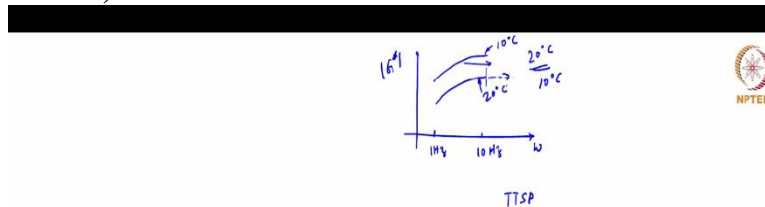


**Mechanical Characterization of Bituminous Materials**  
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**Lecture-14**  
**Master Curve Models**

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Master curve models

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Hello everybody, this is in continuation of a lecture that we have seen before with respect to their master curve constructions. So, this lecture is on the master curve modeling. So, before we start with the master curve model let us recollect what we have seen in the last class, we have seen that the laboratory collected data such as dynamic modulus of material or a relaxation modulus or a creep compliance of the material is a time restricted data. For instant, let us take a dynamic modulus value.

See for a material if you measure a dynamic modulus say for an instant if you measure it at 20 degrees Celsius, we measure a dynamic modulus. Let us assume that we measure a dynamic modulus and to a restricted frequency maybe start from 1 hertz frequency and we go to a 10 hertz frequency, this is what we get it in laboratory scale and we know that the dynamic modulus is increases as a frequency, it is a function of frequency.


So, you have a dynamic modulus here as a function of frequency. So, this is the laboratory measured value and if we wanted to know the dynamic modulus value may be at a higher frequency

or at a high speed of a vehicle at the same temperature, we have to know and do now a predictor high frequency modulus here. So, for this purpose we use a time temperature superposition principle TTSP and measure a modulus or measure a modulus in a laboratory

We will still at a lower temperature, maybe at 10 degree Celsius, just hypothetical value may be at 10 or 0 degrees Celsius when you measure at a different temperature you have an isochronous something like this different temperature dynamic modulus value. So, this is measured at 10 degrees Celsius, maybe at the same frequency range, you can shift this 10 degree Celsius using a shift factor to form an extended to get an extended dynamic modulus value at a reference temperature of 20 degrees Celsius.


So, this time temperature superposition principle you can use it and shift the curves at different temperature to get a master curve at a specific reference temperature say here at 20 degree Celsius. So, this master curve once constructed, I can be used for different purposes. This master curve, we generally use a model forms.

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### List of master curve models

- Jongepier and Kuilman
- Dickinson and Witt
- Dobson
- CA Model ✓
- Sigmoidal ✓
- LCPC model .....



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Master curve model

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Now, let us see what are the model forms available for master curve predictions. So, here are the list of master curve models available which are named based on the scientists and the research papers with. The list is not restricted only to these things, there are many master curve models

available. We will discuss in detail a CA model and a sigmoidal model which is commonly used for a binder and a bituminous mixture master curve predictions.

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## References

- **A structure-related model to describe asphalt linear viscoelasticity**, Didier Lesueur, Jean-François Gerard, Pierre Claudy, Jean-Marie Letoffe, Jean-Pascal Planche, and Didier Martin, Journal of Rheology
- **Modelling the linear viscoelastic rheological properties of bituminous binders**, Nur Izzi Md. Yusoff, Montgomery T. Shaw, Gordon D. Airey, Construction and Building Materials

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Master curve model

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So, for other models or details, details of other models you can refer to a paper a journal paper on a structurally rated model to describe asphalt linear viscoelasticity, Lesueur et al. from a journal of Rheology and another paper from a construction and building materials by Yusoff et al., so, you, this both are review paper which tells, which describes a different model for an asphalt and bituminous material master curves.

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## Master curve model

- CA model (Christensen and Andersen, 1992)

$$|G^*(\omega)| = \frac{G_g}{R} \left[ 1 + \left( \frac{\omega_c}{\omega} \right)^{\frac{\log 2}{R}} \right]^{\frac{-R}{\log 2}}$$

$$\delta(\omega) = \frac{90}{\left[ 1 + \left( \frac{\omega}{\omega_c} \right)^{\frac{\log 2}{R}} \right]}$$

$|G^*(\omega)| \rightarrow$  dynamic modulus  
 $\delta(\omega) \rightarrow$  phase angle  
 $\omega \rightarrow$  frequency  
 $G_g \rightarrow$  glassy modulus  
 $\omega_c \rightarrow$  cross-over frequency  
 $R \rightarrow$  index constant

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Master curve model

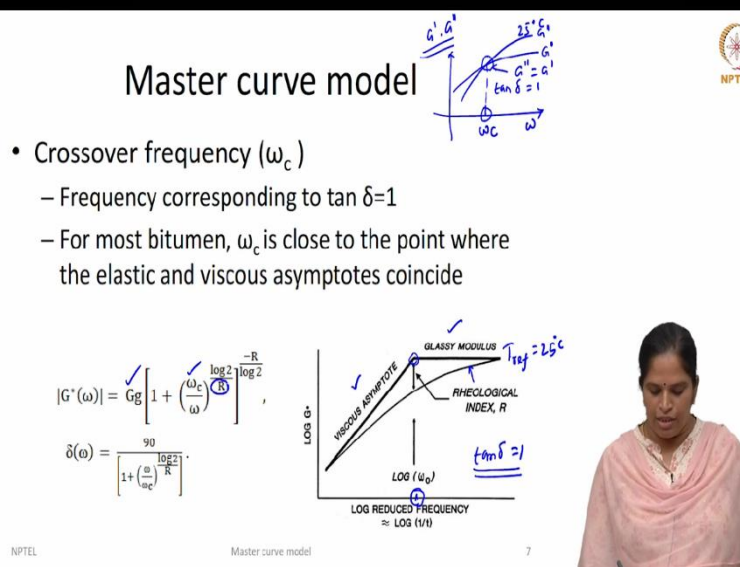
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So now let us discuss in detail a CA model which is based on the research from as a part of SHRP path, Christensen and Anderson given this model CA model, you have 2 equations here describing a master curve for a dynamic modulus and master curve for a phase angle. Now, if you closely look into these 2 equations, so, you can see this either a dynamic modulus or a phase angle is a function of 3 constants, one is  $G_g$ , another one is  $\omega_c$  and the 3rd one is  $R$

Here  $|G^*(\omega)|$  is a dynamic modulus and  $\delta(\omega)$  is a phase angle and this all equation is a function of  $\omega$  here, where  $\omega$  is a frequency we use. So, when you see this equation, it has 3 constants, one is  $G_g$ . So, this  $G_g$  is a glassy modulus we call it as a glassy modulus. Let me explain this term what glassy modulus in the next slide and  $\omega_c$  is is a crossover frequency and

$R$  is a constant or we call it as a rheological index, index constant rheological index. So, which describes a shear susceptibility of a material. Now, let us see in detail how these parameters can be obtained and what is this significance of these parameters.

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So, Glassy Modulus see generally if you see the master curve of dynamic modulus you have a schematic of a master curve here, in this schematic, you can see that the dynamic modulus master curve as plotted as a function of reduced frequency or it can be even an inverse reduced frequency, can also be imagined as a reduced inverse of time. So, you see a dynamic modulus curve here a master curve here at very high frequency.

You can see that the modulus value asymptotically reached a constant value. This value here will be a contribution due to a dynamic storage modulus as well as loss modulus at very high frequency if this loss modulus plot will be insignificant. So, this will insignificant and hence will this dynamic modulus will be contributed most by than elastic modulus or a storage modulus.

At very high frequency the modulus value increased a reached a steady value and this steady state modulus is what we call it as a glassy modulus. So, if you look into a temperature wise. So, this steady state modulus occurs when you test the material at very very low temperature. Otherwise when you in a frequency wise if you do it at very short timing when you test the material at very high frequency you get this glassy modulus. So, this is an elastic modulus. So, we get this most of the binder this we get this value to be close to 1 GPa

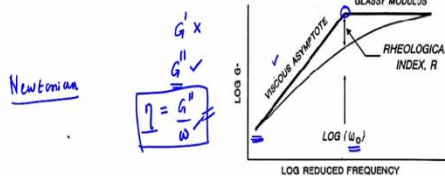
So, this is in case if it is if you test it in an oscillatory mode and you look for a dynamic modulus value, suppose we test it in a timescale and look for the relaxation modulus, this elastic asymptote can be assumed as 3 GPa. So, this is for all binder CA model assumed that this value to be 1 GPa for a dynamic modulus case and 3 GPa for a relaxation modulus. So, now we know what glassy modulus is. So the next term is Crossover frequency.

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## Master curve model



- Steady state viscosity ( $\eta_{ss}$ ) ✓  $\omega_c$ 
  - It is the slope of the master curve when it approaches very low frequencies and is referred to as the viscous asymptote
  - dynamic viscosity nearing  $90^\circ$  Phase angle is taken as the steady state viscosity



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Master curve model

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So, before we define a crossover frequency, let us just look into what a steady state viscosity. So, this steady state viscosity function though this is not directly included in that expression, we need to know what a steady state viscosity to understand what is crossover frequency ( $\omega_c$ ). So, now see

we have a dynamic modulus master curve. Now, let us just relate a high frequency value and the low frequency value.

Now, if you look into it very low frequency, at very low frequency as we discussed a similar to a high frequency result, we have 2 smaller storage modulus and the loss modulus at very low frequency. So, this storage modulus will be insignificant and loss modulus will be always dominating. So, we know this we have seen this already in a small amplitude oscillatory shearing.

So, if this all loss modulus dominating, we can relate this loss modulus to a dynamic viscosity at a frequency using an expression dynamic viscosity to be a function of loss modulus or a slope of this function, loss modulus function with respect to frequency.  $\eta$  is viscosity, dynamic viscosity. So, may this dynamic viscosity can be related to a loss modulus using this relation. So, now this measure this dynamic viscosity value and the slope of this function gives you the dynamic viscosity value.

So, when the dynamic viscosity nearing a 90-degree phase angle why is this 90-degree phase angle is, when phase angle is 90 the response of the material will be Newtonian. So, nearing 90 it may not be exactly 90 it may be nearing 90. So, measure this dynamic viscosity value when the phase angle is nearing 90. So, that gives you the steady state viscosity or the steady state viscosity can also be taken as at very low frequency construct an asymptote, viscous asymptote and the value here can also be obtained using a loss modulus by frequency.

So, now get this value this constructed viscous asymptote. So, you have a 2 asymptote here viscous asymptote and a glassy asymptote. Now with this viscous asymptote and the glassy asymptote will meet at one point and the corresponding frequency is what we call it as a crossover frequency. So, you have a crossover frequency here you have a viscous asymptote and the glassy modulus both meet at one point. The frequency corresponding to that point as what we call it as a crossover frequency.

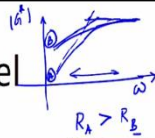
This crossover frequency was found to coincide at a point where  $\tan \delta = 1$  at any temperature say for instant, you are constructing this master curve at a reference temperature of say 25 degrees Celsius. So, now, what you can do is measure a storage modulus loss modulus value in a

laboratory.  $G', G''$  in a laboratory at different frequency at a temperature of 25 degree Celsius, you know that at low frequency loss modulus will be dominating and at very high frequency storage modulus will be dominating.

So, there will be a crossover point here, where this at this crossover point storage modulus and loss modulus will be equal or otherwise  $\tan \delta = 1$ . So, this crossover point which we obtain it in the laboratory will coincide with this crossover point, which we get it from the master curve that is from the meeting point of viscous asymptote and then glassy modulus value. So, you can in a laboratory calculate this crossover point, using a storage modulus loss modulus trend.

So, now we know what is glassy modulus, we know what is crossover frequency. So, we get this glassy modulus you can assume it to be 1 GPa or 3 GPa. Crossover frequency, you can conduct a oscillatory shear experiment at different frequency and find out in a laboratory what this crossover frequency is. Now, the next term here in the master which is nothing but R

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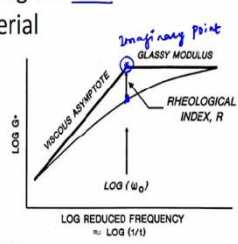
## Master curve model

- Rheological Index (R) ✓
  - difference between the glassy modulus and the dynamic modulus value at the cross-over frequency
  - shape parameter indicating the shear susceptibility of the material


$$|G^*(\omega)| = G_g \left[ 1 + \left( \frac{\omega_c}{\omega} \right)^{\frac{\log 2}{R}} \right]^{\frac{-R}{\log 2}}$$

$$\delta(\omega) = \frac{90}{1 + \left( \frac{\omega}{\omega_c} \right)^{\frac{\log 2}{R}}}$$

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Master curve model



It is called as a rheological index. So, this rheological index is nothing but the difference in the modulus value obtained at the crossover point. So, we have a crossover point here so, modulus value obtained from this peak point to the modulus value which we get it here. So, this point is an imaginary point. So, this is the one which we get it from the master curve. So, difference between the glassy modulus which is a straight line here.

So, this is the difference between the glassy modulus and the modulus from the master curve which exactly measured at the cross over frequency. So, this is exactly measured at the crossover frequency. So, this rheological index parameter gives us the shape of the master curve. So, again depending upon how sensitive the material is when you are shearing the R value varies.

Say for instant, will compare the 2 cases here 1 is a master curve something like this and other is a master curve something like this. So, the glassy modulus is going to remain constant. So, let us consider the case A this to be A binder A and this to be binder B. So, we have a modulus here this is a master curve for a dynamic Modulus and this scale is reduced frequency let us denote it by  $\omega_R$  or here it is  $\omega$ .

So, now, this is a glassy modulus you have a glassy modulus straight line you will get it to be like this. Let us assume the viscous asymptote meeting point is here and for this case the viscous asymptote will be somewhere here. So, exactly at the viscous asymptote or a crossover point, now if you R for binder A will be greater than R for binder B. So rheological index measured for binder A will be greater than binder B in this case.

So, this is something which tells us how this modulus is varying over an entire range of frequency. So, you can call it as a shear susceptible parameter or a shape parameter. So, here as such in this figure we can call it as R to be that is binder B to be more shear resistance compared to binder A. So, knowing these 3 values glassy modulus,  $\omega_c$  and then R value you can predict the value of dynamic modulus and phase angle over a wide range of frequency. So, this CA model is used in an SHRP performance grading further for a binder.

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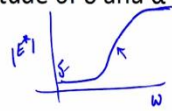
## Master curve model



### • Sigmoidal model

$$\log(E^*) = \delta + \frac{\alpha}{1 + e^{\beta + \gamma(\log f_r)}}$$

- $\delta$  and  $\alpha$  depend on aggregate gradation, binder content and air void content.
- $\beta$  and  $\gamma$  depend on the characteristics of the asphalt binder and the magnitude of  $\delta$  and  $\alpha$



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Master curve model

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So, the next model as we have seen before, as a sigmoidal model, the sigmoidal model we have already seen in the construction of master curve in the constrained fitting. So, we assume the shape of a master curve and we transfer all the data to that assumed shape so as to get a continuous smooth curve. So, the sigmoidal model gives a master curve or gives a model or is of the form for a dynamic modulus to be like this. This model is used in mechanistic empirical pavement design guide for predicting the dynamic modulus value at different frequencies and temperature.

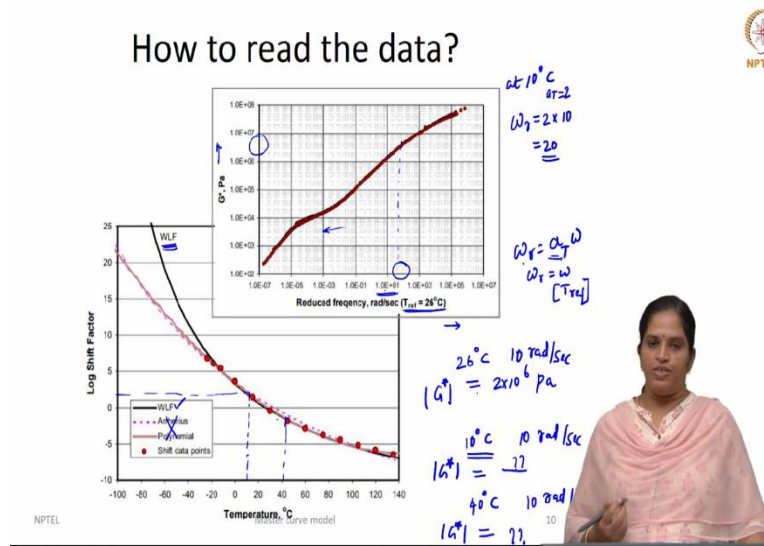
Now, if you closely look into this model, you can see that the dynamic modulus value depends on you have 1 2 3 4 constants  $\delta, \alpha, \beta$  and  $\gamma$ . So,  $\delta$  here is a minimum modulus it takes a sigmoidal form. So, the form of a curve is something like the sigmoidal form. So, this is a dynamic modulus as a function of frequency. So, the minimum for this value is  $\delta$  so, minimum modulus and  $\delta + \alpha$  together gives us the maximum modulus.

So, the maximum value here this point is  $\delta + \alpha$  and this  $\beta$  and  $\gamma$  describes the shape of this line here, it need not necessary to be a straight line. So, this  $\beta$  and  $\gamma$  describes the shape of this modulus. So, this we can see that  $\delta$  and  $\alpha$  will be dependent on aggregate gradation binder content and air void content. So, it depends on the mixture volumetrics and also the binder content which we use.

So, this value is not a constant for all mixtures and  $\beta$  and  $\gamma$  again depends on what is the value of  $\delta$  and what is the value of  $\alpha$  here. So, this is the standard model, master curve model which we

use it for design for a dynamic modulus prediction of a mixtures. So, now, we have seen 2 models. Now, let us take one specific master curve and we will see how to read that master curve.

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So, here is a master curve for a dynamic modulus. So, if Y scale is a dynamic modulus here and X scale is a reduced frequency. So, this master curve is constructed at a reference temperature of 26 degree Celsius. So, you have this master curve and that the master curve you have a corresponding shift factor. Now, let us not worry about the other shift factors, we will just take the WLF equation here, let us assume that the WLF equation holds for this master curve.

Now, we will use only this WLF equation curve to read the data from the master curve. So, now, from this data, suppose we need at 26 degrees Celsius, maybe at 5 hertz frequency, we want to know what is the dynamic modulus value and other is maybe at still lower temperature 10 degrees Celsius, 5 hertz I want to know what is a dynamic modulus value and one more data is maybe at higher temperature than a reference temperature.

So, at 40 degrees Celsius at same 5 hertz, I want to know what is the dynamic modulus value, How to take this data from the master curve. So, at 26 degrees Celsius 5 hertz, what is my modulus value? So, this is at a reference temperature of 26 degrees Celsius. So, at reference temperature if you reduce represent  $\omega_R$  as a reduced to frequency at reference the reference temperature, we know that

The shift factor or  $\omega_R = a_T \omega$  where  $a_T$  is the shift factor which can be obtained from this. So, at reference temperature  $a_T$  value will be 1, or  $\omega_R = \omega$  when T is reference temperature. So, at 26 degrees Celsius you can directly take a frequency to be a reduced frequency to be a frequency. So now 5 hertz frequency so at 5 hertz frequency, so, this is 1E-1, this is 10.

So, this is again in the logarithmic scale let us take it for a 10 hertz frequency, so that we can so 10 hertz frequency at 10 hertz frequency, sorry it is a 10 radians per second. The unit given is radians per seconds, so, we will measure it for 10 radians per second. So, at 10 radians per second the modulus value is somewhere near 2E6. So, the modulus value is 2E6,  $2 \times 10^6$  Pa.

Now at 10 degrees Celsius 10 radians per seconds, so 10 degrees Celsius, I wanted it corresponding to 10 radians per seconds. So, now what is my reduced frequency at 10 degrees Celsius, we want to know find out reduced frequency for 10 radians per second for this we need a shift factor here. So, at 10 degrees Celsius somewhere here the shift factor is, maybe 2. So, shift factor is 2,  $a_T$  value here log of shift factor is 2 log of shift factor here, this is a T value is 2. So, you will get  $\omega_R$  to be 2 into 10 radians. So, it will be like 20.

So, now for corresponding to 20 radians per seconds, so, this is 1, this is 10 you can pick the 20 somewhere here, 20 will be somewhere here, you can pick the 20 frequency 20 radians per second frequency and find out what will be the modulus the modulus value will be somewhere here. So, you can fill this modulus value at 10 radians per second. Likewise, at 40 degree Celsius, pick the corresponding shift factor.

Pick the corresponding shift factor, use it in the shift factor value find the reduced frequency for that reduced frequency if you pick somewhere it may fall in the lower frequency region, because it is in the negative, it is less than your reference temperature. So, it may fall in this region, please do it to yourself and find out what is this value for a dynamic modulus at different temperatures.

So, you can use a single master curve along with the shift factor to predict the modulus value at a different frequency. So far we have seen how to construct a master curve and we also see how to model the master curve data and we have seen a few models for one model that is available for a

binder and one for a mixtures and we have also seen how to read the master curve data. So, with this I will wind up with the master curve session. Thank you for your time.