Design of Masonry Structures
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Module – 04
Lecture – 25
Design of Masonry Components and Systems Part – VII

Good morning, we will look at the P-M design approach.

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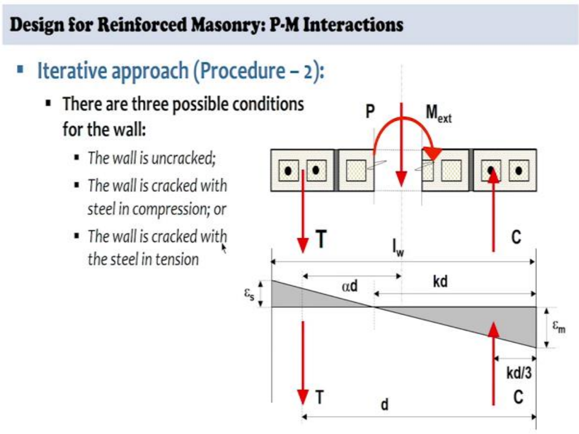

We have been looking at how P-M interaction curves can be made and then we have been talking about within the approach to reinforced masonry design with respect to NBC, how the P-M design is addressed. Since design curves in the form of P-M interaction curves are not developed and readily available for different configurations of reinforced masonry walls. The difficulty is when you deal with design to be able to arrive at the steel required for a specific combination of P and M.

And since one approach would be for each configuration of wall to draw the P-M interaction diagrams, which is not available to you or if you want to do that you are welcome to do that, but you are dealing with several walls in a building. So, that approach may not be the most convenient approach unlike in reinforced concrete design where the P-M curves, design curves are made available to you in design aids.

You do not have that luxury in the reinforced masonry design as of now; plus it is also wall configurations can have different combinations. And so, arriving at design curves is not as straightforward as in reinforced concrete frame cross sections. So, we were talking about an iterative method; we talked about one iterative method in the previous lecture and today we look at the second iterative method. And the next assignment which is going to be dealing with P-M design would then take you through the specifics of how you would arrive at, in which region of the interaction is the design situated and how do you check how much of steel is sufficient for that combination. So, we looked at a first iterative method where the assumption of whether the tension controls or compression controls in the cross section is where you begin with. And then make a set of checks to see whether that is an acceptable assumption and complete the design.

The second technique is similar, but deals with regions in which you may be situated; you make a check to know which region in the interaction curve would you be situated. And for each region you have a maximum moment that the section should be able to carry. And if the design moment is less than the moment that the section can carry for the amount of steel that you have started with and if you are in that region itself, then it completes the design process.

So, the second method is a little different from the first method, but they come from the same requirements of identifying which regions you fall into as far as the P-M interaction curve is concerned. So, basically as we have been examining with the P-M interaction curve development itself you know that there are broadly three possible conditions that the combination of P and M, the applied axial force and the external moment acting on the wall can take the wall to; in three different conditions.

The first condition is that the wall is uncracked you have some amount of bending in the wall, but predominantly you have compression the entire wall is in compression the wall is uncracked; the section is in an uncracked situation. All the bars, if you are providing bars are in compression and the entire wall is in compression.

The second case is where the eccentricities are such that the masonry section is now cracked. But the cracked zone of the wall is such that the length of the cracked zone or the length of the compressed zone is such that all the steel that you are providing is still in compression. So, cracked cross section, but steel is still in compression and as you progress, the steel reinforcement goes into tension. And so, the third zone is where the wall is cracked and the steel is in tension as well. So, these are the 3 regions which you should be able to check a priori. As we discussed earlier the first two regions are simpler because you can actually arrive at a closed form solution to estimate the maximum moment in the first two regions, where basically the masonry compression is controlling. The third region is where the tension is controlling and where the tension controls you need to use an iterative procedure. So, the third region is where iterative procedure becomes important or is required ok.

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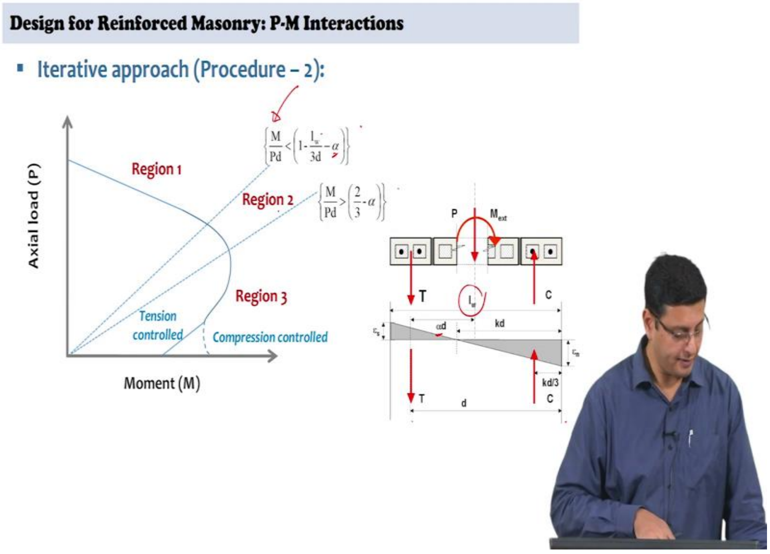

So, the 3 regions that we are talking about; basically you are familiar with the P-M interaction diagram now, regions 1 2 and 3. The first region is up to a point where the wall cracks. So, what we basically do is we use a non dimensional parameter and we will come to define the non dimensional parameter M/Pd as you are aware; M/P has been our definition for eccentricity. So, we are really looking at e/d, so it becomes a non dimensional parameter.

So, it is nothing but the eccentricity divided by the distance from the centroid of the steel reinforcement to the edge compression fiber. So, if you see the way d is defined in the diagram it is the centroid of the steel reinforcement where T is acting to the edge compression fiber that is d, so it is basically effective depth itself.

So, the ratio M/P which is eccentricity e/d is the non-dimensional parameter that you are using. And as you know based on this eccentricity for a linear elastic behavior of a cross section, it is possible for us to determine in which part of the interaction we are. So, with limiting eccentricity of length divided by 6 you are in the uncracked portion and then once cracking takes place, you need to figure out whether the steel reinforcement is now going into tension or steel reinforcement is also in compression.

So, that is the basis with which these two lines, you have these two straight lines expressed in terms of the non dimensional parameter M/Pd with respect to the geometry of the wall; and the length of the wall l_w and the distance from the centroid of the tension reinforcement to the edge compression fiber. And this parameter α that is additionally brought in; this parameter α is actually the eccentricity of the tension resultant with respect to the centroid of the wall itself. So, we are using these parameters to then establish if you are in the cracked region or the uncracked region.

So, these straight lines that go from the origin and the define into which region you would fall. As I said region 1 and region 2 given the linear elastic distribution of stresses in the wall cross section, you can establish the maximum moment. And check it against the moment the external moment that is acting on the wall and see if you are in a certain region. And the amount of steel reinforcement that you have provided or even without steel reinforcement it might satisfy the P-M demand on the wall for regions 1 and 2 in a closed form manner and then you go to region 3 ,where the iteration is required because you could be in the tension controlled region or the compression controlled region in region 3. But in region 3 as per design, it is essential that you have a tension controlled design; you have to ensure that in this region the permissible tensile stress in steel governs the design.

So, it is required that you fall into the tension controlled region, but given the state of stresses you could be in the compression controlled region and therefore, you need to bring it into the tension controlled region. So, that is the iteration that would be required and that is the reason why we do not know where the lie in region 3.

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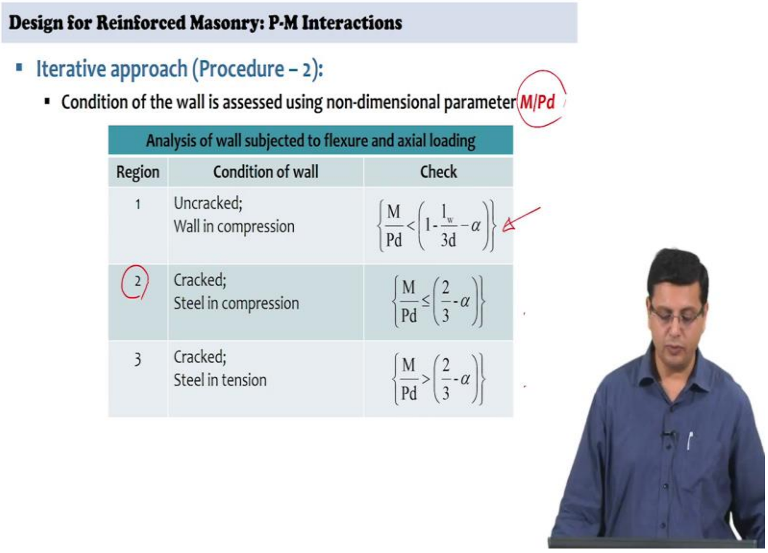

So, basically your first check would be to estimate this non dimensional parameter M/Pd and check in which region the wall would fall. So, if M/Pd is less than this quantity here, the whole wall cross section is uncracked. And you are in region 1 and we will use the maximum moment estimated from equilibrium to get the amount of steel that you should design for the given wall.

In case you then make this estimate of $\frac{2}{3}-\alpha$ for the state of forces and resultants in the wall. And if M/Pd is less than $\frac{2}{3}-\alpha$, then steel is also in compression; the wall is cracked but steel in compression. But if your M/Pd value is greater than $\frac{2}{3}-\alpha$, then it is a condition where now the steel is actually gone into tension, so you know you whether you are in region 2 or region 3.

Region 2 again is compression controlled and you can establish a closed form expression in that region. Region 3; if you fall to region 3 is where the iterative procedure is required. So we will go through the overall procedure for region 3 as well. And you will compare how the 2 iterative procedures work to establish a check for the P-M design in the next assignment.

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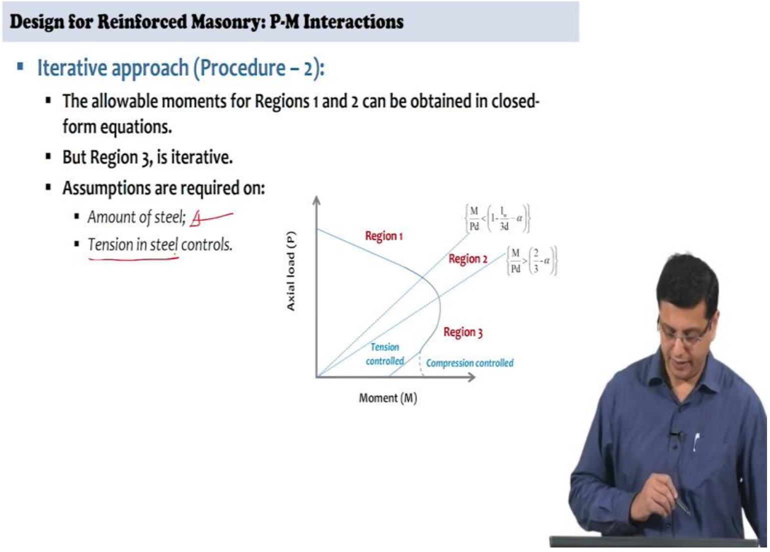

So, the closed form expressions come from the distribution of stresses in the wall. Since region 3 is iterative, you actually have to begin with some assumptions and you are going to be having to start with what distribution, what configuration of steel you want to place and what is the percentage of steel.

And then we assume that tension controls and check if the situation is such that tension controls; if compression controls then you come back and make another round of iteration to ensure that you are still in region 3 and tension controls the design itself.

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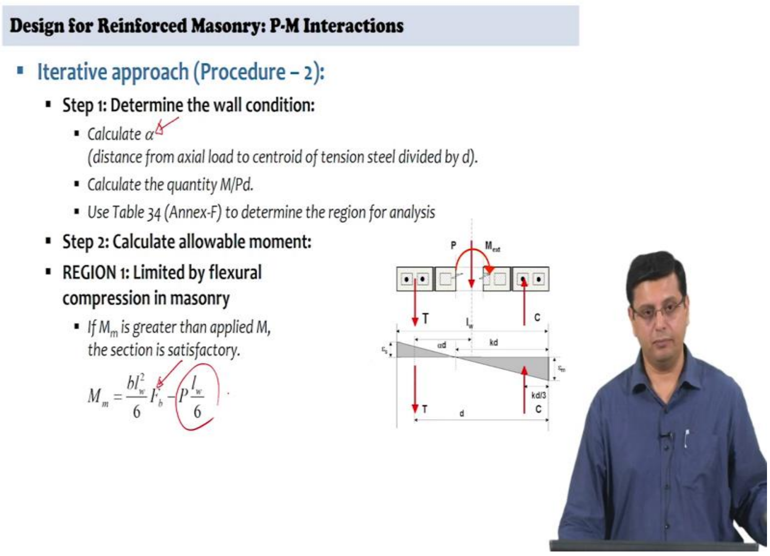

So, that is the overall approach, in step 1 you have to actually estimate based on the P and M that you are looking at. Calculate what is this quantity alpha; this is a new quantity that we are introducing. We were not working with this earlier; this quantity alpha is the distance from where the axial load is acting to the centroid of the tension steel, normalized to the effective depth d which is from the edge compression fiber to the centroid of the tension steel. So, once you estimate alpha, you can estimate the quantity M/ Pd and see in which region you would fall. So, I am making reference to Annex F of the national building code that is where you would find this procedure that we are talking about and the first procedure that we talked about and the previous table that you have seen in the previous slide of the 3 regions you can find that in table 34. So, once you estimate alpha, estimate M/Pd you can then check in which region you fall and based on that use the expression for the maximum moment. So, depending on the region let us say you are in region 1, region 1 fully in compression and you are controlled by the flexural compression in the masonry.

And therefore, you will have F_b which is the permissible compressive stress in masonry due to flexural compression already defined. That is the maximum permissible stress in compression that is allowed if the entire cross section is in compression. So, with respect to the diagram here, there is no tension in the cross section, the entire section is in compression. So, you basically make an estimate of what the maximum moment that the wall cross section can carry. You have a triangular distribution of stresses you have a strain gradient because whole wall is in compression, but you have a strain gradient.

You make an estimate of the maximum moment that the wall can carry based on the limiting permissible compressive stress considering flexural compression.

$$M_m = \frac{bl_w^2}{6} F_b - P\frac{l_w}{6}$$

Of course, you will have to from this reduce the effect of eccentricity of the load itself and that is the part that you see. So, if M_m calculated when you are in region 1, calculated thus is greater than the external moment that the wall is subjected to, then you have you have a wall cross section that is designed sufficiently to address the P-M expected on the wall. So, in reality, there is no reinforcement coming to the picture, here the steel reinforcement is not considered to be effective. Because the entire wall is in crossing is in compression and that is what is governing the flexural compression and the masonry is really governing in this region.

You might still you will you would still have to provide steel reinforcement and that is the minimum steel reinforcement that he will have to provide depending on the type of wall that you are looking at. Your wall Type B and Type C are walls as per NBC which you will go for design as per NBC, but minimum steel as prescribed in NBC.

So, you will have to do this design, let us say you have a combination of P and M which keeps the wall in region 1, it does not require any steel designed in the wall for taking care of the P plus M acting on the wall. However, there is already some minimum steel requirement if it is wall type B or wall type C. So, that has to be provided, but you are not designing steel and placing it in this wall because it is not required for the level of P plus M combination.

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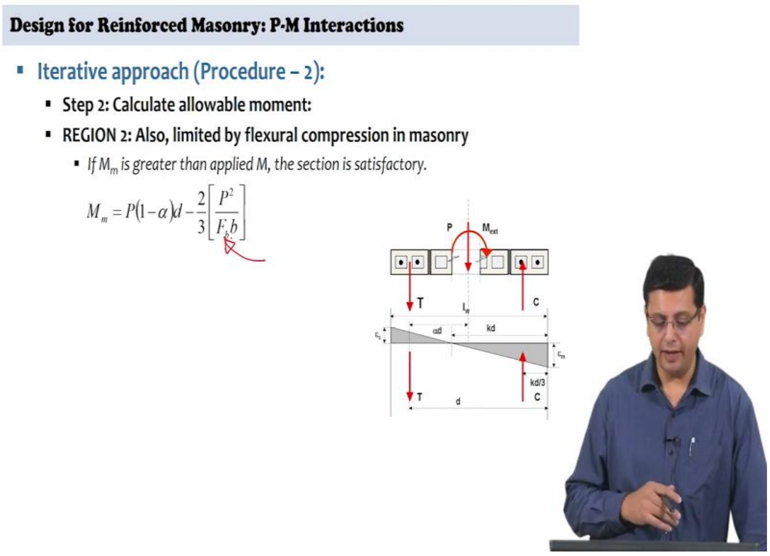

A similar situation is what you will see when you are in the second region, in the second region again wall has cracked, but you are still controlled by the compressive stress in masonry. Therefore, again F_b is something that is governing, here again there is really no requirement of steel as far as the design is concerned.

So, the area of steel, it does not come out of the requirement of P plus M, it comes merely from the minimum steel that is required as per NBC to ensure seismic capacity in the wall. So, in this case again you see that the estimate of the maximum moment comes from triangular distribution in the cracked portion of the wall. But with all the bars still if provided, all the bar still in compression and F_b is the limiting stress again which is the permissible flexural compressive stress in masonry.

$$M_m = P(1-\alpha)d - \frac{2}{3}\left[\frac{P^2}{F_b b}\right]$$

So, regions 1 and regions 2 are simple you have this expression coming from the linear elastic distribution. And you can check M external moment against M_m and the wall itself based on its dimensions should be able to resist the action of axial force plus bending moment. By the way this procedure and the previous procedure that we talked about is valid for both in-plane and out of plane design ok. So, these are procedures that can be used for design checks or design calculations both in case of in-plane bending and out of plane bending ok.

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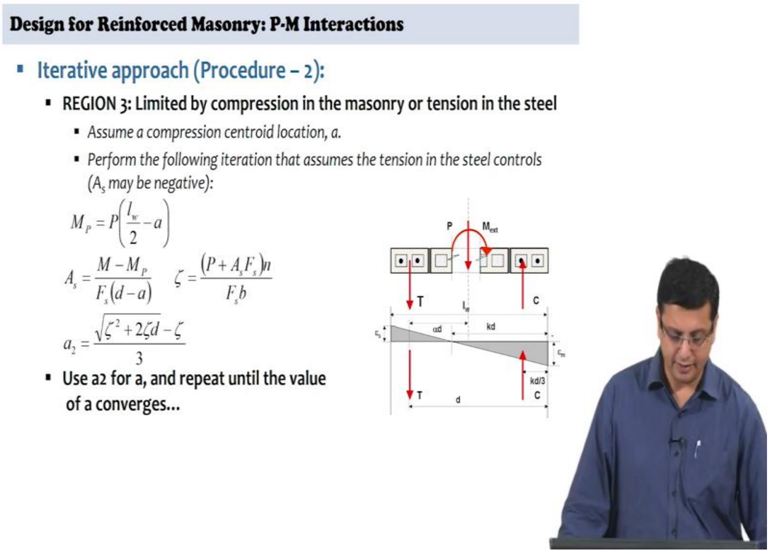

So, region 3 is the one where you are not sure whether you are going to be governed by compression or governed by tension; governed by compression in the masonry, governed by tension in the steel. And therefore, there is an iterative procedure which means we have to begin by some area of steel and then check the original assumption. We could begin with the assumption of the tension in the steel governing and come back and see if that is a valid assumption otherwise make some changes. So, the way we go about is we really do not know what the neutral axis depth is, for the combination of the P plus M that we are looking at for the wall considered.

So, kd is something we still do not know and is the reason why we need to have an iterative approach. So, we begin by assuming that the compression centroid is located at a distance ‘a’ from the edge fiber and then we assume that with that configuration the steel reinforcement which is in tension, is limited by F_s the permissible tensile stress in steel and you start calculating.

Now, you will proceed to estimate what the area of steel would be for this requirement. You might arrive at a state in your calculations where the area of steel may work out to be negative which basically implies that you do not need tension steel in the case of the P+M that you are looking at in the wall. You just have to then go ahead and provide minimum steel that is required as per the code, but there is no design steel required for that particular condition that is what the negative value of the area of steel would imply.

So, here you are going to be estimating what the moment is acting on the wall which is basically due to the cracking of the wall and the eccentricity caused because of the cracking.

$$M_p = P\left(\frac{l_w}{2} - a\right)$$

This is the assumption that you will have to make because you do not know where the neutral axis is actually lying.

To make a first estimate of what the area of steel,

$$A_s = \frac{M - M_p}{F_s\left(d - a\right)}$$

You get an estimate of the area of steel and then proceed with this; is an empirical empirically developed iteration procedure. And therefore, you will make an estimate as you can see of this value a_2.

$$a_2 = \frac{\sqrt{\varsigma^2 + 2\varsigma d} - \varsigma}{3}$$

$$\text{where, } \varsigma = \frac{\left(P + A_s F_s\right) n}{F_s b}$$

And if a_2 is equal to a, your calculation is has reached convergence, if not in the next step take a_2 as a and continue till a_2 and a converge. So, it is an iterative procedure we are trying to see if your assumption of the location of the neutral axis is correct. So, this is a simple iterative procedure, should converge in a few steps, you can keep reducing the difference between a_2 and a that you are assuming in your calculations. And then it will converge quickly and then you need to check whether the tension controlled criterion is correct or not.

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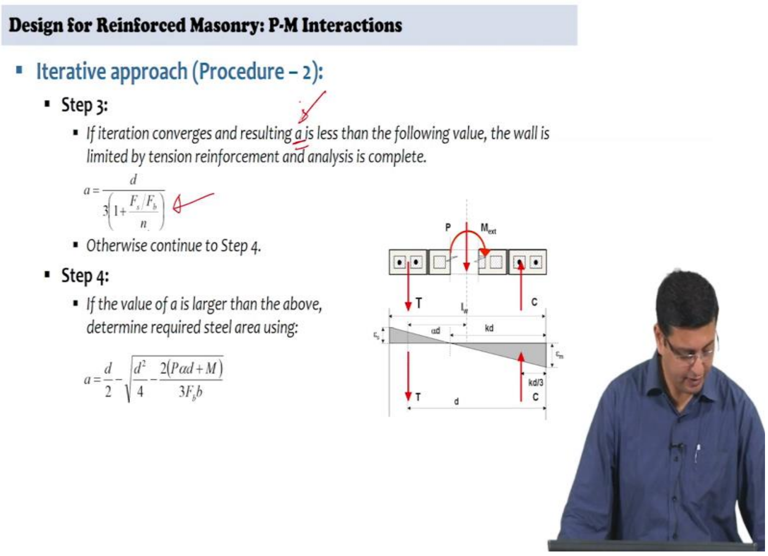

So, if the iterative converges your value of a is correct and therefore, you know what the neutral axis depth is and your calculation is correct the wall is limited by the tension reinforcement the F_s that you have that you have used as the permissible tensile stress in this steel. And you can complete your analysis and the final value of a is estimated and you can use that in your calculations.

And it should work out to be the one that you are seeing on the screen and n is the modular ratio that you have been using in your previous calculations as well. If you calculate this value of a and if the resulting value of a is less than this value; that is the value that you have used in your previous calculations, you get a converged value. If that converged value is less than this limiting value, then your analysis is complete the tension controlled assumption is correct.

And you can proceed to use the area of steel that you have used for that set of calculations as the final area of steel required. In case the value of a that you have estimated by the procedure of convergence is greater than the value, the limiting value of a, then the limiting value of a is the value limited by the stress in steel divided by the stress in the masonry under flexural compression. So, if it is so, you proceed to the next step and then calculate the area of steel corresponding to an a given by the expression here.

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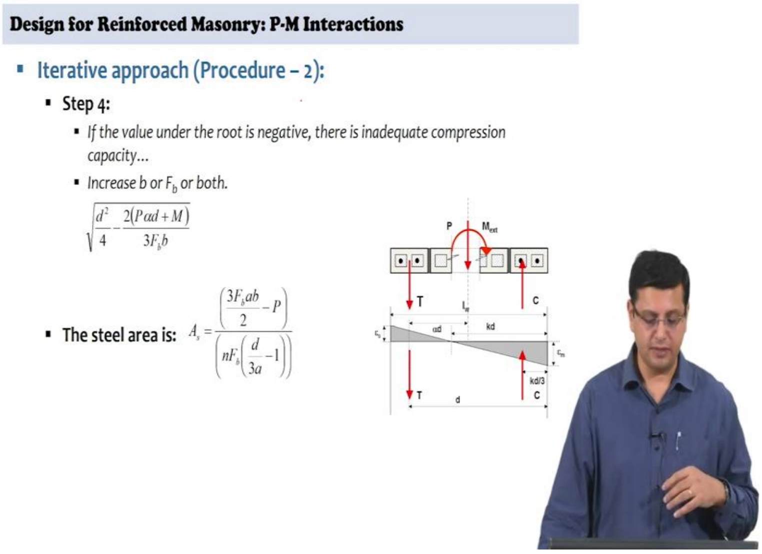

And in this case, you might have a situation where the previous term, the under root in the previous term can become negative. Now, in such a situation it basically implies that the compression capacity of the wall is inadequate. So, if the compression capacity of the wall is inadequate you need to go and basically increase the width of the wall. Meaning now, you have of course, gone into a situation where there is significant moment in the wall; significant section of the wall is cracked and you are designing the steel for that situation.

However, the compression capacity of the wall is compromised, so that state you basically need to go back and change the overall configuration of the wall. So, you can basically increase the strength of masonry which means F_b will change or increase b which is the width of the wall. So, in case the tension controlled criterion is not satisfied when you then proceed to estimate a, if the under root is negative, then there is a physical implication which has to be addressed which is lack of compression capacity which requires a change in the design strength or the cross section of the wall. So, this is something you would need to be careful about; we were talking about the under root term here. And then you proceed to check what is the area of steel for this condition right this we are now in the compression controlled condition it was not the tension controlled condition.

And therefore, we are making a change in the amount of steel that we put into the wall such that we are in the tension control condition. So, the last step here is to ensure that we bring the design back into the tension controlled portion of region 3 and that is your final estimate of area of steel itself. So, assignment 3 will basically take you through the how these calculations evolve, but also the basic assumptions with which these calculations have been made.

But it is iterative and empirical based on tests that have been conducted on reinforced masonry and an understanding of the distribution of stresses in reinforced masonry wall cross sections. But considering a non-linear P-M interaction; we have actually considered a non-linear P-M interaction implicitly in this approach itself. So, that is where I would stop today and what we need to start looking at is, we have looked at design for P+M, we looked at pure axial force design.

We have seen the expressions that are developed for pure axial force design in the reinforced masonry wall. We then have shear design which has to be addressed; with that the component level design is complete and then we will look at system level design of masonry buildings. Then arrive at the demands to walls and then come back to designing the components themselves.