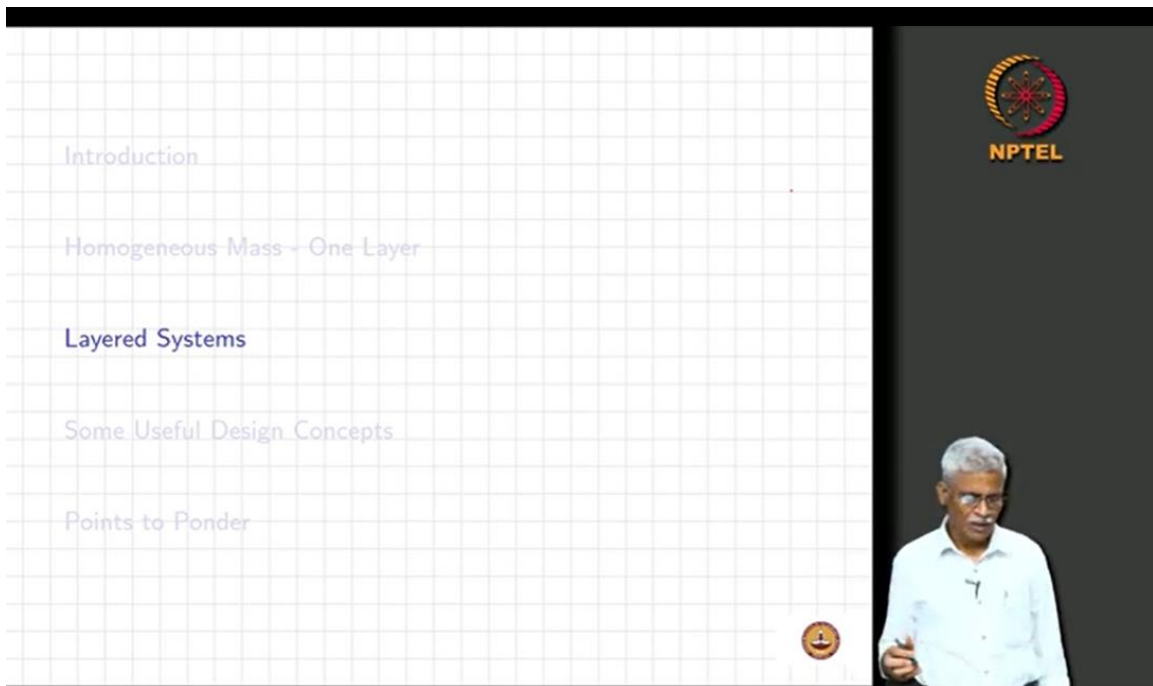


**Analysis and Design of Bituminous Pavements**  
**Prof. J. Murali Krishnan**  
**Department of Civil Engineering**  
**Indian Institute of Technology Madras**  
**Lecture - 05**  
**Stresses and Strains in Bituminous Pavements - II**

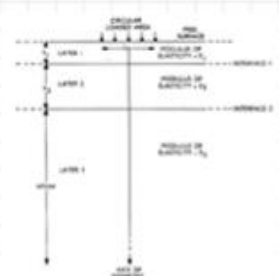
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So, let us continue our discussion on the stress analysis related to the pavement design. So, we will start talking about the actual ones, the layered system. So, the way in which I am going to do it is the following, I am just going to repeat whatever was very clearly articulated by Jones.




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### Pavement Stress Analysis



**Statement of the Problem**

- ▶ A semi-infinite elastic solid bounded at the surface by two finite elastic layers of constant thickness. Find the stresses and strains for a load uniformly applied over a circular area.

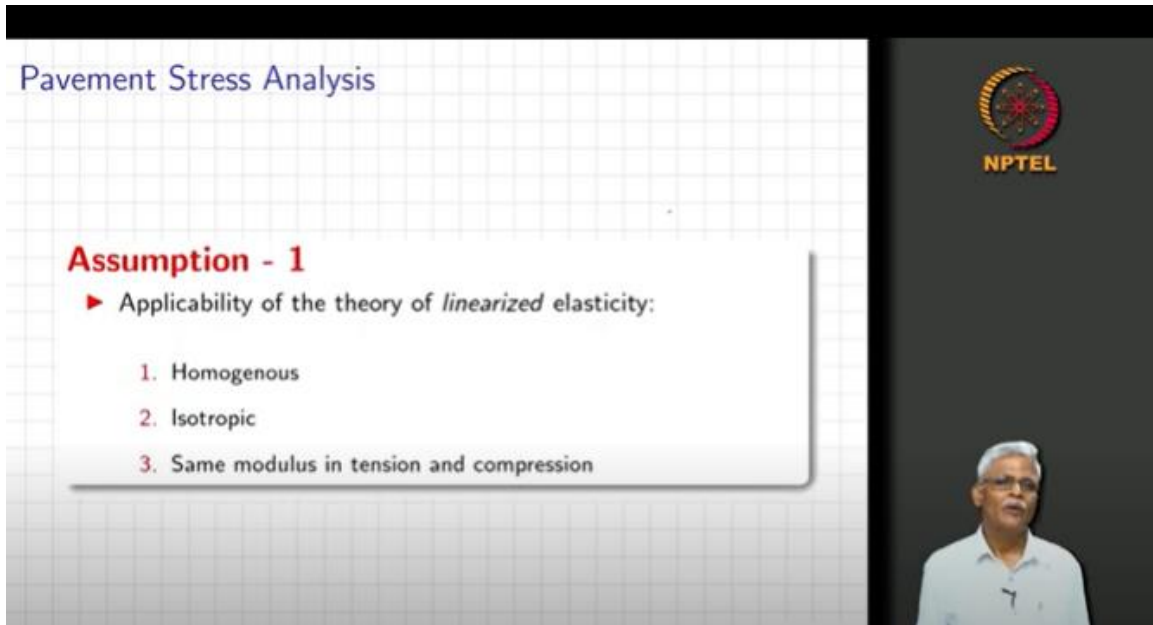


Now, what exactly is the problem that we have in our hand. So, let us assume that we have 3 layers, layer 1, layer 2, and layer 3. And let us assume that the loaded area is circular and this is a free interface. Then the contact is  $2a$ , I have mentioned this before. So this is the terminology that is followed in pavement analysis and design. So this is  $E_1$ ,  $E_2$  and  $E_3$ . So this is interface 1, interface 2 and this you see here is the axis of symmetry. So it is enough if you just find out with respect to one particular radial direction and due to the symmetry, we can find it out at every other point. Now what exactly is the statement of the problem. So if you are watching this video, I suggest you take your notebook and this is what I insist when I teach this course on a face to face, write this statement.

So this is a semi-infinite elastic solid bounded at the surface by two finite elastic layers of constant thickness. Find the stresses and strains for a load uniformly applied over a circular area. So there is it is bound on the surface by two layers here, that is what is mentioned

here and what we really now need to do is to determine the stresses and strains due to a load that is applied uniformly over a circular area.

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The slide is titled "Pavement Stress Analysis" and is set against a grid background. It lists "Assumption - 1" with three sub-points: "1. Homogenous", "2. Isotropic", and "3. Same modulus in tension and compression". The NPTEL logo is visible in the top right corner, and a small video inset of a speaker is in the bottom right corner.

So how do we do it? We need to make some assumptions and these assumptions are very critical and very important and there are many terms that I will not explain and I am going to assume that you all know it from your second year undergraduate level course in mechanics of material or strength of materials. The first and foremost thing that I am going to assume is the theory of linearized elasticity is valid. So that means this entire pavement system can be represented by two parameters  $E$  and  $\nu$ ;  $E$  is Young's modulus, modulus of elasticity, I do not want to call it as Young's modulus, modulus of elasticity and then  $\nu$  is your Poisson's ratio. We also are going to assume that homogeneous isotropic and in addition we are going to assume that there is going to be the same modulus in tension and compression.

Now you all know what exactly is assuming anything to be homogeneous, assuming anything to be isotropic. So I will leave it like that and similarly what exactly is the assumption related to tension and compression that is also I leave it to you because these are all standard assumptions that are made. So this is assumption one.

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## Pavement Stress Analysis

### Assumption - 2

► System Description:

1. Three layers are horizontal and have different elastic properties
2. The top two layers are **weightless** and of infinite extent in the horizontal plane
3. The bottom layer is infinite in the horizontal plane and semi-infinite in the vertical plane



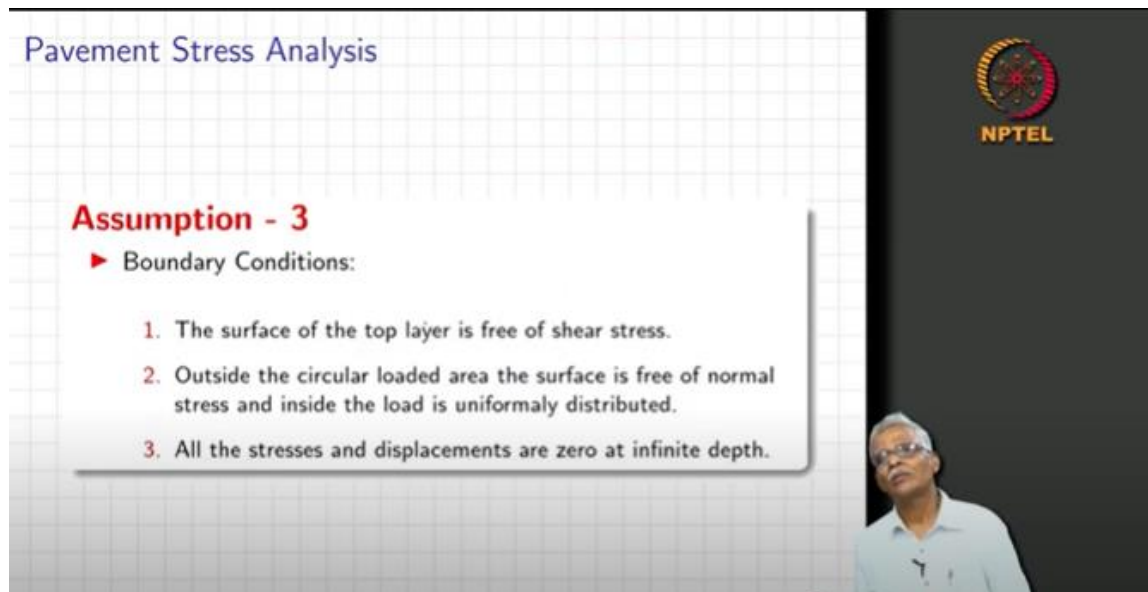
So this is about the model, the material model that you are going to use and in fact I will write it here because we will be encountering this later. What is the constitutive model? In the relation between stress, strain, strain rate, what not that is what it is. Now what is assumption to system description? So what exactly is this system that we are trying to solve? So we are going to make the statement there are three layers. In fact, with KENPAVE and all other softwares that you will be using you can have up to 20 layers but this is the original formulation that was presented in which a lot of tabulated data was given, graphical description was also given. So three layers are horizontal and they have different elastic properties;  $E_1$ ,  $E_2$ ,  $E_3$ ,  $\nu_1$ ,  $\nu_2$  and  $\nu_3$ . And then we are also going to assume that the top two layers are weightless and of infinite extent in the horizontal plane. So when I say weightless what I am trying to do is I am going to assume that the self-weight of these two layers are not going to influence the modulus value of the third layer.

If you actually think about it you will say is that correct? So let us say there is going to be two layers here, layer 1 and layer 2. So let us say you use a 4 cm thick material which is having  $2500 \text{ kg/m}^3$  as the unit weight. So now because of this weight the modulus value that you see here will slightly increase because you know this much of weight is being applied on this material. It becomes it is being kind of confined. So there will be a slight increase in the modulus value, but that we are not going to consider into account at all. And

it is going to be infinite in the horizontal plane and the bottom layer is infinite in the horizontal plane and semi-infinite in the vertical plane. So that is what we are going to assume here.

So and in fact for reasons that will become clear as you go along what we normally will do is we will not actually though this is taken as infinite what we will do for the purposes of computation because otherwise the solutions will keep on going asymptotically. So we need to somehow make it into some point 0. So we will assume that we will provide somewhere a rigid plate and we just take that distance to be something like 50 cm. So that the stresses and strain that you see here can actually come and go to 0 we can force it to 0 at this particular point. So this is not clearly infinite. So you have three layers here these layers are infinite in the horizontal extent and they are all finite. But right now in the way in which I have written it is semi-infinite.

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The slide is titled "Pavement Stress Analysis" and features the NPTEL logo in the top right corner. The main content is "Assumption - 3" which lists three boundary conditions. A small inset image of a man in a light blue shirt is visible in the bottom right corner of the slide area.

**Pavement Stress Analysis**

**Assumption - 3**

▶ Boundary Conditions:


1. The surface of the top layer is free of shear stress.
2. Outside the circular loaded area the surface is free of normal stress and inside the load is uniformly distributed.
3. All the stresses and displacements are zero at infinite depth.

Now the next thing that you really need to do is what is really called as the boundary conditions. What boundary conditions that you really want to write. The surface of the layer is free of shear stress. Now we have discussed this earlier there is going to be a load that is going to be applied in the vertical direction. There is also a possibility of load being applied in the horizontal direction when you are decelerating, accelerating, braking and all

those things. We are not going to consider any of those things that is going to be only one vertical load that is going to be applied.

Now the next thing that we are going to do is outside the circular loaded area the surface is free of normal stress and inside it is going to be uniformly distributed. So that means if this is your surface and this is the extent of your load all the load comes here nothing here 0, 0 only this is where the load is being applied. And all the stresses and the displacement are 0 at infinite depth in our case which I just now explained we restrict them to 0 and sub particular finite depth.

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**Pavement Stress Analysis**


**Assumption - 4**

▶ Continuity Conditions:

1. The three layers are continuously in contact and act together as a composite medium.
2. At the interface, the normal and shear stresses are same in both layers
3. At the interface, the vertical and horizontal displacements are same in both layers.

**Assumption - 5**

▶ The Poisson's ratio is assumed to be 0.5 (incompressible material).



Now comes the most important thing and in fact you can even ask so what really is going to happen between these two layers and if you are familiar with your pavement construction. So what is that you do you have a subgrade then you have a compacted subgrade because your design code will say 10% CBR. So you will kind of make some layer what you call this 100% proctored compacted earth having a required CBR then on top of it what are you going to have you are going to have your subbase. Now what is that you are going to do between these two layers are you going to do anything nothing you are just going to lay it and on top of it you are going to have a base course.

Then when you start here as for layers you will have a small prime coat there and between two bituminous layers there is going to be a tack coat that is going to be applied. Now do this prime coat and tack coat kind of induce some kind of an adhesive nature not necessarily so and in fact the opinion is divided there were many instances in which pavement engineers in US and Europe constructed pavements without a tack coat or without a prime coat and nothing happened. So this is a good feeling and there is also some kind of cleaning the surface ensuring that the surface is clean and there is also what you can say the moisture does not play a critical role and all those things. Other than that the prime coat and tack coat do not have any structural role that is how you have to it they have more like a functional role. So that is what you need to really understand.

Now when you take a look at the continuity conditions there are three layers they are continuously in touch with each other and they act together as a composite medium. So what does that mean? So now what can happen here is when this is subjected to a load all of them depending on the relative stiffness they deflect in more or less as a single composite medium that is the most important thing and at the interface now comes the critical point. So let us take a look at this particular point.

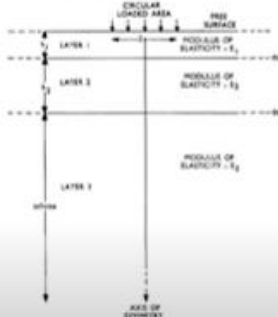
Let us say there is one point here there is another point here. So this point could be considered at the bottom of  $E_1$  means at the bottom of layer 1 having a modulus value of  $E_1$  or at the top of layer 2 having a modulus value of  $E_2$ . Now if you do not enforce these restrictions what can happen there can be slippages that can happen between these two layers. So we are going to assume that at the interface the normal stress and shear stress are the same. So that means I should be able to use  $E_1$  values and compute the normal and shear stress here. I should also be able to use  $E_2$  values and compute the normal and shear stress here and they should really be the same. And similarly at the interface the vertical and the horizontal displacements are the same in both the layers. So this is the continuity condition that you really assume.

Then most important assumption that we make is since we are now looking at giving a closed form solution when I say closed form solutions, solutions in the form of nomographs, charts or something like that and not a computer program. If there is a computer program, I do not really need to work out these things in the form of a closed

form solutions. And because I told you yesterday also in the earlier class there are terms that will come like  $1 - 2\nu$  at many places. So when you the  $\nu$  becomes 0.5 most of this solution simplify they are in a tractable condition and one can use this in a to generate closed form solutions. So we are going to assume that the Poisson's ratio is 0.5 which essentially means incompressible material which essentially means the motions are isochoric. So that means volume preserving. So that is what it means.


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### Pavement Stress Analysis



### Step by step procedure

1. Equations of equilibrium in polar coordinates in 3-D
2. Components of stress and strains in terms of a stress function  $\phi$  satisfying equations of equilibrium
3. Boundary and interface conditions
4. Choice of stress functions (how many? as many as the number of layers)
5. Solve for the stresses and displacements





So what exactly is the step by step procedure and in fact what exactly any step by step procedure for solving any stress analysis problem. So what is that you will first do? You will write strain displacement relationship that is the first thing. Then you will write stress strain relationship which is your constitutive model. Then you will have some kind of compatibility conditions. So what does that compatibility condition means? So you will have 3 displacements  $u$ ,  $v$  and  $w$  that will give rise to 6 strains. Now given any 6 strains I should be able to get back these 3 displacements. So these are what are called as Saint Venant compatibility conditions. Then you will write what are called balance equations.

Now you can have balance equations, balance of mass, balance of linear momentum, balance of angular momentum. We would not get into issues like second law and all but I am just stating it. So these are the balance equations. Then you are going to write the



boundary conditions. So it could be a given attraction what is the displacement or given a displacement what is that state of stress or it could be a mixture displacement conditions. There is Neumann Dirichlet there are many boundary conditions that are available.


So now using this set of equations so strain displacement relationships there are going to be a few equations, stress strain relationship some equations, compatibility conditions, some set of equations, balance equations and using boundary conditions you really need to compute 3 displacements and 4 stresses. That is what we said we are going to appeal to symmetry conditions and 4 strains. So this many this is the set of parameters that we really need to determine from here. So this is the overall stress strain procedure.

Now I am not going to do the derivation of this but I am just going to tell you what is the outline. So first and foremost thing is equations of equilibrium in polar coordinates in 3 dimension we will write it. Then we will write what is really called as a stress function. When I show it to you it will become very clear what exactly I mean as stress function, component of stress and strain in terms of a stress function that satisfies the equilibrium equation which is what I wrote here as balance equations, boundary and interface conditions, choice of stress functions. How many stress functions? How many layers are there? These are what are really called as stress functions.

So one simplest example is the what is really called as the Airy's stress function. So when you go back and most of you who have already taken a course in geotechnical engineering introductory geotechnical engineering, you will be able to see solutions given by related to Boussinesq. And I would like you to go and take a look at the equation related to  $\sigma_{zz}$  and watch on the right hand side whether there are any material properties. You will come back and tell me that no there are no material properties. But you will have material properties coming in in other component but not in  $\sigma_{zz}$ . So that is something that you need to understand. So that is a straightforward case of what is really called as a stress function. In fact, what it means you make an assumption about how the stresses can vary. And if your assumptions are indeed correct because you have to substitute it in the equilibrium equation and solve for it. If the assumption is indeed correct you will be able to meet all the other equilibrium equation.


So it is not really a complication. These are all fairly well developed if you go take a book such as Timoshenko or if you take a look at the some of the classic Russian authors such as Sokolnikov or Mushkilashvili or any of those things. All these equations have been solved derived very cleanly hundred years back. So we do not need to worry too much about it. For a moment since our focus is more on pavement engineering and design and analysis related to that we will restrict our attention to that. And we will be using a software IITPAVE or KENPAVE to compute the stresses and strain. But you need to know that these are all the steps that are followed here. Because unless you assume something like no slip condition unless you assume that I am only going to apply vertical stresses and not worry too much about any traction horizontal traction shear stresses you would not know how you got all these things. So just keep that in mind.

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### Two-layer Systems

- ▶ Flexible pavements are layered systems with better materials on top and cannot be represented by a homogeneous mass.
- ▶ Two layer-system
  - ▶ HMA placed directly on sub-grade (Asphalt Institute's full depth construction).
  - ▶ Asphalt surface course, granular base course, sub-grade - Combine base course and sub-grade into single layer or combine asphalt surface course and base course into single layer
- ▶ Vertical stress on top of sub-grade and vertical surface deflection are two important design criteria.



So what we are going to do is we are just going to now slowly talk about two layer system and in fact what right now I explained is the complete solution for the three layer system. Now what are these two layer system and in fact when Burmister derived this he originally came out with the two layer system.



So two layer systems are what you can say HMA which is placed straight away on the subgrade or it could be see for instance you know you could have your subgrade and then

we just place the HMA layer here. So this is the typical asphalt institute full depth construction I have already shown this to you in earlier slides. So what are the most important thing? The vertical stress on top of the sub grade and the vertical surface deflection are the two important design criteria here. So this is a two layer system I am explaining this for the sake of completion because if you go see pavement analysis and design by Huang or principles of pavement design by the old book classic book Yoder and Witczak they will start with one layer system two layer system and three layer system. Now what I did is I introduced one layer system in terms of how  $\sigma_z$  varies as a function of  $r/a$ ,  $z/a$  and  $\sigma_z/q$  similarly  $\sigma_r/q$ ,  $\sigma_t/q$ ,  $\tau_{rz}/q$  for various  $h/a$  and  $r/a$  we looked into it very carefully. Then I straight away went into three layer system and since I felt that you should also understand what is this two layer system because if you know how to solve for a three layer system what is the big deal about two layer system but still we will take a look at it.

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### Burmister(1958) - Vertical stress distribution

- ▶ Here the thickness of the top layer is equal to the radius of contact area
- ▶ Vertical stress decreases significantly with increase in modular ratio.
- ▶ For the Boussinesq's case, vertical stress is close to 68% of the applied stress.
- ▶ What is the effect of pavement thickness and modular ratio on the stress at interface?

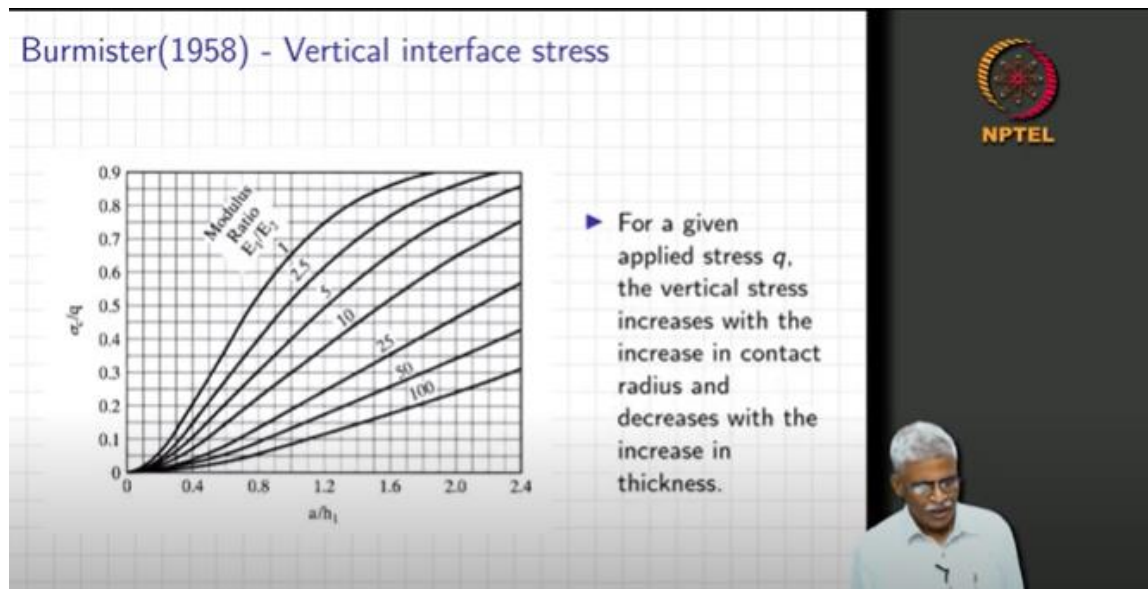



So if you see the vertical stress distribution there are some examples that are given here and interestingly you will see that henceforth especially when you are using some of these charts to compute the stresses and strain we will always be also writing in terms of  $E_1/E_2$ . See when you are using this nomographs you need to understand that everything has to be without any dimension. So you are going to see  $\sigma_z/q$  you are going to see  $z/a$  and you are

going to also see  $E_1/E_2$  the modulus ratios are also being used here. So now when  $E_1/E_2$  is equal to 1 you get your Boussinesq equation.

So the vertical stress that you see here  $\sigma_z/q$  will substantially decrease as the modular ratio keeps increasing and for the Boussinesq case that you see here the vertical stress will be around 68% of the total applied stress. So that is what it is. So there are many combinations that we need to look into it those things and all we will discuss as we go along.

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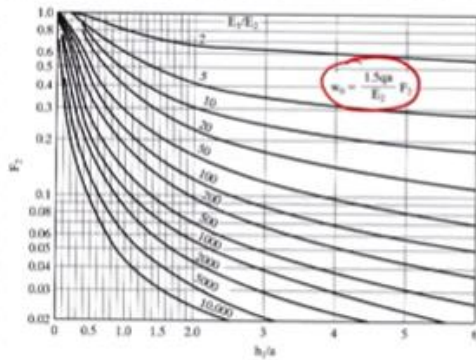


And similarly what is really called as the vertical interface stress. What is this vertical interface stress? So we are talking about  $\sigma_z$  here and  $\sigma_c$  here. So this is what is called as a vertical stress and you have  $a/h_1$  contact radius by  $h_1$  thickness first layer thickness this one and this is and these are all for various ratios of  $E_1/E_2$  and you are going to see that this  $\sigma_c$  for instance if you just take the case of this particular thing. So for if the  $\sigma_c/q$  you want to restrict it to let us say 30% if you use  $E_1/E_2$  as 1 you're  $a/h_1$  that you are going to see here is of this order. So for any given contact radius the thickness becomes sufficiently large, but let us say you use a modulus ratio of 25 you come here and you are going to see that the thickness can be less. So you can increase the modulus value and decrease the thickness of the material and now comes the particle surface deflections.

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### Burmister(1958) - Vertical surface deflections

*FLEXIBLE*



▶ Deflection expressed in terms of deflection factor  $F_2$

▶

$$w_0 = \frac{1.5qa}{E_2} F_2 \quad (1)$$

▶ The deflection factor is a function of  $\frac{E_1}{E_2}$  and  $\frac{h_1}{a}$ .



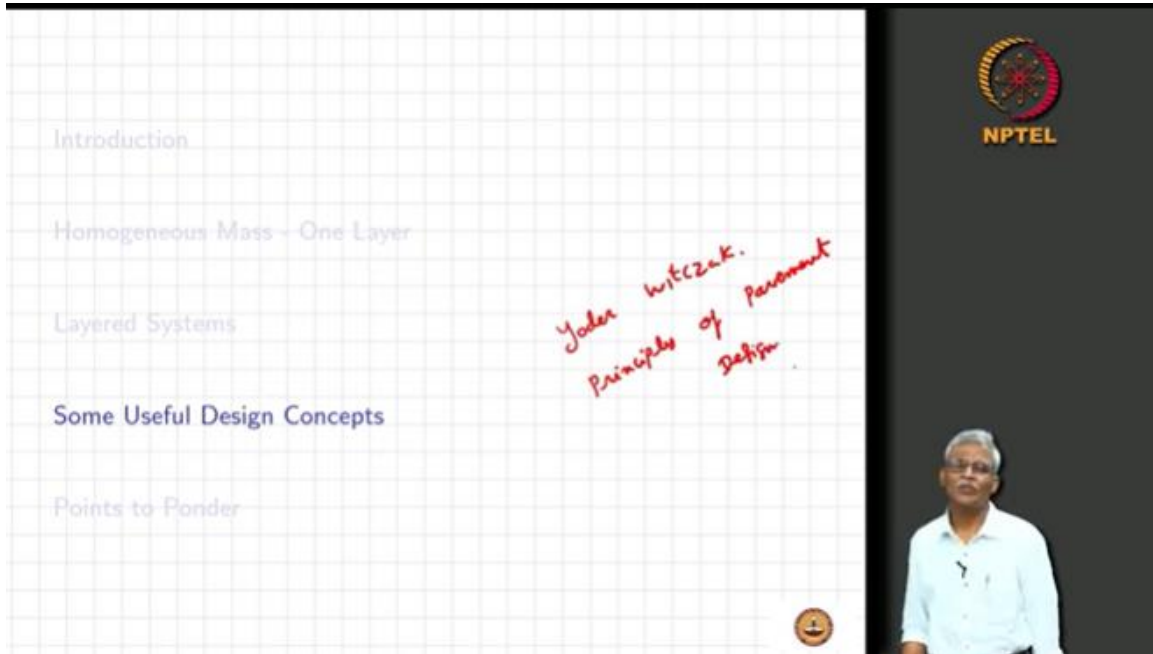
Now you need to see this equation;

$$w_0 = \frac{1.5qa}{E_2} F_2$$

Earlier we used  $qa$  times  $F$  divided by  $E$ , but here we are introducing a 1.5. So this is because of the difference between what is really called as the flexible plate as well as rigid plate. So we will work out some simple problems related to it as we go along. So now this is the deflection so  $w_0$ ;  $u$ ,  $v$ ,  $w$  so  $w_0$  that is at the surface 1.5 times  $qa$  divided by  $E_2$  times  $F_2$ . So what you will only get  $F_2$  this is  $h_1/a$  this is  $E_1/E_2$  this has the ratios that are you are talking about. So let us say you want minimum deflection. So let us say you are talking about minimum deflection of because this deflection is given by this deflection factor here because the rest of the things are constant.  $q$  is the load that is applied,  $a$  is your contact radius  $E_2$  is your second layer modulus. Now you are trying to see how to change it. So you can change the value of  $E_1/E_2$  or you can change the value of  $h_1/a$  since  $a$  is constant you are talking in terms of increasing the thickness of the layer. So if you really want to do it you can keep moving up see this is the most important thing here. So you can have for the same deflection and if your modulus value is ratio is 500 your thickness is going to increase if your modulus ratio is going to be 1000 this is what you are going to get and if

your modular ratio is going to be 2000 this is what you are going to get. So the  $h_1/a$  keeps reducing as you increase your modulus values here.

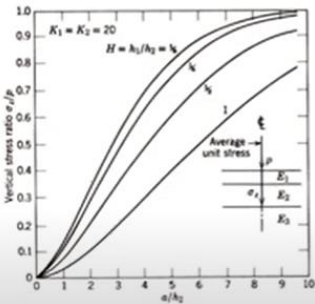
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So let us understand so I am just going to explain few useful design concepts here they are very critical and very important and here I am mostly going to be using some important results that are presented in this book by Yoder Witzack this is a second edition principles of pavement design. If any of you want some portion some chapters of this, please feel free to email me I will be able to share it. So this is a second edition 1975 professor Yoder passed away after that this book was never revised and in between so many other things also happened for instance MEPDG came and most of the concepts that are mentioned in this book suddenly become outdated.

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### Subgrade Stress



- ▶ Influence of changing the pavement thickness upon the vertical compressive stress factor for a three-layer pavement
- ▶ Subgrade stress is greatly decreased by a decrease in the  $\frac{a}{h_2}$  parameter
- ▶ For a given load, this can be accomplished by an increase of the base-course thickness
- ▶ Another efficient method is by increasing the 'rigidity' of the upper pavement layers ( $K_1, K_2$ )



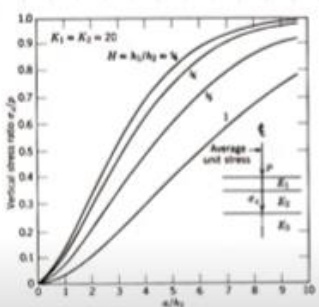
So let us first start talking about subgrade stress. So what exactly is the subgrade stress these are really important design concepts so I want you to pay close attention to some of these concepts. So I have a three layer here layer 1, layer 2 and layer 3 this is the average unit stress given by  $p$ ,  $E_1$ ,  $E_2$  and  $E_3$ . Now what you are seeing here is in the x axis  $a/h_2$  you know what is  $h_2$  this is  $\sigma_z/p$  and in those days especially when these three layer theory exact formulations were introduced the  $E_1/E_2$  and  $E_2/E_3$ ;  $E_1/E_2$  and  $E_2/E_3$  were denoted in terms of  $k_1$  and  $k_2$  etc. So for this particular simulation that is shown here the  $k_1 / k_2$  is taken as 20 and you see here the variation of  $H$ , capital  $H$  this is  $h_1 / h_2$ . So if the thickness of first layer and the second layer are identical  $\sigma_1$  so that means it becomes 1 what exactly is the value.

So what is most important is subgrade stress is greatly decreased by  $a/h_2$  parameter. So that means  $h_2$  is more the denominator is more so this when this number becomes let us say 2 or 1 you are going to see that for various combinations of  $h_1$  and  $h_2$  this is going to be reduced. Now how can you take care of this part you can do it by increasing the base course thickness and one more thing that you can do is right now what you are seeing here is the case corresponding to  $k_1$  and  $k_2$  equal to 20. So that means  $E_1$  by  $E_2$  ratios are taken

and  $E_2/E_3$  ratios are kept constant what you can actually do is to increase the rigidity of the upper pavement layer.



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### Subgrade Stress



The graph shows the vertical stress ratio  $\sigma_z/\sigma_0$  on the y-axis (ranging from 0 to 1.0) versus the depth ratio  $z/h_2$  on the x-axis (ranging from 0 to 10). Three curves are plotted, representing different base course modulus values  $E_2$ . The top curve is for  $E_2 = 20$ , the middle for  $E_2 = 10$ , and the bottom for  $E_2 = 1$ . The curves show that as  $E_2$  increases, the stress ratio decreases more rapidly with depth. A diagram to the right of the graph shows a layered system with a top layer of thickness  $h_1$  and modulus  $E_1$ , a middle layer of thickness  $h_2$  and modulus  $E_2$ , and a bottom layer of modulus  $E_3$ . The average unit stress  $\sigma$  is shown acting on the top surface, and the vertical stress  $\sigma_z$  is shown at depth  $z$ .

- ▶ In a layered system, the base course modulus  $E_2$  has more pronounced effect upon stress reduction
- ▶ In order to reduce the subgrade stress to some tolerable value, increase the base-course thickness or replace the quality of the layered of the material.



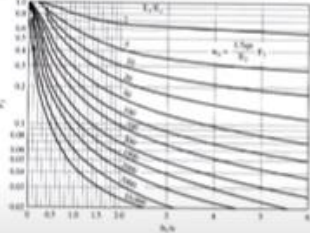
Now when you increase this much higher then what will really happen is you are going to see that the base course modulus  $E_2$  will have more effect on the stress reduction. So now you are debating should I increase the second layer thickness or should I increase the modulus value. So if I have to increase the modulus value you need to use new material slightly expensive material. But if you do not change the material and only increase the thickness there is a cost associated with that. So what you really need to do is to play around with these parameters and find out should I increase the thickness or should I increase the modulus.

So now in order to reduce the subgrade stress to some tolerable value either increase the base course thickness or replace the quality of the layered material. And as I have mentioned now just now increasing the quality of the base course material has more pronounced effect on reducing the stresses compared to increasing the layer thickness.





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### Deflection



- ▶ Most of the deflection (70 to 90%) is caused by the subgrade layer.
- ▶ Same factors that tend to decrease the subgrade vertical compressive strain tend to decrease the pavement deflection.
- ▶ An increase in either the pavement thickness  $h_1$  or stiffness of pavement layer  $E_1$  causes a reduction in the deflection.
- ▶ Greater reduction will occur by increasing the modulus or rigidity of the pavement than by increasing its thickness.



Now comes the next important design concept. So that was the first design concept. What is the next important design concept? What about deflection? So we talked about subgrade stress now what about deflection? Now most of the deflection that you see in the pavement when see so this is this is the most important thing. You know why on a lighter rain after every rain you always see lot of potholes there are problems and there are cartoons and people making fun of highway engineers fairly valid. But there is an interesting and a fairly a complex problem here. Now there are six seven layers here bituminous concrete dense bituminous macadam then let us say wet mix macadam, sand gravel mix compacted subgrade and the existing subgrade. Now when and this is like more like you know you are stacking one on top of each other because we assumed the interface is constant continuously in contact with each other there is no slippage that is happening.

So now when there is a failure that happens let us say in one of the layers let us say third or fourth layer from the top what will happen to the other three layers on the top they all also will fail. What will be the failure that will be seen by you the failure will be seen by you only on the surface. Now do you say oh I think they have not put the required quantity of bitumen. So you go take a core and find out the mixed design mixed design seems to be fine. Then you try to say maybe they did not put the required thickness you take a core again measure the thickness the thickness is correct.

So unless you do a completely take a core or do the cut and see or do some kind of a non-destructive testing you will not be in a position to really say which layer of the pavement has actually failed. So this is the most important point here. But based on our analysis our understanding of the system we know that since the subgrade layer is the weakest layer and you can say that if you protect the subgrade you know if you ensure that most of the deflection that happens is limited you would not have any problem. So the most of the deflection up to 70 to 90% is caused by the subgrade layer. Now the factors that tend to decrease the subgrade vertical compressive strain also tend to decrease the pavement deflection.

Now again take a look at this. So this is the deflection at the surface. So this is  $h_1/a$  this is  $F_2$  and  $E_1/E_2$ . So now if you really want to reduce the deflection you can either increase the thickness  $h_1$  or increase the stiffness. In the same way that I mentioned earlier you can have increased reduction of pavement deflection if you increase the modulus value rather than increasing the thickness.

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### Shear Stress

▶ Notice the Boussinesq

- ▶ Stiff layers exhibit increased shear stress.
- ▶ On any given horizontal plane, the maximum horizontal shear stress  $\tau_{rz}$  occurs directly under the edge of the loaded plate.
- ▶ Please note, the  $\tau_{rz}$  value is zero directly under the plate centerline.
- ▶ Maximum  $\tau_{rz}$  increases greatly in the upper layer as it becomes more rigid.
- ▶ Maximum  $\tau_{rz}$  value occurs about mid-depth in the surface layer.
- ▶ Horizontal shear stress occurring at the first interface is reduced for

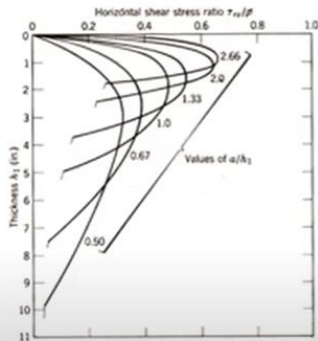
Now when you do all these things then it becomes slightly interesting because the next design concept addresses this issue. When you increase the modulus value of the let us say the saw topmost layer its shear stress will increase.

And in fact the most important thing that you need to understand is the stiff layers will exhibit increased shear stress. So which means now take a look at it. So this is the load per unit area  $E_1$ ,  $E_2$ ,  $E_3$  and in fact these results that you see here corresponds to a layer thickness of 4 inches roughly 10 cm. So this comes to around 11 inches and the contact radius is around 5 inches  $h_1$  is 15 and  $a$  is equal to 5,  $k_2$  is equal to 10. So that is what is really added here. So now what you see here is so this is the center line that you see here.

Now at this particular point  $\tau_{rz}$  is going to be 0 at the center line. And the maximum value of  $\tau_{rz}$  that you are going to see here it will increase greatly in the upper layer as it becomes more and more rigid and in fact you can actually see here when the  $k_1$  value keeps increasing. So  $k_1$  is  $E_1/E_2$  when it becomes higher and higher means that means the top layer becomes more. So you keep the same thickness  $h_1$  and  $h_2$ . So and when you keep increasing it you are going to see that the stresses are going to considerably increase and the maximum value the shear stress that you see here in this case particular case occurs more or less in the middle. That is where you are going to have the shear stress. But the interesting part is at the interface here this is your interface whatever happened maximum it actually becomes the minimum. So in fact you can when you do the simulations with KENPAVE or IITPAVE you will be able to see it and you will also be able to see the Boussinesq solution.

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## Shear Stress



- ▶ What is the influence of the thickness of surface layer on shear stress development?
- ▶ As the thickness of the surface layer  $h_1$  increases, the magnitude of the shear stress is decreased.
- ▶ The maximum value shifts up to app. third point as the thickness increases.



Now if you want to worry about so what exactly is the influence of the layer thickness because you realized that if I have to reduce the subgrade stress I need to play around either with the thickness or the modulus. So now if you increase the modulus value there is a greater reduction. Now the second thing what about the deflection modulus and layer thickness increase the modulus there is a greater reduction and the deflection. But when we come to the shear stress you realize that if you increase the modulus value considerably there is going to be considerable shear stress and the shear stress maximum shear stress is going to be somewhere in the mid depth. So now you are thinking so should I increase the modulus or increase the thickness.


So let us try to see what is the role of the thickness as far as the shear stress is concerned. So when you increase the thickness influence of thickness of the surface layer on shear stress development what will really happen is you can see how drastically the thickness is the shear stress values are changing. So what will really happen as the thickness of the surface layer increases. So what you are going to see is there is going to be a magnitude of the shear stress is going to substantially increase. So what you see here is  $a/h_1$ . So as  $h_1$  keeps increasing for any given  $a$  so this ratio keeps decreasing and you can actually see how much is the horizontal shear stress varying. And the interesting part is the maximum


value now will shift to the one third. In the previous one it was in the middle but here it has shifted slightly up that is what will really happen.

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### Tensile Stress

- ▶ High modular ratio, small  $H$  leads to large buildup of tensile stresses.
- ▶ For  $H < 2$  along with a large  $K_1$  value are prone to large tensile stresses.
- ▶ What will happen for very very small values of  $h_1$ ?



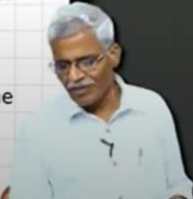



So now we talked about subgrade stress we talked about deflection we talked about shear stress. So now let us talk about the tensile stress. Now there is this also is going to tell you that if you are going to use very high modulus stress value and for a very small thickness what will really happen there will be considerable amount of build up of stresses tensile stresses. So now take a look at this so  $E_1, E_2$  is given  $E_1 h_3$  is infinity. So these are some of the values that are being used here for different  $k_1$  value. So different  $E_1 / E_2$  you see that the  $\sigma_r/p$  is shooting up like anything and for  $h$  thickness less than 2 along with a large  $k_1$  they are going to be prone to large tensile stresses. But if you keep reducing the thickness ultimately what happens is it become compressive in nature but that is a different scenario we would not get into that. So this is something that you need to keep in mind. Now comes the external factors.

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### Effect of Tire Pressure and Total Load

- ▶ Effect of tire pressure is more pronounced in the upper layers.
- ▶ At a depth of 36 inches, the stresses are the same (for same load but different tire pressure)
- ▶ High tire pressure requires high-quality materials in the upper layers but the required total depth does not change.
- ▶ For a constant tire pressure, an increase in total load increases the vertical stress for all depths.



So external factors meaning so these are all pavement cross section related prop matters so that means the thickness and the modulus value. But what about the tire pressure and the load that is being applied here. Now the effect of the tire pressure is more pronounced in the upper layers. So at a let us say a distance of around 36 inches or something they are more or less the same but we need to now focus attention on what is shown here. So there are two different combinations that are given. So 80 kip is nothing but 80000 pounds 200 psi and the same 80000 pounds is now 100 psi. So what do you think is the contact area here you can think about it. So load is the same the tire pressure in 1 is more if the tire pressure in 1 is more the contact area is going to be less. In the other 80000 the tire pressure that you are going to see is less so that means that the contact area is going to be more. So when I am looking at the vertical stress you are going to see that let us say if it is going to be 200 psi you are going to see very clearly. So one is 80000 and 200 psi another is 80000 and 100 psi see how exactly is the variation is happening. So from 200 you are trying to see how much is the stress that is going to happen but they all come and meet at one particular point here.

Now you can look at it in a slightly different way. So let us look at 100 psi and we talk about one is 4000 pound another is 80000 pound. So you can also look at it in that particular



way. So when you try to look at this graph what it means is if you are going to have high tire pressure so these kind of things you are going to have in fact you can actually see high tire pressure so this is what matters. So this is also 200 psi this is also 200 psi high quality material but the required total depth will not necessarily change because most of them will come and reach the same point. So what it means is you need a slightly higher E value there but if you are looking at a constant tire pressure let us say 100 psi or let us say we look at 200 psi the increase in the total load will increase the vertical stress for all the depth. So that is very clear from this graph.

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### Effect of Tire Pressure and Total Load

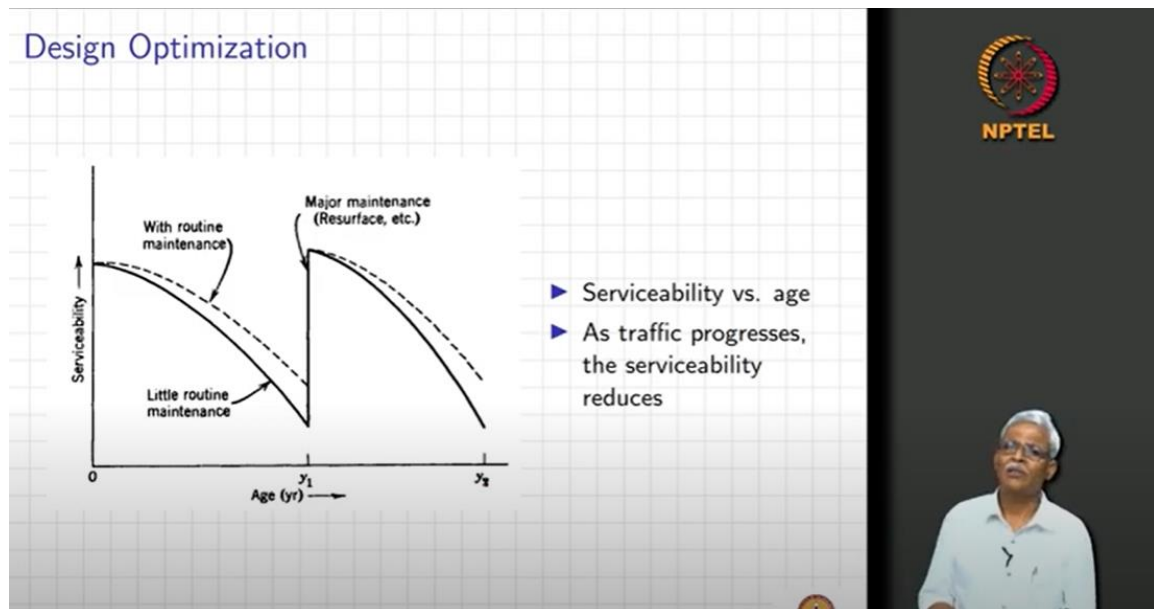
► There is very little influence of the wheel configuration on the calculated stresses.

► Dual wheels, result increased stresses at greater depths as do tandem axles when the pressure bulbs of the tires overlap.



Now whether you are going to do it with single axle single wheel, single axle dual wheel and tandem axial dual wheel. So this kind of things will be tricky to understand because there is going to be some kind of an overlap that will happen and when you do simulations you will be able to understand what is really happening. So dual wheels result increased stresses at greater depths as do tandem axial when the pressure bulb of the tires basically overlap so that will become very clear to you as you go along as of now I am just going to leave it like this. So now what you really understood is so there is an interesting combination of layer thickness and modulus value which plays a critical role here.

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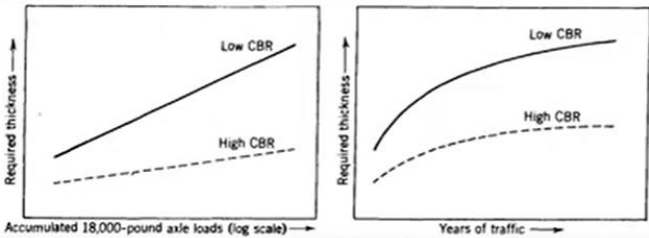


Now we need to do some kind of a design optimization because and this is more or less correspond to the third step that we discussed. So that is the third step that we discussed now what exactly is the third step you have different combinations of layer thickness and modulus value now you want to pick which one you, want normally you will see that we define something called the serviceability as of now think of this serviceability that in a sense of what is the status of the pavement is it a good pavement like you know when I ride on top of it do I feel you know get a good feeling, there is no bumps there, are no ruts the roughness is not there then what you can say there are no visible cracks potholes alligator cracks etc so on and so forth. As the thing is subjected to traffic here the serviceability reduces and somewhere here when you do the maintenance you go back to it so you know you could be a simple chip seal you can do fog seal you can do or you can do an overlay construction you can do and again this keeps repeating. So this is the traffic as the traffic progresses the serviceability reduces and this is what will really happen now.





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### Design Optimization



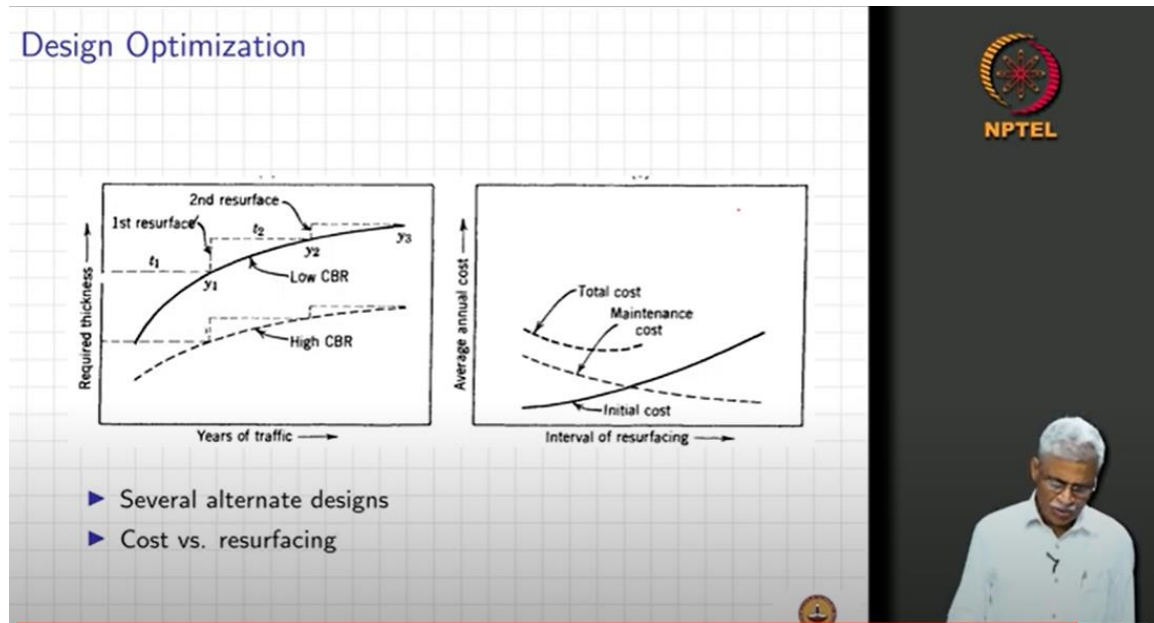
The figure consists of two side-by-side graphs. The left graph plots 'Required thickness' on the y-axis against 'Accumulated 18,000-pound axle loads (log scale)' on the x-axis. It shows two lines: a solid line for 'Low CBR' and a dashed line for 'High CBR'. Both lines show a linear increase in thickness with increasing axle loads, with the 'Low CBR' line being steeper. The right graph plots 'Required thickness' on the y-axis against 'Years of traffic' on the x-axis. It also shows two curves: a solid line for 'Low CBR' and a dashed line for 'High CBR'. Both curves show an initial rapid increase in thickness that levels off as years of traffic increase, with the 'Low CBR' curve being higher and steeper than the 'High CBR' curve.

- ▶ Accumulated equivalent axle load vs. thickness
- ▶ Years of traffic vs. thickness



This all depends on where are you going to construct your road so if it is going to be the thickness is going to depend on me don't worry too much about the CBR you can think of it in terms of the load carrying capacity of the existing subgrade. So if you are going to have a very low CBR the thickness required becomes considerably high and this is the 80 kN standard axle load you will encounter this later and in terms of if you translate it in terms of years of traffic so instead you are going to have something like this the relation is going to be slightly different.

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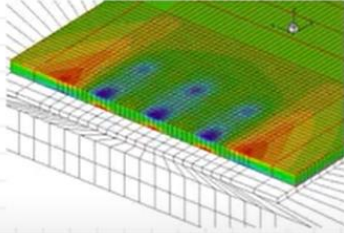


So then the design optimization if you look in terms of it you can do it in different way you can say no I will let me do a stage wise construction so what do you mean by stage wise construction look anyway the pavement is going to fail so why do I do the following why do not I lay only the required thickness and every two years or three years I keep increasing the thickness.



So you can actually have let us say the low CBR value is going here so you can provide the thickness of  $T_1$  at the end of  $y_1$  provide a first over resurface and so on. But you could also provide the both the thickness together in which case you need to provide the maintenance the intervention only at this particular time these are all important things that we need to understand. So if the interval of resurfacing if you are going to you know I do not want to do it every two years or every three years so that means my initial cost of construction is going to increase like this and the maintenance cost is going to decrease like this and the total cost can actually come down because the total cost is going to consist of the cost related to the initial cost as well as the maintenance cost so it might slightly come down and then it can start increasing. So these are some of the combinations that we need to really look into it.

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### Constitutive Models



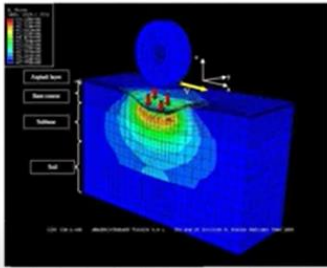
- ▶ Is asphalt concrete linearly elastic?
- ▶ Is the maximum deformation occur instantaneously under the applied load?



So to you need to think few things very carefully here we made lot of assumptions related to the stress analysis procedure we assumed that bituminous material asphalt concrete granular material and all those things are elastic. Now the first thing that you need to understand is they are not elastic we made this assumption because it will give us very interesting nice closed form solutions. So bituminous material asphalt concrete is not elastic it is viscoelastic in nature if it is viscoelastic in nature it will not the deformation is not going to happen immediately under the applied load it is going to happen after a certain time period.

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## Constitutive Models



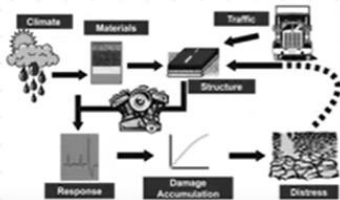
- ▶ Is the granular layer linearly elastic?
- ▶ Is the maximum deformation occur at the point of application of load?



The granular layer is also not going to be elastic it is pressure dependent confinement pressure dependent and the point of application of the maximum deformation is also there is going to be a slight offset here.

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## Dynamic Traffic, dynamic environment



- ▶ What is the influence of vehicle speed on stresses/strains?
- ▶ What is the influence of temperature on stresses/strains?



And what exactly is the influence of vehicle speed on stresses and strains they are going to play a critical role. So if you are driving your truck at 40 kmph the loading time duration we have discussed this earlier is going to be more the deformation is going to be more. And what about the influence of temperature again we have discussed it before. So if the temperature is high the pavement temperature is high or the pavement temperature is very low that is also going to affect the performance of your pavement. So let us stop here and then we will continue in the next session. Thank you.