Mechanical Characterization of Bituminous Materials
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Lecture –8
Small Amplitude Oscillatory Shear – Part 2

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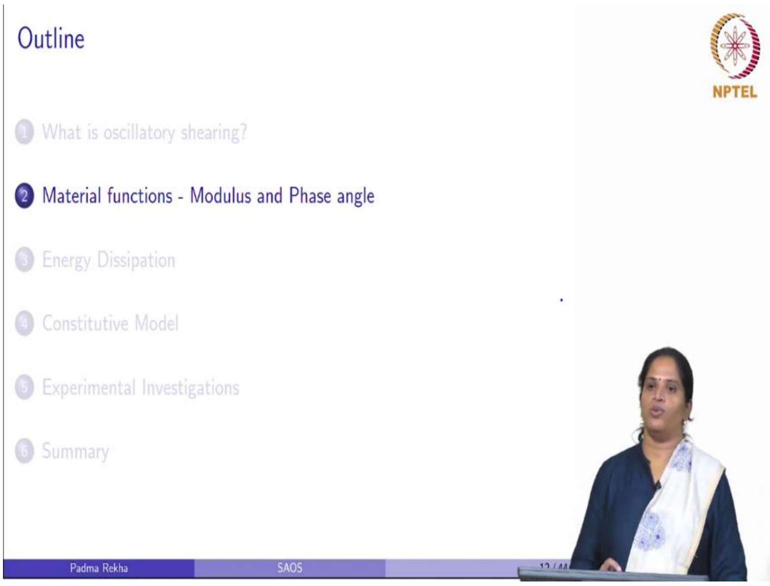

So, if it is an elastic we generally use a simple modulus function and if it is a time-dependent functions you have also been introduced to a relaxation modulus or a creep compliance functions.Now in then oscillatory shearing we use a model is called as an dynamic modulus maybe you are new to this term that you will understand what is dynamic modulus and phase angle you already know what is δ in case of a strain control test.

And creep compliance function and modulus in case of a stress control test.We will see how to determine these modulus functions when we are subjecting the material to an oscillatory shearing.

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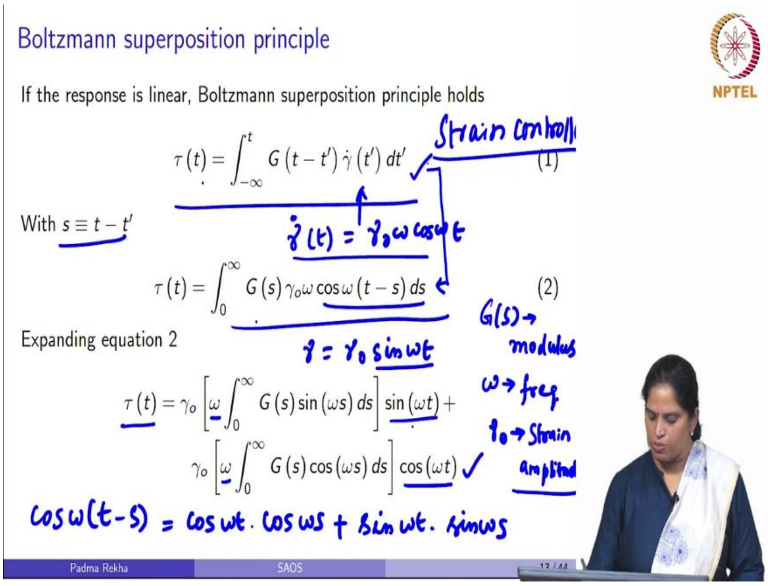

So, we have assumed the response of a material to be linear so to define a material functions we will use a Boltzmann superposition principle so as a material exhibits a linear response Boltzmann superposition principle for stress response let us use a strain control test.Here so for a strain control test when we apply a sinusoidal loading for a Boltzmann superposition integral form of a Boltzmann superposition has given in this equation.

We will get the response stress to be this so this is the same integral form if you put s as $t - t'$ you will get this expression.So, when you look into this expression G(s) is some material function really equivalent to a modulus and **ω** is a frequency as defined earlier andγ_0 is a strain amplitude, you get this function bysubstituting $\dot{\gamma}(t) = \gamma_0\,\omega\cos\omega t$, so when you substitute this in the Boltzmann superposition the this equation simplifies to this.

Now you can expand this equation for cause may be $\omega t - \omega s$ to be equal to let me write it here $\cos\omega(t-s) = \cos\omega t\cos\omega s + \sin\omega t\sin\omega s$, so when you expand this term cos term the equation simplifies to this. Now when you closely look into this expressions the responsestress function has a two component one is a sine component and other is a cos component.

So, the coefficient of sine component again it is a function of frequency and the coefficient of cos component is in it is a function of frequency.So, our input strain function is sinusoidal only it has a sine function which is gamma is equal to$\gamma_0 \sin\omega t$,so we control the strain in the sine pattern in the sine wave form a stress wave form has bothstrain a sine as well as a cos wave form. So, one is in phase component with an input another is an outer phase component with an input.

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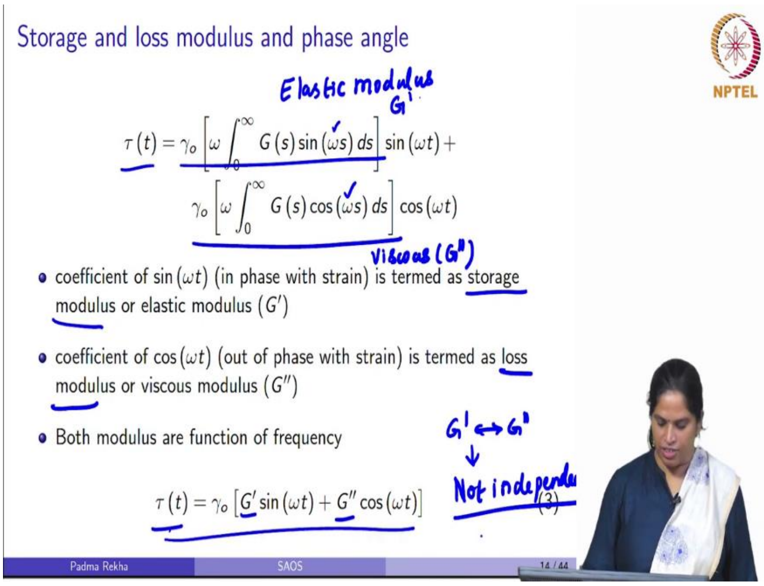

So, this in phase component with an input or the coefficient of term sinωtwe call it as a elastic modulus it is a elastic response so our otherwise storage modulus and it is generally represented by G′.And the coefficient of cos ωtwhich is an outer phase component with a strain we call it as a loss modulus or viscousmodulus viscous modulus or loss modulus and we represent using a term G″.

So, you have 2 functions here one is an in-phase component storage modulus and other out of phase component loss modulus.So, if you want to find as stress you need 2 function storage modulus component and the loss modulus component.You can also see that these modulus functions are a frequency dependent.So, then of the next question is are they storage modulus and loss modulus independent functions or it is relatedto each other.

These functions are not an independent functions they are related to each other they are not an independent function they are related each other.So, you need these 2 functions to define a stress waveform, so stress waveform in simple can be returned as the equation given here in terms of storage modulus and loss modulus.

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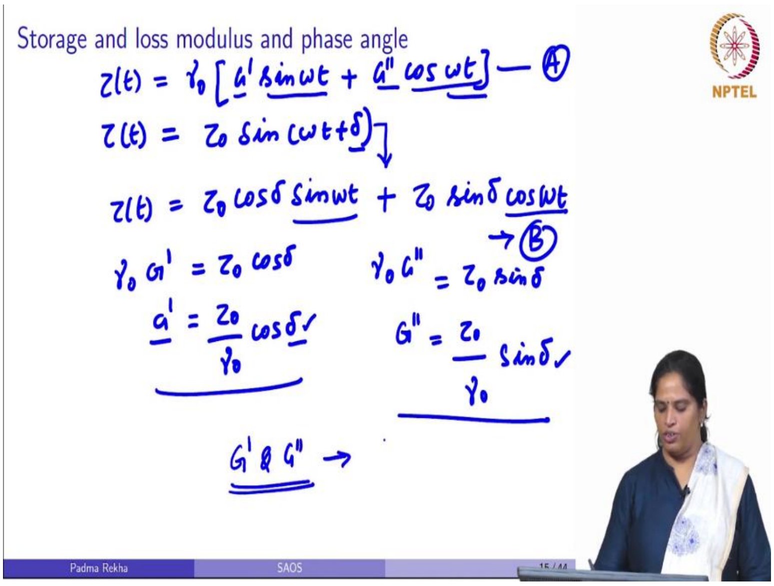

So, now let us summarize the storage modulus loss model as equations that we saw before. So, tau of t can be written as γ_0 which is strain amplitude into G' $\sin\omega t$coefficient is G' $\cos\omega t$coefficient is G''.We have assumed the Viscoelastic material exhibits a stress of the form$\tau_0 \sin\omega t+\delta$.Now we will find a relation between G' $G''$and a phase angle now.So, you can expand this function then you expand this will get$\tau(t) = \tau_0 \cos\delta \sin\omega t + \tau_0 \sin\delta \cos\omega t$

So, comparing an equation suppose if this is A and this is B you can equate the coefficient of $\sin\omega t$term when you equate the coefficient of $\sin\omega t$ you will get $\gamma_0 G' = \tau_0 \cos\delta$ of G' to be equal to τ_0 by γ_0 modulus function into $\cos\delta$.So, loss storage modulus and phase angle are related with this expression.Likewisewhen you equate the cos ωtcoefficient you will get $\gamma_0 G''=\tau_0 \sin\delta$ otherwise $G''=\tau_0/\gamma_0$sine δ.

So storage modulus and loss modulus are related to phase angle both when one is a sine function and other is a cos functions.So, one way of representing material functions for a linear viscoelastic behaviour or when we subject the material to an oscillatory shearing this either using a $G'$and a G''value and both this G'andG''arerelated and you can get the relation between those using a phase angle function.

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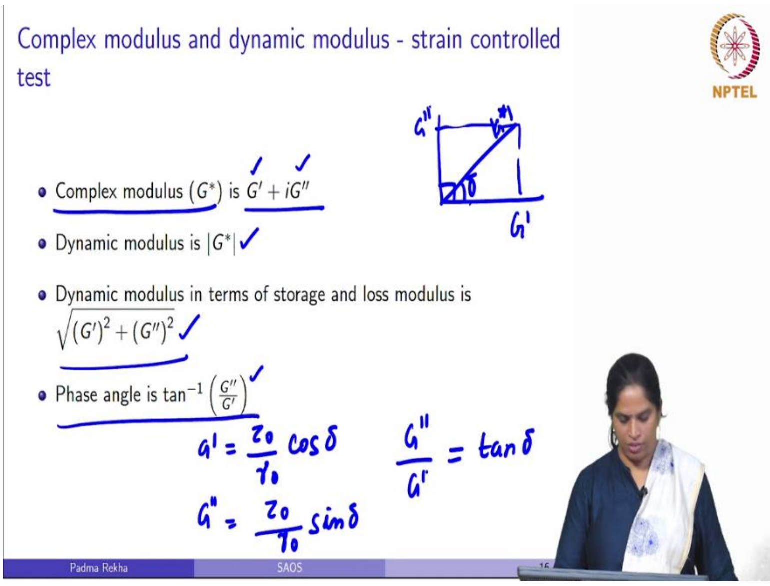

So, in another way of representation is using a complex number we define this complex number as a complex modulus we have a real part of a complex number we use that real part for representing an elastic modulus imaginary part which represents the loss we use it for giving a loss modulus.Now if you see a complex number the real part represents the elastic modulus and the imaginary part represents the loss modulus.So, the mod of the complex number is what we get it as a dynamic modulus and you can determined using this time this expressions G' is related to cos δ and $G''$is related to sine δ.

So the ratio storage modulus lost modulus to storage modulus is a tan δ so if you want to determine a phase angle from a storage modulus and loss modulus you can use this expression.We can also represent in the vectorial form the storage modulus and loss modulus asthis is a dynamic modulus if it is δ when the δ value is 0 we will get a storage modulus and the δ value is maximum we get 90 degree here when it is maximum you will get a loss modulus.

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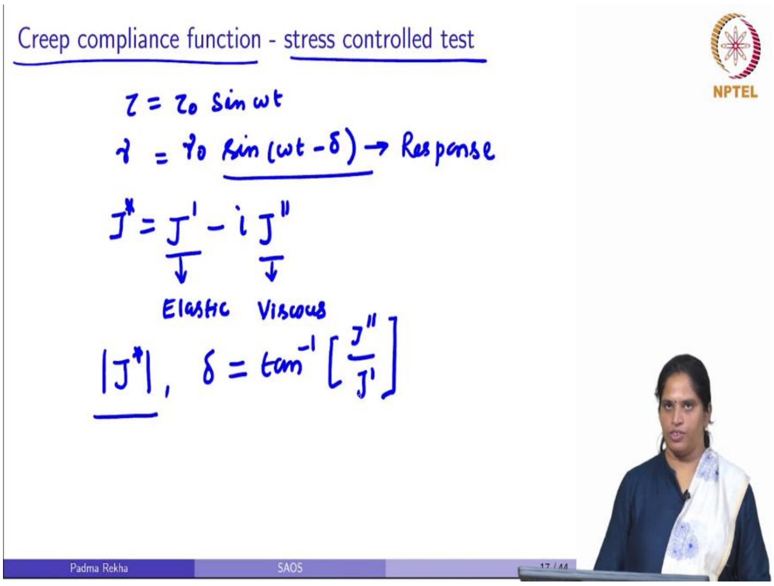

Now next we will define a creep compliance functions for this stress control test.So, for this we will directly use the way in which we define the dynamic modulus we will use that concept and we will define a creep compliance functions for this.So, in the stress control test we control the stress in a sinusoidalpattern and the response strain will also be sinusoidal with a lag this is either viscoelastic material behaviour.

So this is the response function, for this response function with the stress sinωtagain this strain will be a function of sine and a cos ωt,if you expand this and if you proceed in a same way as we proceeded for the dynamic modulus function you will get this function to be a real part function to be like you can separate this with a coefficient of sine term anda coefficient for a cos term and you will get 2 modulus one corresponding to an elastic response.

We use a term here J for defining a creep compliance, so this creep compliance will have 2 component one is a real part of a real part which represents the elastic compliance and other is an imaginary part which represents the viscous or a loss compliance.So, the mod of a creep compliance function is used in defining the material functions.So, similarly as you did it for a dynamic modulusyou will get a delta value to be tan inverse of viscous more compliance by elastic compliance. So this is in case a stress control test.

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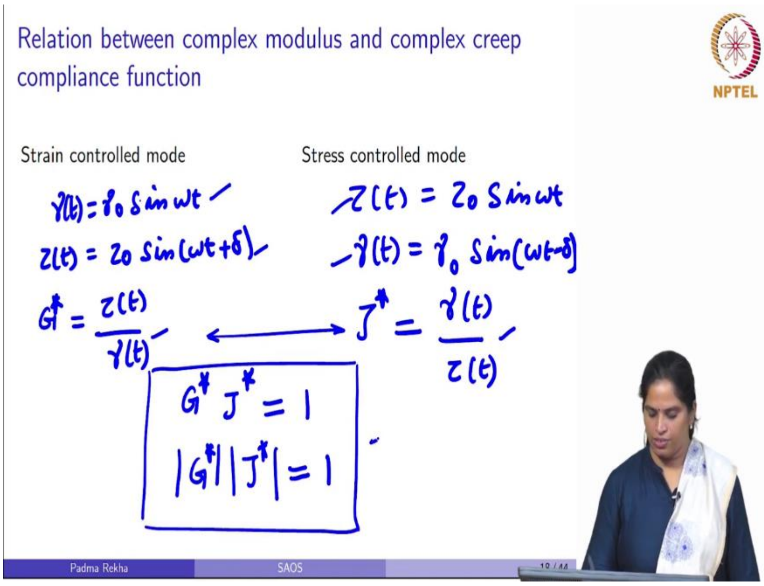

So, let us quickly recollect what we saw for a stress control test and a strain control test in an oscillatory mode and then we will see whether we have defined a material function we will see whether these material functions which we defined for the stress control and a strain control test are related to each other.So, in a strain control test we controlled thestrain and measured stress and the stress for the viscoelastic response is given by $\tau_0 \sin \omega t+\delta$.

We define the modulus here and the modulus is called as a complex modulus this modulus can also be determined from the ratio of $\tau(t)$to γ.Likewise this is also a function of time input is also a function of time so $\tau(t)/\gamma$of t likewise in a stress control test we controlled a stress in a sinusoidal pattern and the stress-strainresponse for the sinusoidal strain for a viscoelastic material will be $\gamma_0 \sin \omega t-\delta$ so we defined a function called as a creep compliance complex creep compliance which is defined as $\gamma_0 (t)/\tau(t)$.

Now if you use this τ function and γ° function in the complex free from creep compliance and if this γ and τ in this dynamic modulus creep compliance and compare both this 2 equations you can prove that the productof complex modulus and the complex creep compliance this one, you can also prove that a product of dynamic modulus and the mod of complex creep compliance we call it as just a creep compliance to be again 1. So, if you need more clarifications on this how to derive please contact us.

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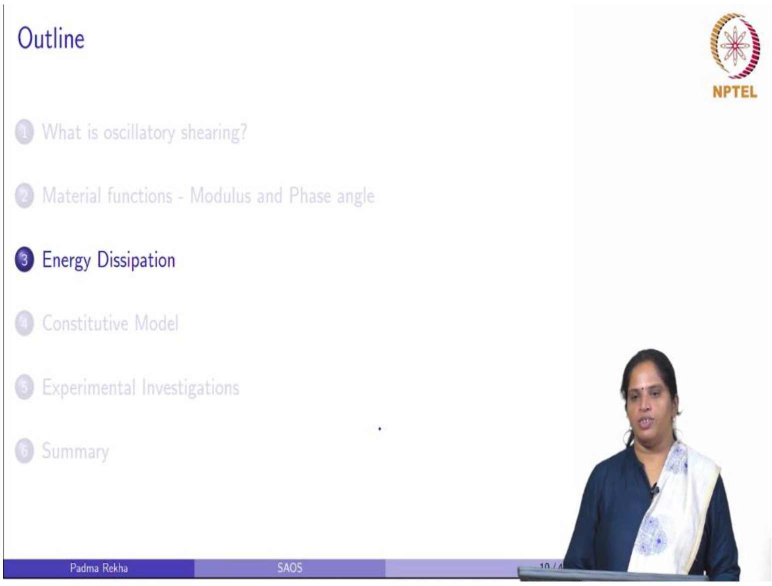

So,we have seen different material functions that we use to quantify that we used to define the viscoelastic behaviour when you test it under oscillatory shearing.Nowwe willcalculate the energy dissipation of a material when it is subjected to an oscillatory shear.

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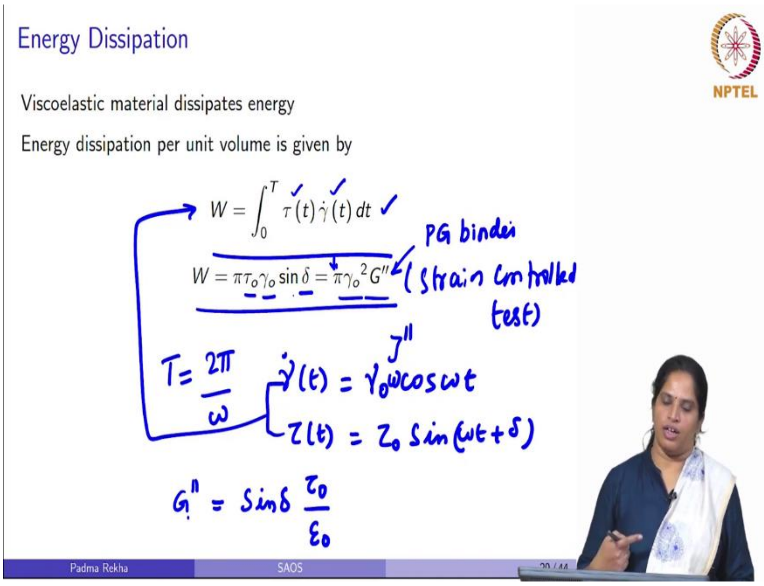

So, energy dissipation in a material can be defined using this work done equation.So, the equation here is per unit volume so you have an integral equations as a function of stress and the strain rate.Now if you solve this integral equations for it is controlled strain sinusoidal strain or a sinusoidal stress you will get the function to be this in this is for a strain control test.If you conduct astress control test you will get this equation in terms of J ″so just a simple way to solve this is use a t value which is one waveform time taken to complete one waveform.

So, it is $2\pi/\omega$ you said t value to be $2\pi/\omega$ with a strain controlled test $\tau_0 \gamma_0 = \gamma_0$ cos γ_0 ω cosωt and τ(t)for a viscoelastic response is $\tau_0$sinωt +δ when you substitute both theseexpressions in the first equation here you will get a energy dissipation per unit volume in a material to be related to a stress amplitude τ_0 strain amplitude γ_0 and phase angle δ.

And we already know that this sine δ is related to loss modulus by the function loss modulus is equal to sine δ into stress amplitude by strain amplitude.So, using this relation if you replace the sine δvalue you will get the energy dissipation in terms of a loss modulus. So, this energy dissipation equation is used in a bitumen testing in performance grading of binder.

So, based on the energy dissipation you can say when the payment or when the binder starts rutting or when the binder starts cracking so this is as energy dissipation varies with the temperature so you can find the energy dissipation values at a different temperature by defining the tolerance limit you can say that when the dissipation exceeds the particular value the payment fails in rutting or when thedissipation takes its extreme values in a low temperature you can say that the payment fails in fatigue cracking.

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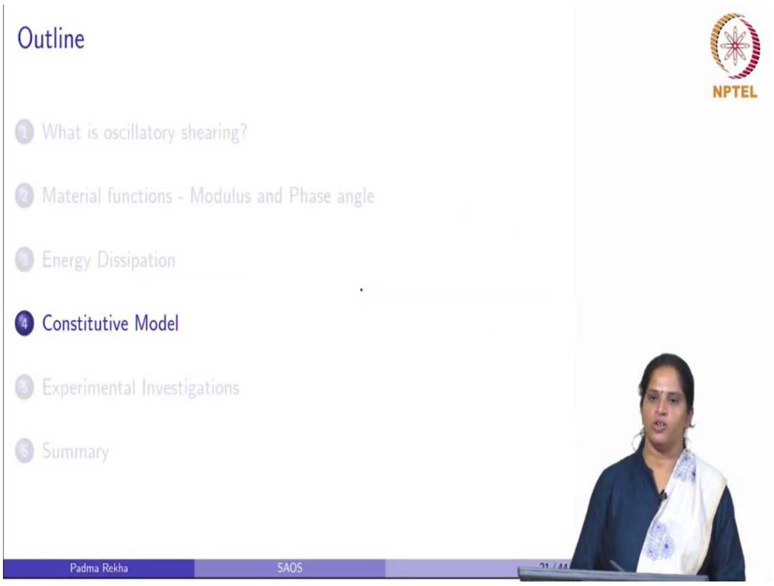

Next is a constitutive modelling we have seen a response of a material to an oscillatory shearing and we have defined a modulus functions and a phase angle functions for an oscillatory shearing.

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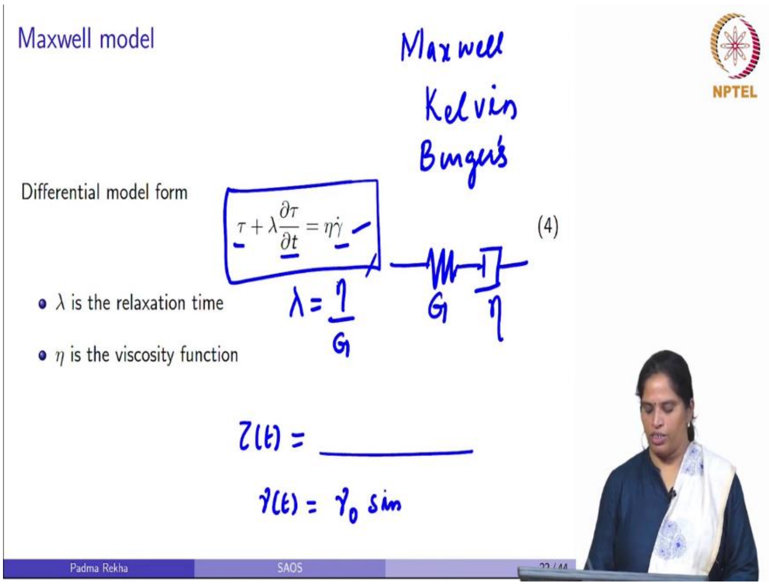

Now we will see how to predict the response of a material to an oscillatory shearing using a simple linear equations so this constitutive model equations are aone-dimensional equations we will focus here with a normal Maxwell model. We have been already introduced to this Maxwell model Kelvin model and Burger model.

So, we will directly use the constitutive relations and substitute the stress strain wave from functions in this and see what will be the response of these functions these equations.So, the differential form of a Maxwell model can be returned as inthese equations.So, the shear stress and the derivative of a shear stress here and gamma dot function here, here Maxwell model as you know can be visualized as a combination of a spring or elastic component and the viscous component in series.

Now if I define the material functions viscous material function as $\eta$and the elastic function as G so the relaxation time gamma can be givenbyη by G so the rate type equation for the Maxwell model is as given in this expression.So, let us use this expressionconstitutive relations to solve the tau function tau of t expression for a controlled strain input of ωtequal to $\gamma_0$sinωt.

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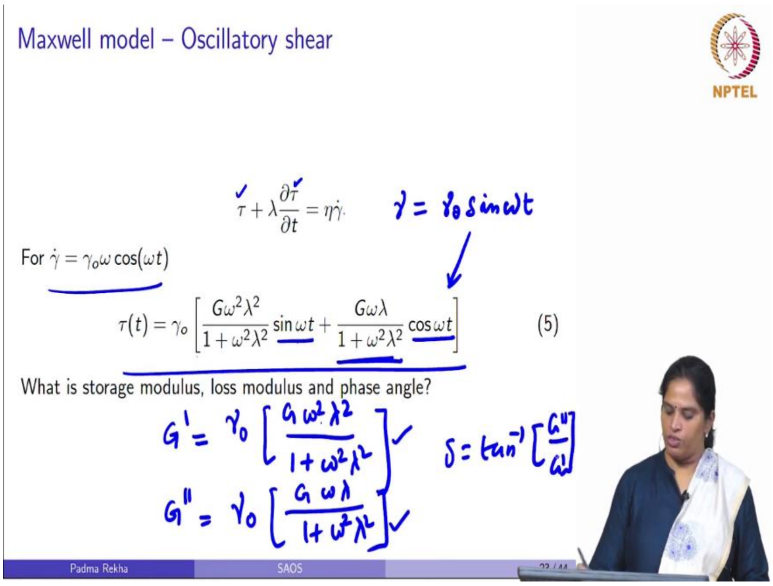

So, we know strain to be $\gamma_0 \sin\omega t$ so if you substitute the strain function in this constitutive relation and solve for τ you will get the relation of τ to be of these form.Now if you closely look into this relations again this equation has a $\sin\omega t \cos\omega t$ term.so, the controlled response which is strainis $\gamma_0 \sin\omega t$ and there are stress responses it's a function of sine and cos term.so, again here if you look into this you have coefficient of sine term and coefficient of cos term you know we defined this coefficient of sine term to be storage modulus.

So, storage modulus here which is γ_0 G $\omega^2\lambda$ / 1 +$\omega^2\lambda^2$likewise loss modulus will be coefficient of $\cos\omega t$which $\gamma_0$into G $\omega^2\lambda$/ 1 +$\omega^2\lambda^2$.So, we have storage modulus and loss modulus for a Maxwell model you see this modulus or frequency dependent now phase angle will be tan inverse of G″/ G′ so you can put a $\tan^{-1} = \frac{G'}{G''}$.

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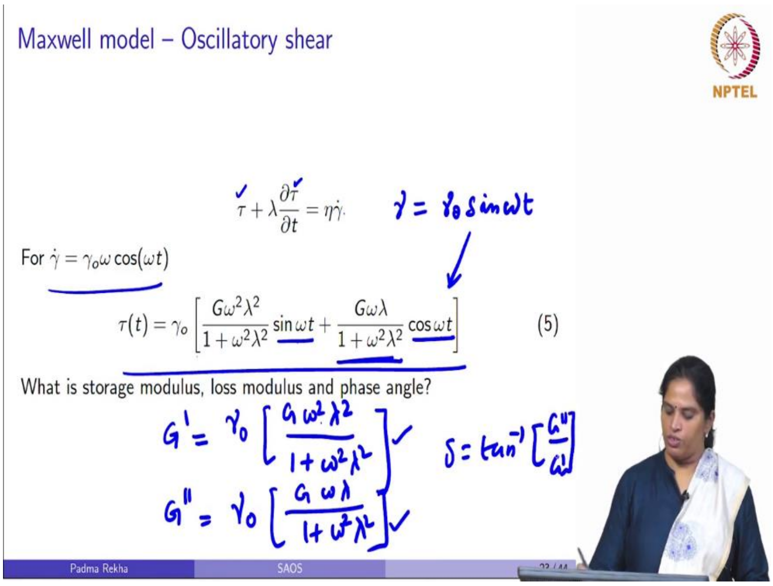

You will get the phase angle to be a function of frequency.So, you have a frequency is also here a time scale and λ here is also a time scale so you have you should know that frequency time scale is an experimental time scale andthe λ or a relaxation time is a material response time scale.So, here we see that the storage modulus and the loss modulus is a function of frequency and the ratio of loss modulus to storage modulus gives a tan δ value.and the tan δ value is linearly it increases changes with again a frequency.

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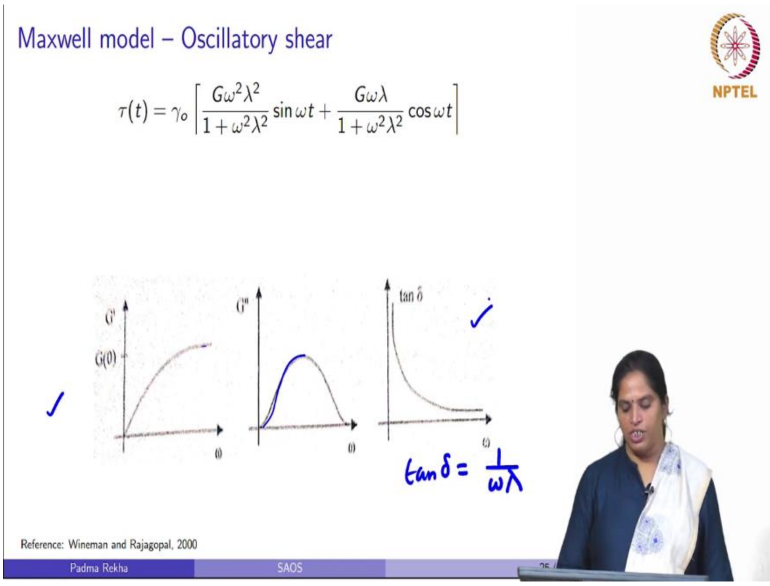

So, this is how the trend is as you test it across a different frequency.So, the modulus value increases as the frequency is increasing.So, you can see theincreasing trend of a storage modulus here consistently increases and reach a steady value when you test it at high frequency.In case of a loss modulus in a you as the frequency increases you can see a increasing value initially and then as the frequency increase further you can see a decreasing functions.

Tan δ value you know as this tan δ is related to a frequency inverse of frequency as 1 by ω into γ so you will get the tan δ function to be related to the inverse of frequency. So, this is a critical Maxwell model response when you subject the material to an oscillatory shearing.So, if the material behaves as a Maxwell model the storage modulus loss modulus and tan δ will vary something like this.

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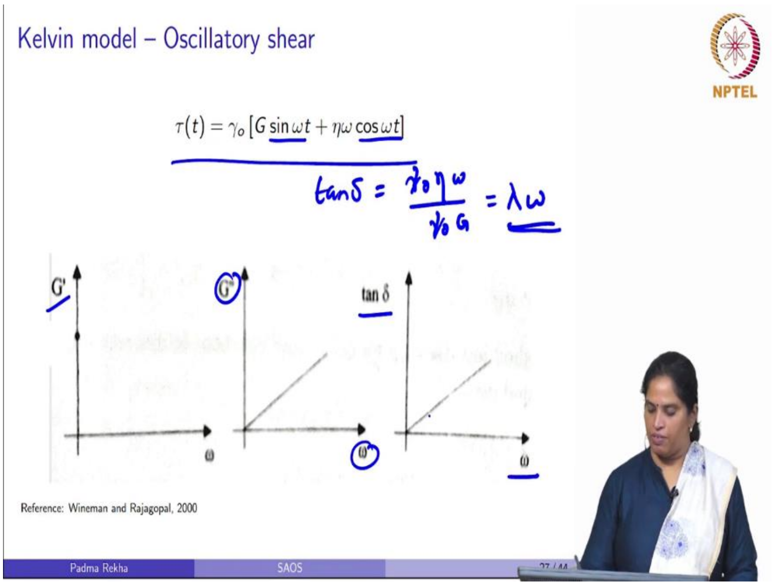

So, next is a Kelvin model we have a differential equation of a differential form of constitutive relation here for a Kelvin model, so τ depends on strain and strain rate so Kelvin model can be physically viewed as a string connected by adashpot in series.So, with a constant spring constant to VG and a dashpot constant to be η,so, when you solve these constant as equations for again $\dot{\gamma}$ to be $\gamma_0\omega\cos\omega t$substitute and γ to be $\gamma_0\sin\omega t$to just substitute in these expressions you will get tau to be of this form.

Again you will have 2 functions one a sinωt function another is the cosωt functions so thecoefficient of sineωtyou can see that it is independent of frequency it was independent of yeah it is independent of frequency and the cosωtcoefficient which is a loss modulus is linearly related to a frequency.and tan δ= $\frac{\gamma_0 \eta\omega}{\gamma_0 G}$which is nothing but by linearly again varies with a frequency γ by G is a relaxation time.

So tan δcan be represented as a product of relaxation time and a frequency.So, the storagemodulus we cannot define it for a different frequency loss modulus varies linearly with a frequency and tan δ a linear variation again with the frequency.This is this will be a typical response of a Kelvin murder.

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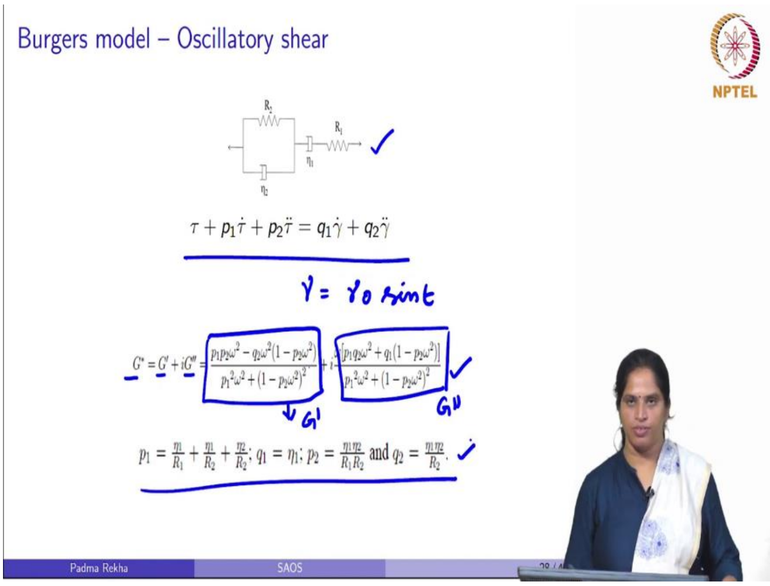

Now Burgers model for parameter model let us start from this constitutive relations.So, now if you solve this constitutive relation for a strain input of γ_0 sinωt,you will get the response function to be this let us use that term here.So, this is acomplex modulus real part of the complex modulus and imaginary part of a complex modulus.So, we have this portion function to be a storage modulus and this function to be a loss modulus.

So this storage modulus term and the lossmodulus term depends on frequency for this constants P_1 P_2 P_3you can refer this.

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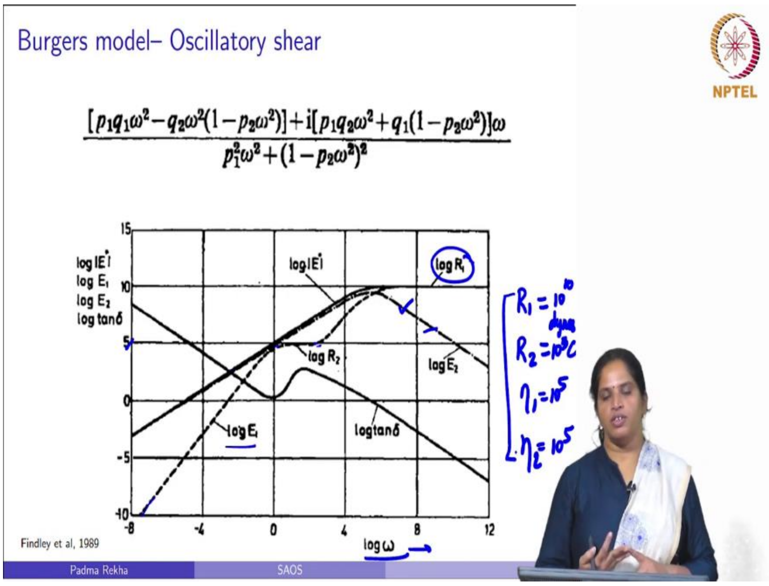

So, when you look into the response function of a Burgers model over a wide range of frequency so you have a frequency scale here it is in the logarithmic scale. This values are this curve is obtained for a R_1 value $R_2\eta_1\eta_2$ value for a fixed values, so R_1 is 10^{10} Dynes/cm^2 and R_2 is 10^5Dynes/cm^{2}and η_1 is both $\eta_1\eta_2$in a poise again $10^5$and 10^5poise.

So R_2 is 10^5, so this is if you see the variation in the modulus across the frequency across a wide range of frequency you can see that the storage modulus it is a logarithmicof a storage modulus the storage modulus increases with a frequency and when it reaches a value of near R_2 value it was constant for some period of time for some frequency and then it started increasing and the asymptotically it reached a constant when the value reached when the modulus value reach the value of R_1 which is R_1 a spring constant here.

The spring constant in the Maxwell portions, so the variation in that loss modulus is something which is given in this curve, initially thereis an increase and after you further increase in the frequency you can note that decrease in their modulus.So, this again trend depends on the material functions R_1 $R_2\eta_1$andη_2.So, we have seen constitutive models the behaviour of the various models that includes a Maxwell model Kelvin models and Burgers model.

Next we will see few experimental investigations that we conducted for a bitumen in an oscillatory shear mode.And we will discuss the results of the bitumen andwe will see which model suits the bitumen behaviour.Let us see it in the next part of a lecture.Thank you.