it? So we need to have different charts for different values of 11.5 inches. So because we do not know what exactly is the influence of the dual wheel spacing on the development of the critical tensile strain. Right now what you see here is this is given independent of we have assumed that the maximum tensile strain here is exactly happening below the loaded area. Now since this is strain and since the variation of strain is not going to be straightforward you are going to need lot more multiple charts here. So what we are going to do is we will not be doing it with chart we will be using ken layer. And in fact in the next class what I will do I will introduce ken layer to you in a systematic way.

And in fact when you are going to have these kind of load say for instance you know you have 1, 2 so you are having a dual tandem axle then this becomes even more complicated.

So we will be talking about all those issues in the next class. So what I am planning to do is to introduce the theory behind KENPAVE and then after that Professor Padma will walk you through the whole KENPAVE software. We will also tell you clearly in what location you can download the URL from for downloading KENPAVE.

So it will be fairly straightforward. So, so far what we have completed is we have completed chapter 1 and chapter 2 of Huang. Exercise problems in chapter 2 should be worked out by you after you understand the logic behind KENPAVE. Okay, thank you very much.