Mechanical Characterization of Bituminous Materials
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Lecture No 41
Dynamic Modulus of Bituminous Mixtures - Part 2

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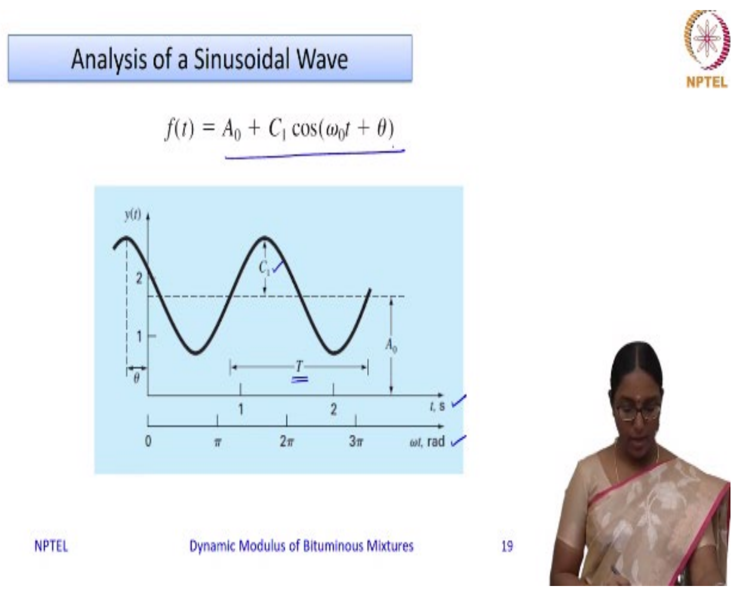

Now how do you analyze a sinusoidal wave? Let us look at it first. The figure shows the sinusoidal wave. As we can see here, this is the amplitude and this is the time period and here the time is given in seconds and here it is, in radians. So this function (this series) can be defined in terms of $A_0 + C_1 \cos(\omega_0 t + \theta)$.

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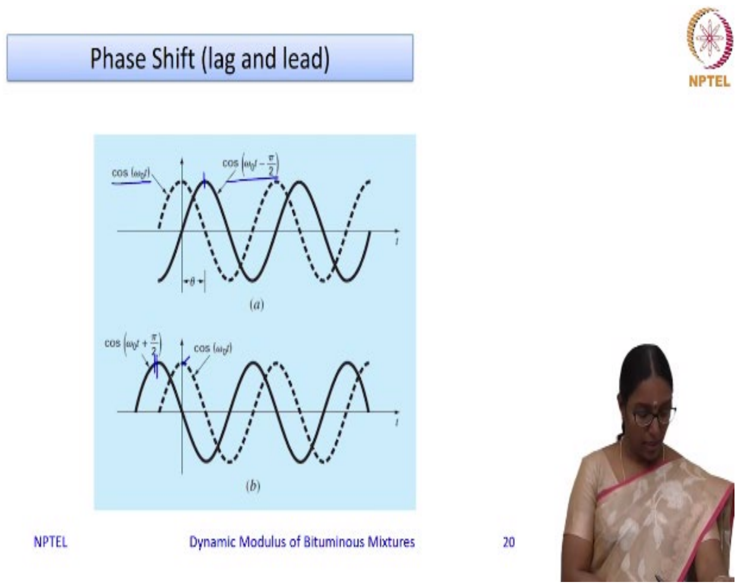

Now the adjacent or the next wave form, say, after stress the strain follows. There could be a lead or a lag. Say in the case of a stress and the strain, so this ($\cos \omega_0 t$) defines the first series. Then the next series will be lagging behind, and becomes $\cos(\omega_0 t - \pi/2)$. Likewise, you can have a leading response.

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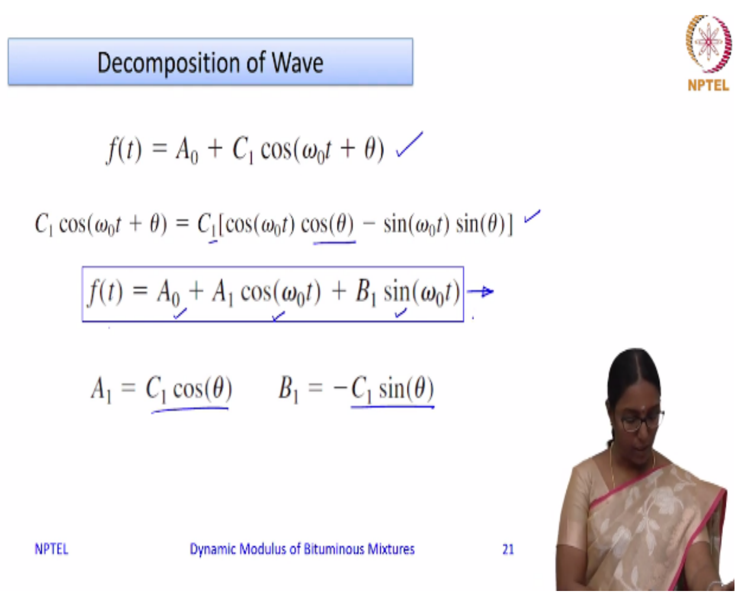

So how you decompose this wave? So as I said a sinusoidal wave can be explained as $A_0 + C_1 \cos(\omega_0 t + \theta)$. Now we can decompose this cos (A + B) as (cos A cos B – sin A sin B) here, so you can rearrange this expression as f(t) is equal to $A_0 + A_1 \cos \omega_0 t + B_1 \sin \omega_0 t$, where A_1 is your $C_1 \cos \theta$ and B_1 is your $-C_1 \sin \theta$. So this representation of the series will actually help you in identifying what is in-phase component of the series and what is the out-of-phase component which will be helpful for you, in determining the Dynamic Modulus and the phase lag? So, a sinusoidal wave is actually in this form.

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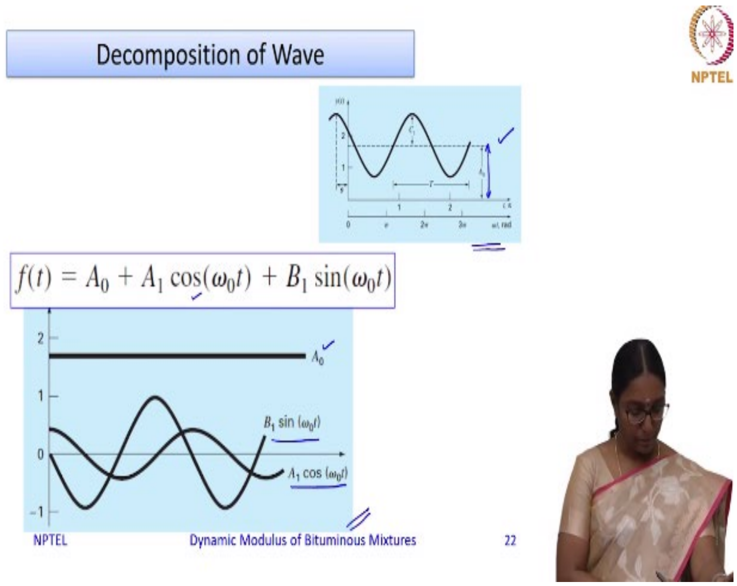

So this is you can see how the decomposition is done. So this was your original wave which is shifted by a distance of A_0 as here. So this A_0 forms the first wave and this is the $A_1 \cos \omega_0 t$ which is your in-phase wave and this is $B_1 \sin \omega_0 t$, this represents the out-of-phase component. So this wave form as you see here in this first figure is now decomposed into 3 forms as shown here.

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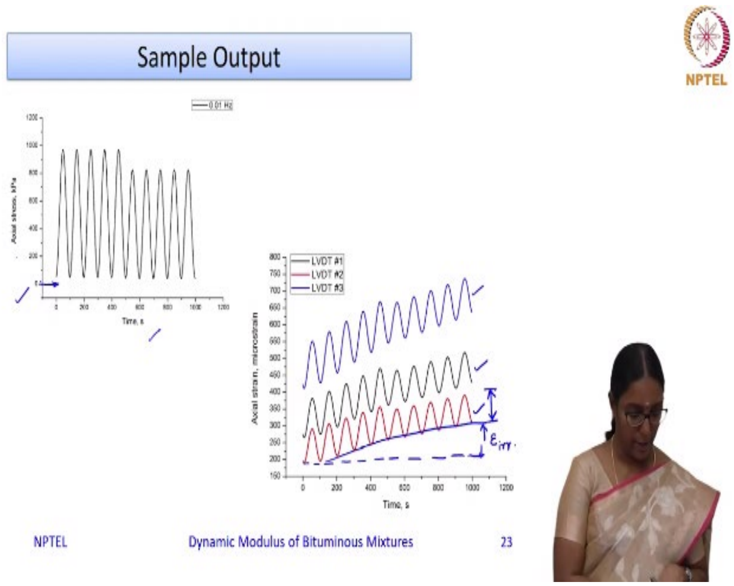

Now in your Dynamic Modulus test, these are some of the typical outputs that you get. The first one is the axial stress versus time. These are some 10 cycles which are randomly picked from the test data in this figure. So you see the axial stress on the y axis and time and you see that this is a haversine loading that is applied.

Now as I said you use 3 LVDTs for the capturing of strain response. These are shown here. The response from the strain LVDTs are shown here in these 3 colors, these are the 3 LVDTs okay. As you see that the response is also has a haversine in nature but it is not horizontal, why because as you know that in every test cycle there is going to be recoverable deformation as well as an irrecoverable deformation. So here, this is the first strain response. As you go further and further you see that there is a certain amount of deformation, this is irrecoverable okay. But what we are interested in this computation is only the recoverable strain. What is recoverable strain? So the peak to peak strain is only what we are interested in, but one should understand that there is always an irrecoverable deformation and your strain is going to shift like this.

And also one should understand from this figure is that all the 3 LVDT may not give you exactly the same value as I said this is highly heterogeneous material. So you see variabilities in data in the 3 LVDTs. Normally what we do is that you average out the data from all the 3 LVDTs.

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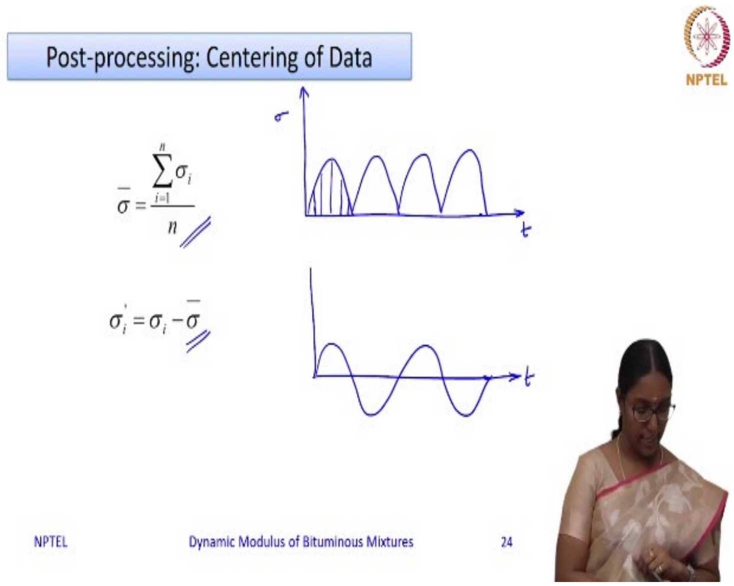

Now, how to process this data? As I said your loading wave is going to be a haversine load pulse. Now, whatever definitions that we have given for Dynamic Modulus and phase lag is in terms of a sinusoidal wave. So I just have to center this data, so that it will look like a sinusoidal wave. So what is done is that you take all the data that is collected for stress.

Say for example I have in one Stress cycle I have collected 50 data points, likewise for whatever stress data you have collected you average it out. So if this is (sum of σ_i)/n, average stress value. Then what you do is that from every data point you subtract that average stress value. So that you have simply shifted origin to a little higher. So that the response remains the same but you can represent it in the form of a sin wave.

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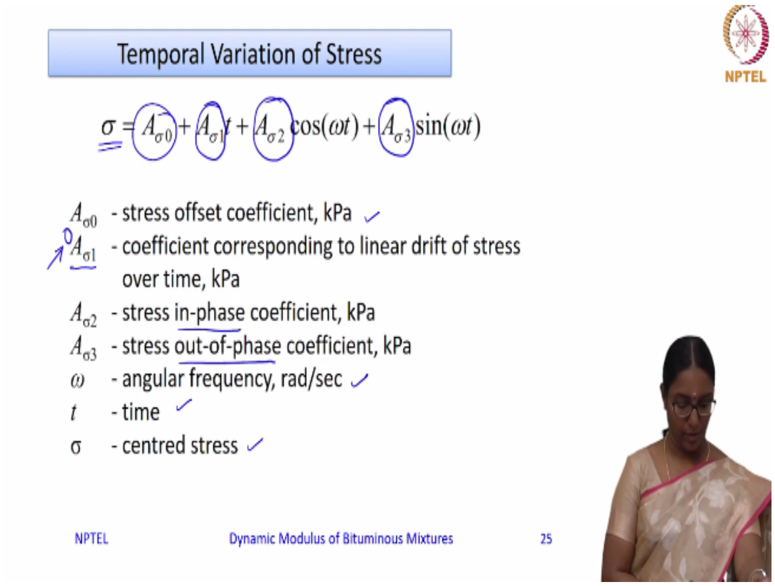

Now, this Temporal Variation of the stress data that you have collected with time and the stress. This you can fit a regression equation for this so it will look like the Fourier series, as we have already explained. So your stress wave can be represented as $A_{\sigma 0}$ and $A_{\sigma 1}$ t, $A_{\sigma 2}$ cos(ωt) and $A_{\sigma 3}$ sin(ωt). So I have 4 different coefficients and these coefficients can be collected using any curve fitting algorithm.

You have to understand that there is no model used in here, it is just a fitting of that curve with a series like this. Now of this $A_{\sigma 0}$ is the first coefficient, which will represent the stress offset coefficient as we have seen when we tried to decompose this stress wave. And the second one is $A_{\sigma 1}$. This is the one which corresponds to the linear drift of stress, for example if there is any drift in the stress data. But normally we apply the data for the 10 cycles we say that it is mostly constant. So this term essentially goes to zero but since, we want to have a similar expression for this strain as well, so it is kept there as such.

And the 3rd term is $A_{\sigma 2}$. This corresponds to the stress-in phase coefficient as I already mentioned and the fourth parameter that is $A_{\sigma 3}$ that corresponds to the out-of-phase coefficient and ω is the angular frequency in radians per second, t is the time and σ is the centered data that we have collected.

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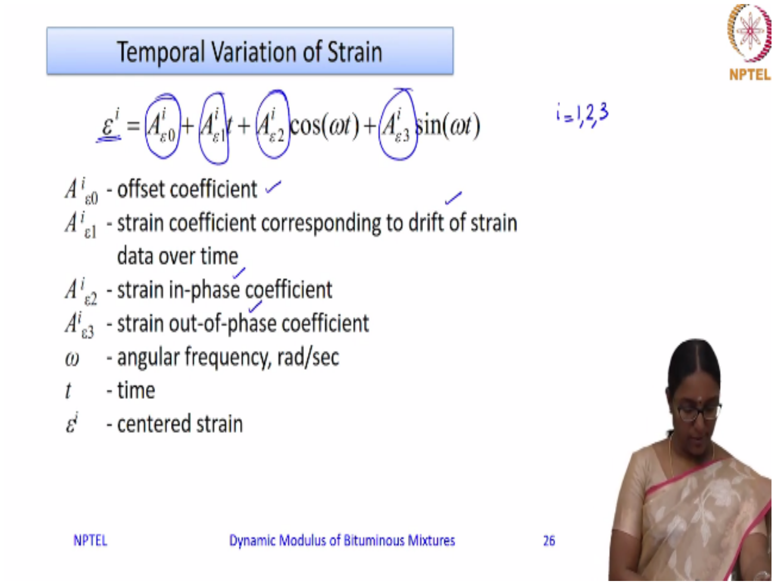

Similarly, for the LVDT data, that is the strain data also you can fit an equation like this. ie., for each LVDT data, we will fit a model, so say, for example ε^i, where i corresponds to the LVDT, so you have i is equal to 1, 2 and 3 for the 3 LVDTs. Again these are the parameters $A_{\varepsilon 0}$ for the i^{th} one, $A_{\varepsilon 1}$ and the $A_{\varepsilon 2}$ and $A_{\varepsilon 3}$ are the parameters. In a similar way, $A_{\varepsilon 0}$ corresponds to the offset coefficient, $A_{\varepsilon 1}$ is the drift of strain. As I said, you have a drift in the strain data. And next is $A_{\varepsilon 2}$ is the in-phase component and $A_{\varepsilon 3}$ is the out-of phase component and the other terms remain the same.

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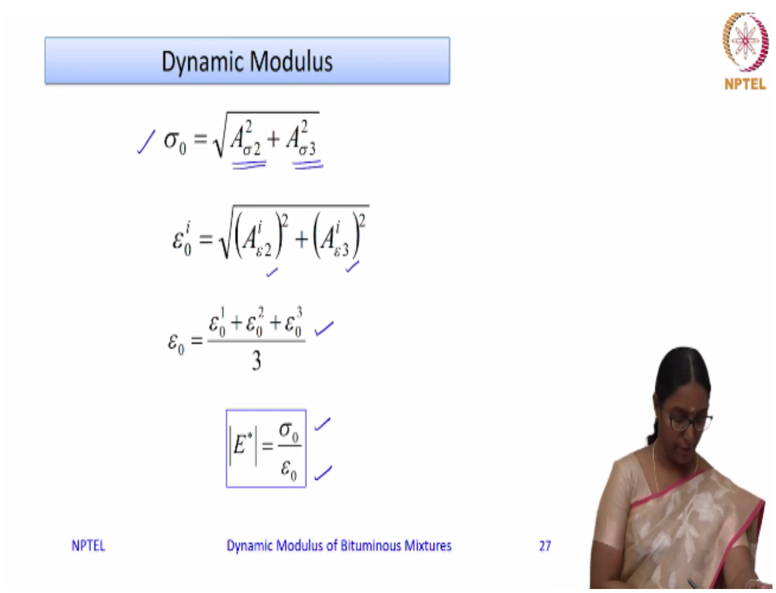

Now from this data, you know, what is the in-phase coefficient and stress out-of-phase coefficient. Root of (square of in-phase coefficient + the square of out-phase coefficient) will give you the peak to peak stress value of the data. Similarly, for the strain you can get the peak to peak strain from the in-phase coefficient and out-of phase coefficient by squaring and adding and taking the root and as I said you have 3 LVDTs. So you will get 3 ε_0 values or the peak to peak strain values from the 3 LVDTs. You can take an average of the 3 values to find out how much is the peak to peak strain value. And as we have already discussed mode of the modulus or the mode of the E* will be given as σ_0 by ε_0, so this gives you a Dynamic Modulus for one particular frequency and one particular temperature of testing.

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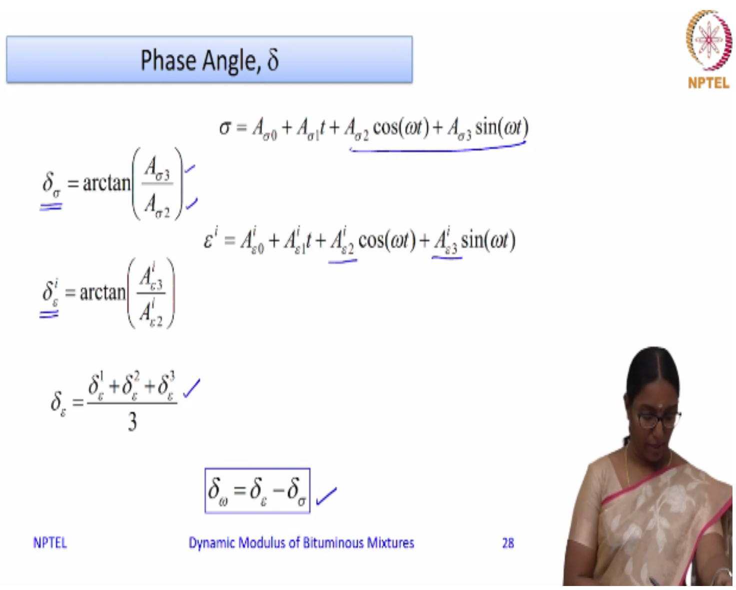

Now coming to the phase angle, see from this equation, you have the in-phase and out-of-phase component for stress, you can take arctan of the out-of-phase component by the in-phase component that will give you the phase lag for stress. Similarly, the phase lag for strain for the i^{th} LVDT can be taken from the in-phase and the out-of-phase components as arctan and for all the 3 LVDTs you take an average to find, what is a phase lag in strain. So once we get the phase lag in stress and the phase angle in strain you can take a difference to find what phase lag between the stress and the strain in terms of ω.

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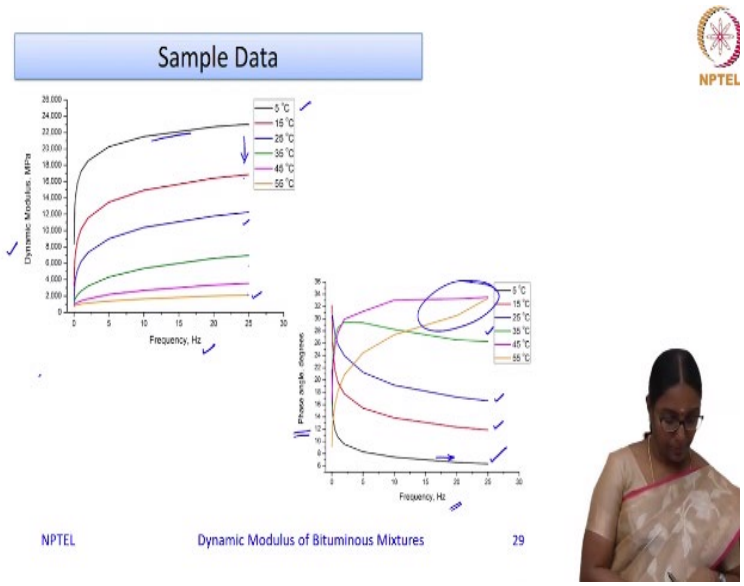

Now this is a sample data from a test that is conducted at IITM on a specimen at 6 different temperatures and we have done it for 10 different frequencies starting from 25 Hz to 0.01 Hz. So here, Dynamic Modulus is in the y axis in Mega Pascals and frequency is in x axis in Hz and each line represents one temperature.

So let us see the 5 degree Celsius data, so this black line is your 5 degree Celsius data. You see that as the frequency increases, the Dynamic Modulus increases. As I said higher frequency, shorter duration of load application and higher will be the modulus response. Material will be behaving more like an elastic material. So your modulus will be very high, as we move to higher temperature, you see that the modulus value has decreased which is quite obvious. In the last one for a temperature of 55 degrees Celsius you start with the 0.01 frequency and then as you go to higher frequency, the modulus has slightly increased.

Now let us look at the phase angle data, so here you have the phase angle in degrees and this is a frequency in Hz provided for all the different temperatures. Say, for example, this is the 5 degree data in black color. As expected, as your frequency increases the material behaves more like an elastic material so the phase angle is going to decrease. At 5 degree Celsius, as the frequency increases you see that the phase angle is actually decreasing. Similar is the case with 15 degrees Celsius 25 degrees Celsius and as well as 35 degrees Celsius.

And between the temperatures as we know that if the temperature keeps on increasing more and more viscous behavior will come into play and your phase angle will be more. So as you can see from here the bottom line starts with 5 degrees Celsius and the phase angle is actually going up when you when the temperature is increasing but what is interesting here is that at 45 degree and 55 degrees Celsius, the variation of phase angle with frequency is not following the trend as we have discussed so far. So this is something which the material response is all about. This is a complicated behavior so at the higher temperatures though we do the test at a lower strain levels of say 75 and 125 micro strains and expect the material is in the linear viscoelastic regime, it shows such behavior as well, so one has to be very careful, in interpreting the data.

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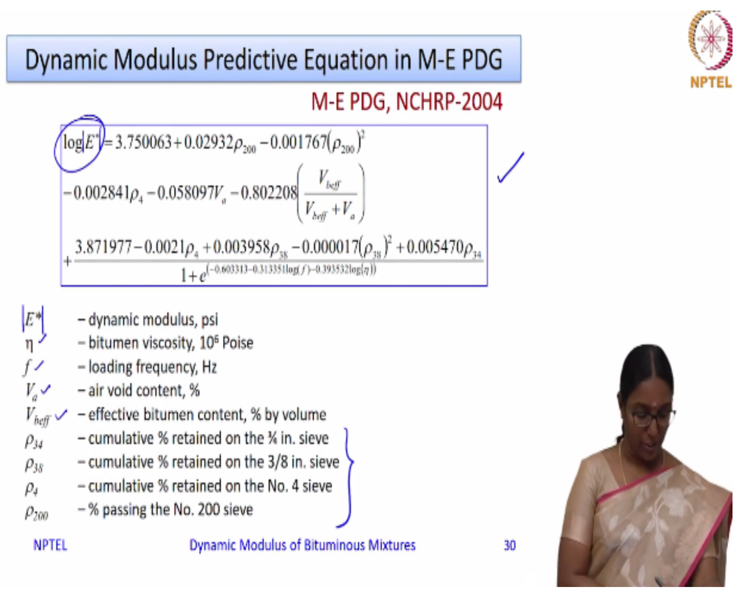

Now, so this is how we estimate the Dynamic Modulus of the material and phase angle of the material in the laboratory. In M-E PDG, they also suggest that if this testing setup is not available you can go for a predictive equation, so they have provided predictive equation to find out E* value. It is actually |E*| value or the Dynamic Modulus value. So this is Dynamic Modulus, though it is written without that modes notation in lines.

So this you can see that it is a function of various parameters like the loading frequency, then air void content of the sample, then the bitumen viscosity and the effective bitumen content and the various aggregate parameters or the mixture parameters. Say for example the percentage of material retained in various sieve sizes or its essentially the gradation of the mix. So in terms of all these parameters, this Dynamic Modulus predictive equation is given by M-E PDG, so one can also use this predictive equation.

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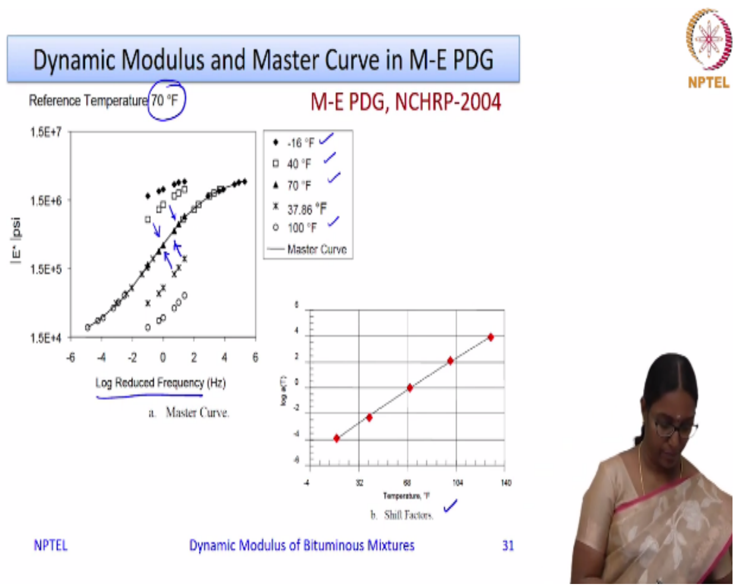

So, how does one uses this Dynamic Modulus and phase lag data in the mechanistic empirical method of analysis? As I said you can do this test at various temperatures. Say in the example that I have shown we have done it for 6 different temperatures and at different frequencies, okay. But in the analysis one may encounter other load frequencies as well as other temperatures. So in order to get that particular value what you normally do is that you prepare what is called as a master curve.

So you Shift this data into a sigmoidal function to represent one general expression or one general curve, which will give you the Dynamic Modulus and phase lag for any particular temperature or frequency that you are looking at. So the preparation of the Dynamic Modulus Master Curve is explained by Professor Padma in her lecture. So we actually use a constrained fitting method so that your data is fitted to one sigmoidal Master Curve.

So as you can see here, this is taken from M-E PDG, so they have taken a reference temperature of 70 degrees Fahrenheit which is approximately 21 degrees Celsius and the data that is collected from different temperatures say -16, 40 70 and 100 degree Fahrenheit, they are actually shifted like this to form one sigmoidal curve and this forms a reduced frequency, which is used to shift the data and the various shift factors which are adopted for each of these temperatures is provided here.

So one can prepare a Master Curve, and it is this Master Curve which goes into the M-E PDG analysis so that for any loading frequency or any temperature that is encountered in the analysis, the corresponding Dynamic Modulus value can be taken from this Master Curve.

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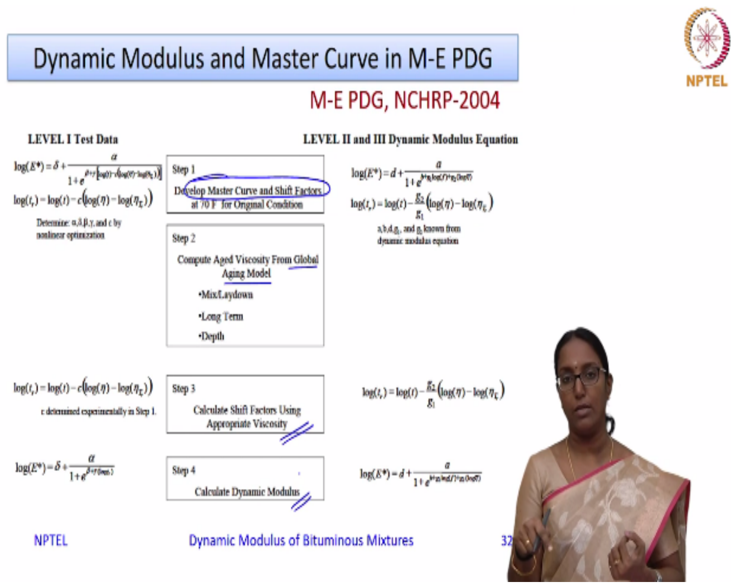

So this is how M-E PDG suggests this methodology works is that, see, you have to develop a Master Curve and the Shift Factors for the reference temperature of 70 Fahrenheit as they have suggested and another important aspect that is considered in M-E PDG analysis is that the aging of the material can also be included in this analysis. As we know, as the service life increases the material will start aging due to many reasons.

As I said, this is already discussed by Professor Nivitha in her lecture. So when the material ages the modulus also will change. So the effect on the pavement will be different if it is an aged material. So this also can be incorporated in the Mechanistic Empirical Analysis, so they use it using a global aging model, which is also incorporated. So using those models you calculate the appropriate viscosity change in the material. If it is aged and then that viscosity will be used to calculate the Dynamic Modulus. So 2 things are here, one is it, Dynamic Modulus is captured and the Master Curve that is prepared which goes in and over a period of time what happens to this dynamic modulus is also incorporated in the form of the temperature or global aging model. So when you do an analysis say 20 years or of 15 years of time, at every instant of time; according to the kind of loading and according to the temperature at that instant the modulus will be taken from the Dynamic Modulus Master Curve and as age passes you have to incorporate the effect of aging also and the modulus value has to be appropriately corrected. So this is a whole process that is being done in M-E PDG.

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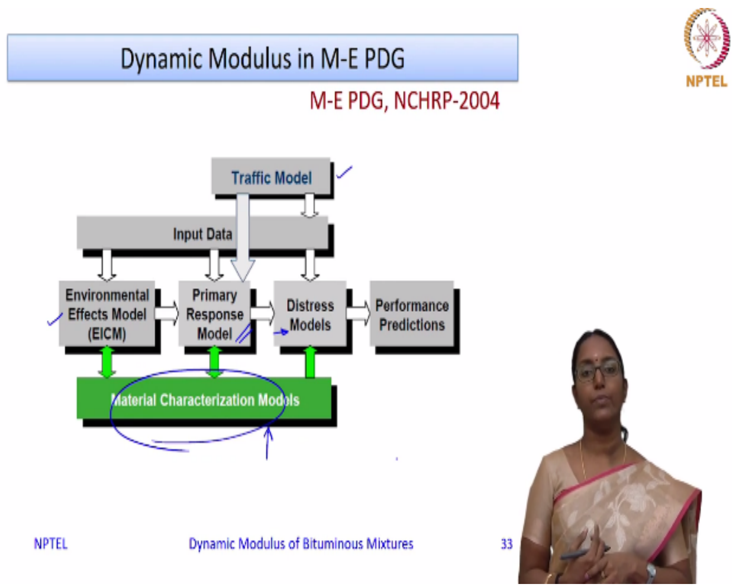

So I will summarize, in M-E PDG, so this is a general framework of how M-E PDG analysis works. So there is a traffic model, you get the data which is Environmental Effect and you do the material characterization. So these material characterization Models will go in to the analysis procedure where the Primary Response Model will compute, what are the stresses, strains and deformations that happen?

And then they are connected to the distress model. As I said, this is a response of the material in the laboratory now this has to be correlated to what happens in the field? So the stresses and the strains developed are actually correlated to the field rutting and fatigue behavior in the form of distress transfer functions and in the distress transfer functions you will check whether the pavement will sustain the type of loading without any failure in the form of rutting or in the form of fatigue cracking.

And if it is satisfactory you will proceed with different layer thicknesses and the materials that you have used otherwise you will repeat the process. So this is the whole outline of M-E PDG though we are not discussing this pavement design in this course. I just wanted to emphasize the need for a modulus, which is time dependent as well as temperature dependent to be used in a payment analysis strategy, so that we can design the pavement according to the actual response of the material in the field.

So to summarize what we have seen is what is a Dynamic Modulus? And what is the stress function? Or how the loading is applied to test this material? What is the data that you collect and how do we process this data in order to arrive at the Dynamic Modulus value as well as the phase angle value? And also at what temperature do you have to do it? And what frequencies that you have to do it, is what we have discussed, thank you.