Mechanical Characterization of Bituminous Materials
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Lecture – 7
Small Amplitude Oscillatory Shear – Part 1

Hello everybody this lecture is an oscillatory shear testing of viscoelastic material. So, this oscillatory shear testing is commonly used for a viscoelastic material if you want to characterize their material response over a wide range of time or we call it as a frequency which is a inverse of time. So, in this study we are going to restrict our response of a material to a very, very small deformations that is why we call it as a small amplitude oscillatory shear for a bitumen.

So, and a very small deformation we assume that the response of the material is linear. So, all this material functions which we are going to define it for the oscillatory shear testing as applicable when the response of the material is in the linear regime.

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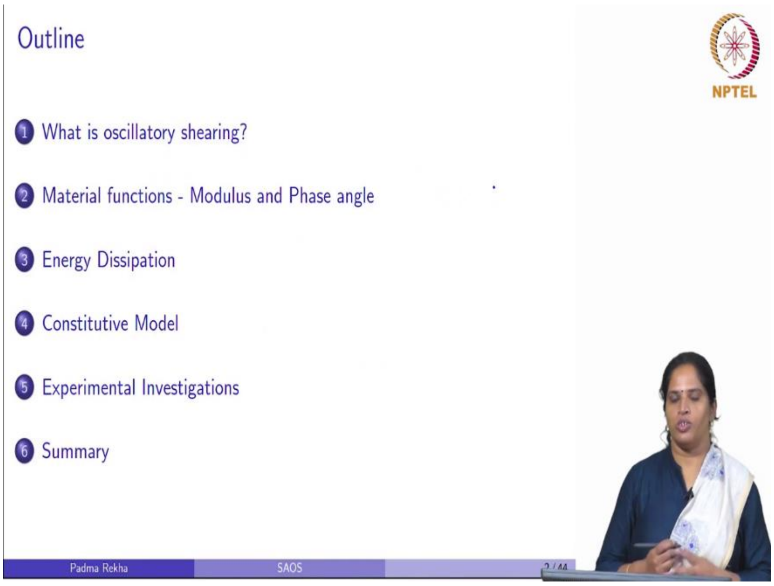

So, this is going to be the outline of a presentations first we will see what is oscillatory shear and you know when we subject the material to an oscillatory what are the material functions that we need to under that we need to define for viscoelastic material followed by we will calculate what is the energy dissipation for an oscillatory shear testing. We will also see a few constitutive model that can be applied for predicting the response of a viscoelastic material under oscillatory shearing.

and finally we will discuss few experimental investigations that we conducted especially for a bitumen and followed by we will summarize what we what we discussed.

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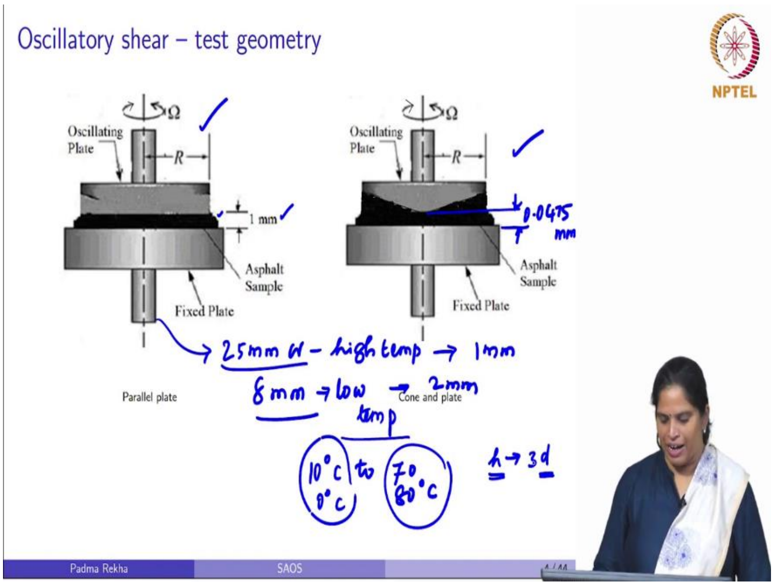

So, oscillatory shearing as we said is it is nothing but a shear flow in inducing a shear flow in a material. So, in an oscillatory shear we keep a material between the two plates maybe a parallel plate as shown in this figure here or between a plate and a cone system and we keep the bottom plate fixed and the top plate oscillate or in case both plate oscillating in a different directions. So, by doing this we are subjecting the material kept between a two plate to a shear flow.

So in a generally for testing of bitumen we use a parallel geometry in which the diameter of the parallel plate wave can be varied or the generally we use 25 mm dia or 8 mm diameter. 8 mm diameter or a small diameter is preferred when we test it at low temperature or when the material is stiff and under high temperature we use a 25mm diameter. This high temperature low temperature we have to be careful in defining there a limit for a high temperature and a low temperature.

So now if this is high temperature is like the material should not flow when you keep between the two plates. So, we cannot use this parallel plate geometry when the material is Newtonian. Likewise you cannot use this parallel plate geometry when the material is very stiff at low temperature. General temperature range for a bitumen to use this parallel plate geometry as in the range of 10 degree Celsius to or maybe as low as 0 degree Celsius to maybe 70^0 or 80^0 Celsius range .

So low temperature range will be in general again it varies depending on the binder stiffness and high temperature range will be like 70 to 80° again it varies with a binder stiffness at this temperature. So, and depending on the geometry we also vary the height of a sample we keep a 1mm height for a 25mm dia and we keep 2 mm height for 8 mm diameter. So, it is oscillating shear is nothing but subjecting a sample to a shear flow.

So, we keep a sample between a two plate we oscillate both the plate or only a top plate in a specific pattern in a cone and blade geometry you see the top system is a conical structure so and the sense it is a conical assembly the gap here is limited. So, it will be in the order of microns the 0.0475 mm so in the order of microns. So, this gap being very small we cannot use for testing a material that has a particle size of a larger distributions.

For example if you want to select a geometry based on the particle size we at least keep a height of a sample to be minimum of 3 times the size of a particle. And d here is the size of a particle size may be a maximum size and h is a thickness of a sample or gap between two plates. So, we keep height to be minimum of 3 times the size of a particle. So, mostly if we test a modified binder we use a parallel plate system.

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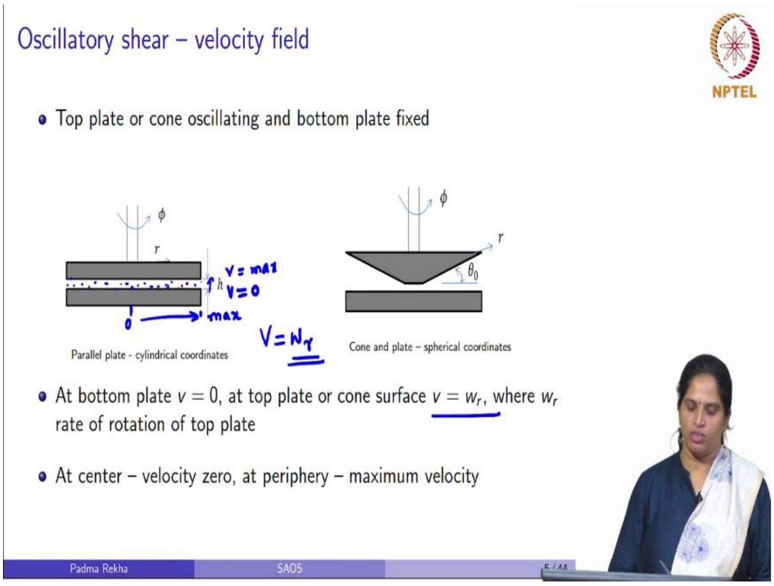

So, in an oscillator a sheared system we keep a sample between two plate and oscillate so while doing this the particle between a two plate is not subjected to a uniform motion. So, if a bottom plate is stationary and the top plate is rotating, so the motion of a particle sticking to the bottom plate will be 0 and the velocity here will be 0 at the bottom plate. So, if the top plate is oscillating so when the velocity will vary from 0 to maximum when the height varies from 0 to maximum from bottom to top.

Likewise the velocity of a particle at the center will be 0 and the velocity of the particle at the periphery will be maximum. So, when you move along the radius the velocity of a particle will vary from 0 to maximum. So, this is in case of a parallel plate system you can see that the velocity of a particle is not uniform, so velocity at the periphery is given by velocity of a particle is given by V is equal to W_r, W_r is nothing but the rate of rotation of a top plate.

So, at the periphery V is given by W_r where W_r is a rate of rotations of the top plate. So, the particles between a two plates is not subjected to a uniform motion, so maximum at the periphery and 0 at the center.

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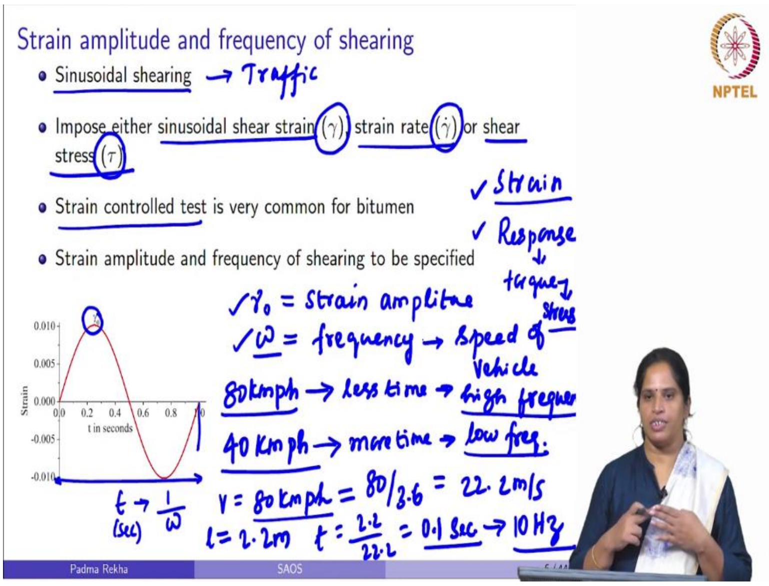

So, by oscillating the top plate we are subjecting the material to a sinusoidal shearing. So, you can control the speed and the deflection of an oscillation so that we will get a controlled sinusoidal shear strain are controlled sinusoidal shear rate are controlled shear stress. So, we control the oscillation in such a way that the material is subjected to a sinusoidal strain or a sinusoidal shear rate or sinusoidal shear stress.

Let us represent a shear strain by $\gamma$and the rate by $\dot{\gamma}$and the shear stress by τ. So, now this give a sinusoidal shearing because sinusoidal shearing closely simulates the traffic conditions loading and unloading conditions in the field. So, we generally use a sinusoidal shearing for testing bitumen. So, it can be when you control a strain we call it as a strain control test or when you control a stress we call it as a stress control test.

We generally prefer strain control test for a bitumen. So, in a strain control test we subject the sample to a sinusoidal strain and we find the response of a material. So, we subject the material to a sinusoidal strain we have to give a 2 inputs one is the speed at which you oscillate so if you control the speed at which you oscillate that defines the frequency or a time for completing one oscillations.

So this time this is inversely related to a frequency let us denote the frequency by ω. So, the time here or time in seconds this is related to the frequency and the maximum deformations you subject when you are oscillating the plate is defined by giving a strain amplitude. So, $\gamma_o$here is a strain amplitude or a peak value and ω here is defined as a frequency.

This frequency can be related to the speed of a vehicle. When vehicle is moving at a faster rate relatively say for instant 80 kilometers per hour and 40 kilometers per hour if you consider both these cases when the vehicle is moving relatively at a faster rate 80 kilometer per hour the time at which the material is loaded will be less, so less time or otherwise high frequency inverse of time as frequency as otherwise it is a high frequency.

In case 40 kilometer per hour compared to 80 kilometer per hour it is loading is more or more time, so we need to test it at low frequency. So, if you want to simulate the response of 40 kilometers per hour use the corresponding low frequency. And if you want to simulate the response of 80 kilometer per hour traffic you test it at high frequency. Now what is the relation between the speed of vehicle and the frequency of a test.

So now for to getting those relations we will take a speed of 80 km per hour. So, 80 kilometer per hour converting to meter per seconds you will get roughly 22.2 meter per seconds. Let us find the time for the vehicle which is running at 80 kilometer per hour time of contact which it has with a payment when the length of contact is say 2.2 meter. So, if the length is 2.2 meter and the speed is 22.2 meter per second the time of contact maybe 2.2 divided by 22.2 this will be approximately 0.1 second.

So this point once again contact time if you convert it you will get 10 Hertz frequency which is nothing but inverse of time. So, if you want to test or simulate the response of a material for it corresponding to 80 kilometer per hour we need to test the sample at a frequency of 10 Hertz. So, we control a frequency and strain amplitude by controlling a strain amplitude and a frequency we subject the material to a controlled sinusoidal strain.

So our strain is an input here we control the strain value now what will be our response of the material if you subject the material to a sinusoidal strain.So, the response is measured in terms of torque or resistance of the material to flow and this torque is converted to stress. Now let us see what will be the stress response when the material exhibits a elastic behaviour or purely viscous behaviour, viscoelastic behaviour which is a combination of an elastic and viscous behaviour. So, we will see what will be the viscoelastic response for the sinusoidal strain.

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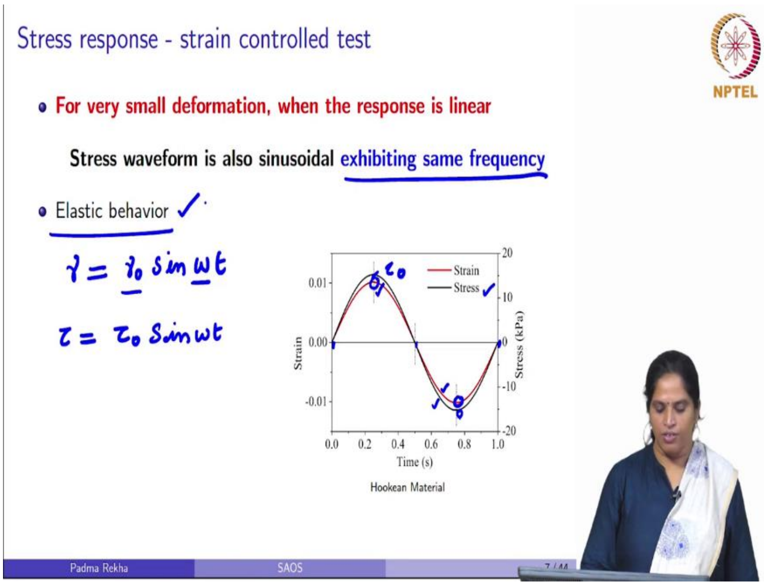

So for any elastic behaviour when we subject the material to a sinusoidal strain the red colour one is a strain waveform. So, we have given an input for this strain amplitude which we defined it as γ_o, strain amplitude. And it is a sinusoidal waveform so if we want to mathematically represent this training sinusoidal waveform we can write it as γ to be γ_o sinω_t it is a sine waveform with amplitude to be γ_o and the frequency as ω.

So, now if the material exhibits a elastic behaviour the response of a wave form will also be sinusoidal. The response wave form is the stress wave form here the stress wave form will also be sinusoidal. The main thing which we have to understand here is the response stress waveform is sinusoidal but with the same frequency. So, here you see the frequency of the waveform the time taken to complete one waveform a same for both stress and strain.

So, you can write the stress waveform for the elastic response to be peak value of stress, we call it as a stress amplitude τ_o into sine ω_t, so the point you have to note here is the peak stress and the peak strain value occurs at the same instant of time . 0 stress and 0 strain value occurs at the same instant of time. So, there is no time lag between stress and strain when the material exhibits elastic behaviour. So, what will happen if the material is viscous.

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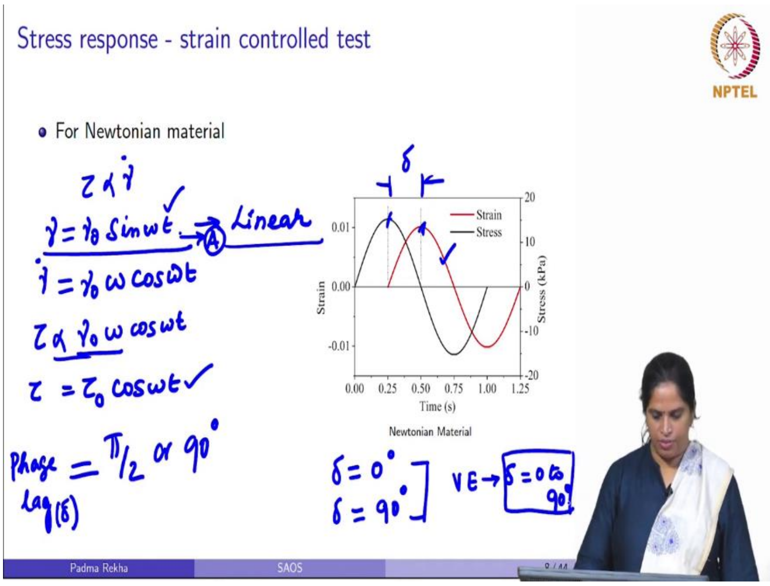

So, in case if the material is viscous or ideally Newtonian behaviour we subject the material to a sinusoidal strain what will be the response of a material. So, we know that for a Newtonian material stress is proportional to strain rate for a controlled strain of $\gamma = \gamma_o\ sin\omega_t$to be strain rate can be obtained by differentiating this equation partial differentiation of this equation. We can do that only if the response of the material is linear at a very, very small deformation. So, γ value especially the peak value here is very, very small.

So that the response is linear. So, $\dot{\gamma}$ if you differentiate this expression it maybe we call it as if you differentiate this expression or do a partial derivative you will $\dot{\gamma} = \gamma_o\omega\ cos\omega t$ so $\tau \propto \gamma_o\omega\ cos\ \omega t$, $\tau = \tau_o cos\omega t.$

So our control function is strain which is a sine function and the response function is a stress which is a cos function. So, we know that the time lag between a sine function and cos function is $\frac{\pi}{2}$or 90^o and we call this as a phase lag or phase angle. Phase angle for a pure viscous material is $\frac{\pi}{2}$ or 90°. So, since the phase lag is 90 which is nothing but the lag peak value lag, lag between stress and strain here. So, you see a stress peak to be at this location and strain peak to be at this location.

The time lag is what we call it as a phase lag and it is represented generally by a δ. So, we will use a δ term here to represent a phase lag or a phase angle. So, this is ideal Newtonian material, so we have a elastic material with a phase angle of 0° no lag and viscous material with the phase angle of 90° maximum lag. Now for a viscoelastic material we can say that it exhibits a combination of an elastic and a viscous behaviour in such case for a viscoelastic material the phase angle δ will fall between 0 and 90^0. So we will keep this concept to frame the stress equation for a viscoelastic material.

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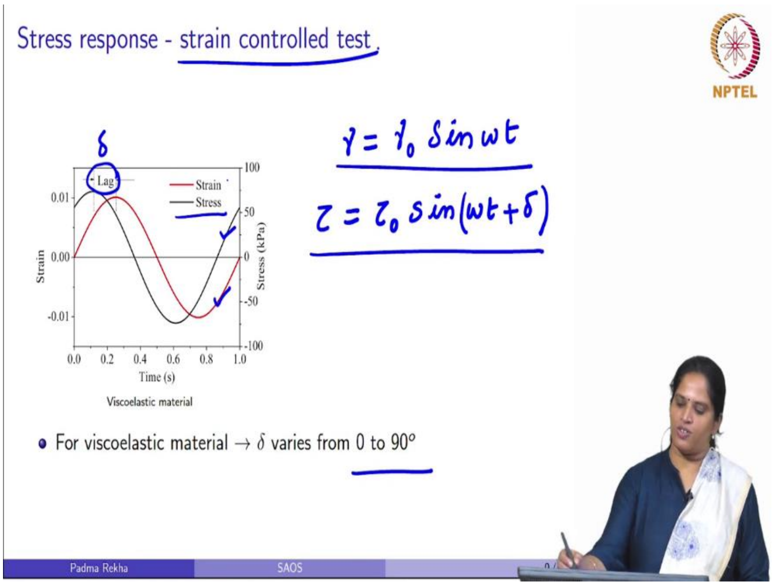

So, you for a sinusoidal strain control test if the material exhibits a viscoelastic behaviour we also get a sinusoidal stress with the same frequency but the lag value here then that is a time lag between peak stress and a peak strain will lie between 0 and 90 degree. So, this is the basic concepts which we are going to use to define a material functions. So, let us write the equation for a strain and stress, so control a strain and can be written as $\gamma_o\ sin\omega t$ sinusoidal strain.

And the stress function is also sinusoidal so with the peak value let us define a stress peak to be τ_o this is also sinusoidal sinωt but there is a lag here so the lag is defined as $\delta$and we know that stress leads the strain value that is you need a minimum load to be subjected to induce a deformation in a material. So, we get $\sin(\omega t + \delta)$, so this is when the material exhibits a linear behaviour and when the material exhibits a viscoelastic behaviour if we control a strain in a pattern of $\gamma_o, sin\omega t$ you will get the stress response in the pattern of $\tau_o sin\omega t + \delta$.

So, this is a stress response in a strain control test. Now what about a stress control test? See we are controlling a strain in a sine waveform now we control a stress in a sinusoidal pattern what will be our strain response?

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So, let us plot the stress response of a stress control test. So, in a stress control test let us assume or let us subject material to a sinusoidal stress because this is a stress control test. So, we will keep a σ_o not to be amplitude of wave form and ω to be a frequency. So, we will represent it by τ because it is a shearing. So τ equal to we are controlling as stress in a sinusoidal pattern with an amplitude τ_o, so tau can be written as this expression.

Now if we have tau to be this if the material is elastic the response is going to remain the same the strain response is going to remind as a same sine waveform with amplitude of a strain to be $\gamma_0 sin\omega t$ where δ is zero. This is in case if the material exhibits elastic behaviour. For the material that exhibits a viscous behaviour γ as we know tau is proportional to γ_o, so the γ is going to be a cos function. So, we can write it as $\gamma_o cos\omega t$, so this is viscous.

If it is a go elastic the γfunction or an response function is going to be$\gamma_0 sin\omega t - \delta$ stress leads strain so it isδ this is viscoelastic. So, here this δ is 90^o for viscous material and δ varies from 90 to 0 when elastic to completely viscous 90^o so viscoelastic there is δ value varies from 0 to 90^0, so now if you sketch their response function let us sketch it on the same line.

The strain response function it will also be sinusoidal with the same frequency. So, if the frequency wave form is Omega, this will also be Omega but there will be a lag , Delta and the stress wave form is given by this expression. This is in case if we conduct a stress control test. So, now just to quickly summarize elastic material if you subject the material to as a strain control or a stress control test it does not exhibit a phase lag or phase angle for this material is 0.

And for a viscous material is phase angle 90 and for a viscoelastic material the phase angle varies between 90 to 0. 90 degree ideally fluid. 0 Degree ideally elastic or a solid so you have a between value which is intermediate value 45 degree, so what happens when the phase angle is above 45 degree or when the phase angle is below 45 degree that let us discuss a few slides down.

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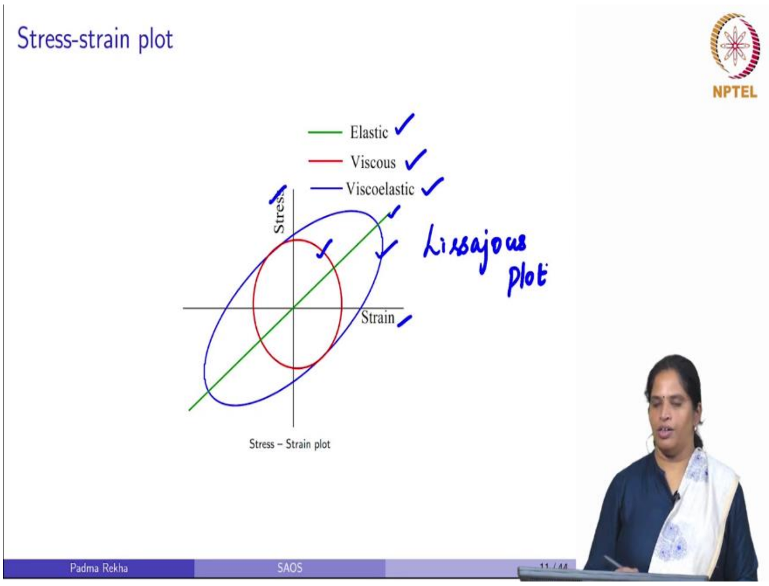

So, this will be an ideally stress strain plot for a different material elastic, viscous and viscoelastic material for a sinusoidal shearing. So, stress strain plot for a elastic material is straight line for a viscous material is perfect circle as a phase angle is 90 degree and for a viscoelastic material it is elliptical. We call this viscoelastic material as stress strain plot and there oscillatory shearing as a lissajous plot.

This Lissajous plot will be more useful when we are trying to find out a energy dissipation in a material. We know that this area enclosed in the stress strain plot is what what we call it as a energy dissipation or we use a stress strain plot area in calculating an energy dissipation.

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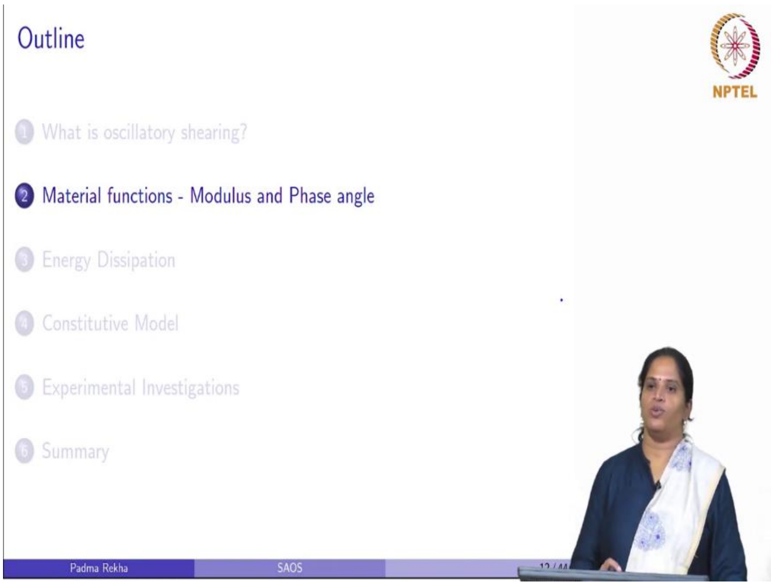

So, we know the response of a material to an oscillatory shearing so we can either do a stress control test or a strain control test. Now what is the material functions we use it for defining the material under oscillator is shearing.