Mechanical Characterization of Bituminous Materials
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Lecture – 11
Linear Viscoelastic Response - Part 05

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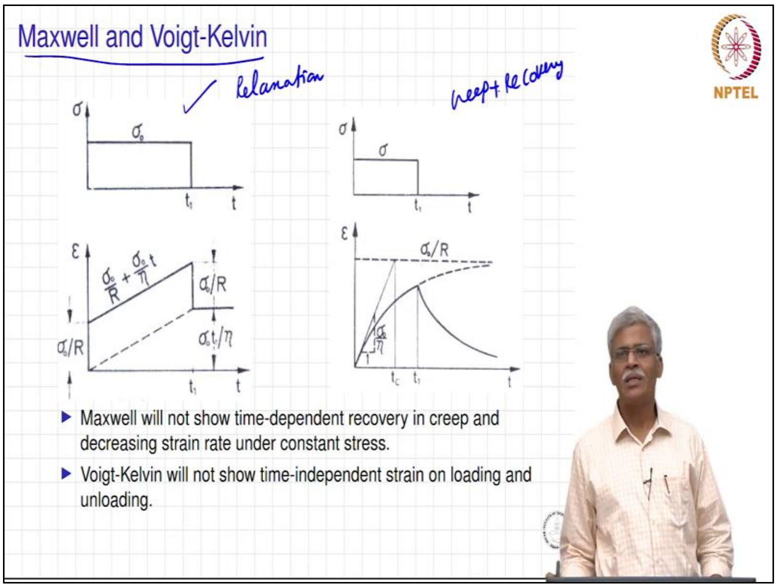

So, hello everyone in the last lecture we stopped with the comparison of the Maxwell and the Kelvin model we also discussed that these two models on their own cannot exhibit all the responses that we want for a viscoelastic material. While Maxwell model can do a very good job of relaxation, it cannot do an excellent job as far as predicting the creep and recovery is concerned as can be seen here. A Kelvin model can do a decent job of predicting creep and recovery but it cannot predict the stress relaxation response.

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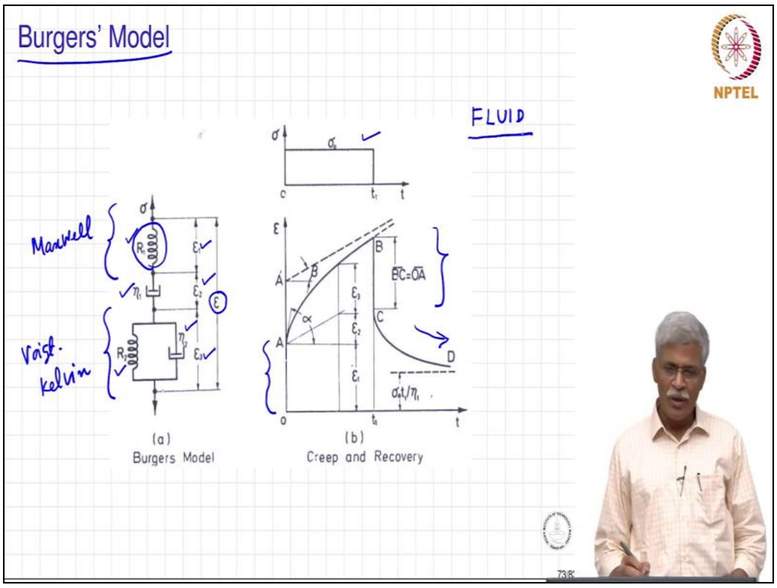

So what we are now going to do is we are going to combine them two. So, we are going to see that there is a Maxwell model and there is a Kelvin model and we are going to combine them in series. This is one of the well-known model that has been used for characterizing the response of bituminous binders, bituminous mixtures, polymer gels, blood and many other fluids that are available.

And you can actually see some of the nomenclature here. So we are going to take *R one* as the spring *Eata 1* as the dashpot and *R two* as the spring and *Eata 2* as the dashpot for the model. So you are having two springs and two dash pots and when this material, when this model is subjected to a creep loading of this form, we are going to see a total strain of *epsilon* and you can actually see that it is *epsilon one, epsilon two and epsilon three.*

In fact, similar to the exercise that we did earlier without even deriving all the expressions, we can try to hypothetically sketch the response of the material. So when we subject this this particular model to this kind of loading, we are going to see that there is going to be and instantaneous elastic response, which is given by this particular jump. And then after that, we are going to see a response which is progressing in this particular way.

This is going to be the combination of the Kelvin model as well as the isolated dashpot. And when we unload it, there is going to be an instantaneous recovery due to the same spring and then there is going to be delayed recovery here. Okay. So now based on the discussion that we had in the earlier lecture, you should be able to find out whether this model shows a fluid like response.

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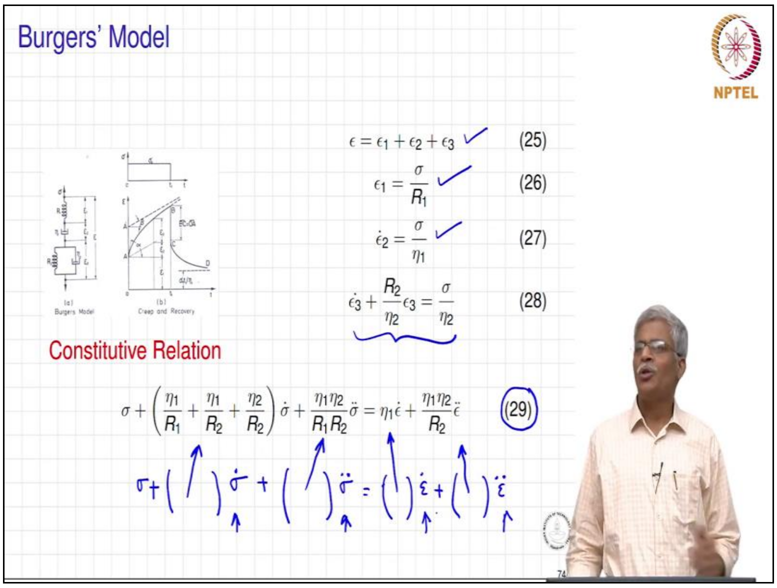

What we will now do is we will try and see whether we can derive the constitutive model for this. The derivation of this will become little bit involved and you are expected to do the required algebra and try and see whether you can get this expression 29. So, *epsilon is epsilon one plus epsilon two plus epsilon three and here epsilon one* is nothing but the strain in the lone spring which is given by *sigma by R one*. *Epsilon dot two* is *sigma by eata one*, what we have done here is to write the expression for the linearly viscous damper.

And we have now right written here is nothing but the expression for the Kelvin model, so the Kelvin model is connected in series. So we have straight away taken and return the expression for this. So now when we play around with all these expressions try to rearrange them, we are going to get an expression of this following form. It will be useful if you could denote this in a slightly different way. So, it is going to be some material constants here of this particular form.

So, you are going to have some material parameters here given by this and some other material parameters here given by this and similarly for the strain. So you are going to have second time derivative of stress here, a first time derivative of stress, similarly a second time derivative of strain and the first time derivative of strain. So you are going to have a second order ordinary differential equation in time for stress as well as in strain.

This solution of this equation is necessary for us to find out how the creep and recovery response is going to be as well as the stress relaxation. It is actually not very difficult to solve this problem.

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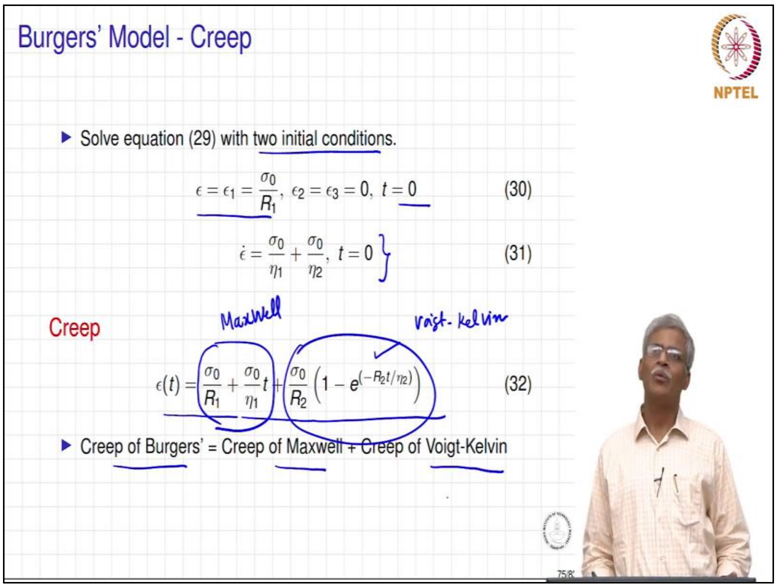

So if we need since we have second order equations, we need two initial conditions when we write the initial conditions like this at t is equal to zero and similarly, we also have to impose this initial condition as for as the Kelvin model is concerned. And when we try and solve it here. This is the expression that we are going to get. I have not made a mention of how this has to be solved.

And if you look into the reference chapter that I have mentioned in the book Findley, Lai and Onaran, you will be able to see the detailed derivation. But as of, as of now for this course it is suffice to say that this is the creep experiment expression that we have got here and in fact you can do the following if you look at it, very carefully closely. This is the creep of the Maxwell model and what you see here is the creep of the Kelvin model.

One should also note that this same burgers model could be combined in different manners. It is not necessary that one should combine this get the same response. In fact, you can have different types of springs and dashboard combinations that can result in equations of this form accept that the material parameters that you are going to see here maybe slightly different. So you will you can generate different types of generalized burgers models which will have second time derivative of stress and strain, first time derivative of stress and strain. But the parameters shall be slightly different. So to summarise the creep of burgers model is creep of Maxwell as well as the creep of Kelvin model.

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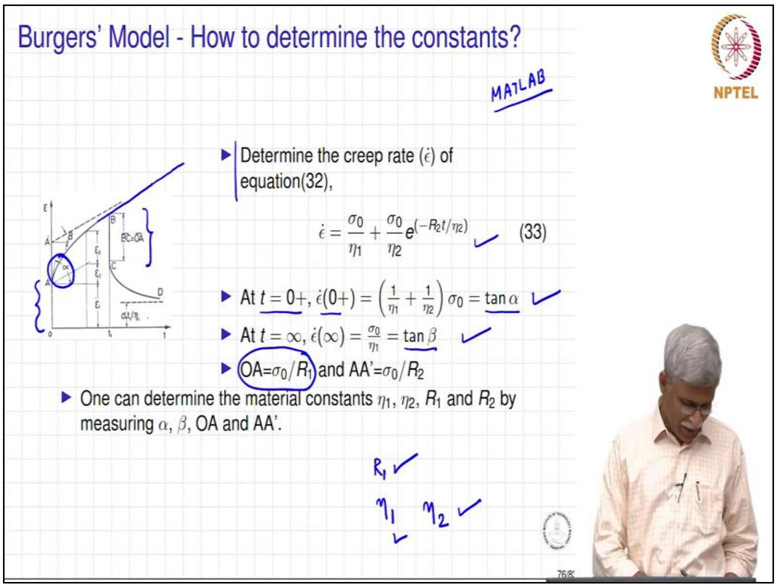

What do we really do with this kind of model? This is at this point of time we should just kind of take a small pass and find out what exactly is this exercise that you are trying to do here? So you are, when we started this lecture on linear viscoelastic model, we wrote on few important points that every viscoelastic model should show. The first and foremost thing is we should have an instantaneous elastic response, a delayed recovery, a delayed creep as well as an instantaneous recovery as well as a delayed recovery and you can have a permanent set or irrecoverable deformation and similarly stress relaxation under constant strain. So these are the essential features that we expect viscoelastic model to have and since we see such kind of response from real life material. So, obviously whenever we sit down and work out some model in a sense it should reflect the true mechanical response of the material.

Now looks like we have finally stumbled upon a model that could more or less follow whatever we were expecting from a linear viscoelastic model. But then the main problem here is how do we really determine the material parameter? We now have four material parameters to be determined. Right. Only if we have this determined these material parameters we will be in a position to even use this model for any simulation, model stress analysis if you really want to do.

There are many ways of determining the material parameters. These days if you are familiar with MATLAB in fact will be doing some exercises on experimental data with MATLAB. So, you will be able to notice that one can actually find out the response the material parameter by writing it within the constraint of optimisation. But as of now with the experimental data that we have got it will be easy for us to make some guesses about the nature of the response of this material as well as the associated parameters.

So first and foremost thing what we can do is we can actually take a look at how do we really find out sum of this material parameter? So, the first parameter which is quite obvious to you, us is OA is equal to *sigma not by R one*. So, that means if we have an experimental data of this particular form, and if you know that this is the stress to which it has been subjected to by just looking at this particular loading as well as the recovery you should be in a position to find out what is *R one*. That is the easiest parameter to find.

Now, initially, when I started discussing the model I showed this spring and dashpot arrangement and then I also mentioned that this model should behave like a fluid. So if this behaves like a fluid, in fact, you should verify whether it behaves like a fluid, you are basically going to see that it is going to asymptotically reach if you keep loading this forever, it is going to reach a constant strain rate.

When it will reach the constant strain rate? We actually do not know because it will reach it only asymptotically. So, so, what we can actually do is, we can take the creep rate of the previous equation, which is equation 32 and this is what you are going to get and at t is equal to zero or what is given here as zero plus we are, when we substitute for it. We can measure that there is going to be some angle *tan alpha*.

And similarly at t is equal to infinity, we are going to get something called as *tan Beta*. And we will be able to see that when we have if we can measure this angle alpha as well as if we can measure this angle beta and looking at the parameters that are associated with that we can actually find out what is *eata one* we can also find out what is a *eata two* and once we have analysed this experimental data like this, you should be in a position to find out all the material parameters.

Normally this is a kind of a thumb rule that is given for finding out the experimental parameters, but one can use these ideas to put some kind of a constraint for *R one* what could be the values for *R one and R two* as well as *eata one and eata two*. And you will also notice here that you can verify what the choice of *eata one* by looking at this particular value here which more or less mentioned either as permanent set and you are going to see that this particular value is independent of the running time.

So you are going to have this expression *sigma not t one divided by eata one*. *t one* is the time duration for which the material is subjected to a load of *sigma not*. So, even by looking at the available deformation after sufficiently longer time, we will be in a position to verify whether this *eata one* that we got using this analysis that was mentioned here. Okay, whether it is verifiable or not. So these are one simple way of trying to determine the material parameters.

There are many ways in which these material parameters can be determined. So now how do we do the recovery?

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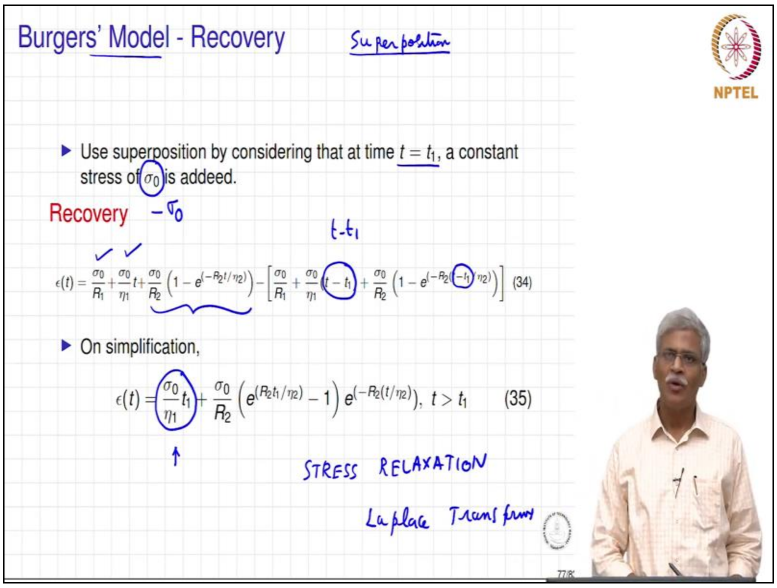

Since this is a linear viscoelastic model, we are going to appeal to superposition. So, when I say superposition what we are going to do is at t is equal to t one, I am going to add a constant stress of minus *sigma not* and so what will really happen if we actually look at the expression you have *sigma not by R one, sigma not by eata one times t* and this is the strain due to the Kelvin element and then *minus sigma not by R one sigma not by eata one*. And this time now is t - t1 because t is the running time t1 is the time after which sigma not is applied and in the same way we skip substituting instead of t we keep substituting t minus t1 and when we simplify the whole thing, you are going to see that this is the expression. The interesting point that you have to notice here is this first term which is actually independent of the running time. And so that means this is going to be the amount of deformation that is what you will call really as permanent set.

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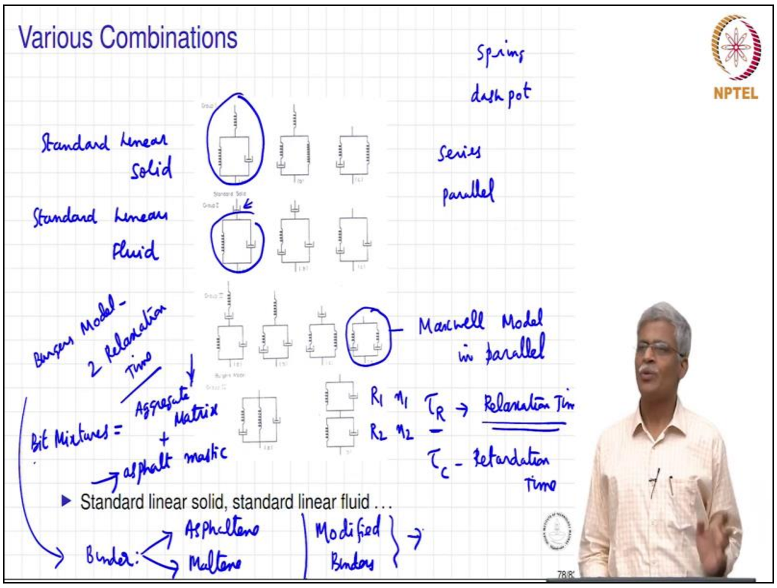

So this is the expression that you are going to get for creep and recovery but what do we do for stress relaxation? So one can do the same thing in the sense you have to solve the constitutive expression that was shown here and do it. It will involve little bit use of Laplace transforms. So if you are familiar with Laplace transforms, you can do it if not you can actually take the expression that is available in the textbook for solving it.

Since the focus of this particular series of lectures is more on mechanical characterization of bituminous mixtures, we will not get into the details related to how to do the Laplace transform and how did we really get it? But if you see the particular chapter you will see the analogous derivations here. So what you now see here is a very general framework that we established here. We identified a spring for an elastic element, a dashpot for a viscous element and then we combined them in series giving raise to Maxwell model in parallel giving rise to a Kelvin model.

And then we realised that these two models can not completely predict the response of a viscoelastic material. So we put the Maxwell and Kelvin model together in series to come out with yet another model, Burger model and it seemed to be doing most of the things. But is this the only unique model that is available? No, you can actually have different types of combinations of the model. So, you can have what are really called as Standard Linear Solid, so as the name suggests, the response of the material is like a solid material in that I will just focus the attention only this.

You will see that this is a Kelvin model with a spring that is attached in series with it. You can actually have Standard linear fluid model and you will see that it is the same Kelvin model with a dashpot is attached in series and you can have these kind of models and incidentally you will see that the response of all this models that you see here will be identical except that the parameters that are associated with the sigma dot sigma double dot can be slightly different.

For instance, you can get the same Burgers model by having two Maxwell models as you see here in parallel. Now it is also time for us to take stock of the situation because we introduced two types of time, one is a relaxation time another is a retardation time. So, considering the response of the Burger’s through its model wherein you have R1, R2, eata1, eata2, you can actually have 2 relaxation times for this material, for this particular model.

Okay. So, Burger’s model you can have two relaxation time. Now, this is very interesting in the following sense. This model will come in very handy when you are trying to model the response of bituminous mixtures while you can assume that your bituminous mixture consists of aggregate matrix plus asphalt mastic. So, you can prescribe one relaxation time to asphalt mastic and other relaxation time to the aggregate matrix.

Aggregate matrix you are talking about is all the aggregate skeleton connected in the joints with the Binder. So, you can assume that it will have one relaxation time. You can have the Asphalt mastics in another relaxation time. You can also use this model for binder because you can actually have asphalt in one relaxation time, Maltene in other relaxation time at least.

It can be used even for modified binders. You can have the polymer phase relaxation time and bituminous phase the other relaxation time. So what we know understand is if we have to explain the behaviour of determiners binders or bituminous mixtures be at least need to have the model that will can exhibit at least two relaxation time, at least two relaxation time that is more or less is the success of the Burgers model.

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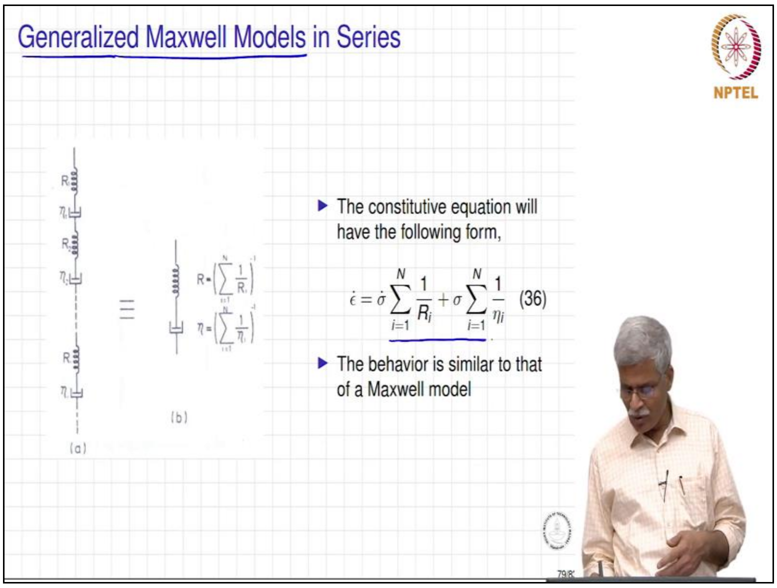

So, in fact, it is not necessary that you know two relaxation times is good enough, we can actually have many relaxation time. And so that is why what we do is we go for such as generalised Maxwell models in series. So that means you can have many Maxwell models having different values for springs and dashpots connected in series and so the expression more or less will be of this particular type.

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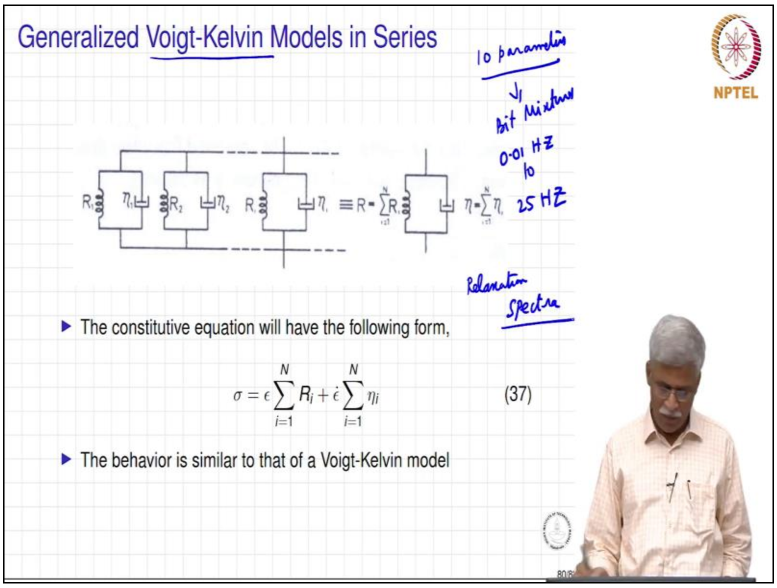

And similarly you can have what is really called as the generalized Voigt-Kelvin models in series and again it can have as many relaxation or as many retardation time as you really want. And what we have seen in most of modelling attempts in fact, when we work with the many of our students doing their PhD, sometimes we have even tried to use sets of more than 10 parameters at least, to explain the response of bituminous mixtures over a range of frequency from 0.01 Hz to 25 Hz.

So this is the most interesting thing about this material at different frequencies, the response of the material will be completely different. And so having a model which just ask two relaxation time may not necessarily be sufficient so we need different relaxation and in fact, these are normally prescribed in the following way in polymer literature, we call it as Relaxation Spectra. Okay.

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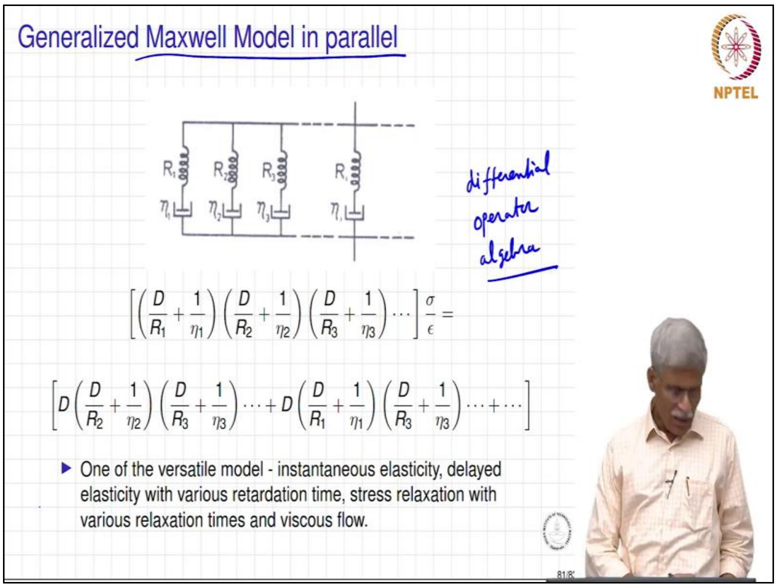

So, you can also have the Generalised Maxwell model in parallel and you need to have what is really called as the, to solve such kind of problems, you need to have what is really called the differential operator algebra, I am mentioning all this things in passing because we are still working with linear viscoelasticity and we are not even talking about nonlinear viscoelasticity.

And in linear viscoelasticity itself, when we want to describe the response of bituminous mixtures, and when we really need more than 4 or 5 relaxation times, the algebra that we need to do to solve a simple problem of stress relaxation or creep and recovery or in oscillatory mode can be really involved. So, we need to be aware of that particular part.

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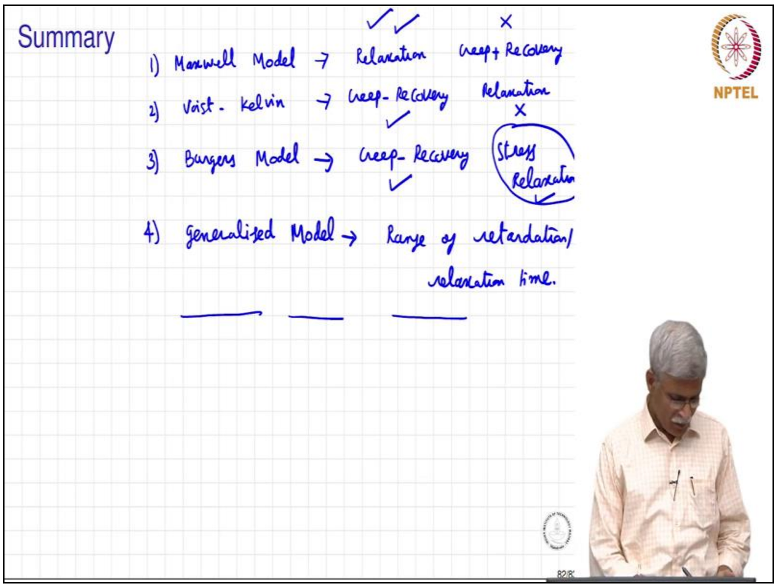

So let us summarise whatever we discussed in this particular series. So, we discussed Maxwell model. We wrote the constitutive expressions for the Maxwell model and we showed that it will do a very good job explaining relaxation but when it come to creep and recovery , whatever we prescribed about what a viscoelastic model should exhibit, it will not do a good job. Then we discussed a Voigt-Kelvin model and we found out that it can do a decent job as far as the creep and recovery because you still will not have a initial jump. So I would if I put a double tick here I will put a single tick here for this.

But when it comes to relaxation, this model cannot do any relaxation because you have a dashpot in parallel with a spring. So it will not do anything. A Burgers fluid model can do creep and recovery, stress relaxation in a much better way. Of course, we did not show the expression for stress relaxation here, but it is understood that based on the structure of this model that we have got we can have it.

We also talked in terms of generalised models especially when we want a range of retardation or relaxation time. So, this is the summary that I wanted to say here. This will more or less help us later to try and see how one can use the response of this material to describe different kind of scenarios. Thank you.