

Colloids and Surfaces
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Lecture - 36
Models of Electrical Double Layer: Gouy Chapman Theory - II

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Potential distribution near planar surfaces –
GOUY-CHAPMAN THEORY – Solution of PB
 equation without the low potential limitation



Consider the Poisson-Boltzmann equation (1D)

$$\frac{d^2\psi}{dx^2} = -\frac{e}{\epsilon} \sum_i z_i n_{i\infty} \exp\left(-\frac{z_i e\psi}{k_B T}\right)$$

For symmetric z:z type electrolyte: The Gouy-Chapman expression for the variation of potential within the double layer is:

$$\psi = \psi_0 \exp(-\kappa x)$$

$$\psi = \left[\frac{\exp\left(\frac{ze\psi_0}{2k_B T}\right) - 1}{\exp\left(\frac{ze\psi_0}{2k_B T}\right) + 1} \right] \text{ and } \psi_0 = \left[\frac{\exp\left(\frac{ze\psi}{2k_B T}\right) - 1}{\exp\left(\frac{ze\psi}{2k_B T}\right) + 1} \right]$$



So now what we will do is we will try and look at simplifying this general expression to some specific cases.

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Potential distribution near planar surfaces – GOUY-
 CHAPMAN THEORY



Special case (1): Low potentials

$$\psi = \psi_0 \exp(-\kappa x)$$



$$\psi = \psi_0 \exp(-\kappa x)$$



What we know is that $z \gamma$ is γ nought exponent - κx is valid for any potential as long as the electrolyte is z is to z . Now so we know that this expression should can be simplified to $\psi = \psi$ nought exponent - κx .

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Potential distribution near planar surfaces – GOUY-CHAPMAN THEORY

Special case (1): Low potentials

$\gamma = \gamma_0 \exp(-\kappa x)$

$\gamma = \frac{\left[\frac{ze\psi}{2k_B T} + 1 - 1 \right]}{\left[\frac{ze\psi}{2k_B T} + 1 + 1 \right]} = \frac{\left[\frac{ze\psi}{2k_B T} \right]}{\left[\frac{ze\psi}{2k_B T} + 2 \right]} = \frac{\left[\frac{ze\psi}{2k_B T} \right]}{[2]} = \frac{ze\psi}{4k_B T}$

$\frac{ze\psi}{4k_B T} = \frac{ze\psi_0}{4k_B T} \exp(-\kappa x)$

$\psi = \psi_0 \exp(-\kappa x)$

Of course, for the case of low potential, so, let us look at how we do that. So, that is the Gouy Chapman solution. And we know that γ is exponent of $z \psi$ divided by $2 k_B T - 1$ divided by exponent of $z e \psi$ divided by $2 k_B T + 1$, that is what, that is a γ nought. That is a γ That is what was here So that is, what is here Now, if we assume the low potential case, so what I can, what one could do is this exponent term, I can write as a, I can neglect the higher order term.

And I can write exponent of $z e \psi$ divided by $2 k_B T$ as $z e \psi$, divided to $k_B T + 1$, and I am going to neglect all the higher order terms. Similarly, denominator also the same thing I am going to only retain the first 2 terms in the series expansion of exponent of $z e \psi$ divided by $2 k_B T$. So, therefore, γ becomes so I can cancel these 2 in the numerator, I have $z e \psi$ divided by $2 k_B T$ divided by in the denominator I have $z e \psi$ divided by $2 k_B T + 2$ because we are taking low potential case, we had.

So, the low potential case is when $z e \psi$ divided by $k_B T$ is less than 1. And because at $e \psi$ divided by $k_B T$ is less than 1 and that divided by 2 is going to be even further less than 1 therefore, I can neglect this term. So, denominator essentially will be just 2 therefore, γ

becomes $z e \psi$ divided by $4 k_B T$. I can do a similar simplification also for γ nought as well, which again turns out to be $z e \psi$ nought divided by $4 k_B T$ and this this gets cancelled this this gets cancelled and you end up with the expression for the low potential limit.

Therefore, the general Gouy Chapman solution you know simplifies it is to the Debye Huckel approximation for the case of low potential.

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Potential distribution near planar surfaces – GOUY-CHAPMAN THEORY

Special case (2): For large value of x at which the potential has fallen to a small value regardless of its initial potential:

$\gamma = \gamma_0 \exp(-\kappa x)$

$\psi = \psi_0 \exp(-\kappa x)$

$\gamma = \frac{\left[\frac{ze\psi}{2k_B T} + 1 \right] - \left[\frac{ze\psi}{2k_B T} + 1 \right]}{\left[\frac{ze\psi}{2k_B T} + 1 \right] + \left[\frac{ze\psi}{2k_B T} + 1 \right]} \Rightarrow \gamma = \frac{\left[\frac{ze\psi}{2k_B T} \right]}{\left[\frac{ze\psi}{2k_B T} + 2 \right]} = \frac{\left[\frac{ze\psi}{2k_B T} \right]}{[2]} = \frac{ze\psi}{4k_B T}$

$\frac{ze\psi}{4k_B T} = \gamma_0 \exp(-\kappa x)$

The slide also features a diagram showing a potential ψ decaying exponentially with distance x from a surface. The NPTEL logo is visible in the top right corner.

So, the other simplification one can do is that is a special case 2 this is for large value of x at which the potential has fallen to small values regardless of its initial potential. Now, we know that potential varies with distance if you go away from if this is ψ nought is initial potential and as we go away from the charge surface, we know that the potential is going to decrease. Now, if we take some distance, that is x such that it is very large that the potential has fallen to a very low value regardless of this initial potential.

So, in such a case, what I can do is I can simplify γ as $z e \psi$ divided by $4 k_B T$ because again I can invoke the condition that at a very large separation distance the potential is low I can again only consider the first 2 terms neglect the higher order terms. So, therefore, γ becomes $z e \psi$ divided by $4 k_B T$. However, because ψ could be larger ψ could be any potential. Therefore, I will have to continue to retain γ nought as it is because I cannot invoke the low potential case like we did for the earlier case.

Because now, gamma could be gamma nought could be significant or gamma nought psi being could be any value. Therefore, I am going to retain gamma as gamma nought therefore, for the case where for a very large value of x such that the potential has fallen to a very small value regardless of the initial potential, I can simplify the Gouy Chapman theory to be to take the form z e psi divided by 4 k_B T that is on the left hand side the right hand side remains the same.

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Potential distribution near planar surfaces – GOUY-CHAPMAN THEORY

Special case (3): For large value of x and large value of initial potential

$$\frac{ze\psi}{4k_B T} = \gamma_0 \exp(-\kappa x)$$

$$\frac{ze\psi}{4k_B T} = \exp(-\kappa x)$$

$$\psi = \frac{4k_B T}{ze} \exp(-\kappa x)$$

TABLE 11.4 Variation of the Parameter γ_0 with ψ_0 at 25°C

ψ_0 (mV)	γ_0
260	0.9874
240	0.9814
220	0.9727
200	0.9600
180	0.9415
160	0.9149
140	0.8765
120	0.8230
100	0.7500
80	0.6528
60	0.5249
40	0.3711
20	0.1968



If you take the third special case again this is for a very large value of x such that the potential has fallen to a low value. Therefore, the simplification that we have done in terms of writing gamma nought as z e psi divided by 4 k_B T will still hold and on the right hand side you had gamma nought exponent - kappa x. What do you see here is a table where gamma nought is tabulated as a function of psi nought that is psi nought in millivolts.

This is gamma nought if you look up this table it turns out that if you look at very large values of psi nought gamma nought essentially is very close to 1 therefore if the surface potential if the initial potential is very large gamma nought will you can it is approximately equal to 1 that is from this table therefore, z e psi divided by 4 k_B T becomes exponent of - kappa x itself because gamma nought = 1. Therefore, from this I can write psi as 4 k_B T divided by z e into exponent of - kappa x. So, these are the 3 simple you know special cases one could derive from starting with the Gouy Chapman theory.

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Comparison of different solutions:

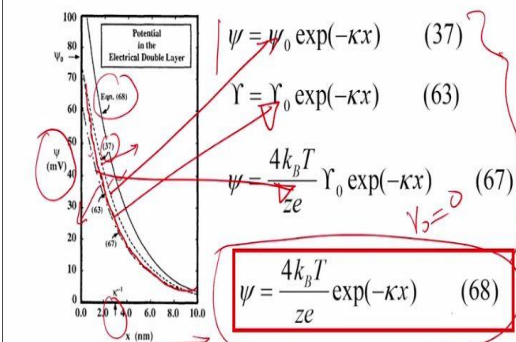
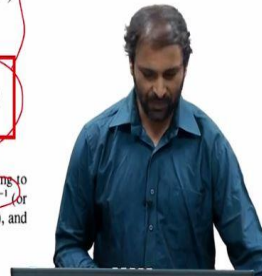


FIG. 11.6 Variation of the double-layer potential versus distance from the surface according to four expressions from this chapter. The figures are for $\psi_0 = 77.1$ mV and $\kappa = 3.29 \cdot 10^8 \text{ m}^{-1}$ for 0.01 M solution of 1 : 1 electrolyte. Curves are drawn according to Equations (37), (63), (67), and (68).



What is plotted here is a potential in millivolts and that is the separation distance and if you look at this plot, there are 4 expressions there are 4 lines a continuous line, a dashed line and dashed and dotted line, there are essentially 4 there are essentially different solutions that is the Debye Huckel approximation. That is the Gouy Chapman theory and this is for the case where the potential has fallen to a very low value regardless of the initial potential and this is the case where the potential is fallen to a low value regardless of the initial potential.

And the initial potential being very high so, if you look at the solution that is plotted for specific case, ψ_0 is 77.1 millivolts. This is a significant potential, this is not the low potential because we said that the potential has to be of the order of 25 millivolts for one to consider the potential to be the low potential case. So therefore, $\psi_0 = 77.1$ is a not a low potential case. And κ is given by this. So κ^{-1} is marked here. That is a κ^{-1} corresponding to this κ value.

And this is the κ value that you have is for 0.1 molar solution of electrolyte and so, the best expression that one could use to predict the variation of potential with separation distance in this particular case is going to be this expression because this is the Gouy Chapman theory, which is valid for the symmetric electrolyte 1 is to 1 in this case and we know that it is applicable for you know any potential because we are not invoking the low potential limit case.

Therefore, this particular line that is this line that I am tracing now, that is the solution that corresponds to the Gouy Chapman theory, but however, if you look at the other 2 lines, that is this line, as well as this line, these are one is for the low potential limit and the other one is for the case, where the potential is fallen to a low value regardless of what is the initial potential, we know that these 2 also do a reasonably good job that means, both that means though the potential is best described by the Gouy Chapman theory that is this expression.

This as well as this still do a very good job of predicting what is the variation of potential with the separation distance. However, because of the fact that the potential that we are considered is 77.1 millivolts, that means, it is not significantly large to take $\gamma = 1$ as it is evident 63 the expression is equation 68 that is the case corresponding to the special case for large value of x and large value of initial potential, it is does not do a good job of predicting how the potential varies with separation distance. Therefore, depending upon the situation, you should be able to use one of these expressions to find out how the potential varies with separation distance.

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Variation of concentration of co-ions and counter ions in the vicinity of the charged surface

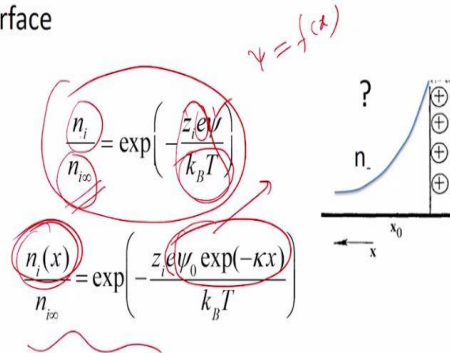
The slide contains two diagrams illustrating ion concentration near a charged surface. The left diagram shows a potential profile $\psi(x)$ for a negatively charged surface (indicated by '-' signs) with counter ions (+) and co-ions (-). The right diagram shows a potential profile for a positively charged surface (indicated by '+' signs) with counter ions (-) and co-ions (+), with a question mark above it. A presenter is visible in the bottom right corner.

Now, that we know how the potential varies with separation distance, we can also look at the variation of the concentration of co-ions and the counter ions as well you know, in the vicinity of the charge surface. So, we know how the potential varies with separation distance, but however, if you want to know how does the concentration of in this case you have positively charged surface therefore, the negatively charged ions are going to be the counter ions. Of course, I could

also have positively charged ions as well in the solution, which are going to be co-ions and how does their concentration vary with separation distances is also of interest?

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Variation of concentration of co-ions and counter ions in the vicinity of the charged surface



So, for that what we are going to do is to again start with the Boltzmann factor. So, we know that n_i divided by $n_{i\infty}$ is exponent of minus $z_i e \psi$ divided by $k_B T$ and this n_i is not a constant and $n_{i\infty}$ is fixed by the concentration of the electrolyte in the solution that is a fixed quantity for a given concentration and this is a constant this as well and z_i is the valence of the ions. So, once that is fixed, so, everything is a constant here except for ψ and n_i varies with separation distance, we know that ψ also varies with separation distance.

Now, because we have derived how does ψ you know depend on various with separation distance? We can substitute relevant expression for ψ . So, if I assume the low potential case, I can replace ψ to be $\psi_0 \exp(-\kappa x)$. Therefore, now, that I know how ψ varies with separation distance I should be able to also calculate what is how does the concentration of the ion varies with separation distance.

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Taken from: Electrokinetic and Colloid Transport Phenomena by Jacob Masliyah and Subir Bhattacharjee

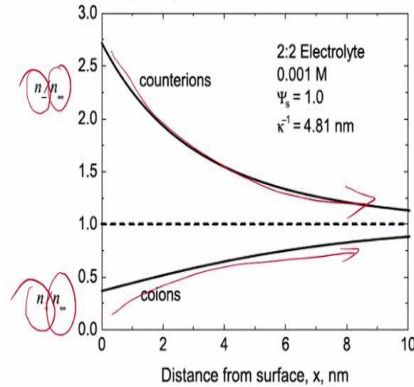


Figure 5.10. Coion and counterion distribution near a charged planar surface.



So, therefore, from this approach, I can actually calculate what is the concentration of the counter ions? And what is the concentration of the co-ions? In this case both of them are scaled with n_{∞} which is the concentration of the electrolyte that I have in the solution. How does the concentration of the counter ion decreases as you go away from the separation distance and how does the concentration of the co-ions again varies with separation distance one could calculate from this approach.

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Taken from: Electrokinetic and Colloid Transport Phenomena by Jacob Masliyah and Subir Bhattacharjee

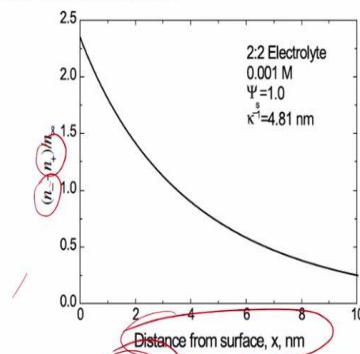


Figure 5.11. Distribution of excess ion concentration normalized with respect to the bulk ion concentration near a planar surface.



What would also be interest would be what is called as the distribution of excess ion concentration that will be given by $n_{-} - n_{+}$ that is the difference in the concentration of the co-ions and the counter ions and how does that vary with separation distance one we should be able to calculate all of this from this simple formulations.