Analysis and Design of Bituminous Pavements

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Lecture - 06

Numerical Problems in One-Layer Theory

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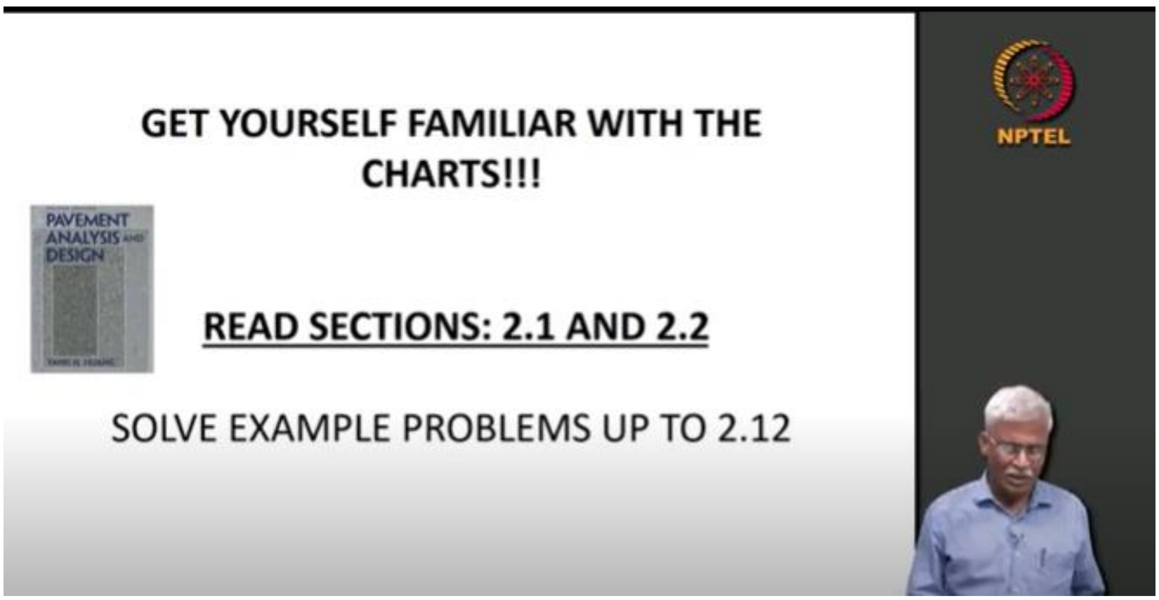

So, let us continue with the discussion that we had related to stress analysis in fact as I mentioned earlier this is going to be your reference book, so if some of you need I can share some individual chapters with you for educational purpose but I suggest that you buy the Indian edition which is quite cheap. Now what I really want you to do is to use this as a basic textbook for this course, so but there will be lot of things in this textbook that you may not really need it. So as and when I find it easier for you I will be suggesting you some of the sections that you should really follow. Now I am going to write here clearly read Sections 2.1, in this 2.1 there is one subsection please ignore 2.1.3 read Section 2.2 ignore 2.3 so that is good enough for you.

So then now there are some example problems that are worked out and there is also some interesting exercise problem that are worked out. So what I am going to do now is to show you some charts and then also try and work out some simple problems. I strongly suggest that you should work out some of the exercise problem and if you have any doubt again feel free to get in touch with me or with my co-teachers or the teaching assistants.

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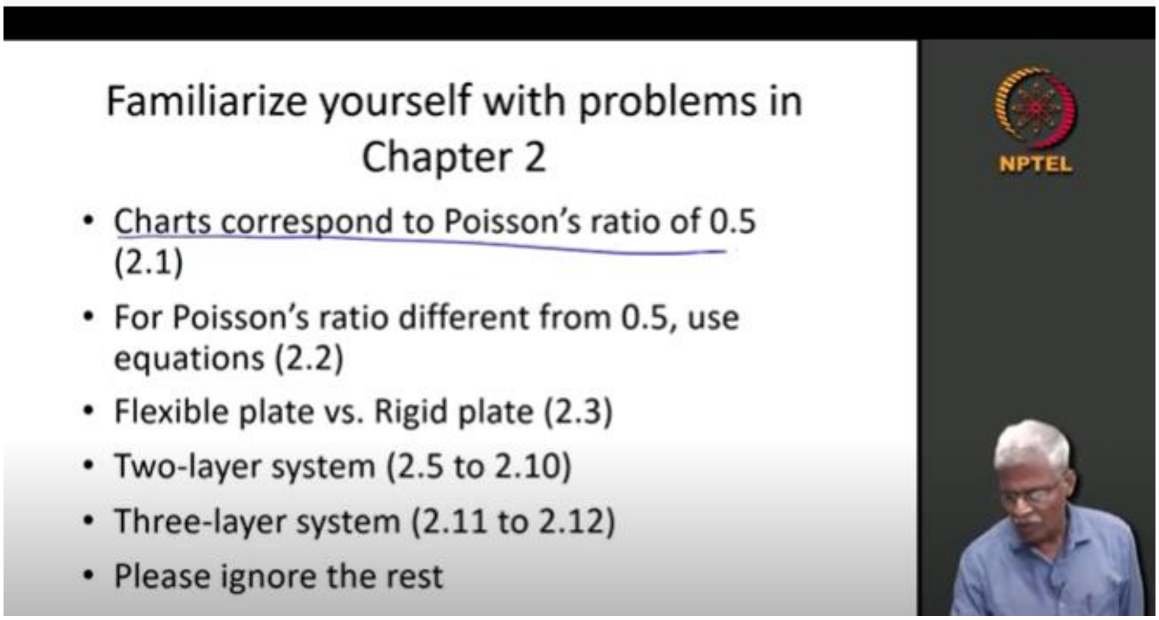

So let us continue here, now these are all the charts that you should really familiarize yourself. So there is a chart corresponding to Poisson's ratio of 0.5 so 2.1. So similarly for having a Poisson's ratio different from 0.5 there is another chart and how to use Equation 2.2, flexible plate versus rigid plate section chart 2.3, 2-layer system there are some 2.5 to 2.10; these are all the problems and 3-layer problem 2.11 to 2.12 and I have written very clearly to ignore the rest.

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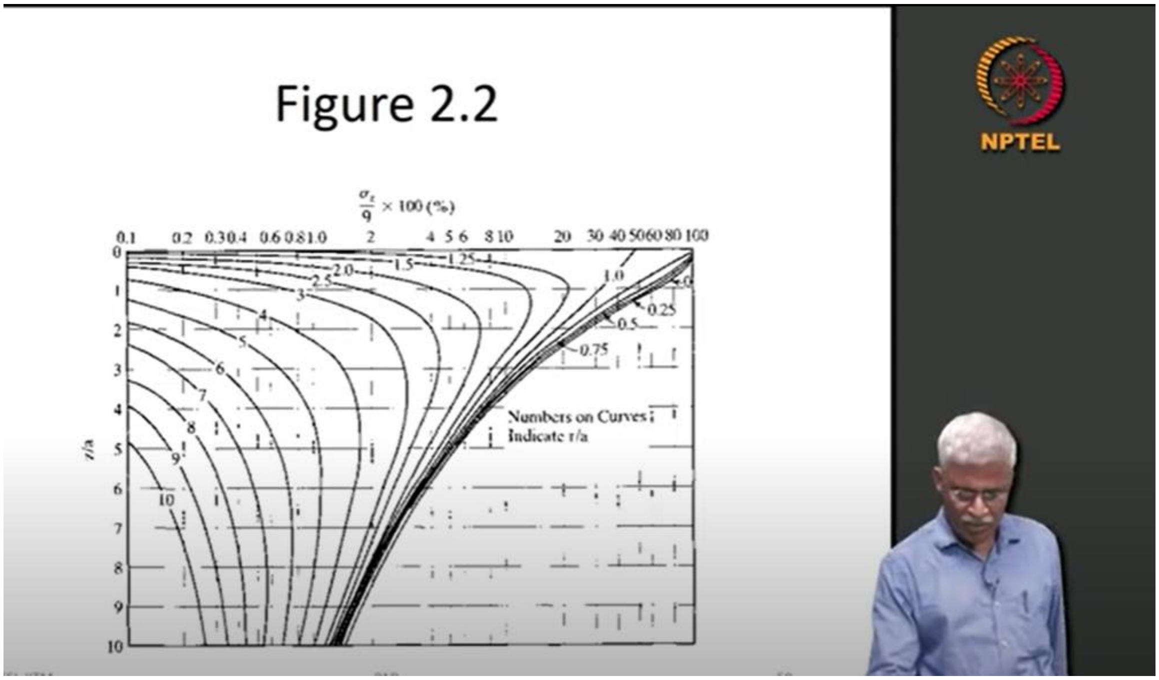

So Figure 2.2 is something that we have already discussed. So this is σ_z /q times 100 in the x axis, z/a in the y axis for various r/a. We have already discussed these things earlier.

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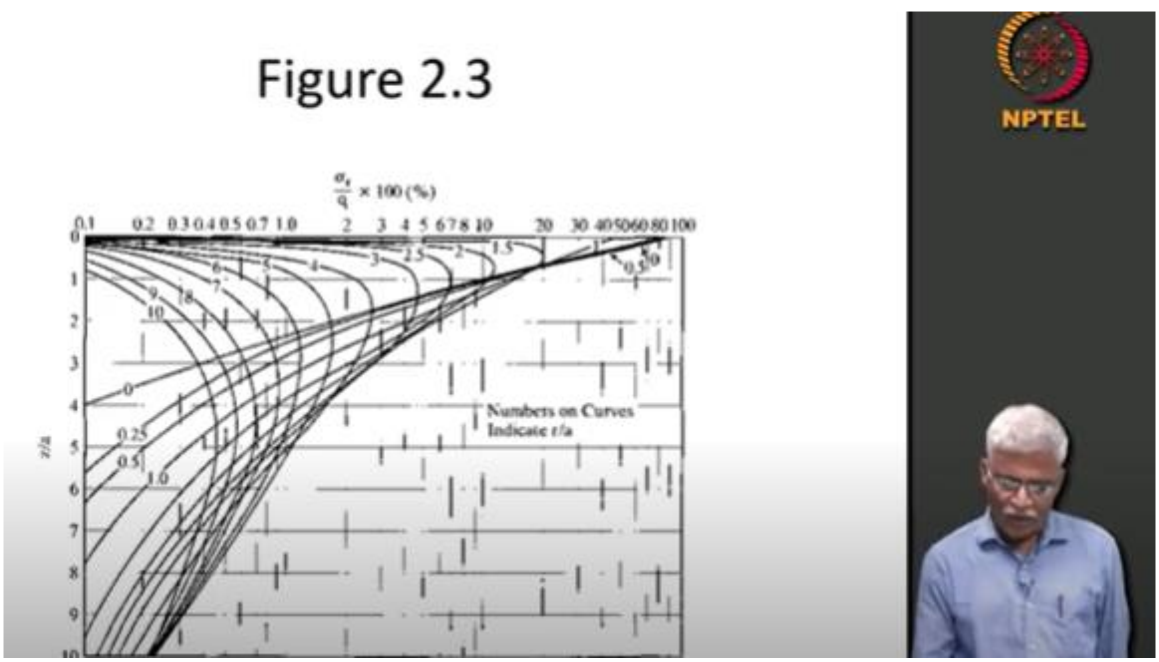

And Figure 2.3 is σ_r /q times 100, z/a and r/a. So this is also something you know very well.

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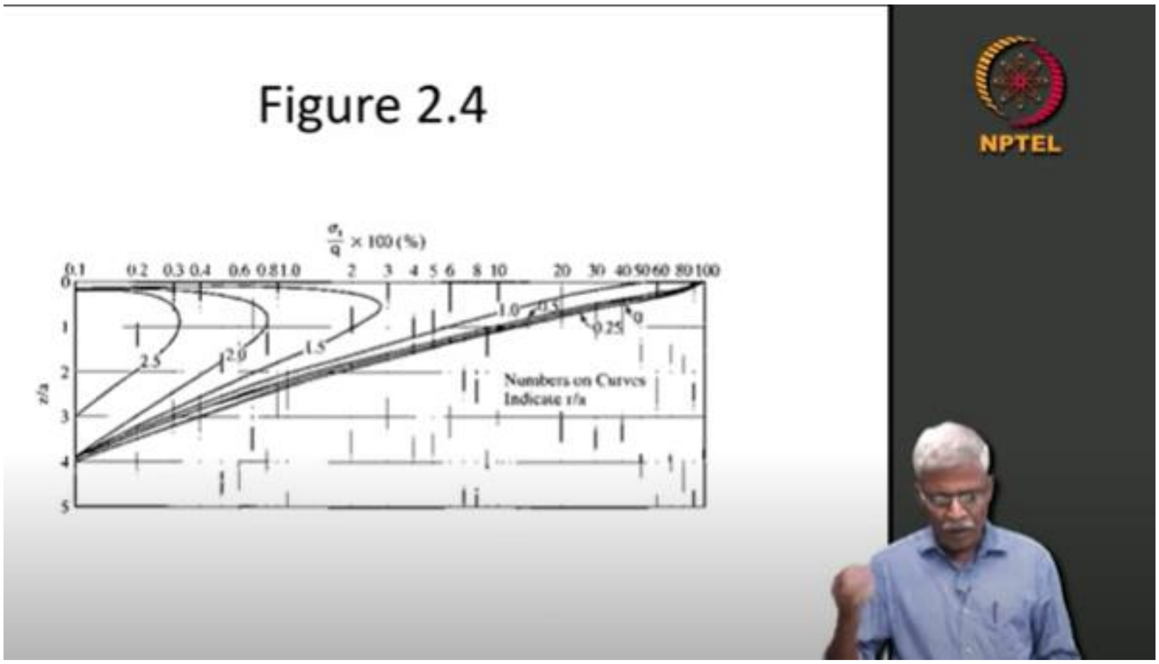

And Figure 2.4 is σ_t /q times 100, z/a for various r/a that is also I have discussed this in detail with you.

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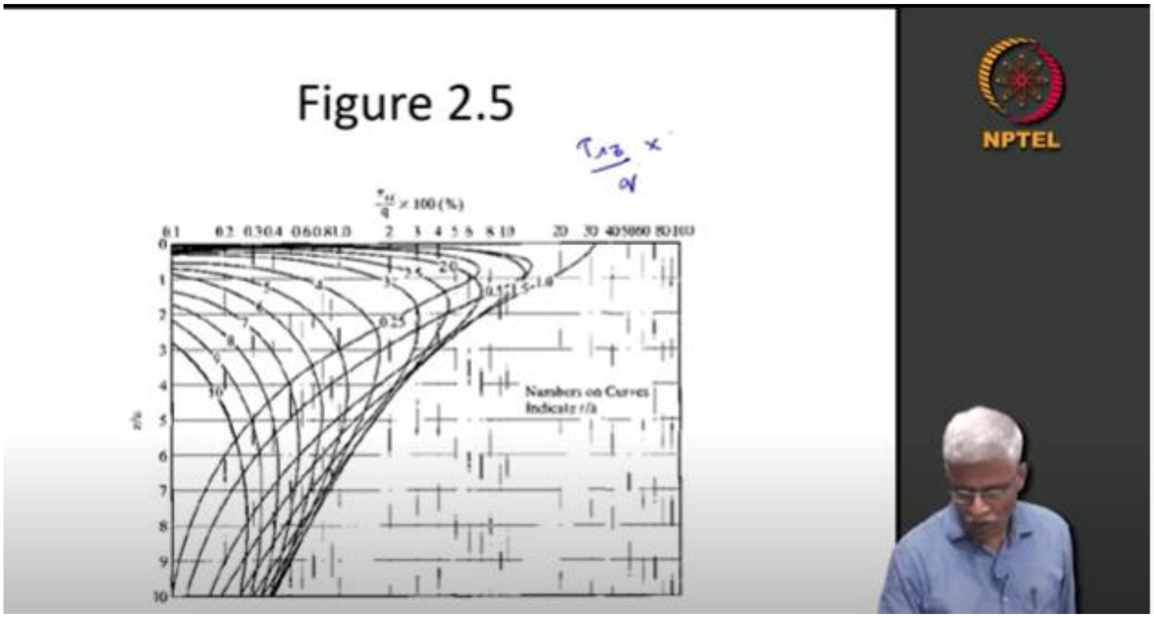

And Figure 2.5 is τ_{rz}/q times 100 for various values of r/a and z/a.

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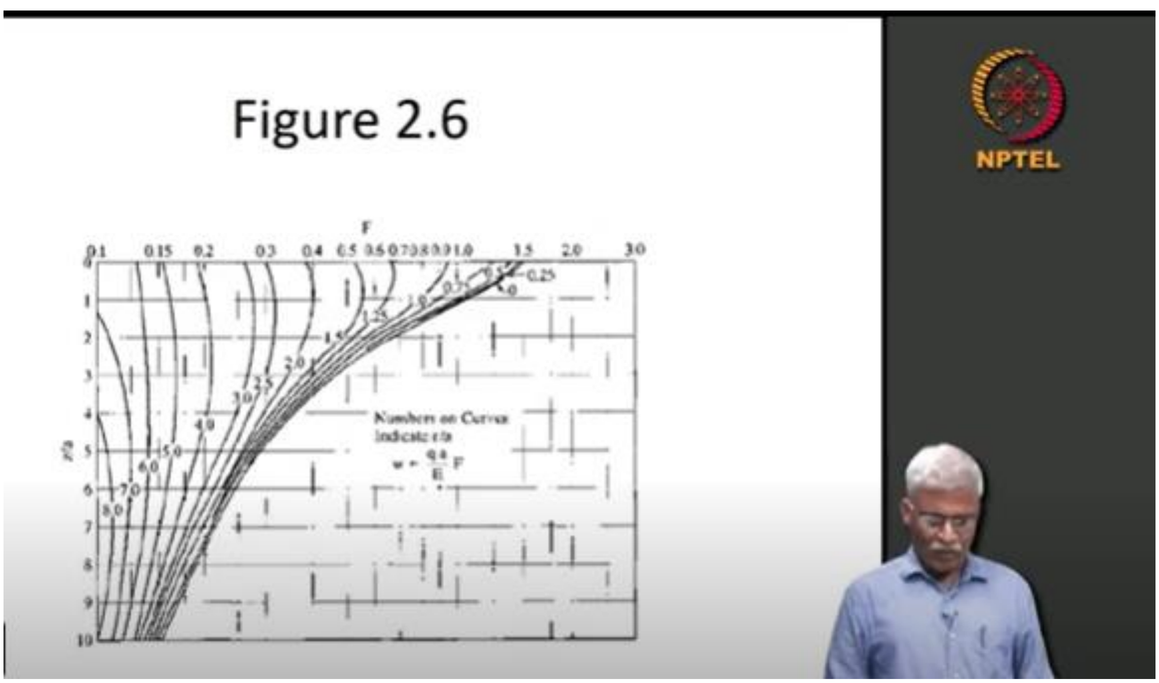

So then Figure 2.6 is related to the deflection factor. So these are all one-layer deflection factors. So we use the formula below;

$$w = \frac{qa}{E} F$$

So this is given here for different cases of z/a for various values of r/a. You compute this deflection factor, read this deflection factor from this chart, substitute it in this equation to get your interface deflection factor.

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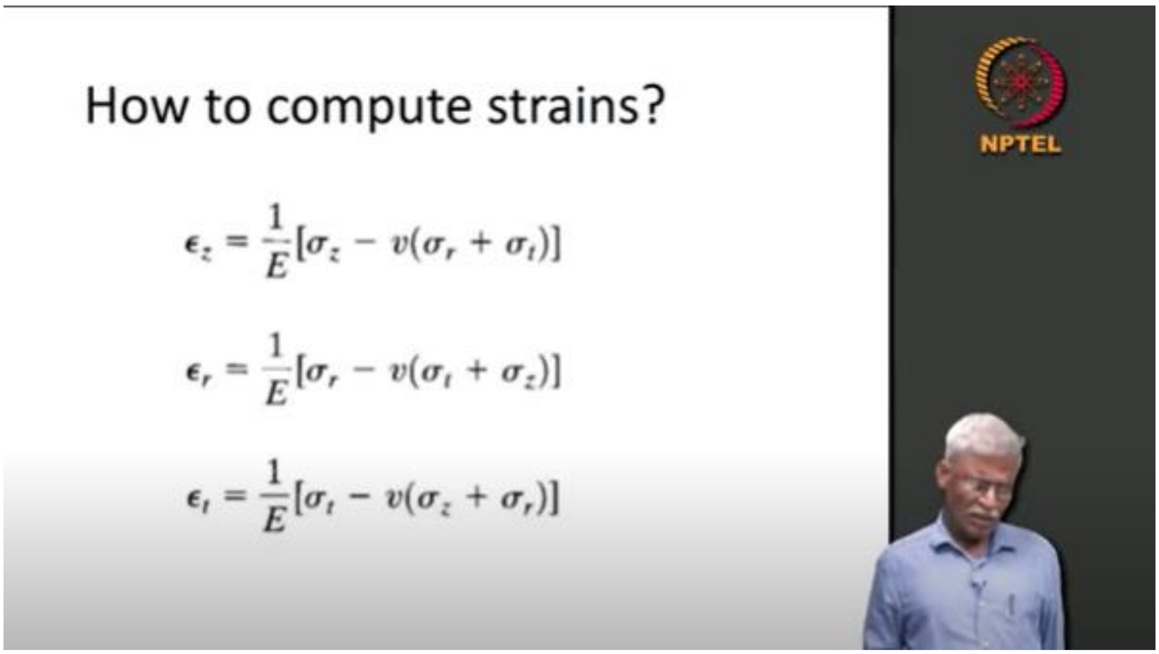

So then how do we really compute the strains? It is a very simple formula that you must have learnt already in your strength of material. Though I have written these equations here for you, I strongly suggest that you should also write it on your own. So this is how you have to compute the strains.

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_r + \sigma_t)\right]$$

$$\varepsilon_r = \frac{1}{E}\left[\sigma_r - \nu(\sigma_z + \sigma_t)\right]$$

$$\varepsilon_t = \frac{1}{E}\left[\sigma_t - \nu(\sigma_r + \sigma_z)\right]$$

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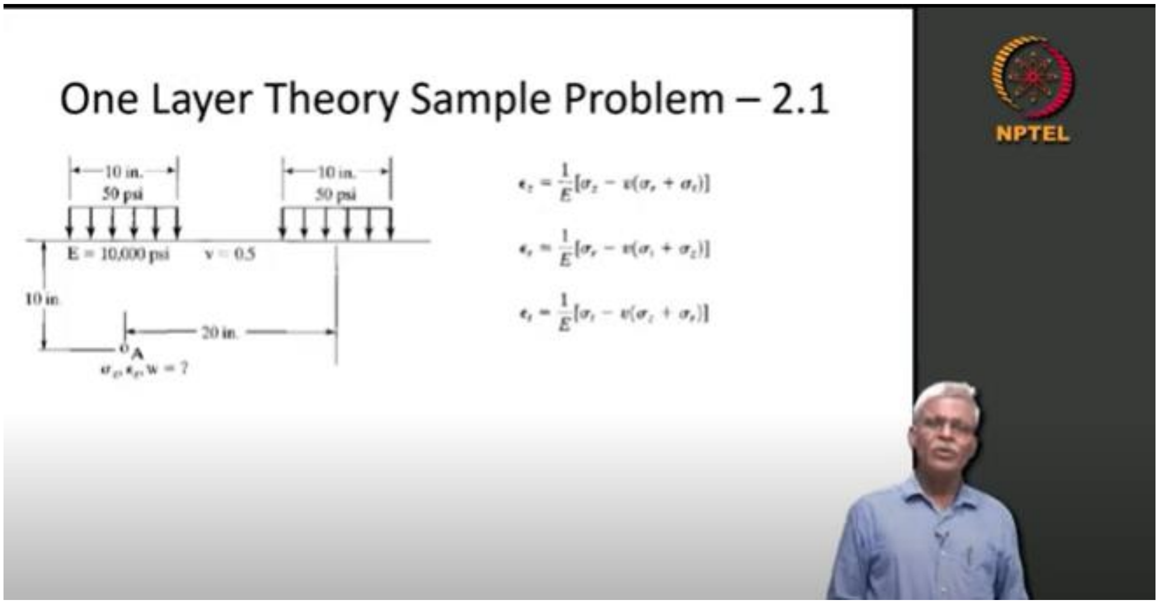

So now we will look at a very simple problem here. So now what is given here if you actually look at it? So since the units are given in you know the pounds/inches^2 and inches, so do not allow this to completely distract you. So there is not really a problem and in fact this is probably one of the finest textbook available for pavement analysis and design which is used to throughout the world. If you actually go and take a look at the textbook for every unit in inches in the bracket you will also see in centimeters for pounds/inches^2, you will also see it in kPa. So when you are working out the problem use those units but when I am explaining it since these Figures are shown here, I am going to use this in this following way.

In fact, the problem is fairly straightforward. It says there are two wheels here 10 inch and 10 inch with contact radius 2a and the load that is applied here is 50 psi. This is the E value given, this is the ν value that is given here. Now you are expected to find out σ_z, ε_z as well as w. This is what you are expected to do. So what I am going to do is going to explain you how to do that, then after that I will write those values, I suggest that you read it from the chart and convince yourself. Sometimes later in the course Professor Padmarekha will introduce you to IITPAVE and KENPAVE and you will see that you can just input these values in the software and straight away get the results. In fact, you can use this as an exercise problem to solve all the things that are given in Chapter 2. I hope it is clear. So let us do this following now.

Now the thing that you should understand is at any given distance a, the stress that you are going to see is going to come from this as well as due to this. So there is going to be an effect of the right hand side wheel at this point. So that means we need to find out the stresses at a due to the left hand side wheel. We need to find out the stresses at a due to the right hand side wheel and left hand side wheel and then we just need to add them together.

So if you take a look at these things, so let us write r/a. So what exactly, so you call this as a, let us call this as b. So for a, r/a is 0, what about for b? It is going to be 20/5. This is the coordinate I am writing. So 2a is 10, a is 5, so this is going to be 20 by 5, z/a here is this is 10 and this is 5, so this is going to be 10/5 and this is also going to be 10/5, because the z position is basically the same. Now what you are going to do, use Figure 2.2, 2.3 and 2.4 and compute σ_z, σ_r and σ_t. Now σ_r = σ_t that will become obvious to you. So and you need to compute it for both a and b. So now let us just go back and try and see maybe, you know let us look at Figure 2.2. So what is that you see here? You see, you know what is r/a, let us say for this location it is going to be 0, for location a and for location b it is going to be 4. So this is r/a and similarly z/a is going to be 2. So all you really need to do is to come to this, find out what is the value for 0 as well as what is the value that you think for 4, read it from this, so that gives you $\sigma_z\ /q$. Then you know what is q, so you just need to multiply it to get your corresponding value. So when you do that σ_z ,when you do that what you are going to get here is, so let me write here for location a as well as for location b; σ_z and σ_r .

Now why did I say σ_r = σ_t ? Please read one or two sentences in this book, you will be able to find out and if you are not able to find out send an email to me, because I would like you to use this lectures as well as also have the textbook in front of you when you are going through this course, so that you will have a connection between what is being taught here and what is discussed in the book. So that is why purposefully I am not saying some of the things. So this is going to be in psi, so I have taken $\sigma_z\ /q$ × 100, I know what is q here, what is q? It is 50 psi, I multiply it and then I get exactly what is needed, so this is going to be 0.38 and similarly this is going to be 0.1 and 1.3, so you add it up you get 14.38 psi and similarly you are going to get 2.10 psi. So what you did it is a simple one-layer problem, now where is the connection? Let us make the connection with real world. You have a single axle dual wheel, so now this single axle dual wheel is sitting on top of a pavement, you idealized the pavement as one layer and then you want to find out what will really happen. So you are now taking the case of this as the reference and trying to add these two things to superpose it.

Now the important thing that you would really want to ask is the following and this will be useful to you later. Now where is the maximum stress that is going to be there? Is it going to be at the location A or is it going to be somewhere at the center? So for this position you can find out what is r/a, what is z/a and then you can compute. My suggestion to all of you is to work out this and find out where exactly it is. You know what you can now find out what is the location, the coordinates here and maybe take one more point here. Maybe you can actually take this tire edge and compute, so you might get a three points here which you connect, you will be able to find out the location of the maximum stress. So I leave it to you to find out how to do that. So now you have actually computed σ_z and σ_r . Now knowing σ_z and σ_r what is expected from you? You are expected to compute ε_z. So you can actually compute ε_z because you know σ_r . σ_r and σ_t are the same. You take Poisson's ratio as 0.5, you have computed already σ_z , you know what is the E and then you can do that. Now you are also expected to find out the deflection. So to find the deflection, maybe I will use a different color, find the deflection, use Figure 2.6 which says w is equal to q times a by E times F. So now what you need to do? Again you need to do the superposition. So when you try to do the superposition of the left hand side wheel as well as the right hand side wheel and compute the value of F, you are going to get something like 0.89. So please do that and once when you do this for 0.89 substitute it here, you might get the correct value as around 0.02 inches. So this is typically how you are expected to solve this particular problem.

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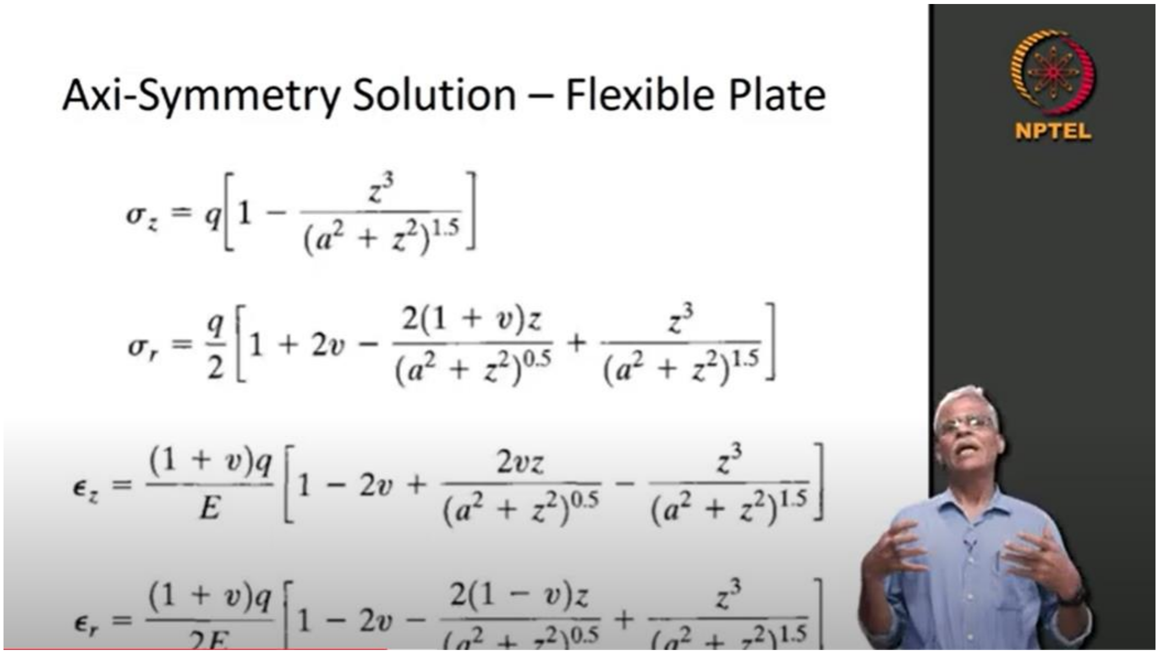

So this is just a demonstration of it. So now let us take slightly go to the next level. Now all this that you have seen here is what is really called as an axisymmetric solution. So that means you assumed your contact area to be a circle and so you did all the work here. Now this is called as a flexible plate solution. Why this is a flexible plate solution? It will become clear to you. So do not need to confuse with the tire, oh is it because it is flexible, no nothing like that. It all depends on the load that is being applied in that particular area whether that load per unit area distribution is it constant or is it varying. That is what it really matters. Now comes the most important thing.

So if you take a look at this particular slide, you know what you see the equations,

$$\sigma_z = q\left[1 - \frac{z^3}{(a^2 + z^2)}\right]$$

$$\sigma_r = \frac{q}{2}\left[1 + 2\nu - \frac{2(1+\nu)z}{(a^2 + z^2)^{0.5}} + \frac{z^3}{(a^2 + z^2)^{1.5}}\right]$$

$$\varepsilon_z = \frac{(1 + \nu)q}{E}\left[1 - 2\nu + \frac{2\nu z}{(a^2 + z^2)^{0.5}} + \frac{z^3}{(a^2 + z^2)^{1.5}}\right]$$

$$\varepsilon_r = \frac{(1+\ \nu)q}{2E}\left[1 - 2\nu + \frac{2\nu z}{(a^2 +\ z^2)^{0.5}} + \frac{z^3}{(a^2 +\ z^2)^{1.5}}\right]$$

Now you must be thinking so when I take this exam should I remember this formula, you do not need to remember the formula. Even at IIT Madras when I teach this, I just give a separately at the end of the examination paper a formula sheet in which some of these formulas are already given. But what I do is the following is I just remove this and I give this formula and the students are expected to look at it and understand okay so this is σ_z, so this is σ_r, this is ε_z and this is ε_r. But when you spend time looking at all these solutions, it will become very clear to you.

Now please take a look at this particular one wherein you see σ_z is given which is given in the following form. Now where is the modulus value, where is the Poisson's ratio? You see that it is independent of E and ν. If it is independent of E and ν, in a sense this is the case of a stress function. Now take a look at σ_r it is dependent on Poisson's ratio but still it is independent of E. So you just need to remember this very carefully. So this is the relation for ε_z and ε_r.

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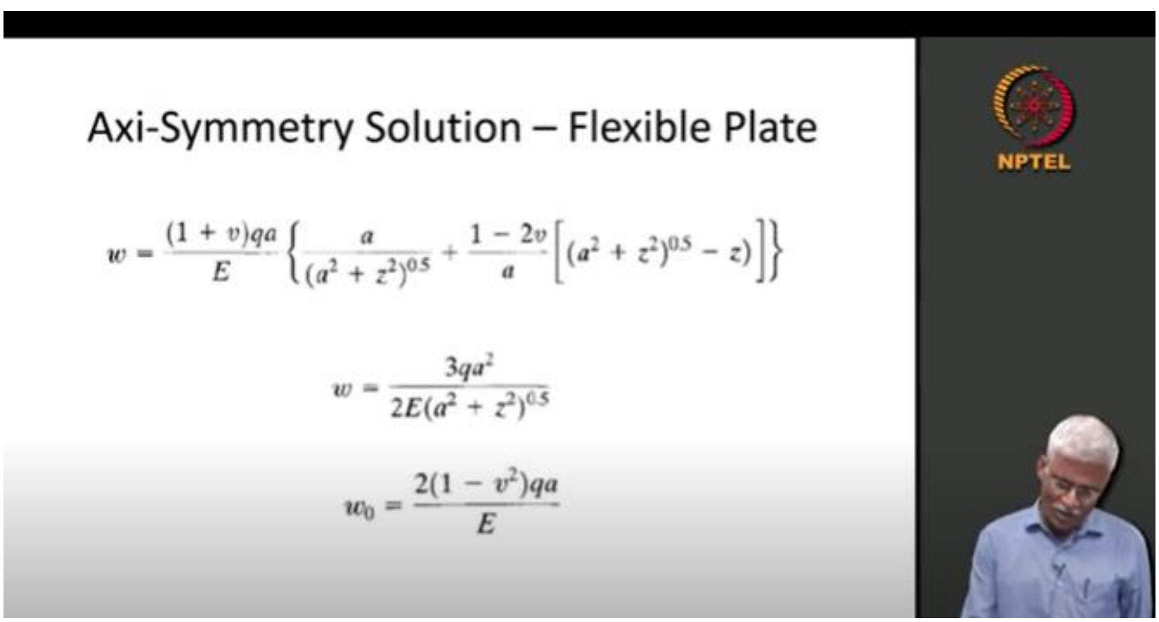

Now let us continue our discussion, so this is what is given here for w here. So in fact what did we discuss in the earlier slide? In the earlier slide we showed σ_z, σ_r, ε_z and ε_r; and now we write it for w.

$$w = \frac{(1+\nu)qa}{E}\left\{\frac{a}{(a^2+z^2)^{0.5}} + \frac{1-2\nu}{a}\left[(a^2+z^2)^{0.5} - z\right]\right\}$$

$$w = \frac{3qa^2}{2E(a^2+z^2)^{0.5}}$$

$$w_0 = \frac{2(1-\nu^2)qa}{E}$$

Now you can actually simplify this. What is the easiest way in which you can actually simplify; if ν is equal to 0.5? So what can happen? This will completely go 1 - 2ν is 0, so this whole term goes. So if you simplify the whole thing, so you are going to get 1.5 that is 3/2 qa^2 divided by E $E(a^2+z^2)^{0.5}$. So this is the simple form that you are going to get. And if you are interested in finding out the solution when z = 0, so that means you are interested to find out where is the, what is the deflection at the surface, then you know wherever you see here. So when you knock them out and simplify, you are basically going to get a fairly straightforward formula. So 1 + ν and you can actually cancel these things out, so you are going to get $2(1-\nu^2)qa$ divided by E, so this is what you are going to get as far as the flexible plate is concerned.

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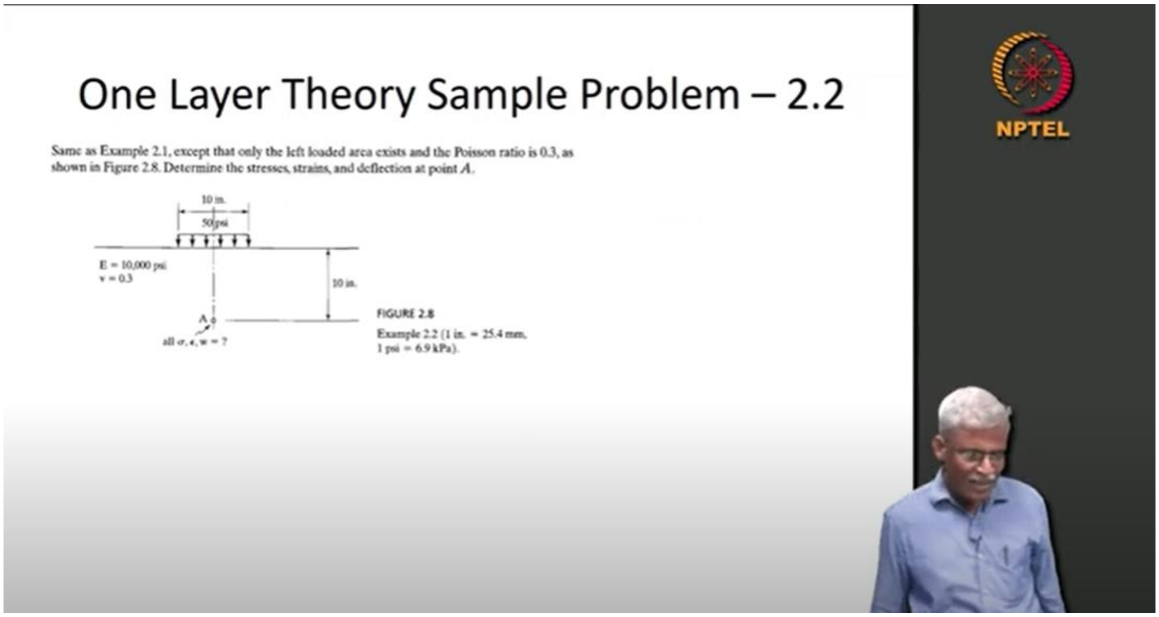

So now let us try to solve one simple problem. So this is the following, it is more or less the same as the example 2.1 except that what it says only the left loaded area exist and now Poisson's ratio is not 0.5 and so what you really need to do is you need to use the equation. Now all these charts that were shown to you was for a case when Poisson's ratio is 0.5, so that is why you are able to put them together in this nice chart form. Since the Poisson's ratio is not equal to 0.5, what you really need to do is to solve the whole thing.

So let us start writing what is known to us, a = 10 inch, q = 50 psi. Now I am going to use the equation numbers, so if you have the textbook in handy you will be able to find out what are these equation numbers and all these equations have been already shown to you and z = 10 inches, ν = 0.3. So use Equation 2.2, if you use Equation 2.2 you will get this and in fact to give you a flavor of what that Equation 2.2 is you can see that this first one. So this is what you need to find out, so you know what is q, you know what is z, you know what is a, you can actually find out what it is. Anyway σ_z is independent of E and ν, so you are not really going to have any problem. So you are going to get something like 14.2 psi. Then let us use Equation 2.3 and you are going to get -0.25. Now what exactly this minus means uses the same sign convention you are talking in terms of tension. So this is the sign convention that is followed here and interestingly what will happen if Poisson's ratio is 0.5

what you are going to do is you are going to get a compression. So just need to understand because when you use Poisson's ratio of 0.5 you do not need to use this lengthy formula you can just read it from the chart and when you use the software you do not need to worry about whether it is 0.3 or 0.5 because you just plug in the value and then write what are all the coordinates in which I need you will be able to get all the solutions.

So this is what you need to really do that. So now let us write Equation 2.4 if you use you will get ε_z and that is going to be 0.00144 and if you use Equation 2.5 you are going to get obviously negative. So what does that mean? If you get a negative value think about it and similarly Equation 2.6 you are going to get w = 0.0176 inch. So this is how you actually solve this particular problem.

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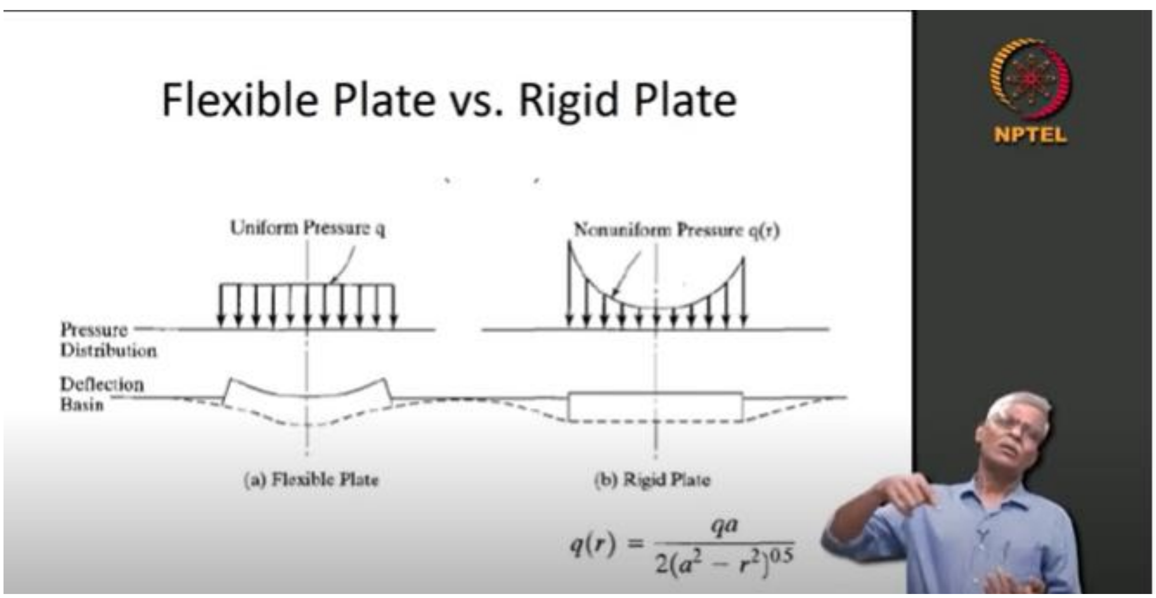

So let us keep moving to the next one. Now this is where I want to talk to you little bit about the difference between the flexible plate and the rigid plate. So now if you really want to think in terms of tire contact what I will assume is that the tire contact is the actual load that is transferred from the tire to the pavement, it can be visualized as a flexible plate which in a sense gives rise to your flexible pavement and all those things. So if you think it is convenient for you to do that you can do that but what it basically means is, there is a uniform pressure that is applied. And since it is a deflection basin is going to be like this.

On the other hand, if you have let us say a rigid plate, now where is this rigid plate coming from? So let us say you go to the site and you would like to find out what is the modulus value of the existing subgrade what will you do? You will do a plate load test. So you will basically put a plate and then try to apply the load and see how much the plate is moving inside. So the plate is not going to deform the whole thing is going to move. So that means this is a rigid plate so you are going to see deformation; it is sinking in here. If this is sinking in here, like this the pressure cannot be uniform. So there is going to be a non-uniform pressure. Normally you need to think of it this way what we will do is take a trench put a plate here and dial gauge and then we just keep applying and measure how much the plate is moving inside as a whole because this is a rigid plate. So you are going to have something as a non-uniform pressure that is going to really happen. And now when there is a non-uniform pressure you can show that I am not going to show that but you can show that the q the variation is going to be something like this.

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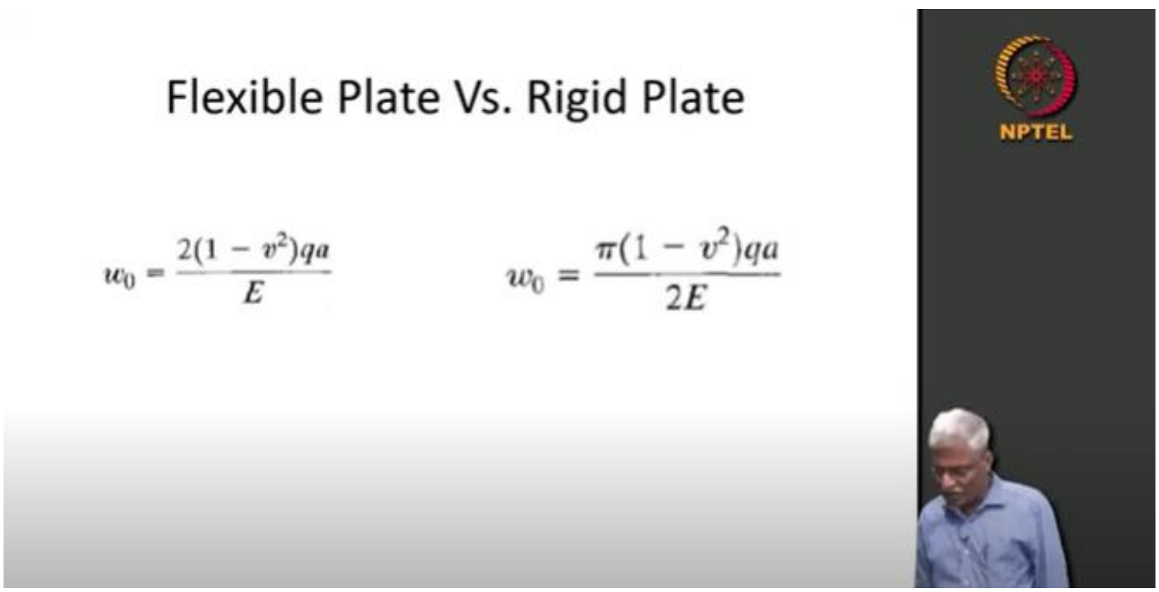

So now we will compare this flexible plate versus rigid plate and this is the final equation that you will get.

$$w_0 = \frac{2(1 - \nu^2)qa}{E}$$

$$w_0 = \frac{\pi(1 - \nu^2)qa}{2E}$$

Now what exactly is going to be the difference here? The surface deflection under the rigid plate is easily you can show that 79% of that under the center of a uniformly distributed load. So there is going to be this difference is going to be there. So let us try and see whether we can solve some simple problem here.

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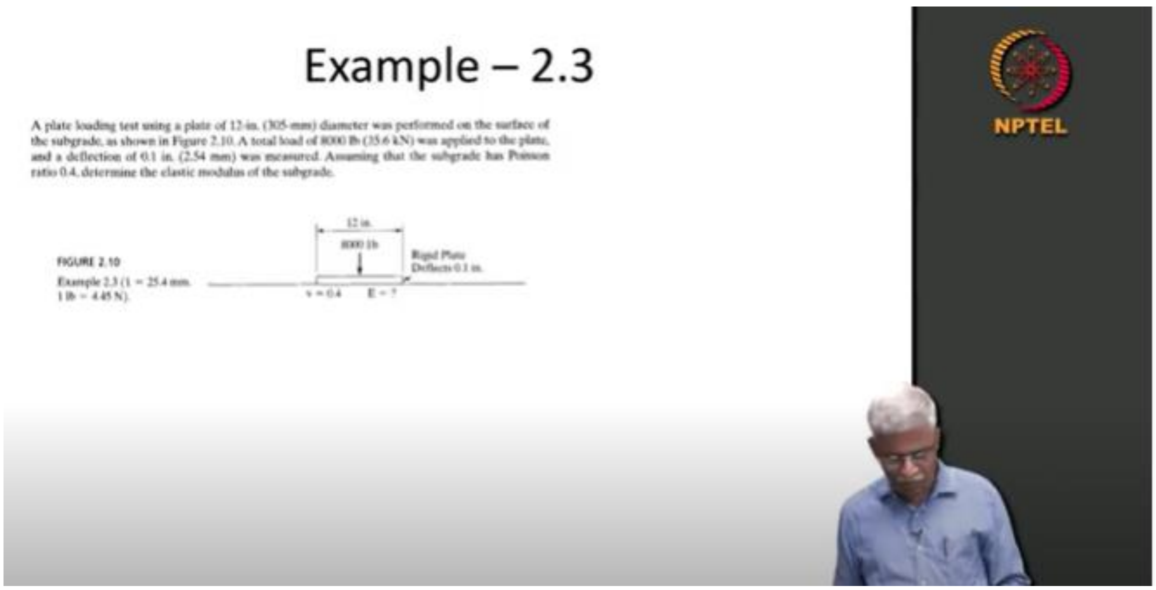

So what is this problem saying? This is Example 2.3, a plate load test using a plate of 12 inch diameter was performed on the surface of the subgrade as shown in Figure 2.10. A total load of 8000lb corresponds to 35 kN was applied to the plate and a deflection of 0.1 inch was measured. Now assuming that the subgrade has a Poisson's ratio of 0.35, determine the elastic modulus of the subgrade. In fact, this is the problem that you really want to find out. What you really want to find out is, what is the modulus of the subgrade and for that you do your plate load test. So the only parameter that you need to assume is what is the Poisson's ratio of the elastic modulus.

So what you can now you need to do is you need to compute q. What is q? q is going to be 8000/36 pi and that is going to be 70.74 psi. Now all you really need to do is use the appropriate equation. Which equation you should use? You should use the equation with the flexible plate or rigid plate because you are using the rigid plate. You know what is the deflection of the rigid plate and now you know w is known to you, q is known to you. So exactly let us go back to this. So q is known to you, a is known to you, ν is known to you, w_0 is known to you. All you really need to do is to use this formula and compute what is really is the E. So when you use the above formula you are going to get this as something as 5600 psi. So use, I am giving the reference to the equation. So get yourself familiar with this system of equations so you will not really have any problem.

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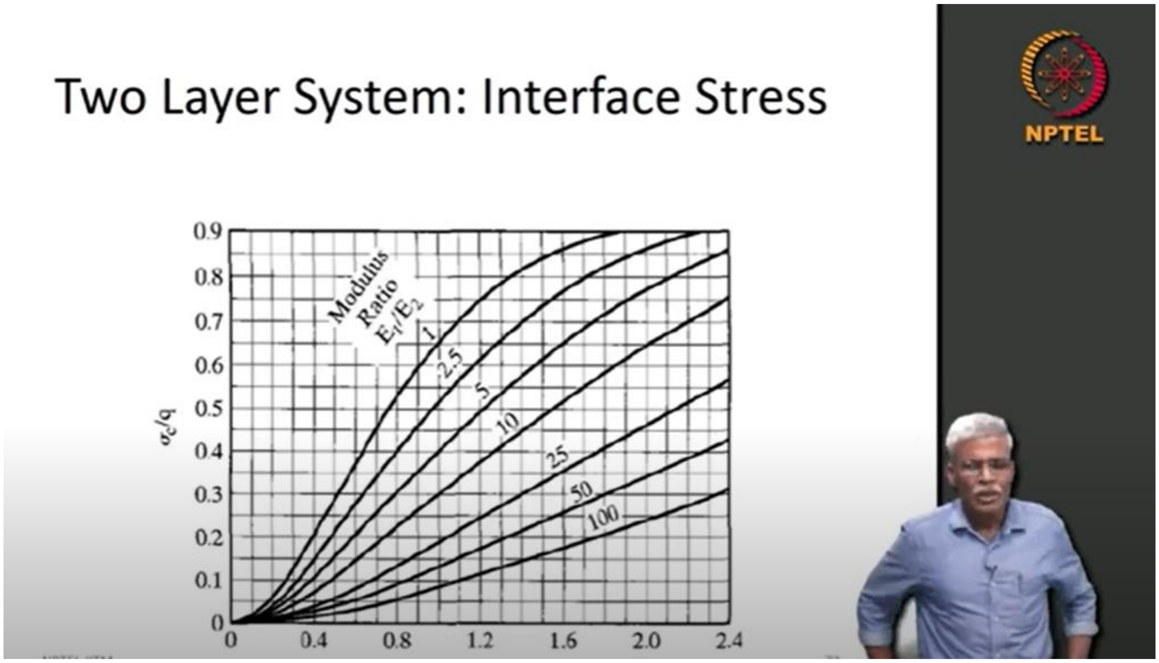

So let us go to the next one which is now what is called as the two-layer system. So we are going to have the interface stress and I think I have already shown similar things here to you. So what is this interface stress? We are talking about the stress here. So what is that you see here? σ_c/q, this is modular ratio E_1/ E_2 and this is a/h_1. So you need E_1, h_1, ν_1, E_2, h_2 and ν_2; this is what you need and we can use some of these charts to determine the interface stress, deflection and what not.

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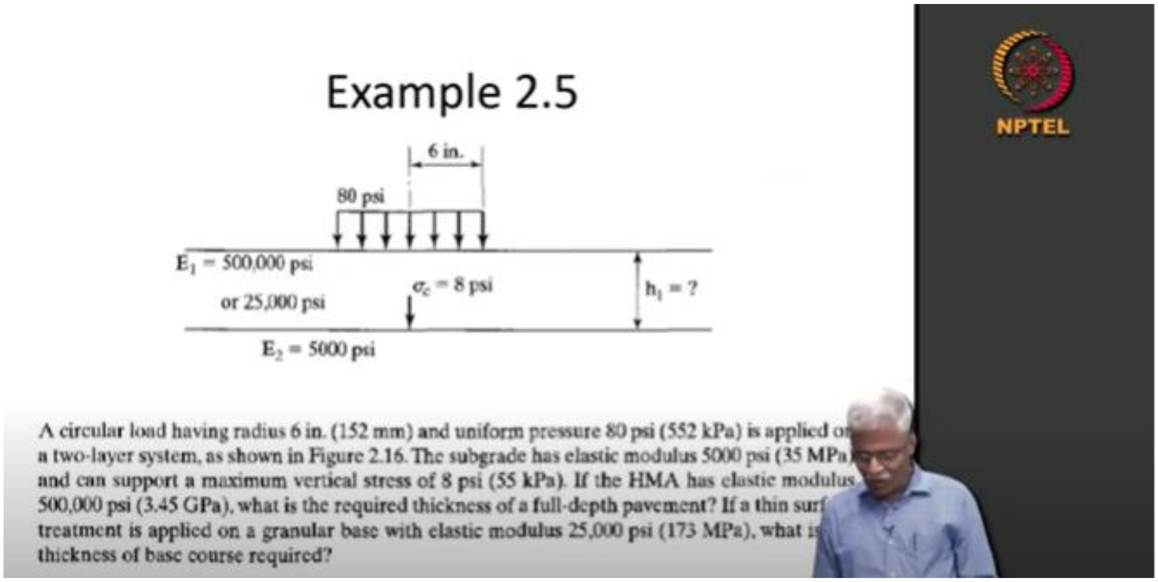

So now let us understand this problem very carefully. So what you see here you should be able to look at this and carefully understand what needs to be done here. So a circular load having a radius of 6 inches and uniform pressure of 80 psi is applied on a two-layer system. The subgrade modulus is given. So E_2 is given and it can support, this is the interfacial stress σ_c.

You remember that is what I indicated. If the HMA has an elastic modulus of 500,000 psi what is the required thickness? If a thin surface treatment with an elastic modulus of 25,000 psi what is the thickness that is needed? In fact, this is the problem that I was motivating you in the last few classes. So that means you fix the interface stress because you know that this interface stress is going to result into some kind of a deflection. Now you have a choice of using a very expensive layer which probably will have 500,000 psi modulus or you can actually have around 25,000 psi. So in which case what you really need to do is you need to ask the question what is the thickness that is needed.

So let us solve this problem. So now what is E_1/ E_2? So let us do it for the first case. The E_1/ E_2 is going to be 500,000/ 5000 and that gives you 100. Now the next thing that you need to write is what is σ_c/q? What is σ_c/q? σ_c is 8, q is 80. So it is going to be 8/ 80 is 0.1. It is a dimensionless ratio. Now what you really need to do from Figure 2.15, so let us go back to Figure 2.15. This is here Figure 2.15. So you know very well what is the E_1/E_2 ratio here. So this is the ratio that you are looking at. You know what is σ_c/q. This is what you are looking at this particular point. You can actually find out what is the a/h_1 ratio. Now roughly this a/h_1 ratio you are going to get it around 1.15. So let us go back to our problem. So you are going to get a/h_1 is going to be 1.15. If a/h_1 is going to be 1.15, what exactly is the value of a? a is 6 inches. So h_1 is going to be 5.2 inches. So this is what you are going to get. Now let us solve for the next case wherein it is E_1/E_2 is equal to 25000/5000 and this is 5. Rest of the things are same. σ_c/q is equal to 0.1 and again from Figure 2.15 a/h_1 is equal to, so let us go to a/h_1 chart. So this is the a/h_1 from the chart. So what is your E_1/E_2 is 5 and σ_c/q is the same it is 0.1. So it is going to be somewhere here. So how much it is going to be? It is going to be 0.4. Now if this is going to be 0.4, your thickness is going to be 15 inches. Now what this simple problem tells you? The simple problem tells you that for any given interface stress if you use a very high modulus material it is enough if you provide some thickness 5 inches. On the other hand, if the modulus value substantially reduces you need to provide 3 times the thickness that is required. So it is going to be 15 inches. So that is what this particular problem tells you.

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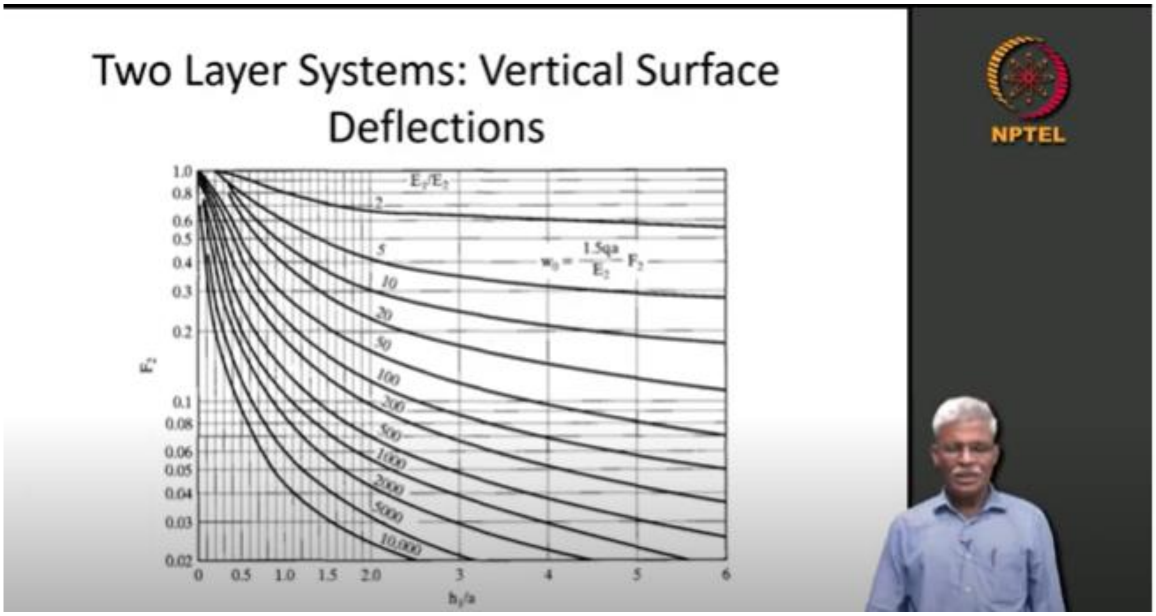

Now let us talk about in the same 2-layer system, particle deflections. So these are surface deflections. So now take a look at these things. The equation for surface deflection is,

$$w_0 = \frac{1.5qa}{E_2} F_2$$

So this is h_1/a, this is what you are going to need and these are all for various E_1, E_2 values. So let us try and see whether we can work out some simple problem.

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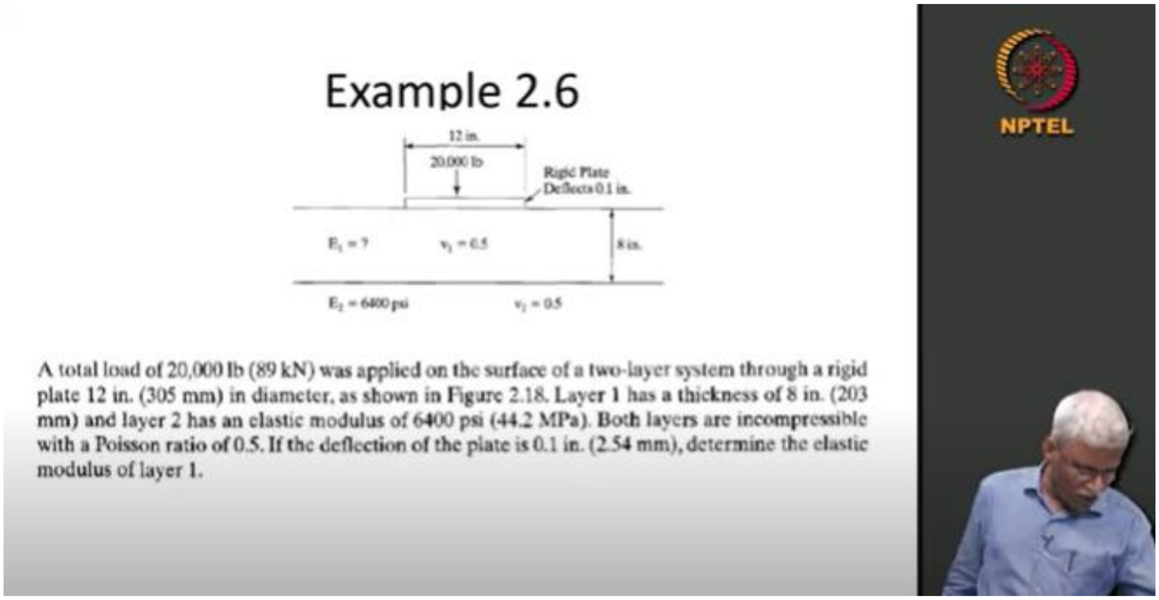

So let us work out this particular problem and what you are going to see here is a total load of 20000lb was applied on the surface of a 2-layer system through a rigid plate. So you need to be careful here 12 inches in diameter as shown. Now layer 1 has a thickness of 8 inches. So they have given the layer 1 thickness, the layer 2 modulus is given here. Both layers are incompressible, so your Poisson's ratio is 0.5, life becomes very simple, you can read it from the chart.

If the deflection of the plate is 0.1 inch, determine the elastic modulus of layer 1. So you need to appropriately use some of these expressions. So let us try to do it here carefully. Now first we will write what is the pressure. Now you need to understand this carefully, I am talking in terms of the average pressure because your rigid plate might actually have a pressure distribution.

So I am going to write it this way, so it is going to be 20000/36 psi and that gives to 176.8 psi. So now w_0 is 1.5qa / E_2 × F_2, this is what we have written. Where is this written? This is written in the chart and this is basically for what is really called as the flexible plate. Now when h_1/a = 0, what can happen? F_2 can actually become 1, because this becomes basically a single layer problem now. So on the other hand we will use this formula which is for the qa. This is the same thing that you are going to get. So if you recollect this particular one. So let us go back, where did we see this formula, this is what it is, $2(1 - \nu^2)qa$ divided by E.

Now we also wrote something here for rigid plate. Now we are going to use this here. Now for rigid plate w0 is going to be $\pi(1 - \nu^2)qa$ divided by 2E and if you write it, solve it, you are going to get 1.18qa / E_2 × F_2. In fact, what it tells you is the following. If you use the chart, this is the formula that you use. If you use the same chart, but when it is for a rigid plate, you are going to get 1.18. Now it is very interesting if you try to work out the ratio between, you will get an interesting number and that number we have already discussed. So now let us solve the problem.

So what you are going to do is, you are going to compute what is really called as F_2 here. So it is going to be 0.1× 6400. So why is 0.1? This is what it is. And 6400 is your E_2 value. So it is going to be 0.1 × 6400/1.18 × q is 176.8 × a is 6. So this is the deflection factor that you are going to get. So now this deflection factor is the same whether you are using rigid plate or a flexible plate. Now that you have found out the deflection factor, what you now need to do, in fact what does it say? It only tells you how to find out E_1. So let us go back to the chart and take a look at it. What is that you see here? You see E_1/ E_2. Now you know F_2, you know h1 by a, you need to find out the appropriate E_1/ $E_2$and since you know E_2 you can find out what is E_1. So given h_1/a = 8/6 which = 1.333, use Figure 2.17 and what you are going to get is E_1/ E_2 = 5 or E_1 = 5 × 6400 which is 32000 psi. So this is what we have done till now. So let me stop here. This is a good place to stop and then we will start talking about vertical interface deflection. Thank you.