Mechanical Characterization of Bituminous Material
Prof. J Murali Krishnan
Department of Civil engineering
Indian Institute of Technology Madras

Lecture No 3
Linear Viscoelastic Response Part-01

Hello everyone, what we are going to do in this lecture is to talk in detail about linear viscoelastic response. If you recollect in this course, we are more interested in the mechanical characterization of bituminous mixtures. So, when we use the word mechanical characterization, we really mean the models relating to the stress-strain-time response as well as the experiments that can be conducted to elicit such response.

So, this is an iterative process. What kind of experiments we do and what kind of models can explain such behavior? In the earlier days at least before 1960s - 50s during those time most of the models that were used for characterizing the response of bituminous mixtures where linearly elastic in nature. This has something to do with the lack of sophistication that we had in the experimental characterizations.

Over a period of time in 60s -70s and especially during the late 80s, when SHRP came in substantial amount of intros have been made in terms of the mechanical characterization of the bituminous mixtures. So, if we really need to understand how to interpret the experimental results we need to have some tacit understanding of how this material actually behave and in this lecture or in this series of lectures we will be talking about the linear viscoelastic response

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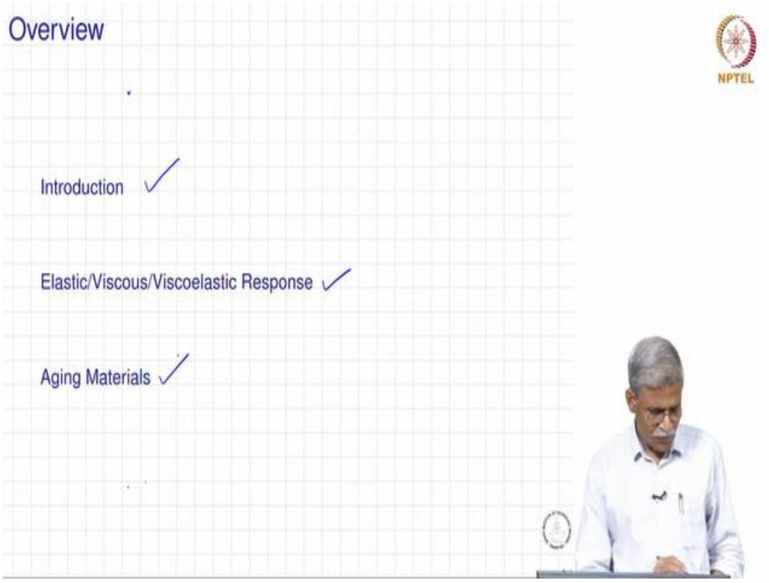

And the overview we will have an introduction, we will be talking about elastic/viscous/ viscoelastic response and then we will also introduce in a nutshell what is aging material though we won't be spending too much time on this aging material. Okay?

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And the study material for this will be the book by Wineman and Rajagopal "Mechanical response of polymers- An introduction" Cambridge University Press, 2000.

There are few chapters in this book which are used in this particular lecture. And in fact, in this lecture, we will be introducing the concept of creep compliance and stress relaxation. In the next lecture, we will be looking at what exactly means to be a linear elastic material, the ideas related to scaling, superposition as well as the Boltzmann superposition will be introduced and then we will also look at few differential rate type models. So, this will more or less conclude our discussion about linear viscoelastic response. Okay? Right?

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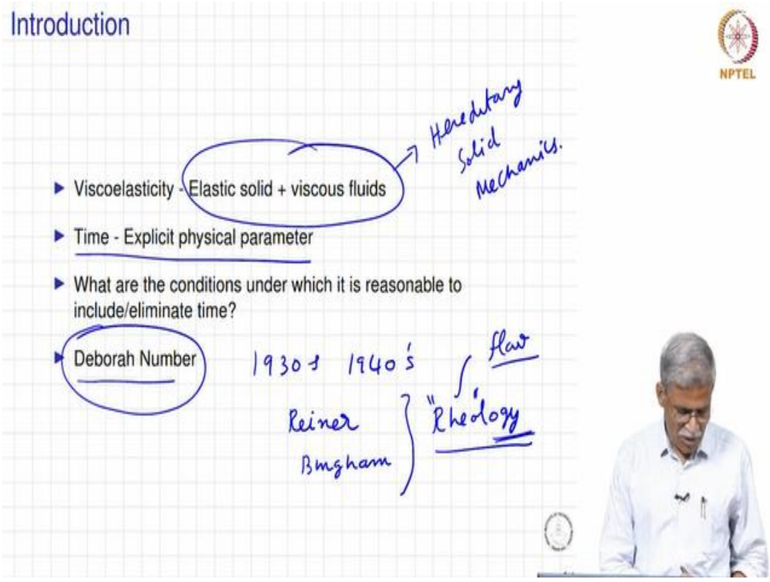

So, what exactly is this viscoelasticity or what it means is very simple in fact a very simplistic definition of viscoelastic is as if it is a additive of elastic solid and viscous fluid. This is not really the correct way of interpreting this response. In fact, the correct way of doing it will be to take into account the hereditary behavior of the material; there are hereditary solid mechanics fluid mechanics textbooks that handle such kind of things.

But as far as this course is concerned and since our focus is going to be mostly on bituminous mixtures and we are going to use viscoelastic response as a tool to understand this material. So, we will be introducing the subject matter as much is essential. There are graduate level courses maybe three or four courses that one should take to completely understand the linear as well as the non-linear viscoelastic response of this material.

And if you actually search through our NPTEL archives, there are many courses that are already offered or available in which one can develop substantial understanding. Right? first and foremost thing is we need to understand that time is an explicit parameter and we now want to ask this question under what circumstances one can include the response of the time or exclude the response of the time.

So, in the earlier days around 1930s - 1940s, we had interest development with more some of the most famous scientists by name Marcus Reiner and Bingham, in fact these 2 people were credited with coining this word Rheology. So ‘Rheo’ essentially means flow. okay and this is the science of flow and the starting point for all these things is a fictitious number that they called it as Deborah number.

This is a very critical point here when you want to really include the response of the time and when you conduct an experiment how much time one should even do an experiment. Okay? So that is the most critical point here. Right?

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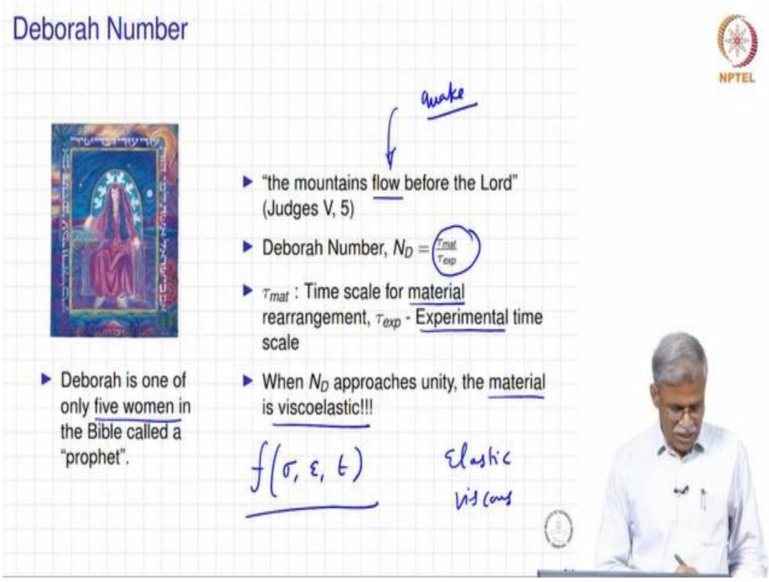

So, Deborah is credited to be one of the five women in the Bible called as a prophetess. I have written it as prophet because that is what it was given there and she is supposedly quoted to have said mountains flow before the Lord and in fact a later translation says mountains quake in the presence of Lord and that is from the older Testament right. So, this Deborah number is taken as the ratio of the time scale for material rearrangement to the experimental time scale.

So that means you are taking a material you do not know what that material is and you are trying to do an experiment there is going to be some time before the material can completely respond and reach a steady state if your experimental time scale on the time required for the material to reach a steady state response are identical then it essentially means that the material shows what is really called as a viscoelastic response.

If you take a material and if you apply a load and if the material instantaneously responds, so that means if the material rearrangement timescale is very, very small compared to the experimental timescale you are going to call those material as elastic. On the other hand, if the material is keeping on it is never reaching any steady state and it is keeping on deforming; okay? In which case you are going to say that the material response is viscous?

A viscoelastic material in a sense says that the material rearrangement scale as well as the experimental scale are of the same order and so the Deborah number is supposed to be 1, in which case you cannot exclude the influence of time. Okay? So, we need to carry out experiments always as a function of time. So that means if you really want to express the response of the material you always have to write something like this. Okay? It is always going to be the function of the stress, the strain as well as the time.

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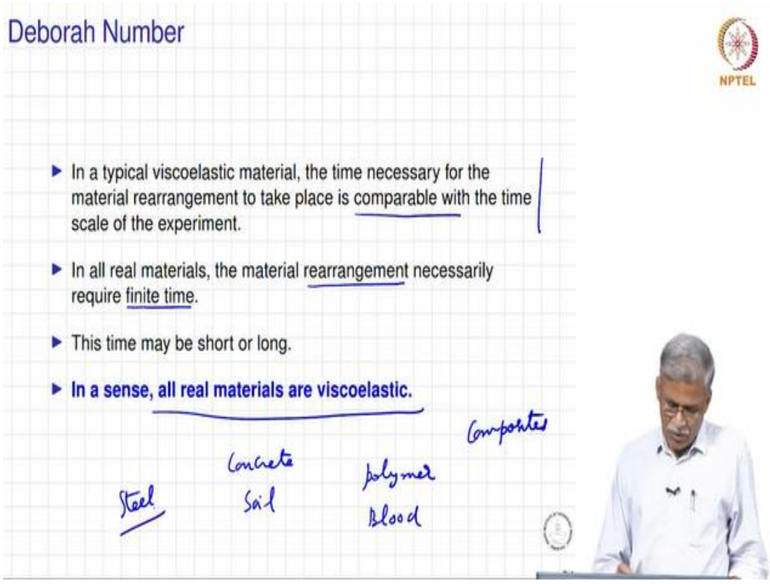

So, to re-emphasize again, in a typical viscoelastic material the time necessary for the material rearrangement to take place is comparable with the time scale of the experiment. In real life, all the material the rearrangement takes some finite time based depending on the boundary value problem that we want to solve us choose to include or exclude the influence of time. Because by adding time as another variable in all your calculations in a very simple language all the ordinary differential equations, now become partial differential equation.

The integral you have to solve integral equations, and these are not reasonably easy integral equations to solve. So, most of the time what practitioners do is to look at the boundary value problem that they are trying to solve and find out whether it is essential to include the influence of time. If it is not necessary, we just exclude the influence of time and then we solve it in a reasonably simpler way.

And in fact to emphasize, all the real-life material are always viscoelastic in nature and we choose to ignore the influence of time. In fact, to give an example, you have seen concrete behaving like a viscoelastic material, soil behaving like a viscoelastic material, almost all the polymeric composites are viscoelastic material, human blood is a viscoelastic material, most of the composites that you see are viscoelastic in response and under very high temperature even conclude that steel will behave like a viscoelastic material. Right?

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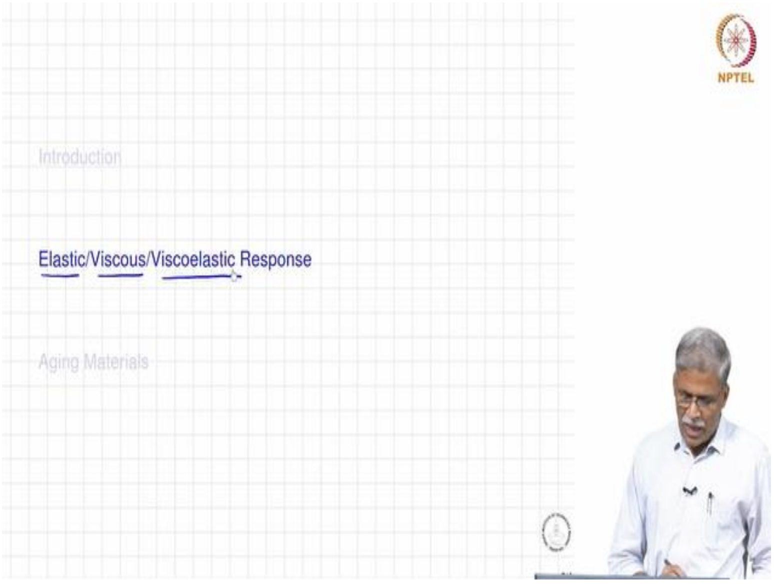

So now what we are going to do is we are going to look at what is elastic response? What is viscous response? And what is viscoelastic response? And most of the motivation that I am going to present now in this particular section, can be found in detail in Chapter 1 of the reference that has been given earlier. Right?

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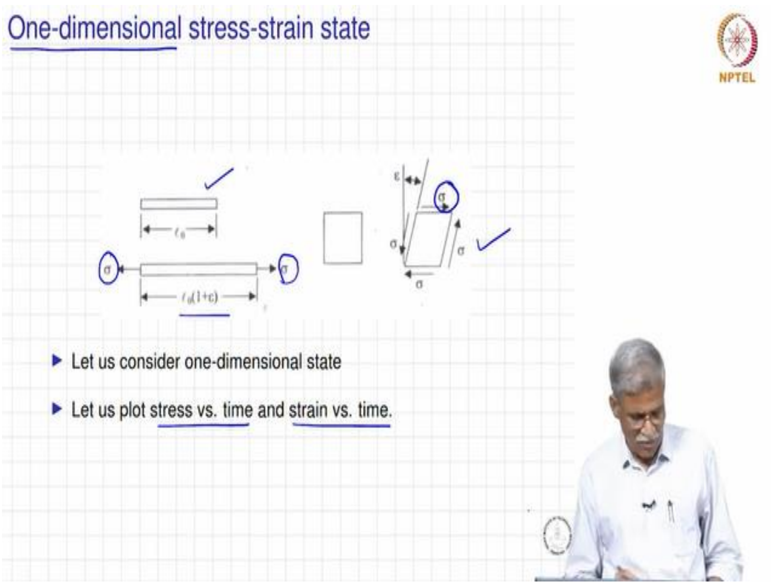

So, we are going to now do a very simple experiment, what we are going to do is we are only going to do let me underline this is going to be the one-dimensional stress-strain state. So, we are going to find out where the stress versus time as well as strain versus time. Since we say that almost all the material exhibit responses that depend on time irrespective of whatever be the material be we will include the influence of time and then later depend on the uniqueness of the stress-strain relationship we will choose to include or exclude the influence of time.

So, for illustration purpose we are going to take a steel rod or anything having a length L naught and it is subjected to that results in a stress of Sigma and this is the length. Okay? So, I am not here really very careful here I am since I am talking about one dimensional stress state. I am not going to be too much worried about what kind of traction is going to be applied that will result in this kind of a stress.

So, it could be a uniaxial extension, or it could be something like shear. Here also I am not differentiating between the shear stress as well as the normal stress and the same notation of Sigma is used here. Okay? Right?

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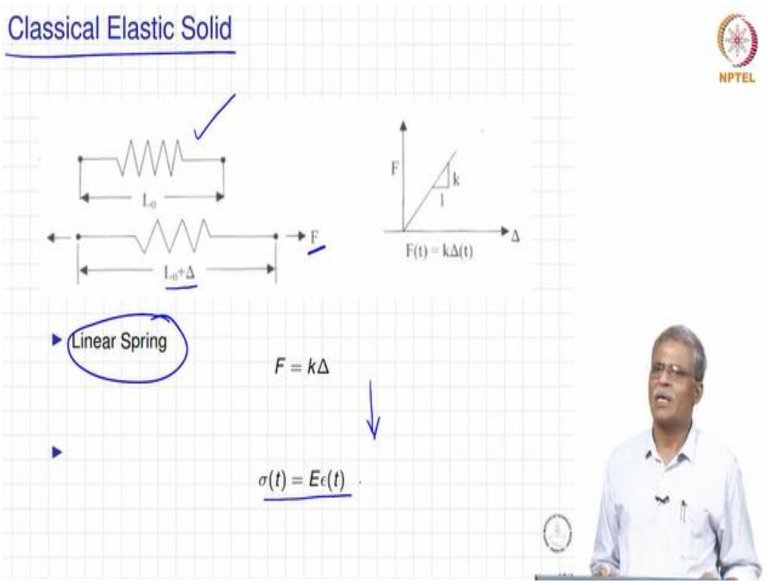

So, let us go to the thing so we will start now looking at the classical elastic solid. So, by classical elastic solid now we have to give some kind of a model to it. So, we will take the ubiquitous spring having a length of L_0 and it is subjected to a force F and if you extend it by $L_0 + \Delta$ and this is a linear spring. Okay? So, since this is a linear spring, one can straightaway write the relation between F and Δ as $F = k\Delta$F.

And so now what I am going to do is I am going to straightaway write the relation between stress and strain for this material. So, I am writing it as $\sigma(t) = E\ \varepsilon(t)$. Now this is an elastic spring, linear elastic spring we know very well that it can be safely concluded that you know it will not depend as a function of time but let us explore it little more carefully. Okay?

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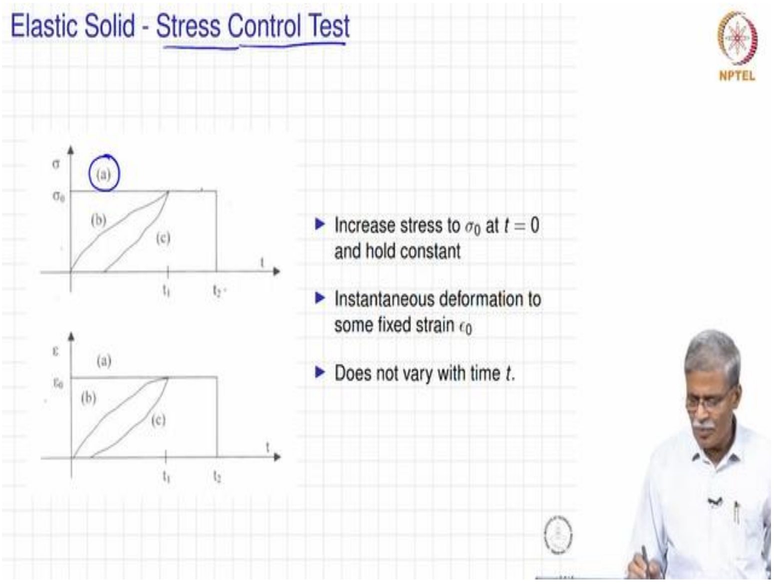

Now what we are going to do is, we are going to do some simple experiments thought experiments. So, what do I mean by, that I am going to do a stress control test? So, if you see here in the top figure this is σ and this is time. So, I am applying a stress given by (a), in which I apply a σ_0 here and when I apply this stress

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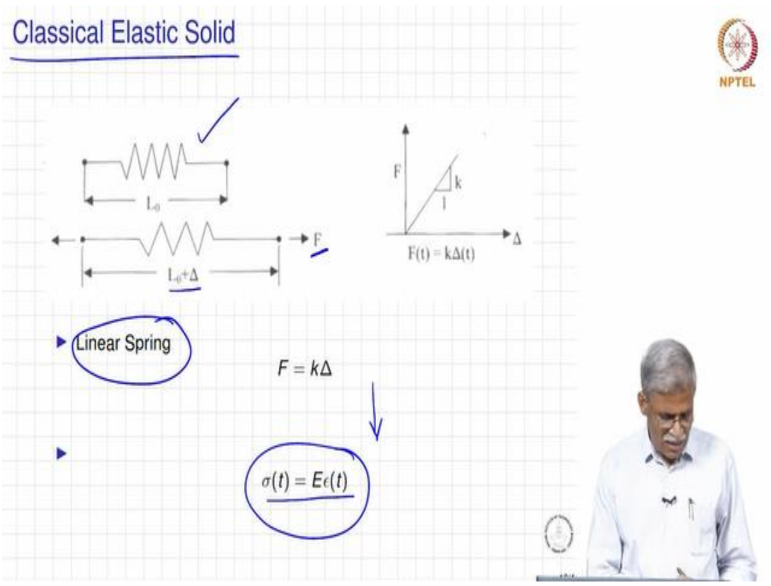

Since the response of the material in fact if you look at this equation, $\sigma(t) = E\ \varepsilon(t)$ and since E is the constant.

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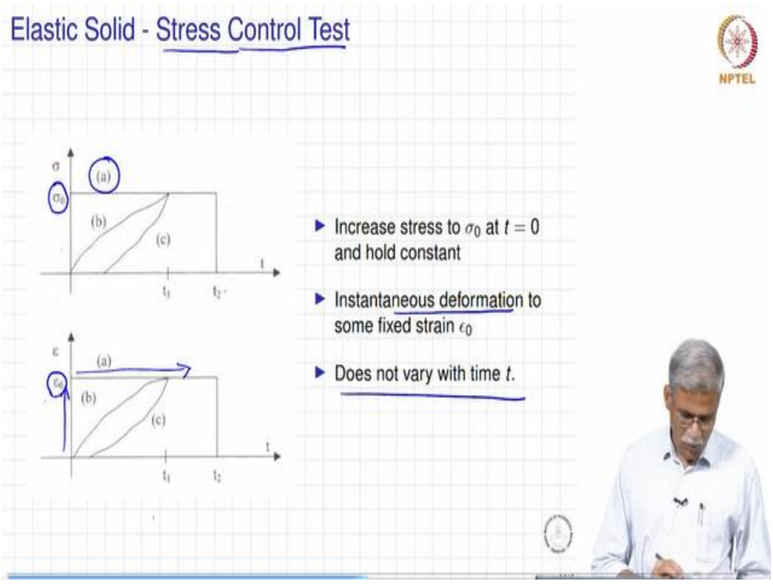

What we are going to get is the response of the strain is also identical in nature to the stress. That we have given here so there is going to be a jump here and then the stress is constant it is going to be here something like this, so there is going to be an instantaneous deformation to some fixed value ε_0 which will basically depend on the value of σ_0 that we applied and this is not varying with time t. Okay? Very nice!

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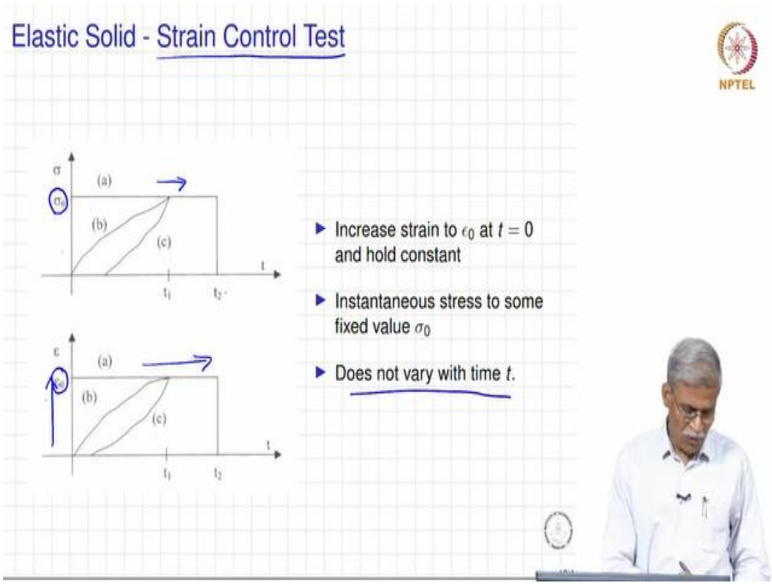

So, let us go to the next step, we will do the same test instead of stress control test? We will do this strain control test. What do we mean by the strain control test? We are going to take this steel rod, whatever be the object and subject it to an instantaneous strain of ε_0 that is shown here and because of this what will really happen the stress also reaches an instantaneous value or to put it in a correct way to get a strain of ε_0 one needs to apply a stress of value σ_0 here and since the strain is kept constant. Okay? And the stress that is required to keep the strain constant is also going to be of this particular order and this is also not varying with time. Okay? Right?

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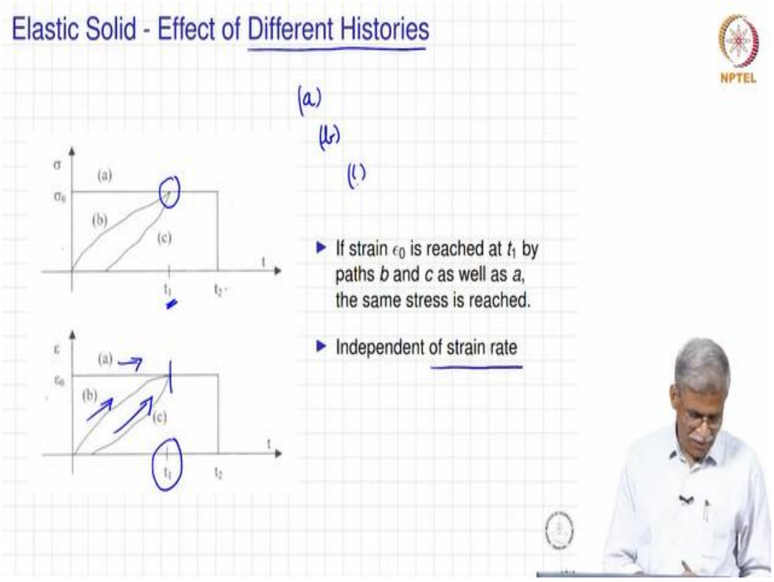

Then what we want to do is we want to see whether the effect of different histories will play a critical role? what are all the different histories, we are not only talking about stress history (a), we are talking about something like (b) and we are talking about something like that (c). so now, if you reach this strain ε_0 by in fact you can actually see we reach this strain ε_0 at time $t1$ and you can reach it through (a), you can reach it through (b) and you can reach it through (c).

And you will see that more or the same stress is reached at $t1$ and this is going to be independent of the strain rate. So, what is the strain rate here? This is the strain, and this is the time. So, the derivative of this is your strain rate but since we have a constant material constant here which is independent of the time even the stress is also going to be of the same way and this is going to be independent of the strain rate.

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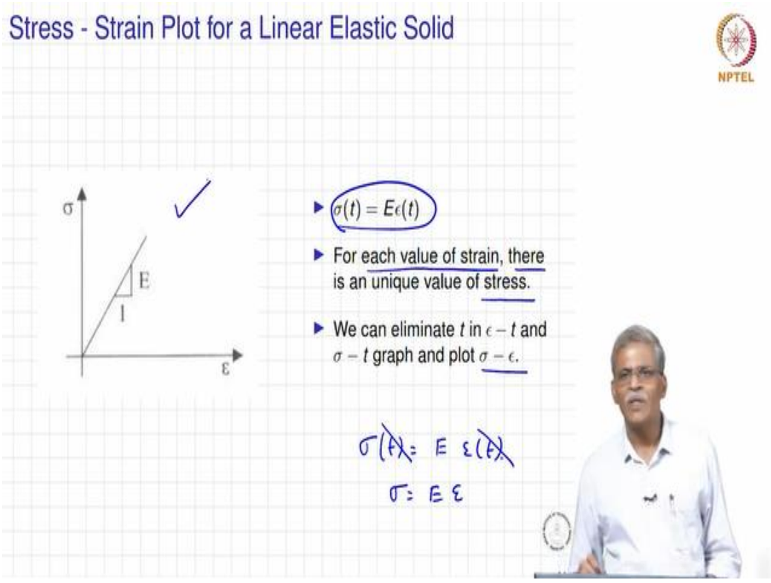

So what now we can do is, we have now seen here that we gave as we did the stress control test, we did a strain control test, we also looked into the influence of different stress histories and once we have done all these things we realized that for each value of strain or stress there is an unique value of stress or strain. Okay? So, this equation that we see here we realize that it is not necessary to write in this particular way;

So, what we are going to do is we do not want to do this we can straight away write this without explicit mention of time here and so we can plot σ and ε, and which is what you see here. So, this holds good for a linear elastic solid.

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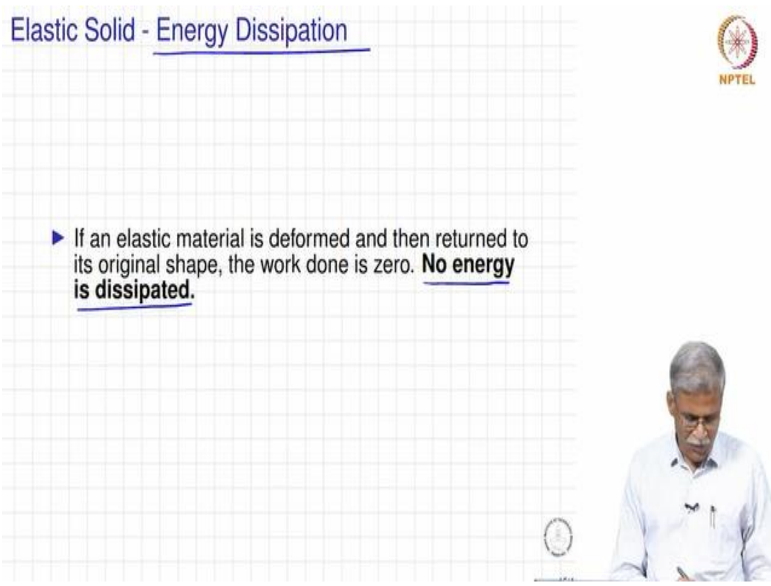

The next thing that we want to also say is and in fact, if we, if you take a good course on mechanics the big continuum mechanics course most of the time the formulations will start with the thermodynamics. So that means, the manner in which a body stores and dissipate energy essentially tells us something about what is the constitution of that particular body.

So most of the thermo dynamics based modelling will start with the reduced energy dissipation equation and then we prescribe constitutive assumptions on how the material stores energy and how the material dissipate energy and subject to some appropriate physical constraints, derive the equation for the model, for the constitutive model for the material.

So, it is always very, very useful to keep in mind something about what is really called as the energy dissipation. And as far as bituminous materials are concerned. it is going to be necessary to understand that some amount of energy is going to be dissipated, whenever there is a vehicle passage in real life, be it for the case of rutting which you must have heard many times during this lecture series or be it in terms of the fatigue damage.

That is always going to be energy dissipation and the quantification of the energy dissipation only gives us some reasonable and rigorous specification criteria. Okay? So now let us understand what happens to an elastic solid when we subject to this kind of a deformation, we know for a fact that there is no energy that is dissipated. So that means if you take an elastic material and if you deform it and if you retain it to the original shape then absolutely the work done is 0. So, there is no energy dissipated.

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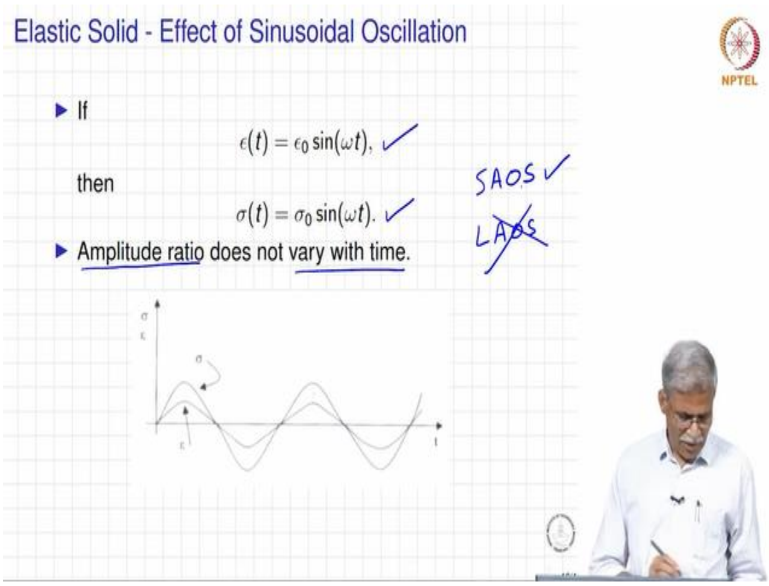

Then we will also look into quickly into what happens in terms of the sinusoidal oscillation. So if this is the history, strain history that you are prescribing and you will also going to see that, this is the stress history that is going to be necessary for you to make the material deform like this and you will also see that this amplitude ratio, which is nothing but $\frac{\sigma_0}{\varepsilon_0}$, it is not varying with time, it is not an explicit parameter of the frequency.

So, the effect of sinusoidal oscillation will be the same and in fact when we come to viscoelasticity, we differentiate between what is called as small amplitude oscillatory shear and large amplitude oscillatory shear. We will not be discussing about large amplitude oscillatory shear in this course we will be talking about small amplitude oscillatory shear and this will be discussed in detail by Dr. Padma Rekha. Right?

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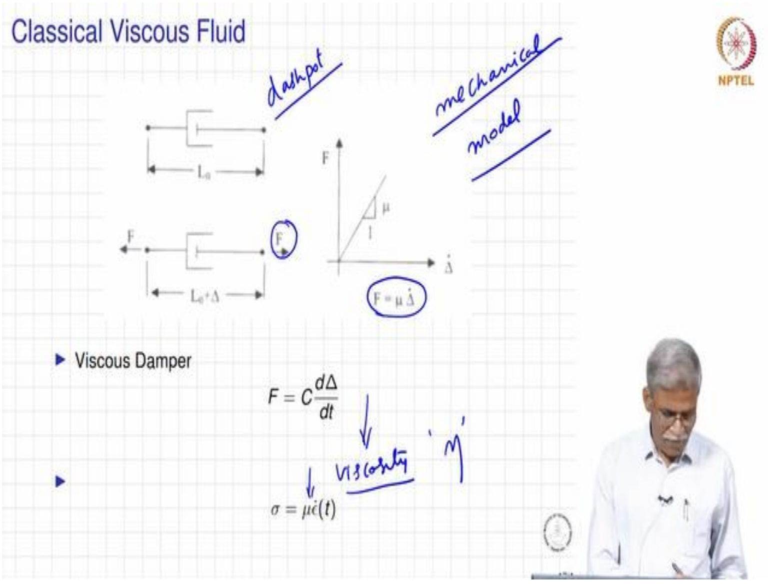

So now let us go to the viscous portion, so we to recap we said the viscoelastic material response could be taken as some kind of a sum of elastic solid and viscous fluid and we wanted to explicitly consider the influence of time. So, we assumed spring to be a model for elastic solid and we did some thought experiments basically such as stress control test, strain control test as well as different strain histories and we found out that one can eliminate the influence of time.

So, an elastic solid you can draw the strain response in a σ versus ε graph. Now what we are going to now do it is we are going to look at the classical viscous fluid and we are going to prescribe some mechanical model here. So, these kind of mechanical models in a sense helps us to visualize the response of the material. If you go and take a good look at some of the rigorous textbooks in viscoelasticity.

For instance, Christensen or many of the other books they normally do not encourage the use of these kind of models, analogous models but for the sake of pedagogy here we are going to prescribe and use this kind of a models here because it helps us to visualize how this material can actually responds. So, what is the model that we are going to use for a classical viscous fluid we are going to use a dashpot.

So we all know what is a dashpot, so no explanation will be given, you can actually see that the interstices of this dashpot are filled with a Newtonian fluid so we subjected to the force F and you are going to see this extension and then interestingly you are going to see that what we draw here is F versus $\dot{\Delta}$ (time derivative of Delta) and we will see that the equation that is given is $F = \mu \dot{\Delta}$, this is nothing but the Newton's equation of viscosity that most of you are familiar with.

So, what we are going to do it the same thing we did whatever in the earlier times. So, we are going to write it as $\sigma = \mu \dot{\varepsilon}$, and here you can actually see this is nothing but your viscosity. I am following the notation that is provided in the reference book for this material you will also notice that at many times in this course I have myself and my core teachers have used η for viscosity they are all one and the same.
They do not really think there should be any reason for confusion. Right?

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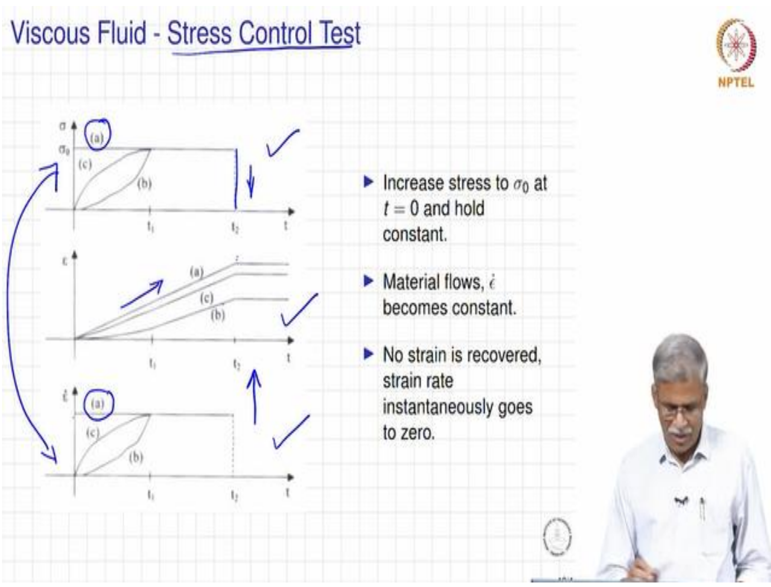

So, we will do the same thought experiments, so we will go for the stress control test. now instead of 2 graphs which is σ versus t as well as the ε versus t. that we do earlier for the elastic response we are going to have one more graph which is the $\dot{\varepsilon}$ and in fact it makes sense to connect these two things because our equation that you saw earlier is connected only in terms of σ and $\dot{\varepsilon}$.

So, knowing epsilon dot we will be trying to see what kind of response we will get for ε versus t. So, we do this stress control test, so we increase the stress to σ_0; please focus your attention on the stress path (a) and we hold it at a constant. Now what will really happen if you use the equation show earlier, this (a) it is also going to be constant and the material starts flowing.

So, $\dot{\varepsilon}$ remains the constant, there is not going to be, no strain is recovered so if you unload it here that is not going to be any strain recovery and the strain rate instantaneously will go to zero. Now you can also try and get what will be the variation of $\varepsilon(t)$ knowing the $\dot{\varepsilon}$ here. Since, $\dot{\varepsilon}$ is constant this is going to increase at a constant rate here as can be seen here. Right.

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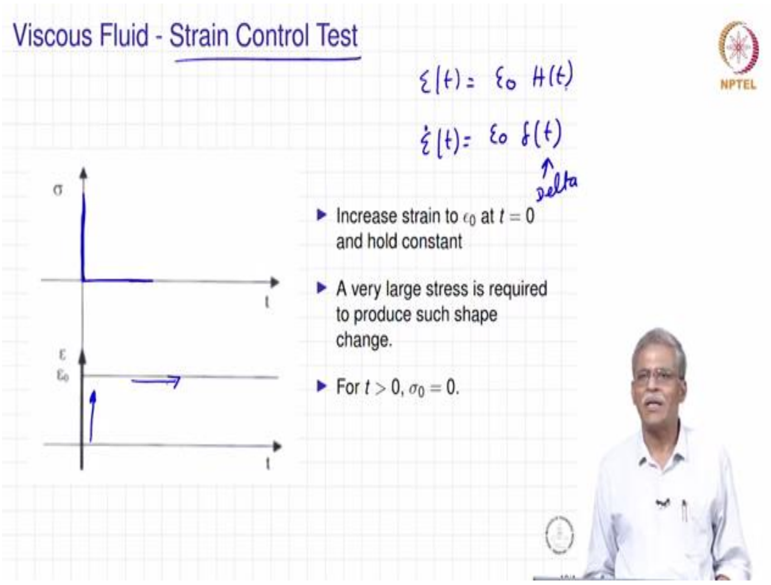

So, this is the stress control test. But what about the strain control test for that we need to understand a little bit carefully. So how should one write this particular test that we did if you really want to write it carefully, we need to write it as in this form. Okay? And this is nothing but your Heaviside function. Okay? You should be able to understand what a Heaviside function is, any basic textbook of calculus book will tell you what is a Heaviside function. Okay?

So, what is this essentially means is for $t < 0$, $\sigma_0 = 0$ and for $t > 0$, $\sigma(t) = \sigma_0$. That is what this operator essentially says. now what is that we are going to do here? So, if you write the same thing here if in a strain control test this is what it is okay? Now if we want to take the derivative of $\sigma(t)$ you are going to get because again from elementary calculus.

You know that if you take the time derivative of the $H(t)$ you are going to get a Dirac Delta ($\delta(t)$) and $\delta(t) = 1$ at $t = 0$, and elsewhere it is zero.
So, what it means is if I have to conduct an experiment like this on a viscous fluid the force that is required to cause that the deformation is infinite. Okay? And it is going to shoot up like this and then come back at the same thing and it is going to be remained to be zero.

Okay? While this kind of analogy are theoretically possible in practical sense of the word what it means is it is not possible for you to conduct a strain control test experiment on linearly viscous fluid. Because there is no way you can mobilize infinite force at $t = 0$, so that the material can have a constant strain. So, these things are not really possible;
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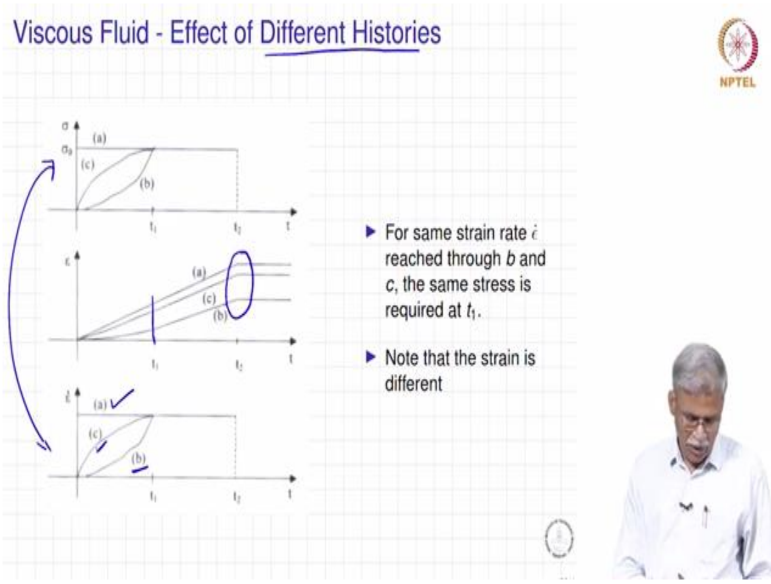

So, now if you are looking at the effect of different histories, so let us talk about trying to reach the same strain rate through different histories so (a) is what we talked about now let us take a look at (b), so this has a different strain history, strain rate. (c) has different strain rate, so while you might reach Sigma naught at the same as far as if you look at these two graph, but since the strain rate is going to be completely different.

The actual strain that you reach at t1 is not the same and the strains are going to be completely different and even here also the strains are going to be completely different. Okay?
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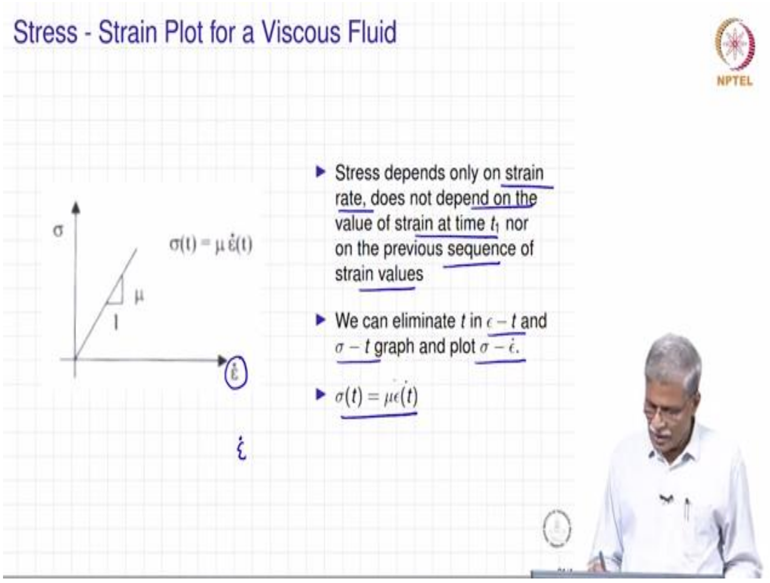

So, what it means is the time plays a very critical role here and if you want to really characterize the response of a viscous fluid there is no way you can ignore the influence of the time. So, the stress depends only on the strain rate, it does not depend on the value of strain at a time $t1$ or on the previous sequence of the strain values. So, if you eliminate t in $\varepsilon(t)$ and $\sigma(t)$ and we can plot σ versus $\varepsilon(t)$ if you want you can do it, but you need to understand that the x-axis is $\dot{\varepsilon}$.

So, you can eliminate time and plot as long as $\sigma(t) = \mu\, \dot{\varepsilon(t)}$ that is what it is okay? Right?

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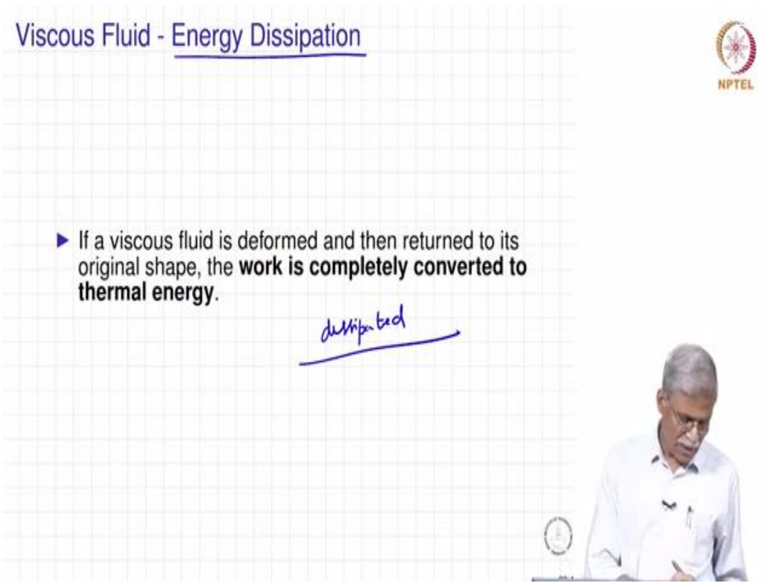

So, if now let us continue our discussion talk about energy dissipation. So, what this essentially means is if you take a viscous fluid and if you deform it the work is completely converted to thermal energy. So that means everything is dissipated. Okay? So that means you recover completely nothing out of it.

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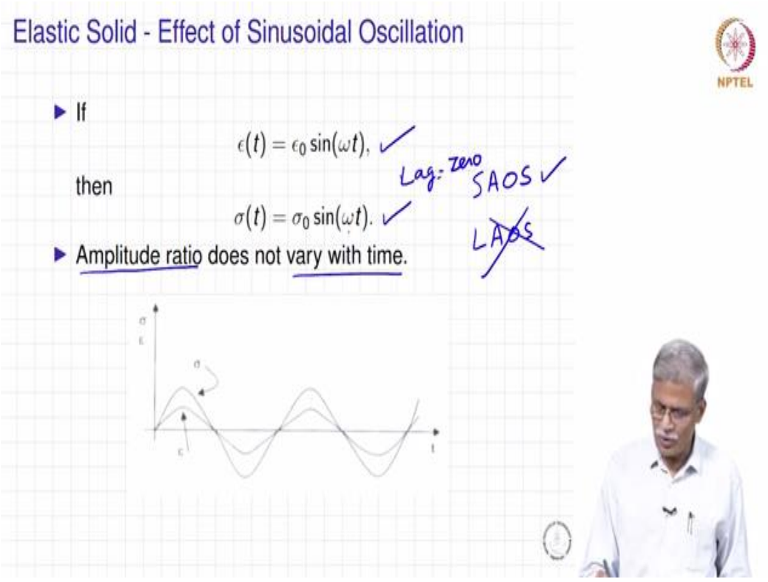

And if you talk in terms of sinusoidal oscillation; if you subject it to this particular strain history and when you take the derivative of it and substitute it here you are going to so if you have if you take the derivative of it you know what you are going to get so you will have mu if you write it in terms of $\sigma(t) = \mu\, \dot{\varepsilon}(t)$ this is what you are going to get.

So you take the derivative of it and then when you substitute it here you are going to have μ here $\dot{\varepsilon}$ here $\omega \sin \omega t$, $cos\ \omega t$ you are going to get and then you rewrite the $cos\ \omega t$ in terms of $\sin \omega t + \pi/2$, there is a reason to rewrite it in this particular way because it essentially tells you that there seems to be a lag between the stress and strain response and this lag for a linearly viscous fluid is 90.

Now what was the lag as far as the elastic solid is concerned if you recollect this particular expression you are going to see that the lag was zero. There was nothing here so for your elastic solid the lag is zero, whereas for a viscous fluid the lag is 90. So, we need to remember this because when we are talking about the viscoelastic response of the material the lag can vary from 0 to 90.

If the viscoelastic material exhibits a response close to that of an elastic solid the lag may approach close to 0 and when it behaves like a viscous fluid the lag can actually reach 90. So, it is a very simple and interesting way of interpreting the response of the viscoelastic material and this will come in handy when we are talking about master curves for bitumen or bituminous mixtures.