Mechanical Characterization of Bituminous Material
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Lecture No. 5
Linear Viscoelastic Response Part-03

Hello everyone, we restart our discussion on the linear viscoelastic response and in fact in the last lecture that we had.

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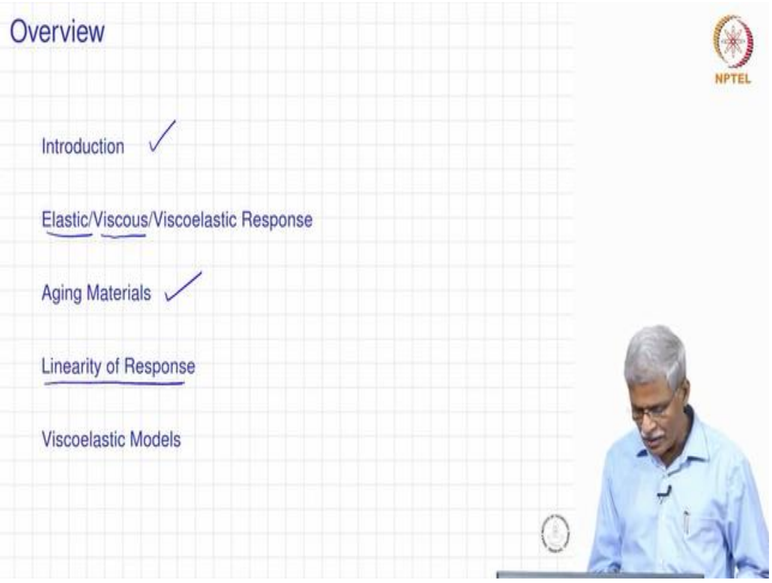

We introduced what exactly is meant as viscoelastic response and we basically were looking at the viscoelastic response as sum total of elastic as well as viscous response. We also discussed in detail about aging materials. Now what we now need to do is to talk in detail about what exactly is meant as the linearity of the response.

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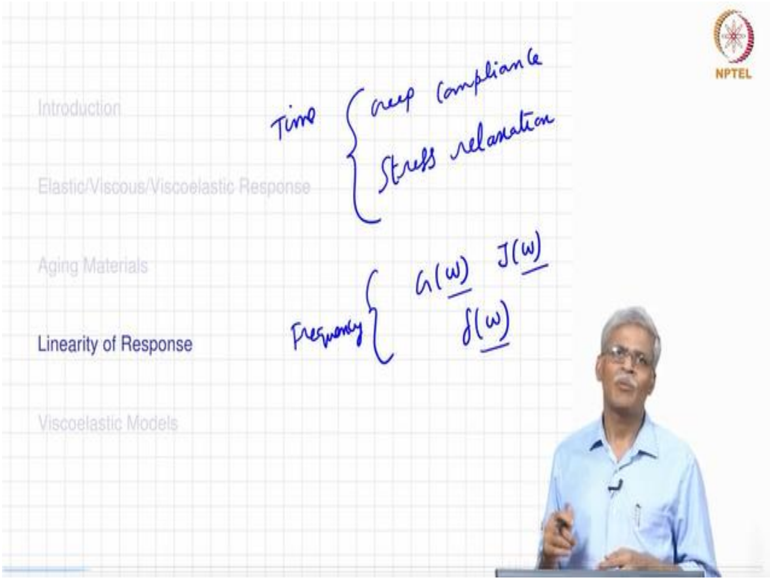

So if you recollect the discussion that we had, we talked about creep compliance, we talked about stress relaxation; all of them in time domain and in frequency domain. We talked about these parameters storage modulus, the same stress relaxation as well as the creep complaints in the frequency domain. So now what we need to clearly understand these how do we even define these functions.

Apparently, these functions are defined only for a specific class of mechanical response and that mechanical response is prescribed in the linear viscoelasticity. The linearity that we see in a viscoelastic material can be slightly different compared to the linearity that we are familiar with in elasticity. The reason is since we explicitly take into account the influence of time even some of the definitions that we thought as a simple especially for linearity can become very involved. It can be easily related in the time domain but in the frequency domain it becomes little more complicated.

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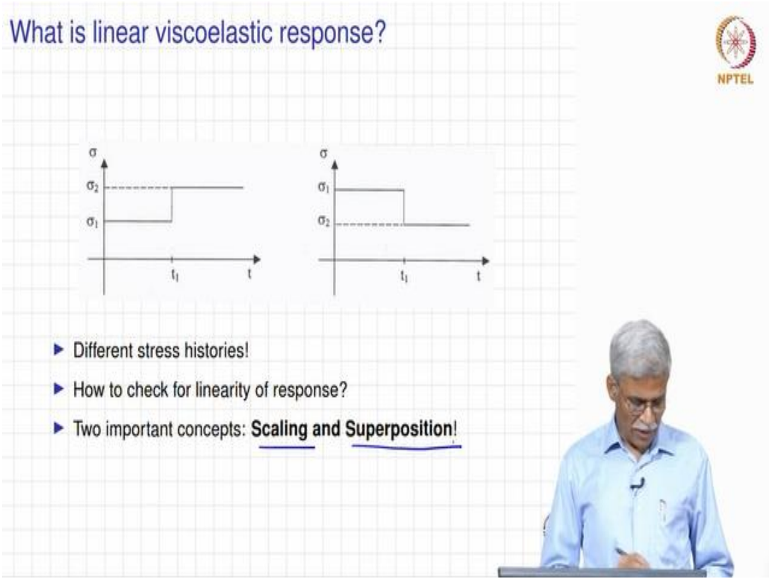

So let us first now start looking at what is really called as the linear viscoelastic response. Now what you see in this picture is two different stress histories and we need to clearly see how to check for the linearity of response. So we are going to introduce two important concepts what is really called as scaling and superposition. Right?

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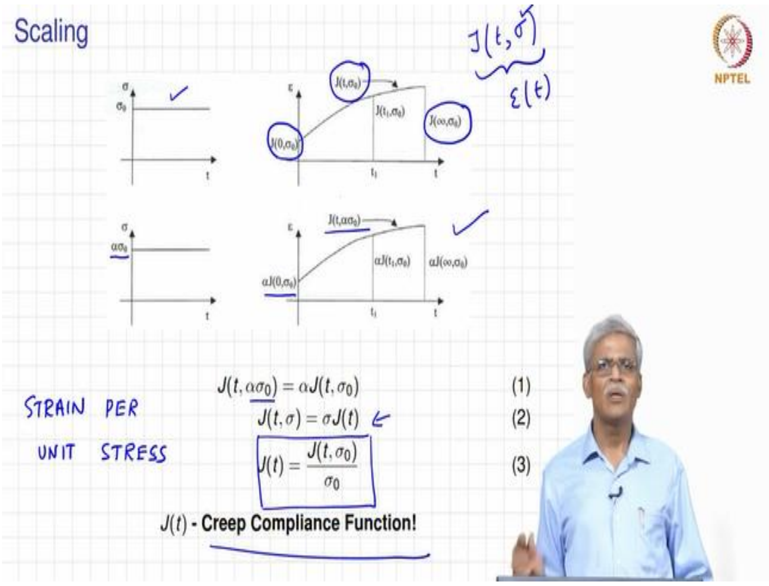

So let us keep looking at each of them as we go alone. So right now let us get back to our older definitions we defined a function called creep compliance, which had two parameters t and σ. So that means, for any given stress level for any given t this function returns what is called as the strain at that particular instant for any applied load given by σ. Right? So now let us take a look at this first picture wherein we are applying Sigma of σ_0.

And you will immediately understand that at each and every specific point the response is given in this following form. Now let us apply one more loading which is given by alpha (α) times σ_0, alpha is any scalar and you are going to see that one can actually write this in this specific way and in fact please focus your attention on equation one in which we have written clearly that this function creep compliance function is J (t, $\alpha\sigma_0$).

And since alpha is scalar, we just take it out and we write it as α times J (t, σ_0) and which is what is essentially represented in this figure. Now, since σ_0 is constant and alpha can vary in whatever way we want one can actually write this in the specific way and so finally one can define creep compliance in the following manner. Creep complaints function; so that is nothing but J (t) is J (t, σ_0) divided by σ_0.

So what is that we have done now; we have eliminated the presence of σ here within this function and we are going to divide each and every strain as with respect to the stress that is applied. So that means this creep compliance come function can be written as strain per unit stress. Now it is not very clear to us whether one can define such function for all the class of response. So we need to have some restrictions on the class of mechanical response for which such definitions are applicable.

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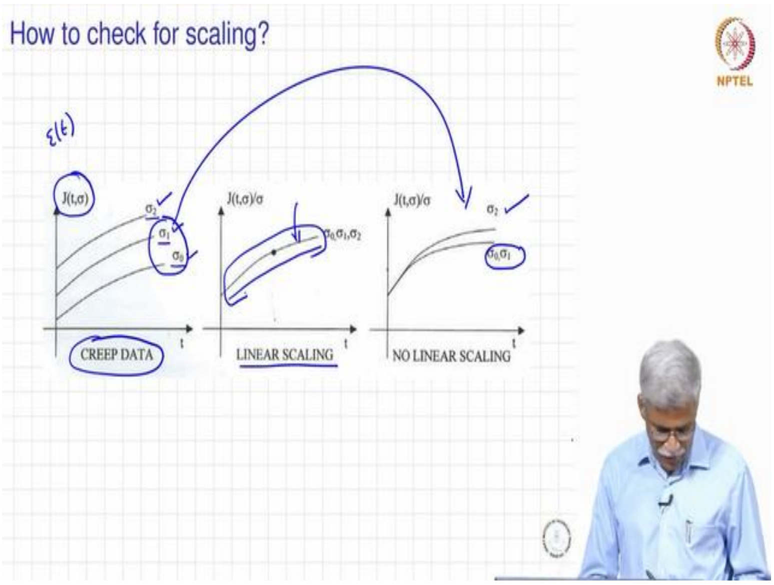

So how do we really check for this scaling? For instance, take a look at this picture this is the creep data and you can actually see that there are three different types of stress levels that have been applied σ_0, σ_1 and σ_2. This is as a function of time and you will also notice that this is nothing but $\varepsilon(t)$.

So that means we get back to our old way of prescribing this creep compliance function; so J (t, σ). Now what we are now going to do is to divide each and every strain of this with the appropriate stress and so we are going to say that if all these lines fall on one specific segment then we are going to say that the response of the material is scalable or we really call it as linear scaling.

But on the other hand, if let us say σ_0 on σ_1 fall on top of each other whereas σ_2 that means the strain corresponding to σ_2 is deviating you are essentially going to say that there is no linear scaling. So we need to be also very clear to say that up to a response of σ_1, so that means you can take this material has given here and up to σ_1 the response of the material is scalable but beyond it the response of the material is not necessarily scalable. Okay!

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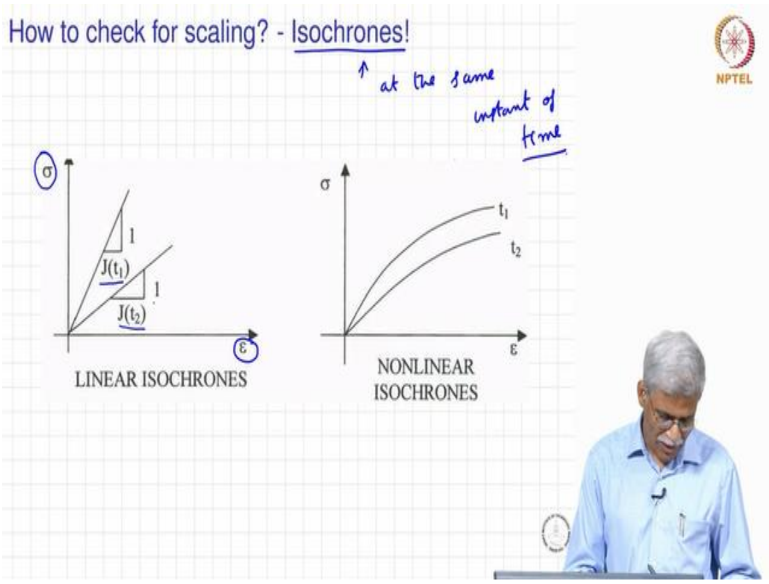

So now we will look at various ways of interpreting this scaling. What we define as isochrones. So what exactly the meaning of isochrones; Iso is same Chrone is time at the same instant of time. So now you can actually take draw a picture in which you plot σ on the y-axis, ε that is the strain on the x-axis and you try and plot the isochrones that is J (t, 1) or J (t, 2) at any instant of time. So how do we do that let us go to the previous graph.

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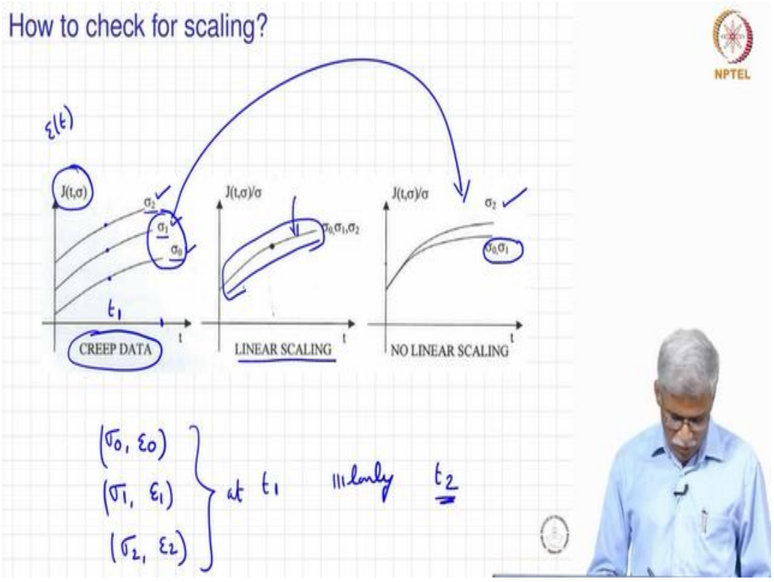

So we basically take a look at this is your creep compliance, so this is your strain, this is your strain and this is your strain. so let us take t1, so you are going to get (σ_0, ε_0) as one pair similarly (σ_1, ε_1) as 1 pair and (σ_2, ε_2) as another pair all of them at t1 and you can always do something like this. Similarly one can do it for t2 also.

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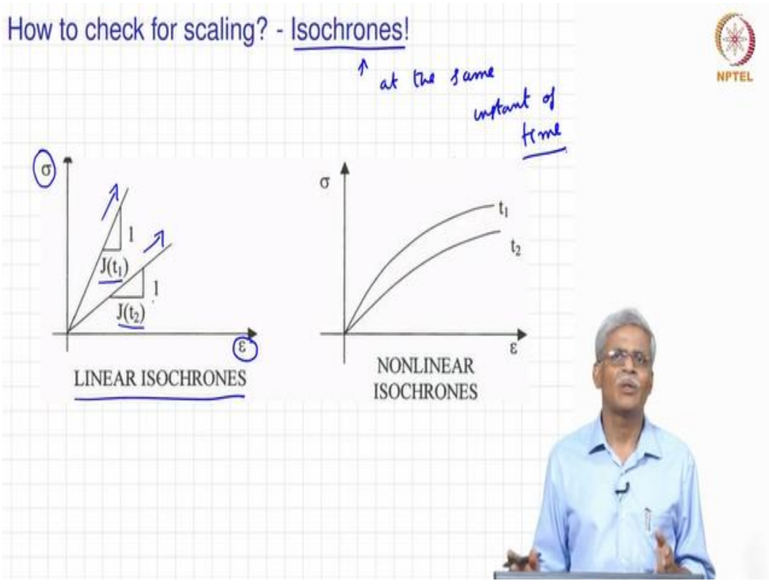

So when we do that and when we plot this particular line, you are going to see if you are going to get a straight line. The way in which it is seen here you are going to say that this is the response of the material is linear or what you really call here as linear isochrones. Now I need to make a clarification based on the statement that I made in the last lecture. In the last lecture I very clearly emphasized that one should not plot stress versus strain graph for a viscoelastic material. Especially when you are trying to plot the creep and recovery and we also showed the absurdity of some of the graphs. But here what we are doing is,

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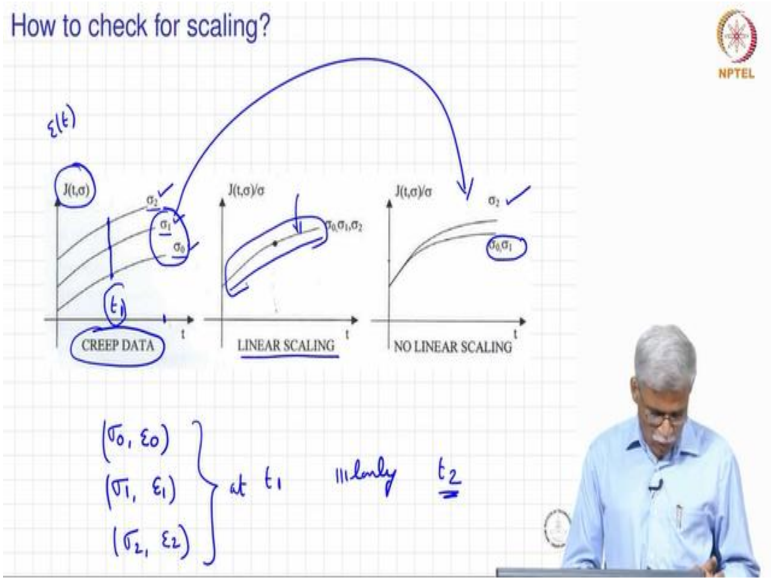

We are picking one particular time t1 and we are taking this (σ, ε) pairs and trying to plot those graphs to get the check for the linearity.

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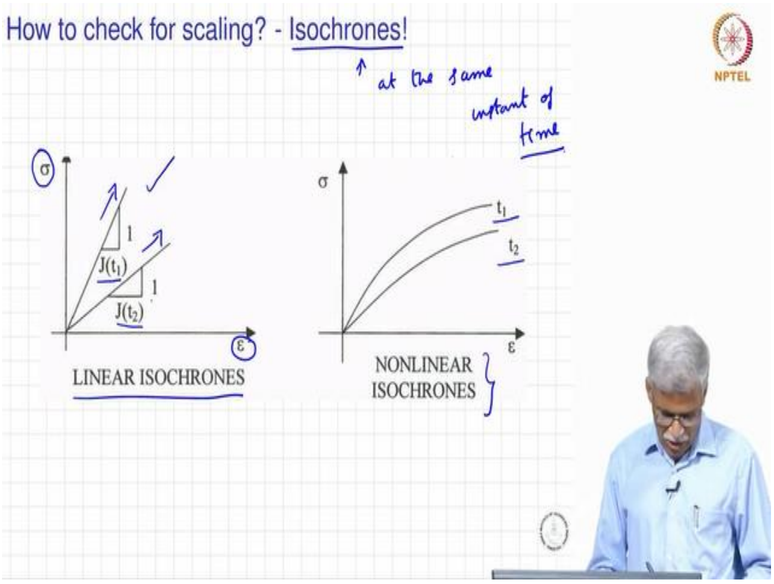

So these specific graphs have a special meaning, especially when you want to characterize the linear response of the materials. On the other hand you could if you are going to have the response of the material as t1 and t2 something like this you are going to say that this is a nonlinear isochrones. There is also one more statement that I need to make here, which will become amply clear as we go to the next set of analysis.

The linearity as far as viscoelastic response of the material is concerned does not necessarily mean that the response of the material in the stress-strain space is has to be a straight line. It so happened that is in the manner in which we have defined the linear isochrones these responses seems to be a straight line and hence you are calling it as linear isochrones. So let us not confuse our motion related to linearity as a straight line as far as linear viscoelastic response is concerned and this will become very clear when we start talking discuss few concept in the later portion of the lecture.

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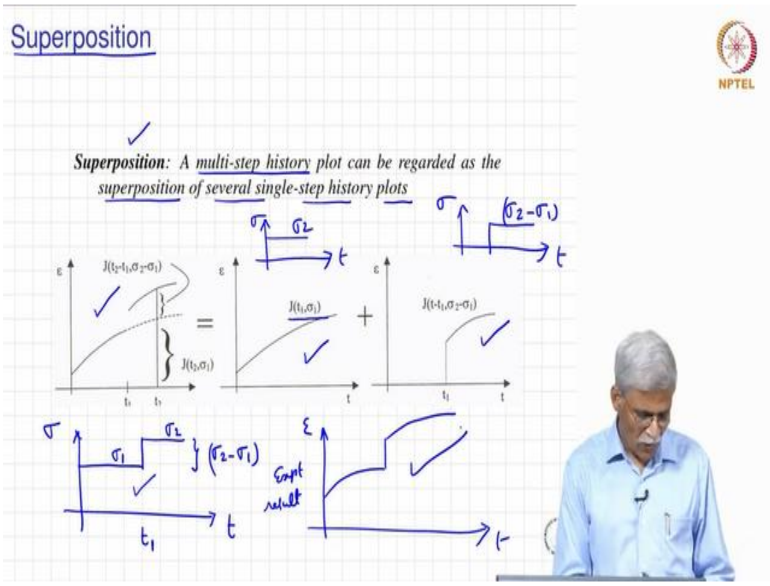

Now let us start take say something about superposition. Now the definition of superposition can be given in different ways, few slides down the line we will be talking about the Boltzmann's superposition principle which is a more precise statement. So as far as this particular discussion is concerned what we are going to say is a multi-step history plot if you can regard it as the superposition of several single step history plots then you basically you are going to say that one can superpose.

So what is the stress history that you are really looking at, so let us say that you know you are applying a stress of the following form. So this is σ, this is t, t1 and it keeps going. So probably you know you are talking about σ_1 and σ_2 so this difference is (σ_2 - σ_1). So when you apply the stress history of this particular response here whatever is the you can actually see that there is a J (t1, σ_1) plus.

So how do we really write this so you are going to have t1 here and t is the running time. So J[t-t1, (σ_2 - σ_1)]. So if I could separately take the response of this and take the response of this and add it up and if I could get the response as given here then we are going to say that these responses are super possible. So what it means is if I take a stress history like this and if this is the response that I am going to get for instance this could be the experimental result; ε versus t and if it is going to vary like this and if you could do separately do another experiment in which you apply a load only for this particular time and another one for this particular time of (σ_2 - σ_1) and σ_2 and when you add them together if you get this particular response you are going to say that this response is super possible. Okay!

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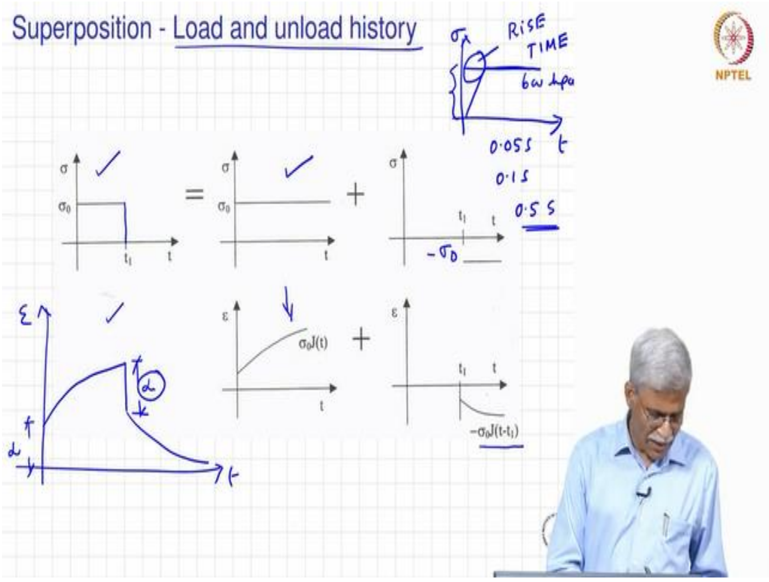

So where are the real applications for this? The real applications for this is to check, how to do the loading under unloading history. And we have seen this in the thought experiment on creep and recovery that we carried out in the earlier discussion. For instance let us say you are going to apply a load of this specific form in which you apply a σ_0 up to t1 and then after that it is zero, so what we can do we can write it as a superposition of a σ_0 that runs forever and a minus σ_0 that starts running from t1. So when you add them together you get this correct response. So now when you write the response of this material like this so that is σ_0 times J (t) and then here it is going to be - σ_0 J (t - t1), t1 is the time in which you are going to apply your -σ_0 and if you could get a response of this in this particular fashion, so I could draw a strain response something like this. Okay.

So then if you could if you get exactly the same as the sum here for the actual experimental data then you are going to say then this is super possible. Now what is the understanding that you get here? The understanding that you get here is whatever be the recovery that you are the instantaneous recovery that you are going to get and whatever is the jump that you are going to get here are identical.

It is not necessary in any experimental data to get such kind of a response but within the experimental variability that one normally encounters especially when we apply this kind of loading for bituminous mixes, these things are very, very useful to select a class of viscoelastic models for our analysis. We also need to understand few things here I will initiate the discussion here later when we are talking about flow number and flow time these things will become lot more easier. Now please see that, there is a response that is given here in this following way. It is practically not possible to apply such kind of loading during any experimental protocol. Normally always this, it will take some finite time to reach this particular stress level. So whatever is the time that it takes to apply a load of this specific magnitude is normally called in experimental mechanics as rise time and this rise time depends on the sophistication of the gadgets that you have got here. Depending on the capacity of the power pack depending on the sensitivity of the loading electronics that you have got one can reach let us say, 600 kilo Pascal, let it be the 600 kilo Pascal you can reach this loading in 0.05 seconds, 0.1 second, sometimes even 0.5 second. Since, we have discussed the earlier that the response of the material depends on the loading history, you are going to see different responses for the same material for the same 600 kilo Pascal loading based on the loading history to which it has been subjected to and this is a caution that we should exercise when we interpret the data for this kind of a material.

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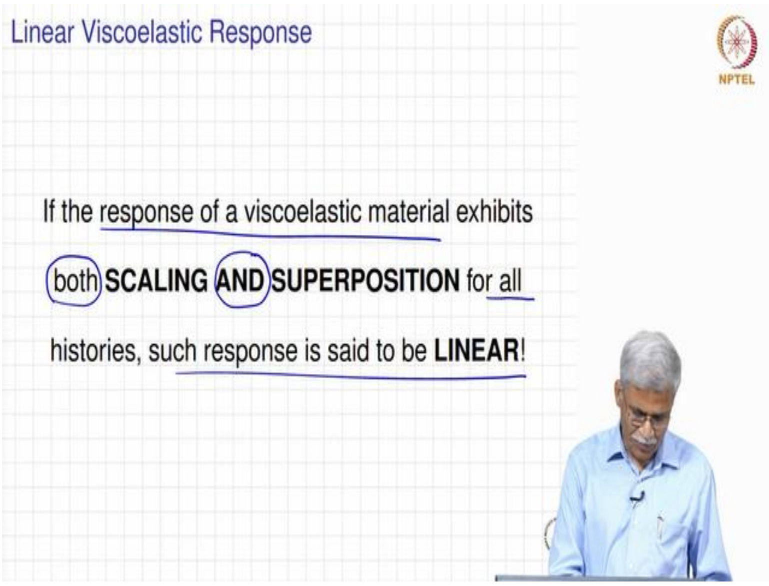

So let us make this statement very clear if the response of a viscoelastic material exhibits both this is the most important thing scaling and superposition for all stress histories such response is said to be linear. So that means when you do a creep and recovery or when you do a stress relaxation or when you test the material in frequency domain in oscillatory shear, you need to ensure that the response of the material is scalable and also superposition is valid.

If only it meets both the criteria then only you can say that the response of the material is linear and only then you can actually use any of the linear viscoelastic models. The asset in all this observation is the fact that you should also be able to include the variability associated with the experimental data acquisition. For instance in many of the bituminous mixtures testing, we allow for plus or minus 10 percent variation in the experimental data that is collected.

So within this plus or minus 10 percent variation you should be able to meet the scaling as well as superposition and only then such response could be modelled as a linear viscoelastic response and you can use any of the linear viscoelastic material. Okay?

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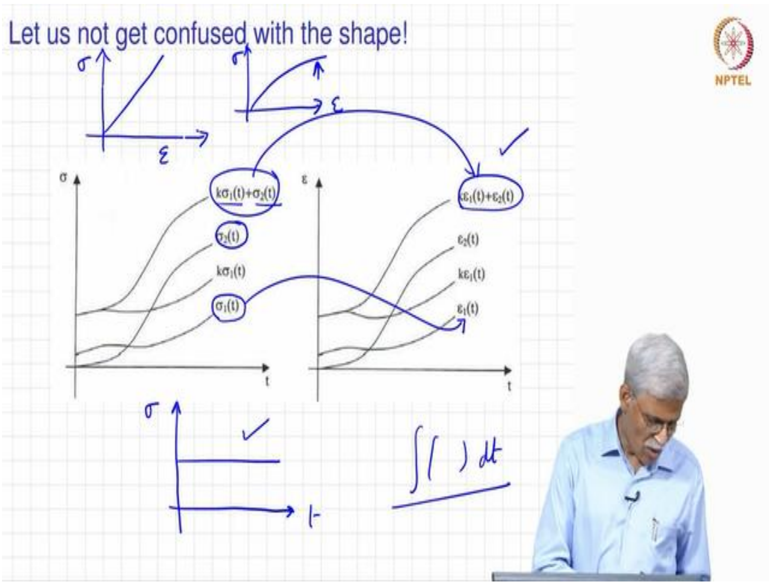

And now to emphasize more on this as of know we have been only talking in terms of stress histories which are fairly straightforward stress histories or strain histories in which we just applied something like this. Now what will really happen if you are going to apply a stress history of this particular form or this particular form? Okay, any arbitrary stress history can be applied.

Now we need to understand that, there is a reason why we adopted this kind of histories because we could use straight forward a rare type model because there is the stress history is means if you take a derivative of this it is zero because the load way there is no variation. But if it is going to be an arbitrary history we need basically an integral kind of a model which we will discuss in the next slide.

But we need to make one statement here clearly. So this is σ_1 as a function of time and this is nothing but k times σ_1. Similarly there is σ_2 and you can actually see that this is K times (σ_1 + σ_2). Now when we are really talking about superposition here so this for the σ_1t, if this is ε_1t and whatever you see here if you are going to get this as the summation of separately k times (ε_1t + ε_2t). Then you are going to say that this is super possible.

Now if I show this picture as it is ε versus t and asks you whether the response of the material is linear or not, most of you will basically say that oh it seems to be varying it is not a straight line. So if the response of the material is nonlinear. No it is it has nothing to do with the shape of the strain curve in the strain time domain.

What it means is since there is a variability associated with the time we need to all check for scaling as well as superposition and only then we can conclude that whether the response of the material is linear or non-linear. Recollect the discussion that we had about isochrones, wherein we drew pictures something like this. So this is a linear isochrones, this is isochrones in which the response goes like this again let us not get confused just because the stress was a strain space this seems to be nonlinear.

This is basically because of the manner in which we carried out we defined our isochrones to be of this particular way. So if we emphasis once more you can call the response of the material linear only if it meets scaling and superposition and not anything else. Right?

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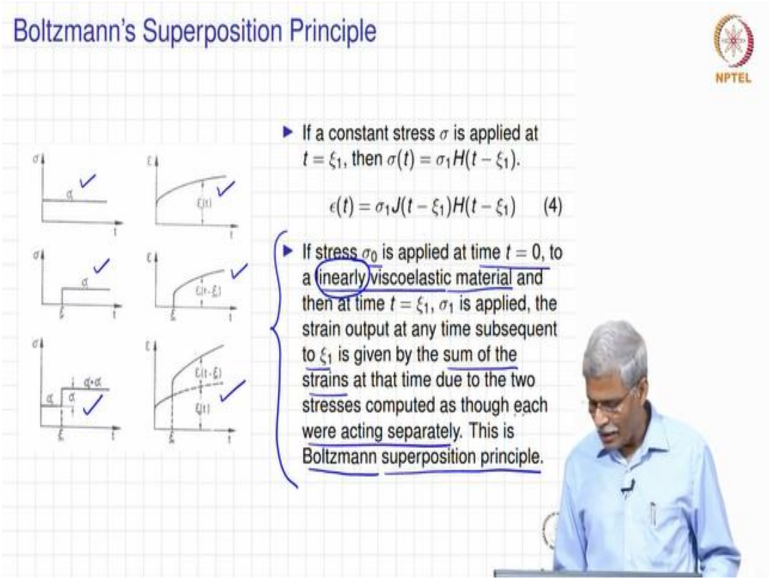

So I have actually written a lot of text here, for the benefit of the students and this is the statement of the Boltzmann superposition principle. In fact what we presented here is some arbitrary stress history. So if you are going to have an arbitrary stress history and if you really want to model, the response of this material especially within the linear viscoelastic regime it actually makes sense to use an integral model of expression of this particular type.

So how are we going to do that here so let us see that here. okay. First and foremost thing is we will take a look at this particular picture so we apply $\sigma(\sigma_0)$ and this is what it is going to be so you can apply σ_1 at this particular time and this is the response is going to be. So if you have a step stress this is the response and we discussed it here. So what it means is if we read it here clearly and I suggested that all the students especially at the undergraduate and graduate level spend some time thinking about this statement very carefully.

If stress σ_0 is applied at a time t is equal to zero, to a linearly viscoelastic material the emphasis is on linearly viscoelastic material. Then at a time t is equal to ξ_1, σ_1 is applied, the strain output at any time subsequent to ξ_1 is given by the sum of the strains at that time due to the stress is computed as though they were acting separately. So this is the Boltzmann's superposition principle.

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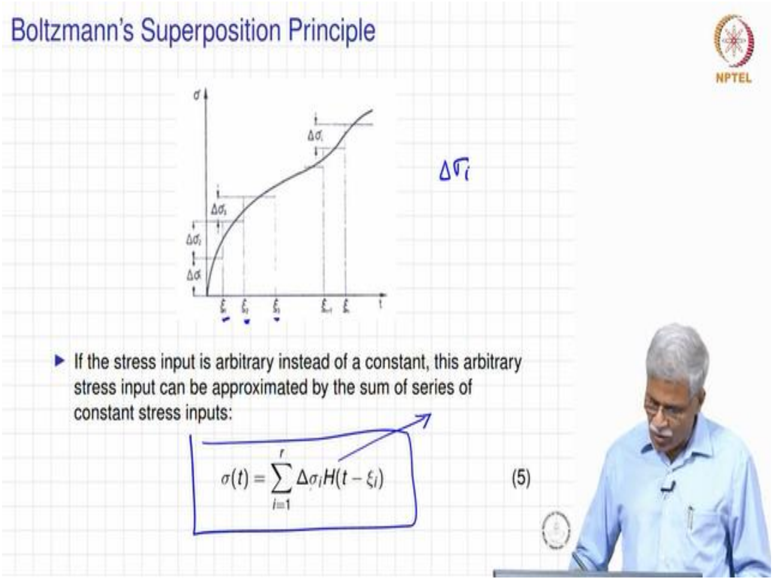

So let us use this to see how we can model the response of the material for any arbitrary stress history. So let us assume that you know you are applying a stress history of this particular form. So now what we are going to do is we are going to kind of split this into very small $\delta\sigma_i$, that is given here. Okay? And for each time step it is going to be given as ξ_1, ξ_2, ξ_3 like this.

So what is so we are going to approximate this stress history that is given here in the following form? So $\sigma(t)$ is the summation of $\delta\sigma_i$ and H is nothing but the Heaviside function that was introduced in the earlier.

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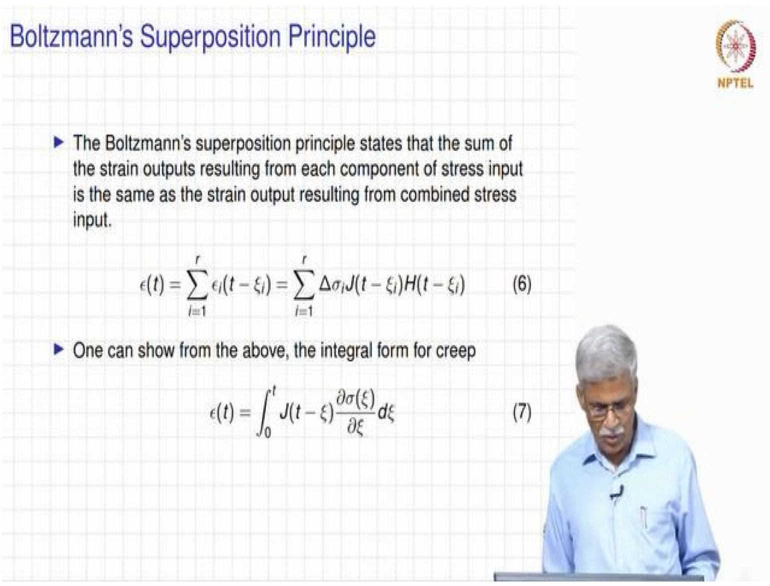

So if that is going to be the stress history to which you are going to subject it to then the strain response ε(t) is going to be nothing other than. So this is going to be so if you just take a look at it so this is going to be $\delta\sigma_i$ H (t- ξ_i) summed for all the steps i is equal to 1 to r.

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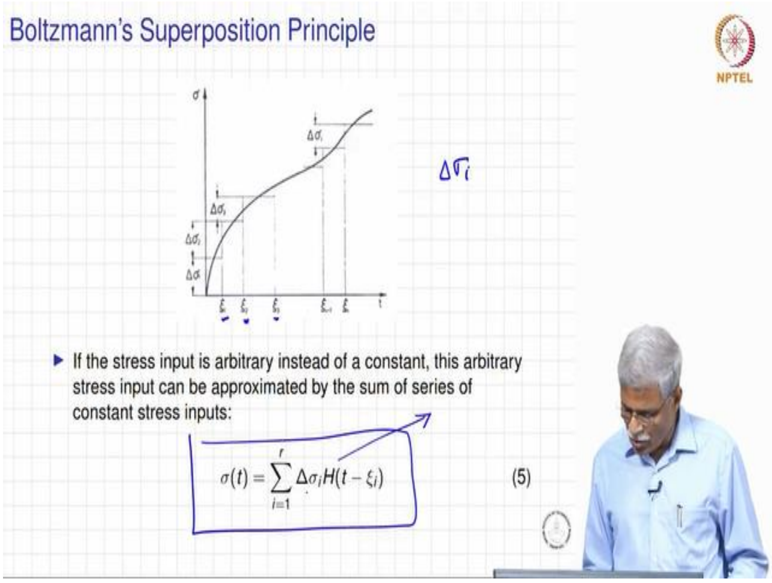

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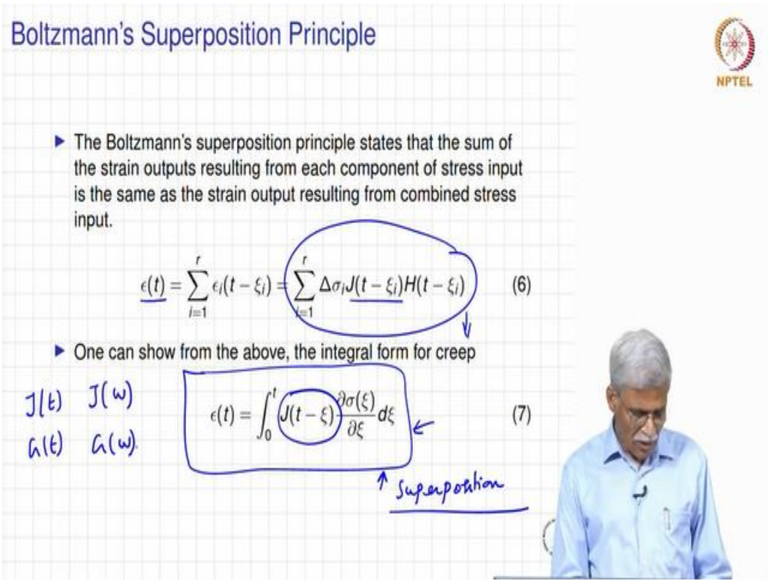

So it is going to be the creep function that is given here your creep compliant function given as J (t- ξ_i) H (t- ξ_i). So I won't spend time on how one takes this summation to an integral elementary a calculus book should tell you that. So basically when we write this in the integral form this is the expression that you are going to get. Now ε(t) is integral 0 to t J (t - ξ_i) d σ (ξ_i) divided by d (ξ) times d (ξ).

So this is the familiar form of integral expression for creep that is used here. Now we need to understand that this takes care of the superposition. We do not yet know whether for this arbitrary stress history scaling will work or not and that is something that we need to do it based on the rules that we explained earlier as far as the scaling is concerned. Now we are stuck here, we yet do not know what kind of form that one should use for J.

Because right now what we introduce we have introduced J (t), we have introduced G (t) also we have also introduced J (ω), we have also introduced G (ω). So what should be the structure for this we need to spend some time thinking about it so let us do that first, okay?