

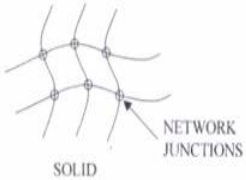
**Mechanical Characterization of Bituminous Material**  
**Prof. J Murali Krishnan**  
**Department of Civil engineering**  
**Indian Institute of Technology Madras**

**Lecture No 4**  
**Linear Viscoelastic Response Part-02**

(Refer Slide Time: 00:14)

**Material Microstructure**

- ▶ **Metals:** Strong interatomic force - Atomic crystalline structure
- ▶ Elastic response is due to small deformation of crystalline structure



The diagram illustrates a crystalline structure with a grid of atoms connected by lines representing interatomic forces. The label 'SOLID' is placed below the grid, and 'NETWORK JUNCTIONS' is placed to the right of the grid, with an arrow pointing to one of the junctions.

NPTEL

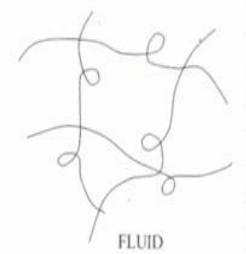
Okay? Right! So where is the understanding when there are many ways in which these things can be motivated, but I just thought that I will share some of this information that is already available in many textbooks. So if you are looking at a solid, there are going to be strong inter atomic forces, and there is a precisely defined crystalline structure and whatever is the elastic response that you are going to see is especially, we are talking about small deformation; this is going to be the small deformation that you see for the crystalline structure.

(Refer Slide Time: 00:50)


**Material Microstructure**

*Van der Waals*

- ▶ **Fluids:** Weak attractive forces
- ▶ Continuous movement of particles - fluid flow



FLUID

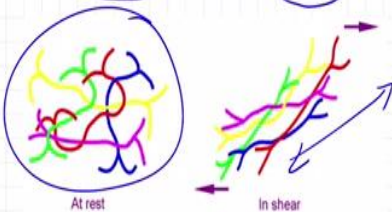


But, if you are going to talk in terms of the fluids, mostly you are talking in terms of Van der Waal what kind of your forces and there is always some kind of a wiggly noodles or spaghetti kind of structure that you are going to see.

(Refer Slide Time: 01:12)


**Viscoelasticity**

- ▶ Viscoelasticity = Elastic solid + viscous fluid ✓
- ▶ FRP, polymer, polymer composite, soil, concrete, asphalt, alloys  
...
- ▶ Viscoelastic solid-(like) and viscoelastic fluid-(like)



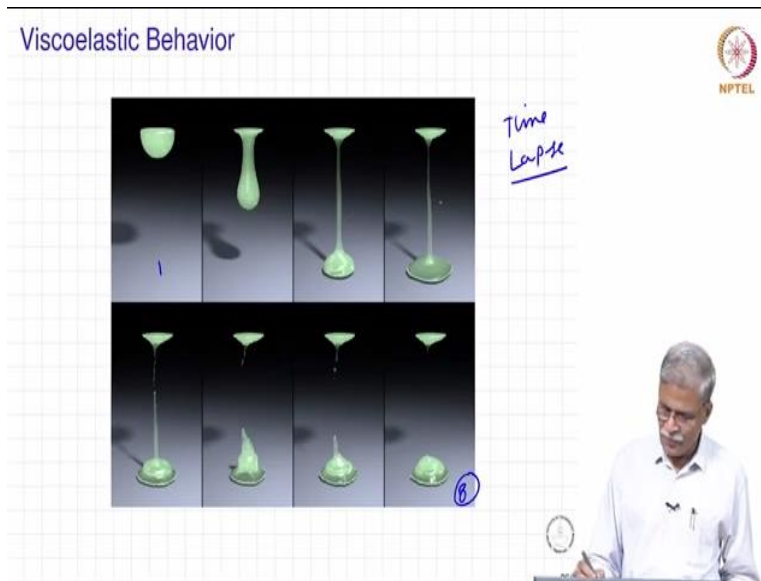
At rest

In shear



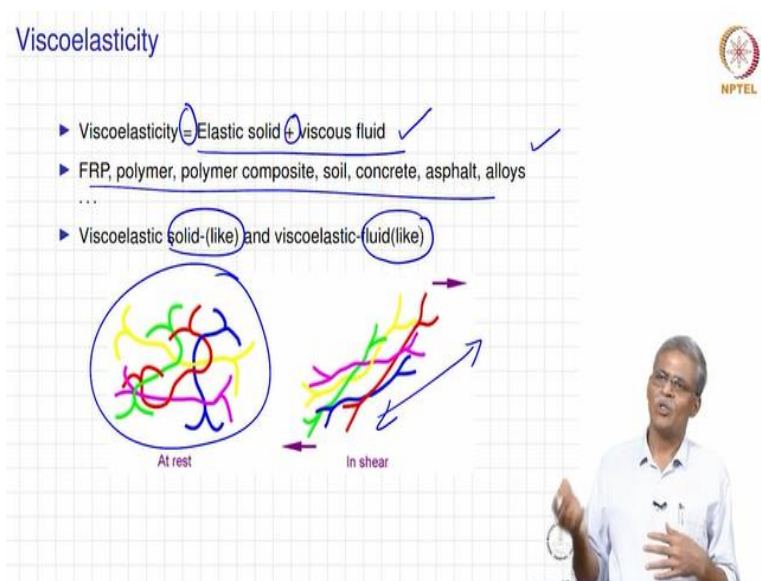
And when and in fact you can actually see when you have a structure like this. And when you are trying to subject it to some shear, that is going to be some kind of straightening motion that you can say,

(Refer Slide Time: 01:25)



And in fact it can be very nicely explained by means of this kind of time lapse picture of a nice viscoelastic gel, actually you can see, if you go from 1 and all the way up to 8. You can see how the gel starts flowing, the thread gets cut. And then finally you can actually see how the shape exactly tries to recover, over a period of time. So this is more or less the response that we are going to see here.

(Refer Slide Time: 01:54)



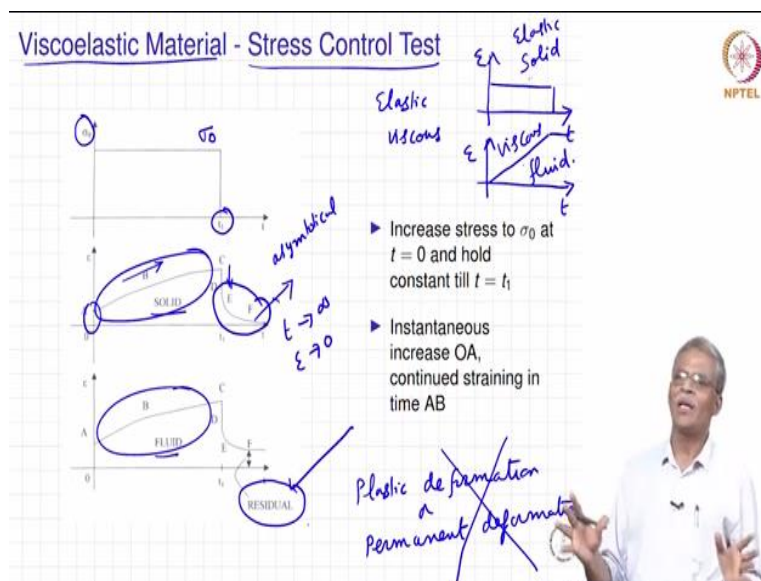
So to reiterate viscoelastic behavior can be considered. Again, this equality has to be very carefully done. These also have to be very carefully understood. As of now for this course, this is taken to hold true, but this is not the decomposition that one looks for this is a typical additive

decomposition that we are doing. It is not necessary that one should have such kind of decomposition.

It is very simple and it helps us to understand our material which is bitumen and bituminous material. So, we can actually have many of these materials behave in this way and I have mentioned it earlier also on the now we are going to go a little more further and try and see whether we can differentiate this in little more in terms of viscoelastic solid like and viscoelastic fluid like.

It becomes very tedious, when we start using the word, like, a very well, but henceforth when we say viscoelastic solid what we really mean is this elastic solid like. If it is going to be viscoelastic fluid it is going to be viscoelastic fluid like because these divisions are arbitrary and they cannot be carefully delineated, even in a sophisticated experiment. Because most of the time, the experimental artifacts will play a critical role here. And you will never be able to conclude whether the response of the material is solid or a fluid and as we have all discussed now time plays a critical role here.

(Refer Slide Time: 03:32)



Okay, so now let us talk in terms of the same thing viscoelastic material. We do a stress control test. So what we are going to do it so let us do this very carefully. So, I applied a  $\sigma_0$  here for time  $t_1$ . So which means I applying  $\sigma_0$  under time  $t_1$ , I bring it to zero. Now, this is the hypothetical

response that I am expecting for a solid as well as a fluid, irrespective of whether it is for solid or a fluid, please understand that this is a combination of elastic and viscous.

And as I mentioned in the earlier slide this combination need not have to be additive also. So that is going to be an instantaneous jump, OA here and as the load is sustained there is going to be a continuous deformation here. If it was an elastic solid, if you recollect the response it was the same. The stress strain relation was independent of time and you had a constant as the modulus value. If you are talking in terms of a viscous fluid, you essentially mentioned that it was going something like this because you are  $\epsilon_0$  was constant.

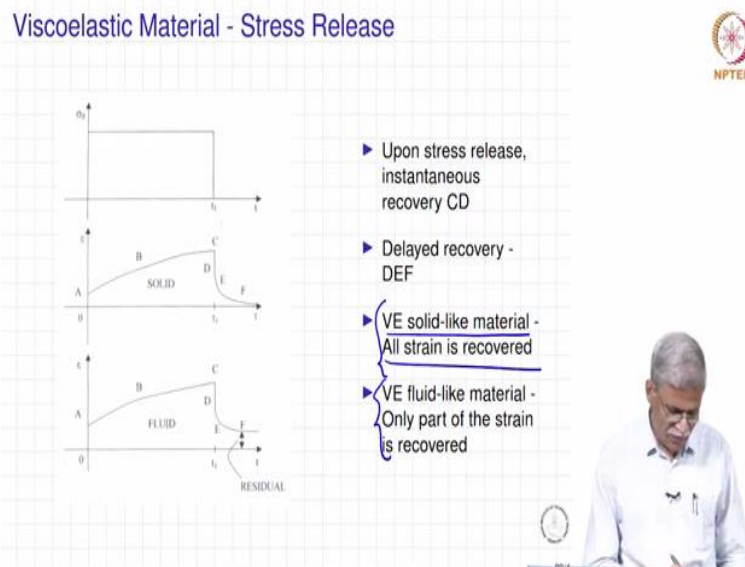
Now what we need to do is we need to understand these two things together and try and see how the response of the material be. So it can be something like this and at C that is at a time  $t_1$  when they unload this material there is going to be a drop here. Similar to the drop that I am going to get, but of course what you are going to see here, this is going to be exactly the same there is not going to be any recovery there. So this is for the elastic solid and this is for the viscous fluid.

And so there is going to be a jump here, in terms of C and D which is the recovery. And there is going to be some kind of a time dependent recovery here, given in terms of D, E, F. Now if we do the same experiment, and you can actually see that there is a difference that is given in terms of a solid, in terms of a fluid. Since a fluid, as can be seen from the viscous fluid has a deformation which is residual. Please make a note of the terminology that is used to here this is called as residual deformation, do not confuse this with the plastic deformation or permanent deformation. This is the correct way of representing it. This is called as the residual deformation. Now, how should we say that this is a solid and this is a fluid? We know that after unloading, a solid will recover all the strain, and it will go to a state in which the strains are zero.

But how long it is going to take here, this strain recovery is basically going to be asymptotical. So that means, as time goes to infinity,  $\epsilon$  will go to zero. That is what is going to be. And whereas, in a fluid at finite time you will be able to see your residual deformation which may not vary with respect to time. And this is where now you need to understand the point that I emphasized earlier on, what is the role of the experimental time.

But what we can also do is and this will become very apparent when we start looking at spring and dashpot kind of a model is, we can actually look at the loading portion. Here this portion where the strain increases and come to some conclusion about whether the response of the material is solid or fluid; viscoelastic solid on viscoelastic fluid and for that we need some kind of a model but at this point in time we do not really have a model we are only talking in terms of some hypothetical structure. So we need to help those models we will discuss later.

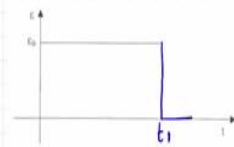
**(Refer Slide Time: 08:06)**



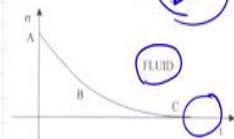
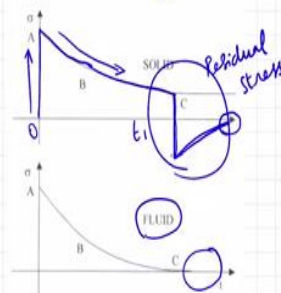
Now, let us look at the same thing is the stress release. And it is very clear the viscoelastic solid like material, all the strain is recovered asymptotically. Similarly, only part of the strain is recovered.

(Refer Slide Time: 08:22)

### Viscoelastic Material - Strain Control Test



- Increase strain to  $\epsilon_0$  at  $t = 0$
- Instantaneous stress jump to OA
- Stress required to maintain  $\epsilon_0$  reduces gradually, ABC
- See the difference between solid-like and fluid-like material.



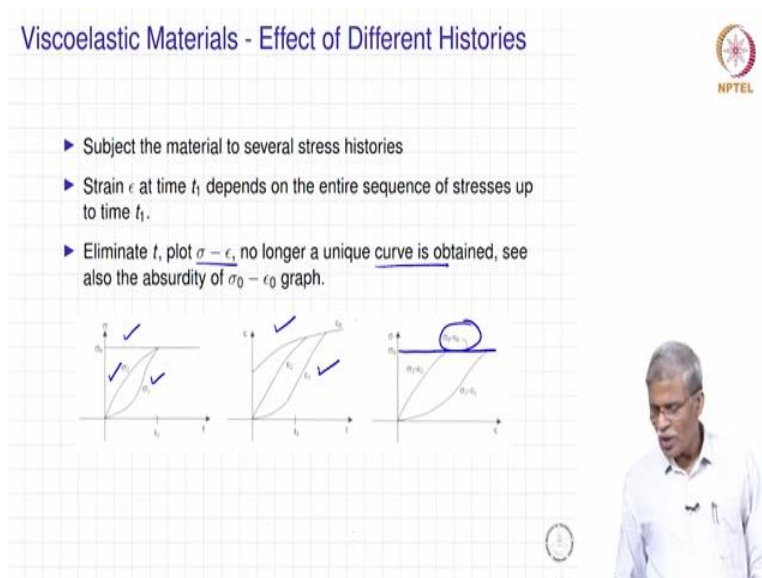
Now let us do the strain control test. You increase the strain instantaneously and then you hold it here and what will really happen if it is going to be. So there is a jump that you are going to see here OA, and it reduces gradually in this particular way and there is also a difference between the solid and the fluid material there is a residual stress here and from your undergraduate level definition on fluid, you are going to see that the stress is zero here, close to zero again asymptotically going to zero.

So we need to keep that in mind and here you are talking in terms of the fluid kind of a response here. Right? Now, one thing that we really need to understand here is we did not talk in terms of what happens when the strain goes to zero. Now what will really happen if the strain goes to zero? I am just going to illustrate only for the solid and I will leave it to you to find out how the response will be, and you also need to understand how to explain this.

So if you take this as  $t_1$ , you are going to see that it is going to go down all the way to the other side, and then slowly react. So that means if you are going to have this is the response that you are going to have when you are going to remove the strain and make it into zero here, and this will be not very intuitive if you are not familiar with the response of this material. So what it means is, as you remove the strain here, the stress that is relaxing will more or less; If you take this OA, more or less go to the other side and then from here it starts relaxing. Now, whether it

reaches a constant value or whether it reaches the zero, will depend on whether the response of the material is solid or fluid. So this is something that we need to keep in mind.

**(Refer Slide Time: 10:40)**



Now, what we can do is we can talk in terms of different stress histories, for instance you know you can have  $\sigma_0$  like this you can have  $\sigma_1$  like this you can do,  $\sigma_2$  like this, and obviously depending on the stress histories, the strain histories also will keep changing. So now if you try to do the same thing that you did the earlier, of trying to eliminate  $t$ , and see whether you can plot,  $\sigma_0$  versus  $\epsilon$ ; you are not going to get a unique curve and in fact you can actually see the absurdity of this  $\sigma_0, \epsilon_0$ . So what really happens if  $\sigma_0$  is constant and  $\epsilon_0$  is varying, so you are getting a line, something like this. Which actually makes very little sense, so what it means is if you are going to be a viscoelastic studying viscoelastic material unless otherwise stated drawing a sigma versus epsilon plot for viscoelastic material in time domain will give rise to incorrect interpretations.



(Refer Slide Time: 11:41)

### Viscoelastic Materials - Main Goal



- ▶ To determine a physically reasonable relation involving stress, strain and time so that we can find  $\sigma(t)$  if the sequence of values of  $\epsilon$  up to time  $t$  are known or
- ▶ To find  $\epsilon(t)$  if the sequence of values of  $\sigma$  up to time  $t$  are known
- ▶ Find  $\sigma(t)$  knowing preceding strain history or find  $\epsilon(t)$  if the preceding stress history is known.



So a student should be very, very careful and aware of not making such kind of a plot. So basically what is the main goal as far as the linear viscoelastic response is concerned is, to determine a physically reasonable relation involving stress, strain and time. So, that one can find  $\sigma(t)$  if we know  $\epsilon$  up to time  $t$  or the other one to find  $\epsilon(t)$  if the sequence of values up to of  $\sigma(t)$  or  $\sigma(t_0)$ . So, that means, if you know the strain history, you should be able to find out the stress history, and vice versa. So that is the basic goal of viscoelastic material.

(Refer Slide Time: 12:29)

### Viscoelastic Materials - Energy Dissipation



- ▶ If the specimen is deformed and then returned to its original shape, **some but not all of work is converted to thermal energy.**




Now, it is quite obvious as well as intuitive to say that, if I am talking in terms of energy dissipation and if I subject a material to deformation and returns to its original shape some but

not all of the work is converted into thermal energy. So that means I am going to lose something and I will get back something. What will they get back will basically depend on the elastic nature of the material, what will they lose will basically depend on the viscous response of the material. Right?

(Refer Slide Time: 13:04)

Viscoelastic Materials - Effect of Sinusoidal Oscillation



► If


$$\epsilon(t) = \epsilon_0 \sin(\omega t),$$

then

$$\sigma(t) = \epsilon_0 G(\omega) \sin(\omega t + \delta(\omega)).$$

► Amplitude ratio and phase lag vary with frequency.

- 1) Stress Control Test —
- 2) Strain Control Test
- 3) Oscillation



Now finally when we talk in terms of the sinusoidal oscillation and if I am going to give a strain history like this. I am not going to get something straightforward, I am going to get something interesting here. And in fact, it will be discussed these issues will be discussed in detail later. So I am going to get one material function, something like a modulus value, and I am also going to get phase lag.

We introduce the phase lag for the viscous fluid in which we said that it is going to be off the order of 90, whereas for the elastic solid it is going to be 0. So here, this is going to be some value. And what is interesting here is the amplitude ratio that is the ratio of  $\sigma$  to  $\epsilon$ , which is given as this  $G$  is dependent on the frequency and similarly the phase lag also is going to be dependent on the frequency. Okay?

(Refer Slide Time: 14:03)

### Viscoelastic Materials - Effect of Sinusoidal Oscillation



► If

$$\epsilon(t) = \epsilon_0 \sin(\omega t),$$

then

$$\sigma(t) = \epsilon_0 G(\omega) \sin(\omega t + \delta(\omega)).$$

► Amplitude ratio and phase lag vary with frequency.

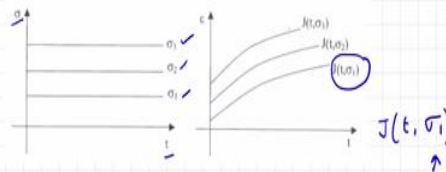
- 1) stress control Test -
- 2) strain control Test
- 3) oscillation



So now we need to start prescribing some material functions for it. We did three types of test here. One is the stress control test. Another is the strain control test. And the third is oscillation. So there are going to be separate material functions for this. And we will first define them. Then, in the next lecture, we will be talking about what makes this, what gives us some unique identity to this material function.

(Refer Slide Time: 14:52)

### Viscoelastic Material - Material Properties - Creep Test



- For each stress value, there is a corresponding creep curve
- Let  $J(t, \sigma)$  denote the strain at time  $t$  at stress  $\sigma$
- $J(t, \sigma)$  - response function for the material

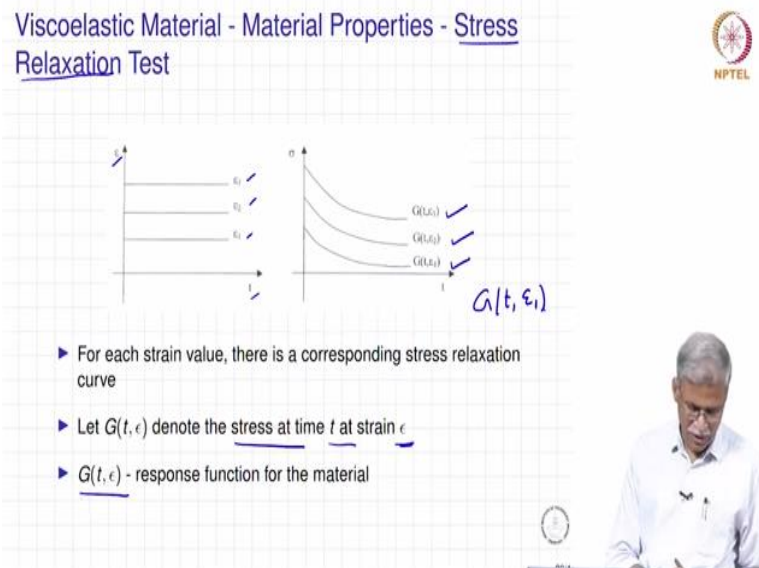


So we will do this Creep test. In fact, this particular test in which you apply a load  $\sigma$ , for a constant time  $t$  is  $\sigma_1, \sigma_2, \sigma_3$  is the creep test. And we are looking at the response of the strain. So I want you to take a close look at this particular function,  $J(t, \sigma_1)$ . So this function, basically tells

you that what is the strain in this material when it is subjected to a stress history, stress of  $\sigma_1$  at any given time  $t$ .

So this is the first response function of this material. So to rephrase again if I apply a stress of  $\sigma$ , this function  $J$  tells me what is the strain at any given time  $t$ . Right?

(Refer Slide Time: 15:48)



The next function that I am going to talk about is what is really called us the stress relaxation, just under associated stress relaxation function here. So I am subjecting this material to  $\epsilon(t)$ . So constants strain is applied  $\epsilon_1, \epsilon_2, \epsilon_3$  and these are the stress relaxation response that is given and focus your attention on this specific function. So this denotes the stress at time  $t$ , due to an application of strain  $\epsilon$  and this is another response function for the material.

(Refer Slide Time: 16:31)

Viscoelastic Material - Material Properties - Oscillatory (small amplitude) Test

Stress is controlled  $\rightarrow J(\omega)$   
 $\delta(\omega)$   
 $\epsilon(t) = \epsilon_0 \sin(\omega t)$   
 then  
 $\sigma(t) = \epsilon_0 G(\omega) \sin(\omega t + \delta(\omega))$   
 Strain is controlled  $\rightarrow G(\omega)$   
 $\delta(\omega)$

►  $G(\omega)$  and  $\delta(\omega)$  are response functions for the material.

► Prof. Padmarekha will discuss in detail SAOS!

So, the third one is the oscillatory mode. And here we are talking in terms of  $G(\omega)$  as well as  $\delta(\omega)$ . So, let us write all the functions that we are looking at. One is this, another is this. This is when strain is control. And one can have when stress is control. So, these two things are in time domain. This as well as these two things are in the frequency domain. So the issues related to the frequency domain will be discussed later in detail with my co teacher, Professor Padma.

(Refer Slide Time: 17:48)

Introduction	
Elastic/Viscous/Viscoelastic Response	
Aging Materials	

But now what I want to do is to very briefly introduce something about aging of the material. This basically is not straightforward useful to this particular course, but as students taking this

course, you should understand that the time comes in different, different ways. There are many materials in which the structure keeps changing as it is subjected to different kind of environmental conditions.

And in fact, the human body is consisting of an aging material. As I am speaking and as you are listening, the cells are being born they are dying, in fact, our human bodies as classic example for aging material. So whatever response that we see now is, essentially because of the aging mechanism that is happening here. So you can actually have not one time dependence, we can have many, many time dependence that can take place.

(Refer Slide Time: 18:53)

Aging

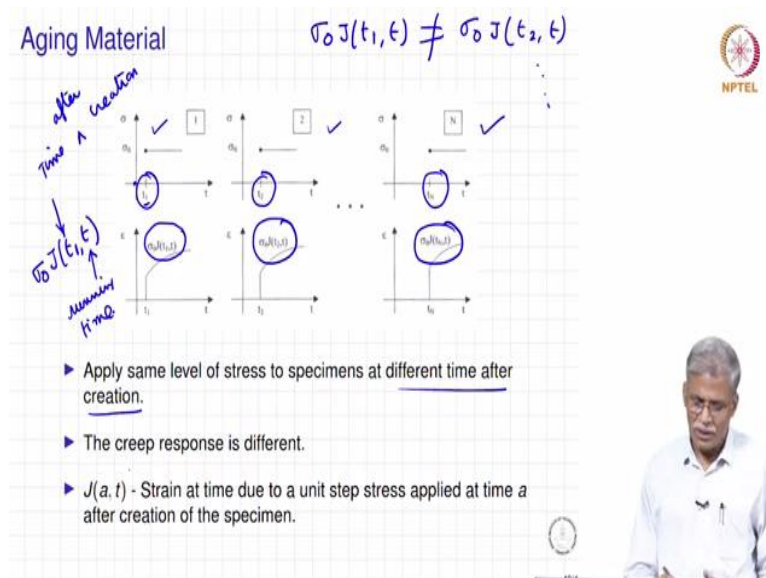
- ▶ The response of several real life material depends on 'when' the stress history is applied.
- ▶ What is creation time? *Steric Hardening - curing Time*
- ▶ This is due to 'aged', the properly depend on when they are loaded relative to the time they are created. *↑*

NPTEL

To illustrate this point, let's take a very simple example. The response of several real life materials basically depends on when the stress history is applied and it is also something to do with what is called as creation time. Later, in this lecture you will be hearing something on Steric Hardening and there will be a term of curing time that will be used by Dr. Nivitha. And you will notice that it will depend on how much time you keep the material at room temperature without doing anything. So if you take the material and subjected to test the response is going to be completely different. So that is time counter that keeps running in the material, and whenever you do a test on this, that particular response at the time depends on when the time clock actually started. As far as bitumen or bituminous mixtures is concerned when we do really start the clock? Is it the time in which the bituminous tanker leaves the refinery or is it the time in which the bituminous

roller leaves the pavement; we do not know. But what it means is, there is always a progression of the structure that takes place from some fixed time and whenever we take the responsibility. We are going to see some other response often. And this is basically due to the aging. And when they are loaded relative to the time they are created. It is extremely difficult to qualify this kind of term for most of the material. But it is much easier if you start with thinking in terms of a concrete for which is seven day cubes strength is different compared to 14 days compared to 28 days.

(Refer Slide Time: 20:52)



So if one could do a test on the material at a different, different time intervals, you are going to see a different response. There is a very interesting discussion that is available at the end of this first chapter in this reference material you should be able to read it. And so what it means is I take the same type of stress that I applied to the material at different time after creation. That is the most important thing. Okay?

Now you are talking in terms of  $t_1$ , you are talking in terms of  $t_2$  and you are talking in terms of  $t_n$ . If you take  $t_1$  as this particular point as the creation time, when I apply a load  $\sigma_0$  of this type after time,  $t_1$  from the time of creation. The response that I am going to see here is given as the following. Now, this is the time after creation and this  $t$  is the running time.



So, you are going to see another response function here and another response function here. So what this tells you is not the same as and so on. So we need to understand that how an aging material, basically response.

(Refer Slide Time: 22:35)

### Maxwell's Observation



► "What is required to alter the form of a soft solid is a sufficient force, and, this when applied produces its effect at once. In the case of viscous fluid it is time which is required, and if enough time is given, the very smallest force will produce a sensible effect, such as would require a very large force if suddenly applied. Thus a block of pitch may be so hard that you cannot make a dent in it by striking it with your knuckles; and yet it will in course of time, flatten itself by its own weight, and glide down hill like a stream of water."

J. C. Maxwell, Theory of Heat, Ninth Edition, 1888



And in this context, we need to clearly identify some of these issues and more or less in a nutshell, by the famous statement made by Maxwell, and all you really need to do is to read this very carefully. So, if you take any soft solid and subjected to sufficient force, and it will produce its effect at once. The brilliance of this statement is the simple manner in which the response of the material be it elastic, viscous or viscoelastic is explained by him, in theory of heat.




In the case of viscous fluid, you will notice that it is the time, which is required. And if enough time is given the smallest force can produce a sensible effect and such as would require a very large force if suddenly applied. Okay. I think this is a very preformed statement which needs to be understood very carefully. So you can actually have a block of pitch, which may be so hard that you cannot make a dent by striking it with your knuckles, but over a period of time flatten itself and glide down the hill, just like a stream of water absolutely brilliantly written. Okay.



(Refer Slide Time: 24:05)

### Pitch Drop Experiment




- ▶ One of the oldest running experiments in the history of science
- ▶ Professor Parnel started this experiment in 1927 in University of Queensland



I just want to show you to end lecture by showing one very famous experiment that is going on from 1927. Please go such and find out on your own on the pitch drop experiment. This basically this pitch is nothing but a refinery pitch that you get; means if you take the vacuum residue and then if you subjected to propane treatment you are going to get a pitch. This pitch is going to be very, very hard at room temperature. But if you allow it enough time, it can actually start flowing like a fluid.

(Refer Slide Time: 24:38)

### Pitch Drop Experiment



Year	Event
1927	The slom was cut
1938 (December)	1st drop
1947 (February)	2nd drop
1954 (April)	3rd drop
1962 (May)	4th drop
1970 (August)	5th drop
1979 (April)	6th drop
1988 (July)	7th drop
2000 (November)	8th drop
2014 (April)	9th drop
2020 (?)	10th drop watch!

For instance, if you take this pitch and hit it with a hammer, this is going to break like an elastic solid. But, if the same pitch was filled in a funnel and then allowed to flow gently. You can actually see the interesting history here. From 1930 till 2014 nine drop has fallen. And there is a website called the 10<sup>th</sup> drop watch, and they are expecting that by 2020 May or June, the 10th the drop may also make is likely to fall down. Okay.

**(Refer Slide Time: 25:15)**

The slide has a light gray grid background. In the center-left is a video frame showing a single drop of liquid hanging from a glass funnel, poised to fall into a glass dish below. To the right of the video, handwritten in blue ink, are the words "Elastic ✓" and "Viscous ✓" at the top. Below them, the functions  $J(t)$  and  $G(t)$  are listed and bracketed together. An upward-pointing arrow is drawn below the bracket, followed by the question "Linear?". Below this, another handwritten note asks "How one should conclude linearity?". In the top right corner is the NPTEL logo. At the bottom left, the text "Thank you for your time!!!" is displayed above the lecturer's name and email: "Murali Krishnan", "IIT Madras", and "jmk@iitm.ac.in". On the bottom right is a small inset photo of the lecturer, Murali Krishnan, a man with glasses wearing a light blue shirt.

So, thank you very much for your time. And what we did in this lecture is to introduce in a very simple way, the viscoelastic response as a sum total of elastic and viscous response, we introduce different material response functions in time domain, as well as in frequency domain. Now, whether such responses are linear or how one should conclude the linearity will be discussed in the next lecture. Thank you very much.