Design of Masonry Structures
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Module – 04
Lecture – 34
Design of Masonry Components and Systems
Example - II

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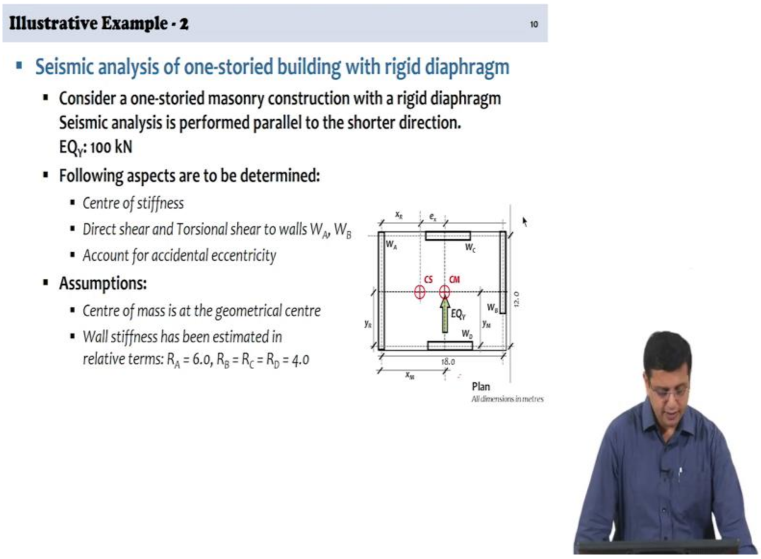

Let me move on to the second illustrative example, and the second illustrative example is one that deals with a single storied building again, but with a rigid diaphragm. And in this condition, I would like to arrive at the design forces for the different walls. So, this week you are actually making calculations on the distribution of shear force to walls when you have rigid diaphragm assumption. So, now, you will see an example which is embedded within a design approach ok.

So, what are we looking at here, we are considering a one storied masonry construction with a rigid diaphragm. And rigid diaphragm it could be a reinforced concrete, roof slab for the single storied structure. And this should perform as a rigid diaphragm as long as the aspect ratio is, aspect ratio of the plan is not greater than 3. When you have aspect ratios where the length of the building is 3 times or more than the breadth of the building, you could have conditions of a semi rigid diaphragm. And, that cannot be treated in the way we have treated, we are treating this particular case over rigid diaphragm action.

Seismic analysis is going to performed again parallel to the shorter direction and the earthquake force in the y direction that we are considering here is 100 kN. The following aspects are to be determined. So, that is the plan that we are looking at, I am looking at centerline dimensions in this particular problem, we are working on already estimated stiffnesses. So, it is convenient to work on centerline dimensions.

So, the plan shows you, how in the longer dimension the length of the building is 18 meters and in the shorter direction, breadth of the building B is 12 meters. And so, that is the building you have four walls; you have two walls in the direction of the earthquake, and two walls that are perpendicular to the direction of the earthquake. And you can see that there is no symmetry in one direction; whereas, there is symmetry in the y direction.

So, this particular example, we will use these examples to estimate what the eccentricity is, and then from the eccentricity arrive at what the design forces are going to be to the walls in plane direction. In this particular case we are interested in looking at, wall A and wall B and estimate the design forces in wall A and wall B.

So, the things to be calculated, the center of stiffness; we need to estimate then, what the direct shear component is and the torsional shear component is. And since we are looking at earthquake Y, I am interested in wall A and wall B. If you consider the earthquake force in the x direction, then you can look at in addition the design force is coming on to wall C and wall D.

We also need to account for accidental eccentricity. So, in the exercise that you have been doing, you have not accounted for, we have neglected the effect of accidental eccentricity; but within a design framework you cannot do that, and the code requires that accidental eccentricity be considered. So, this is an example where we look at how much the accidental eccentricity is and how do you, then use the accidental eccentricity to look at the worst case situation for the shear force that you are interested in wall A or wall B.

Assumptions, the center of mass is at the geometrical center. So, we are assuming that, the center of mass is at the geometrical center of the plan 18 meters x 12 meters. And, the wall stiffness is estimated for you, so you have already gone through the process of calculating the stiffness of a single wall considering the boundary conditions; whether it is fixed-fixed at the top and the bottom or it has pinned-pinned condition.

So, here in this particular case, we are we already have the estimates of the stiffnesses of the walls; but they are given to us in relative terms. Here wall A has a stiffness of 6 units; whereas wall B, C and D have a stiffness of 4 units. So, we have the stiffnesses given to us in relative terms; in a real problem you would actually have to then also take into account what the actual stiffness is for to arrive at the relative stiffnesses also. So, this is what is given to us and these are the assumptions that we are making.

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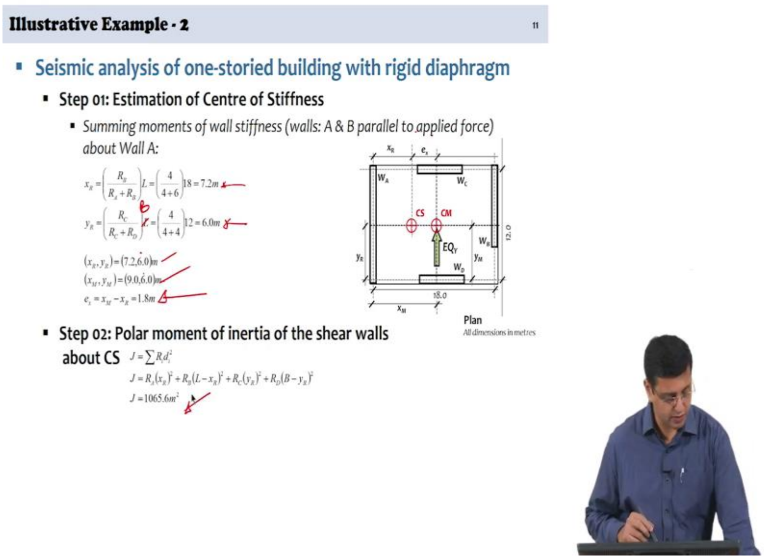

So, how do we go about examining this problem; the first step is the estimation of the center of stiffness. In estimating the center of stiffness, we are interested in looking at the center of stiffness coordinates x and y coordinates, x_R and y_R, given the symmetrical layout of walls C and D; along the y direction there is a certain there is symmetry and therefore, y_R will coincide with the center of mass, that something of a simplification in this particular case. But in the other direction, the center of stiffness and the center of mass along the x axis are not coincident; and therefore, that eccentricity is what is going to cause the torsion in the system when earthquake action in the y direction is considered.

So, we need to estimate the center of stiffness, you can calculate that by summing the moments of the wall stiffness; wall A and wall B parallel to the applied force about wall A. And so, I am making this estimate of x R and y R, as

$$x_R = \left(\frac{R_B}{R_A + R_B}\right)L = \left(\frac{4}{4+6}\right)18 = 7.2\text{m}$$

$$y_R = \left(\frac{R_C}{R_C + R_D}\right)B = \left(\frac{4}{4+4}\right)12 = 6.0\text{m}$$

So, the coordinates of the center of stiffness, estimated for this given plan configuration (x_R , y_R) is (7.2 ,6); and x_M, y_M we originally talked about this, we consider that it is at the geometrical center. So, 18 and 12, (9,6) is (x_M, y_M). You see that, along the y direction you do not have eccentricity; the point is 6 and 6 there; and therefore, the coordinates are 6 and 6 there. So, there is no eccentricity in that direction, but you have eccentricity in the x direction.

So, $e_x = x_M - x_R$ and therefore, we estimate the eccentricity as 1.8 meter. So, that is the eccentricity which is causing torsion, and that significant eccentricity, it is 10 percent of the plan dimension perpendicular to the direction of the earthquake which is significant. We typically use about 5 percent of the plan dimension as an estimate of accidental torsion and we have here a case where it is double. So, it is significant eccentricity ok.

To be able to then estimate the shear forces, we need the polar moment of inertia, of the shear walls about the center of stiffness designated as CS here. And therefore, the polar moment of inertia is a summation of the stiffnesses of each wall into the square of the distance, centroidal distance of the wall and that then translates as relative stiffness of wall A.

$$J = R_A(x_R)^2 + R_B(L - x_R)^2 + R_C(y_R)^2 + R_D(B - y_R)^2$$

The polar moment of inertia in this case works out to about 1065.6 meter square.

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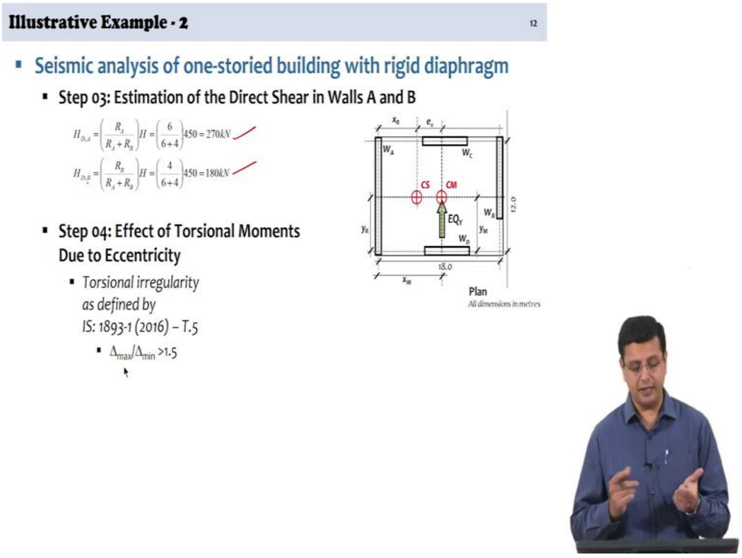

So, step 2, we have estimated the polar moment of inertia and therefore, we are in the right direction to be able to estimate the torsional moment. Now step 3, let us estimate the direct shear in the two walls that we are interested in; and since it is a rigid diaphragm problem with a rigid diaphragm, we are going to be looking at distribution of the shear forces proportionate to the stiffnesses. And therefore, the direct shear forces here designated as $H_{D,A}$; direct shear force corresponding to wall A is nothing but the distribution factor; so, in this case $\left(\frac{R_A}{R_A+R_B}\right)H_Y$ which is the shear force that we are considering in the problem, which is 450 kN. So, $\left(\frac{R_A}{R_A+R_B}\right)H_Y=\left(\frac{6}{6+4}\right)450=270\text{kN}$ in wall A. Similarly, the direct shear force corresponding to wall B, $H_{D,B}$ is estimated which is $\left(\frac{R_B}{R_A+R_B}\right)H_Y=\left(\frac{4}{6+4}\right)450=180\text{kN}$. So, that is the direct shear force distributed between the two walls.

Now, do we have eccentricity, yes, and significant, and there is going to be torsional moments when the earthquake force is acting in the y direction; and therefore, the effect of the torsional moment due to the eccentricity needs to be calculated. At this point, I would like to remind you of what the code IS 1893 part 1 2016 talks off in terms of plan irregularity coming from torsional irregularity.

So, plan irregularity can come from different sources like having a cutout in a plan,, in the plan of the building, it can be due to the shape of the building and it can be because of the torsional irregularity due to the irregular disposition of the resisting shear walls. So, torsional irregularity, how does the IS code define; this is with reference to Table 5 of IS 1893 part 1; it requires us to estimate the maximum displacement in the plane, the maximum drift of the wall divided by the minimum drift.

And if this exceeds, so if you look at a plan dimension and we are looking at the two walls, wall A and wall B; if the drift of wall A due to the in plane shear force and the drift of wall B due to the in plane shear force is calculated, you have one maximum value and one minimum value. The ratio if it is greater than 1.5, then it is a torsionally irregular building. So, this is the definition of torsional irregularity, we will make an estimate to check if this building is torsionally irregular; but to be able to do that, to be able to estimate what the drift is; we need an initial estimate of the shear force acting on wall A and wall B.

So, we need to make an initial estimate of the torsional shear and then the total shear, due to a combination of the direct shear and the torsional shear. So, before we do that we need to understand depending on where the center of stiffness is located with respect to the center of mass of a building, how is the torsional moment expected to act on the building, because of which on different walls located around the center of mass and center of stiffness; how is the effect of the torsional shear and the direct shear considered.

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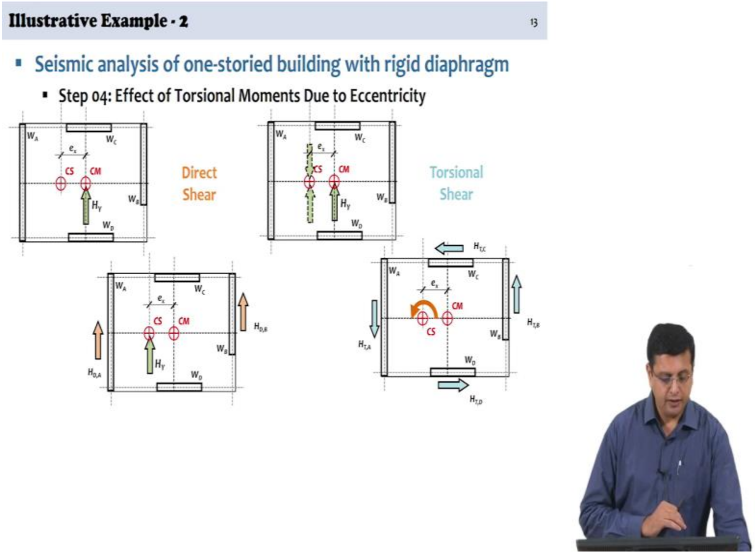

So, for that let us look at what is happening in terms of the earthquake force acting in the y direction, I will examine them separately as direct shear and torsional shear. So, we have the plan configuration and the four walls, disposition of the four walls around it. We have an eccentricity e_x between the center of mass and the center of stiffness. Direct shear force H_y is acting at the center of mass and the resistance is coming from the direct shear force which goes on to wall A and wall B, which is the value that we have estimated already.

What about the torsional shear? You have an eccentricity and therefore, it is about the center of stiffness that the plan will try to twist; and therefore, you have this additional torsional moment which comes as a demand to the structure because of the eccentricity. So, we need to estimate what is the torsional shear coming from this torsional moment and what is the value that we need to attribute of the torsional shear to the different walls; wall A, wall B, wall C and wall D.

The direct shear is counteracted by the two walls; and therefore, we estimated $H_{D,A}$, which is the direct shear force corresponding to wall A based on the distribution factor coming from the stiffnesses. And $H_{D,B}$ is the shear force corresponding to wall B which again is multiplied by the, is by multiplying H_y with the distribution factor, the stiffness of the wall divided by the sum of the stiffnesses. So, we did that, we have the direct shear force.

Now, this is added to what happens as far as the torsional moment and the shear coming from the torsional moment. So, what happens here, the torsional moment is acting about the center of stiffness. Since in this particular example, the center of stiffness is placed to the left of the center of mass. And we are looking at a shear force acting at the center of mass of H_y, the directions of the shear forces coming from the torsional moment is critical.

It is important for us to look at the position of the center of stiffness with respect to the center of mass, the direction of the earthquake at the center of mass, and then estimate what the shear demand on these walls would be, the direction of the shear demand on these walls. So, in this particular case, the torsional moment is acting about the center of stiffness; the blue arrow marks that you see in the four walls correspond to the torsional shear due to this effect.

So, the demand torsional shear on the four walls is in the anti-clockwise direction; and this is because of the positioning of the center of stiffness with respect to the center of mass. So, which means that, rigorous estimate of the center of stiffness is essential to be able to check the directionality of the torsional shear as well. So, the four torsional shear forces involve a $H_{t, A}$ which is a torsional shear corresponding to wall A; $H_{t,B}$, $H_{t,C}$ and $H_{t,D}$ now need to be estimated.

Then you have the direct shear and the torsional shear working together. Now when you look at these two figures at the bottom, you can see that on wall A the direct shear and the torsional shear in this particular case are in opposing directions; and therefore, the torsion will have a beneficial effect in this particular case, in reducing the total shear force coming on to wall A. However, if you look at wall B, then you see that the direct shear and the torsional shear are in the same direction. So, there is an augmentation of the shear force, the total shear force is now a summation of $H_{D,B}$ and $H_{t,B}$.

So, this is important. So, it is important for you to work this out; of course, with respect to the axis that you are considering, you have to ensure that you are not making errors in calculation. And this is the basis for us to understand, are you going to be adding the direct shear and torsion shear or are you going to be subtracting the torsional shear from the direct shear.

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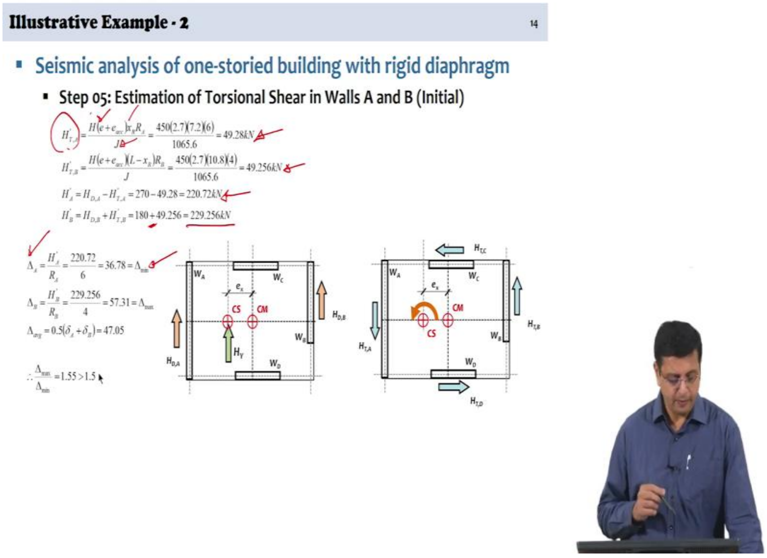

So, let us look at an initial estimate, we said we need an initial estimate; because with this initial estimate we will check if you are looking at an irregular configuration. And once we know that we are looking at an irregular configuration, we can then estimate the accidental eccentricity and add the accidental eccentricity within a design framework. So, this is initial estimate I have not made an estimate, there is no estimate of the accidental eccentricity yet.

So, let us use a thumb rule to calculate the accidental eccentricity before we go to the code estimate of the accidental eccentricity. So, in this particular case, since we are making an initial estimate of the torsional shear; I am using the notation $H'_{t,A}$, which is the initial estimate of torsional shear in wall A, and initial estimate of torsional shear in wall B. So, the estimate of the torsional shear is the shear force in the y direction H into the eccentricity that we have estimated e_x plus an accidental eccentricity.

We will assume as a thumb rule the accidental eccentricity is 5 percent of the side dimension of the building, in the dimension of the building perpendicular to the direction of the earthquake. So, eccentricity will be 5 percent of the 18 meters that we have as the length of this building multiplied by x_R and into R_A which is the stiffness of wall A. x_R is the distance between the center of stiffness and the centroid of wall A itself it is divided by the polar moment of inertia that we estimated in our step 2. So, make an estimate, we see that the initial estimate of torsional shear is about 49 kN, 49.28 kilo

Newtons. Similarly we make an estimate of $H_{t,B}$ and here to be careful that we are using the right distances, this is the distance from wall B to the center of stiffness.

So, L - x_R and we again have an estimate of the torsional shear, you see that these two values are rather close to each other. However, as I had mentioned earlier, the direction of the torsional shear matters, you have to be careful. So, you have H_A' because it is an initial estimate, you have H_A' is equal to the estimate of the direct shear $H_{D,A}$ - $H'_{t,A}$. So, $H_{D,A}$ is opposed in direction to $H'_{t,A}$; and therefore, I take 270 the direct shear component minus 49.28 and I get the initial estimate of the total shear in wall A.

Whereas in wall B, I said both $H_{D,B}$ and $H'_{t,B}$ are in the same direction and therefore, we add these two effects and we get at the estimate of the. So, 18 plus 49.256 gives us 229.256 as the total shear initial estimate of the total shear in wall B. With this known, we can quickly make a calculation to see, I know the shear force now, I know the stiffness of the wall; and therefore, an estimate of the displacement the drift in wall A and drift in wall B is possible. And from the drifts, since you are looking at two walls, from the minimum drift and the maximum drift we will be able to check if the code prescribed limit is exceeded or not.

So, I am estimating Δ_A here as, $\frac{H'_A}{R_A}$; mind you R_A is in relative terms here, it is a relative stiffness and therefore, the displacements do not have a unit here, but it is not essential for us. So, the estimate of the minimum displacement is there, the stiffness of wall R A is lower than stiffness of wall B. And therefore, Δ_A would be Δ_{min} and Δ_B would be Δ_{max} in this case, we get two values here; the average of the values is estimated.

And we are looking at $\Delta_{max}/\ \Delta_{min}$ which is 57.31/36.78 gives us a value which is greater than 1.5; that is the definition of a torsionally irregular building or a building which has plan irregularity coming from torsional irregularity. So, this is a torsionally irregular building. So, what does the code prescribe in terms of accidental eccentricity?

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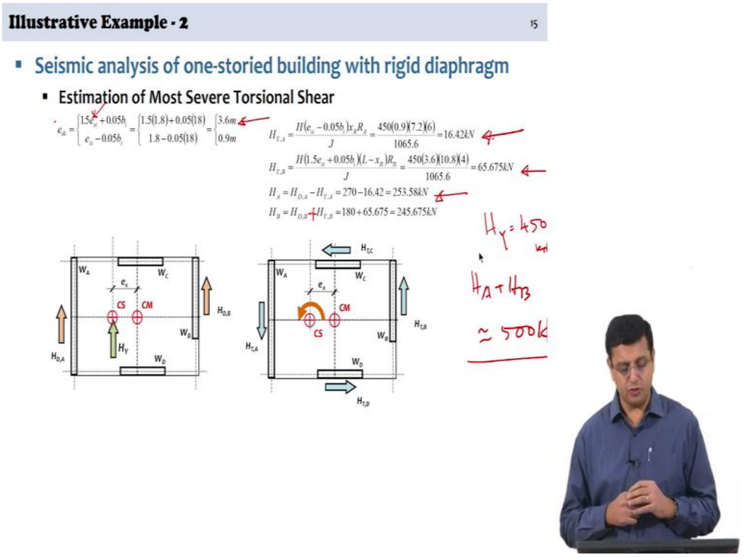

So, keeping in mind what we will supposed to do now, we are supposed to estimate the accidental eccentricity and use the accidental eccentricity to estimate the most severe torsional shear and calculate what the total shear on wall A and wall B are going to be. So, IS 1893 prescribes that the total accidental torsion considering dynamic effects, considering dynamic amplification and that is why you have the subscript d. So, the total eccentricity considering dynamic amplification should be considered as 1.5 times the static eccentricity plus 5 percent of the width, 5 percent of the dimension perpendicular to the direction of the earthquake action.

So, here e_{si}, e_{si} is nothing but the static estimate of the eccentricity which is what we have been doing all along. We have actually made an estimate of the eccentricity, but this is the static eccentricity. Under dynamic conditions larger eccentricity is expected, because a calculation does not implicitly assume dynamic effects; the code requires that the dynamic eccentricity is estimated in our calculations for design forces.

So, $1.5e_{si} + 0.05b_i$. And you have another estimate which is $e_{si} - 0.05b_i$. So, typically as a thumb rule you would look at, eccentricity estimated plus or minus 5 percent accidental eccentricity. So, in this particular case that is what we are doing, and the e_{si} minus 0.05 b would give us a worse scenario and that is how the code prescribes the amplification using a dynamic amplification in one case, and we do not use the additional effect of the dynamic amplification in the one below.

So, this estimate is made and the estimate is eccentricity including the accidental eccentricity dynamic amplification is 3.6 meters in one case, and 0.9 meters in the other case. The code requires that, for each wall that you are considering you must look at what will cause the most severe situation of total shear; it could be 3.6 in one case, it could be 0.9 in the other case.

So, we make an estimate for both, I would like to mention that; if you are doing time history analysis and there is an explicit consideration of the dynamic effects, you need not consider the dynamic amplification eccentricity through the dynamic amplification that you see here 1.5 times the static eccentricity. However, if you are basing your design force calculations on the response spectrum method or equivalent static force method, then it is essential to calculate the dynamic eccentricity and use that to estimate the total force.

So, we have made an estimate of the eccentricities we can see that it is 3.6 meters, which is 20 percent of the dimension L of the building. So, this is significant. So, eccentricity is 20 percent of the side dimension. So, with that, if you actually look at estimating $H_{t,A}$, the torsional shear on wall A and torsional shear on wall B; if you remember again, we need to consider that the torsional shear in wall A will reduce the total shear; whereas, will increase the total shear in wall B. With that in mind, we are going to reduce the least eccentricity effect of eccentricity for wall A and add the maximum effect of eccentricity in wall B.

And therefore, I use the second expression in estimating the torsional shear for wall A. So, $H_{t,A} = \frac{H\left(e_{si} - 0.05b_i\right)x_R R_A}{J}$. The estimate of the estimate of the torsional shear for wall A is 16.42 kN; whereas, the estimate in the other case wall B, since it is additive we use the higher eccentricity and therefore $H_{t,B} = \frac{H\left(1.5e_{si} + 0.05b_i\right)\left(L - x_R\right)R_B}{J}$. The torsional shear component for wall B is 65.675 kN.

Therefore now coming to H_A and H_B which are the total shear forces; in the first case direct shear force minus $H_{t,A}$ will be the total shear force 270 minus 16.42 kN, 253.58 kN is the worst case torsional shear for this wall. In the second expression H_B which is the shear force coming, the total shear force to wall B, $H_B = H_{D,B} + H_{t,B}$ and that is 180 plus this value that we have estimated now 65.675 is 245.675 kN.

So, you see that we have two arrow on the conservative side, we have actually used the worst case situation for both wall A and wall B and estimated the total shear force. You can see that now you can add up the two shear forces 253.58 and 245.67 and the value is close to 500 kN, which is different from what H_y was. H_y was 450 kN; whereas, $H_A + H_B$ is almost 500 kN.

So, that, so the extra 50 kN is because of the additional effects that you have that is because of the eccentricity and the accidental eccentricity which is also built in from the design perspective. So, with that we have the design forces for wall A and wall B and you can go ahead and design it as a reinforced shear wall, standard design approach to be followed there.