Design of Masonry Structures
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Module – 04
Lecture – 27
Design of Masonry Components and Systems Part – VI

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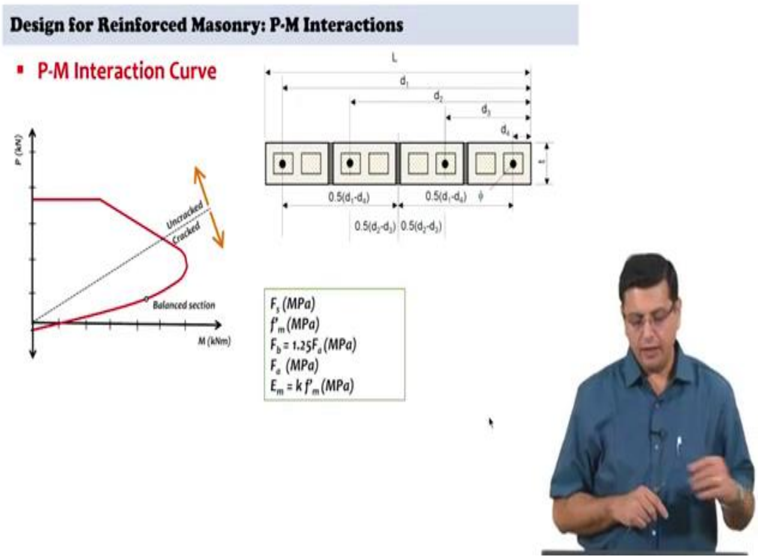

So, good morning we continue looking at the P-M interactions, we looked at yesterday how the P-M interaction curves for in-plane bending of masonry walls, reinforced masonry walls, the set of expressions can be developed with the working stress approach consideration. And we looked at four different zones for the interaction curve and primarily it is important to understand that we consider there is a portion where the masonry cross section is uncracked and then a portion where the masonry cross section is cracked.

And in this region where the masonry portion is uncracked and once cracking occurs till you reach the balanced section the masonry compressive stresses governing. It is controlling; beyond that you have the situation where it could be controlled by compression or could be controlled by the permissible tensile stress in steel.

And that part of the interaction curve where beyond the balance section, depending on the state of stress in the wall you would have to either take a compression-controlled state or a tension-controlled state. So, that is the portion of the interaction curve where a priori you would not know where the tension controls, or compression controls. The actual distribution of states of stresses is essential to be able to establish that.

So, you will see that the reason for introducing an iterative approach as far as the design is concerned is primarily because of this region where you do not know a priori whether the tension controls or compression controls. So, we will come to that. This was the interaction curve for the working stress approach. It is instructive to examine the interaction surface that you will get with the working stress approach and compare it to the interaction curve that you will get for the limit state approach.

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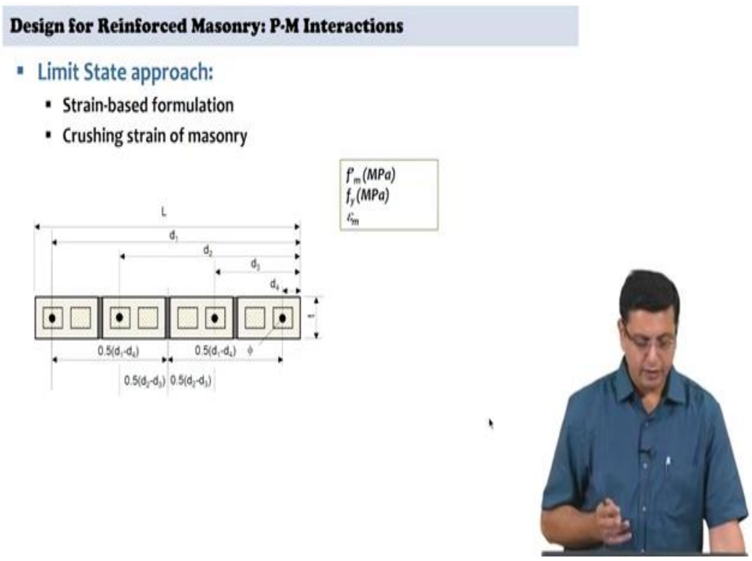

So, the limit state approach is what we will look at; what considerations you will have to make; of course, our design is completely in the working stress approach. So, this is only an exercise that you will do as far as the interaction curves are concerned. So, if you are adopting the limit state approach, then it is a question of reformulating those expressions in which we had four different zones as far as the working stress approach was concerned.

Here given the limit state approach we can make our formulations completely with respect to strains, so a strain-based formulation is what would be ideal if you are adopting the limit state approach. So, you will see that the formulation is based on assumptions of strain in the cross section under the in-plane loading of P plus M.

Of course, you need to make an estimate; you need to use an estimate of the crushing strain of masonry. Now, you are familiar with crushing strains in concrete epsilon cu the ultimate crushing strain of concrete as 0.003; 0.003 is a value that is also typically used for masonry. So, this is a value that is again established based on experiments that look at crushing failure under flexural compression.

So, the crushing strain of masonry an assumption of 0.003 is again an acceptable assumption and used for calculations in masonry as well. So, if you remember the cross section that we were looking at yesterday, we had a symmetrical distribution of reinforcement. It is important to make these a priori assumptions on what the diameter of the bars that you want to use are and the placement.

And therefore, you established what these values of d_i, d_1, d_2, d_3, and d_4 which is the distance of the centroid of the steel reinforcement bar numbers 1, 2, 3, and 4 with respect to the edge fiber. Again, our assumption is that the architectural layout is going to set what are the dimensions of the wall, the overall dimensions of the wall.

However, the configuration of the grouted masonry whether it is fully grouted, or partially grouted where you are placing steel reinforcement is again left to you. But, importantly they would make differences; they would bring about differences in your actual values in the geometry. So, it would be an iterative process to go and change the layout of steel, the bar diameters, and then look at the values d_1, d_2, d_3, d_4 or whatever those steel reinforcement locations and sizes are.

So, as far as the limit state approach the values that you would require, the crushing strength of masonry is required f'm and then you need the yield strength of steel and the crushing strain in masonry. Earlier we had our permissible stresses in compression (in flexural compression) and permissible tensile or compressive stress in steel. And here we are going to be working with the ultimate strengths of the masonry and the steel itself.

So, the formulation is far simpler than the four regions approach that we had earlier and you get a more continuous P-M interaction curve in the limit state approach. Because, it is possible to work completely in strains and keep increasing the strains and estimate the stresses at different stages.

Whereas, what we have been doing in the earlier situation is working from the stresses, but then the continuity and stresses cannot be guaranteed you start taking off sections. You will have to make amends for the cracked uncrack situation and then once the control comes from the steel permissible stress as well.

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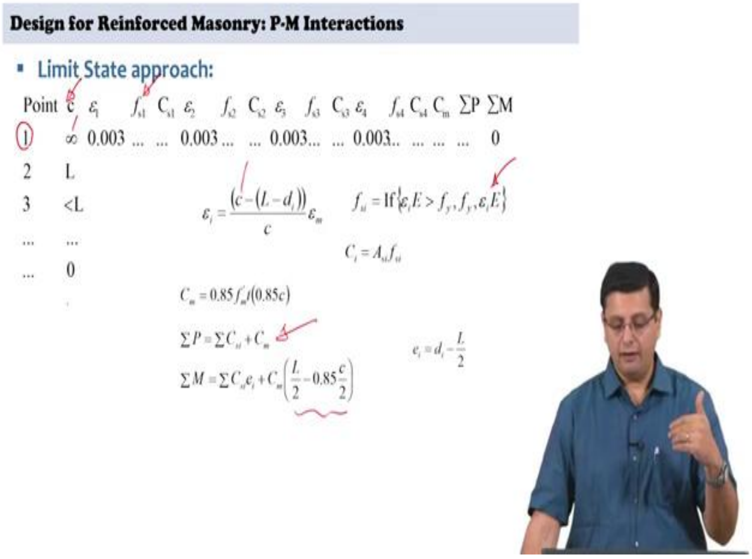

So, the formulation as far as the P-M interaction diagram here is far simpler. So, we will come to the table in a moment. You need to make an estimate, you need to start from a state of strain and then look at introducing eccentricities such that there is a strain gradient. So, when you begin you begin with the assumption that we are at the crushing strain in all the entire cross section.

So, the first point is actually looking at a state of pure compression, the first stage is stage of pure compression ε_1, ε_2, ε_3, and ε_4 are the strains corresponding to the 4 bars. And we are keeping that because of the compatibility of strains equal to the crushing strain of the concrete itself. So, you have got 0.003 at all those four locations; your moment capacity is 0 at this stage, the axial load capacity is your maximum axial load capacity in the wall.

So, that is your starting point of course, for that the notation c that is used here is the compressed length of the wall ok. The compressed length of the wall implying in this case, the first case I have put infinity here implying that the position of the neutral axis when you have full compression is infinity. So, you keep reducing the position of the neutral axis in the subsequent stages.

So, how do you formulate, so in the first stage you are fixing the values of ε_1, ε_2, ε_3, and ε_4 as 0.003 and estimating corresponding stresses. And the corresponding compression resultant in each bar C_{s1}, C_{s2}, C_{s3}, C_{s4} and estimating the compression carried by the masonry, getting ΣP and ΣM based on corresponding eccentricities ok.

So, how do you estimate the strain at each location, the strain at each location is related to the compressed length c; c is the compressed length L is the total length of the wall. And d_i is the distance from the edge fiber to the centroid of the cross section of the bar that you are considering 1, 2, 3, or 4 and ε_m is the value of crushing strain that you have assumed.

$$\varepsilon_i = \frac{\left(c-\left(L-d_i\right)\right)}{c}\varepsilon_m$$

So, merely by the distribution of strains which is remaining triangular you are able to make your estimate of what the strain is in each bar; correspondingly you will estimate what is the stress with an assumption of what the modulus of elasticity is. So, here modulus of elasticity of steel has to be assumed and you will estimate what is the force in each bar. Mind you were in the limit state approach the concept of modular ratio is out of the picture now in we are working purely on steel stresses and we are working on strains and then moving on to the steel stresses and the masonry stresses.

So, here E you will use directly the modulus of elasticity of the steel itself, but then you limit the value of the steel stress to the yield stress and so, you will make a check if $\varepsilon_i E$ is greater than f_y, if it is so then f_y is the value that you are limited to steel has yielded. And you will continue using f_y if it is not yielded, then you will use the value that you get out of the calculation of $\varepsilon_i E$ itself.

So, with the steel stress known, so you know f_{s1}, f_{s2}, f_{s3}, f_{s4}, you can then estimate based on the areas of cross section of the steel bars what the compression resultants are: $C_i = A_{si} f_{si}$.

Once that is done we now have to look at what is the contribution and compression coming from the masonry. But we started by examining the crushing strain stage in the masonry, that is where we are beginning.

And therefore, the compressed end of the masonry is at its crushing strain, is at it is ultimate point. And therefore, the stress block has to be brought in; here we have already reached crushing strain and therefore, the masonry compressive stress cannot be estimated like we had done earlier. We use 0.85 f'_m where we are using the stress block parameters now. And therefore, the compression resultant C_m in the masonry is 0.85 $f'_m t(0.85\ c)$; 0.85 x 0.85 gives you the rectangular stress block parameters and c is again the compressed length of the wall in masonry.

So, C_m is estimated; you get your estimate of the total force resultant axial force resultant and the total moment resultant.

$$\sum P = \sum C_{si} + C_m$$
$$\sum M = \sum C_{si} e_i + C_m \left(\frac{L}{2} - 0.85 \frac{c}{2} \right)$$

The lever arm for the steel reinforcement bars has to be estimated for each bar as nothing but the distance from the edge fiber minus half the length of the wall. Whereas, for the masonry portion it is centroid of the rectangular stress block with respect to half the length of the wall. So, you get the two eccentricities, you have your estimate of M.

So, what you do next is to start reducing, let us assume that the first stage is over. The second stage you are estimating that the eccentricities are such that the compressed length is equal to the length of the wall right. So, it is at that limiting stage where beyond that point you start getting tension in the cross section.

So, now, all the four bars are in compression, entire length is compressed; compressed length is equal to L of the wall. For that state keeping the masonry compression compressive strain at the maximum 0.003; you will then estimate what is the corresponding strain from the geometry at bar 1, 2, 3, and 4.

So, you are you are fixing the eccentricity and then estimating the corresponding strains from the linear distribution of strains assumption. Estimate ε_1, ε_2, ε_3, and ε_4 and proceed;

then reduce the compressed length you assume cracking in the cross section and keep going further till the section is completely cracked.

Or compressed length comes down to 0; the beauty of the formulation based on strain is that the same set of expressions hold right from the first stage full compression to the last stage where it is going to be fully in tension. Where depending on the geometry possibly all the four bars have yielded or few bars have yielded and few bars have not yielded.

And then with the last stage you will have no axial load capacity, only moment capacity and establish the P-M curve for the limit state approach. So, that is rather straight-forward and this is what you would be doing as far as comparing the P-M interaction diagram from the limit state approach to the P-M interaction diagram that you have that you have seen yesterday.

So, things that you will be comparing in the assignment that follows is to look at the effect of compression reinforcement in our calculations. You can do that even here; even here where the contribution of compression reinforcement can just be neglected. Put a 0 wherever you have contribution of compression C_{s1} in compression C_{s2} in compression and once they are in tension use those values only as you did for the working stress approach.

So, the first one is to look at a comparison between the working stress approach and the limit state approach in terms of the P-M interactions. The second would be to look at how effective is consideration of compression reinforcement in the P-M interactions; what is the percentage increase that you get in the surface itself.

And the third would be to see if tensile strength of masonry, if you actually make use of the tensile strength of masonry. It is so marginal that you can actually neglect it in your calculations, should you be considering even that the permissible tensile stress in masonry towards estimating the P-M interaction surface itself.

And finally, you can look at different arrangements of steel reinforcement, if you have the concentrated steel reinforcement at the ends versus a distributed steel reinforcement how do the P-M interactions pan out as far as the same geometry, and the same material strengths are concerned. So, these are iterations that you can do and get an understanding of the capacity the bending capacity affected by the axial load levels in masonry in reinforced masonry walls ok. With that I think it is at this stage important to link up to design.

Now, as you know in reinforced concrete design you have design aids where particularly for different types of cross sections and different layout of steel reinforcement. You have P-M interaction diagrams design curves which are readily available to you in the design aids right.

Now, we do not have design aids here and that is something we could look at coming at some point of time. However, masonry wall configurations can be more random than regular reinforced concrete column cross sections. So, it is not going to be so simple to be able to arrive at a set of design aids for different lengths and different configurations, it is going to be too exhausting, too voluminous in reality.

And therefore, as far as design is concerned it is not going to be possible, it is not going to be feasible for you to prepare a P-M interaction curve for every reinforced wall in the masonry structure right. That may be the ideal thing to do and if you do that you have complete control in terms of your design loads and the interaction surface itself.

However, that may be too much to ask for and therefore, since you will not be using a P-M interaction surface, you need to know what is the for the combination of moments and axial forces at a given wall, you need to know what is the state of stress in the wall. Are you in the cracked condition of the wall, are you in the uncracked condition of the wall is compression controlling if you are beyond the balanced section. And that will determine how much steel you are going to put in the wall itself to ensure the wall design satisfies the load combinations.

So, since as I mentioned earlier, beyond the balanced section you really do not know a priori whether you are in the compression control state or the tension control state. You need an iterative procedure to understand in which part of the interaction curve do you does the combination fall in and use appropriate expressions to estimate where the neutral access depth is lying.

Because that is something you do not know at all, you have to estimate where the neutral access depth is lying. And then based on that go back and check the state of stresses in the masonry and the state of stress in the steel and ensure that they do not go beyond the permissible stresses. So, that aspect of not knowing where you are in the interaction surface and whether it is going to be a tension-controlled estimate or compression controlled estimate.

And therefore, what is the neutral access depth is the unknown portion of the design process. So, the code when you are looking at design for P plus M, the code actually gives you two iterative procedures ok. The description of the iterative procedures are available in the annex of the section that deals with reinforced masonry in the national building code.

So, you could look at using either of these procedures and as I said we look at one of the iterative procedures today and examine the second iterative procedure as well. So, that we are familiar with the approaches to iteratively estimate the state of stresses in the wall, and do the necessary design check.

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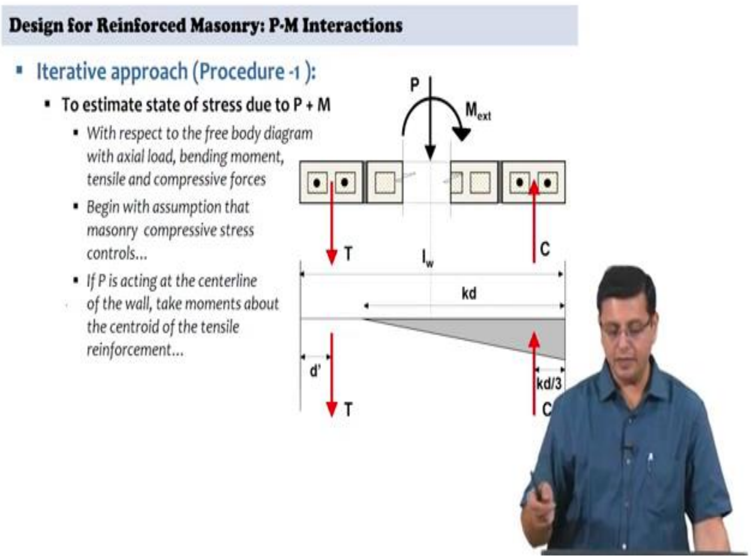

So, that is where the first iterative procedure is what I am looking at. Procedure 1 makes an assumption of where you are sitting in the interaction surface. And then tries to arrive at what expressions should we use when you are in the compression-controlled assumption or tension-controlled assumption.

So, we look at the free body diagram here, we are looking at a wall made out of hollow blocks, it has a certain arrangement of the steel reinforcement. Here we have an arrangement with the steel reinforcement concentrated at the two ends ok. So, again total length of the wall is l_w and you have the steel on the two ends, they are the flexural steel.

You have an axial load acting about the centroid of the wall and you have a moment coming from external forces acting on the wall. Now, what you really need to establish is what is kd or the depth of the neutral axis and this is still in the working stress approach. And therefore, the triangular distribution of stresses is what we are going to be basing these calculations on.

So, you have the free body diagram at the bottom and then you have the bending moment and the axial forces, the tensile and compressive forces that we will have to estimate. We are assuming that part of the wall is cracked, and the tensile force is acting at the resultant of the region which has the reinforcement in tension.

So, we are taking the centroid of the two bars and that is where the tension resultant has been placed compression resultant is of course, sitting at the centroid of the triangular distribution of compressive stresses. So, you need to make some basic assumptions when you start and then go back and check if for the distribution of stresses and the values of stresses is there assumption right.

If that assumption is incorrect then you go back and make a change to that basic assumption which is on whether we are in the compressed compression-controlled zone of the interaction surface or in the tension-controlled zone of the interaction surface. We begin by making an assumption that we are in the compression-controlled zone of the interaction surface. And we assume, this again depends if you have got an eccentricity of the axial load with respect to the wall.

This is the axial load coming from gravity, if there is an eccentricity that is something that you need to consider. But if P is acting at the centerline of the wall you can then take moments about the centroid of the tension reinforcement. So, you have the tension reinforcement on this side, on the left side you take the centroid of the tension reinforcement. And take your moments about the centroid of the tension reinforcement itself.

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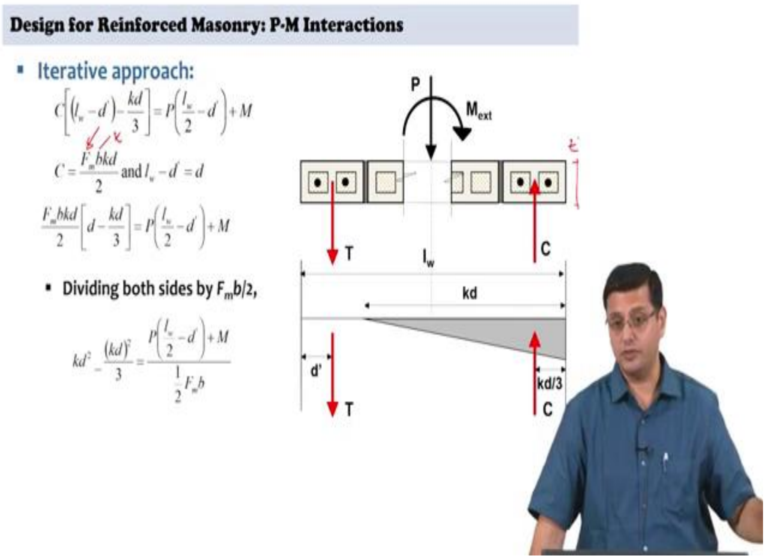

So, with respect to the notations in the drawing here, we take the equilibrium of moments. And then the moment due to the load because there is an eccentricity that is coming because of the cracking in the wall. Because of the cracking in the wall the eccentricity would then imply that there is an additional or secondary moment that occurs because of the cracking in the wall.

$$C\left((l_w - d') - \frac{kd}{3}\right) = P\left(\frac{l_w}{2} - d'\right) + M$$

So, with that written you can basically write down the compression resultant from the triangular distribution of stresses in terms of F_m. Because since we are assuming that compression controls, we are at the permissible compressive stress.

$$C = \frac{F_m bkd}{2} \text{ and } l_w - d' = d$$

$$\frac{F_m bkd}{2}\left[d - \frac{kd}{3}\right] = P\left(\frac{l_w}{2} - d'\right) + M$$

$$kd^2 - \frac{(kd)^2}{3} = \frac{P\left(\frac{l_w}{2} - d'\right) + M}{\frac{F_m b}{2}}$$

$$(kd)^2 - 3d(kd) + \frac{3\left[P\left(\frac{l_w}{2} - d'\right) + M\right]}{\frac{F_m b}{2}} = 0$$

You will see that you get a quadratic equation in kd.

So, you basically need to estimate kd for the assumptions that you have made, and for the kd that you establish you want to check what the state of stresses are. So, we now have an expression that can give us a quadratic in kd which you can solve and get values for kd.

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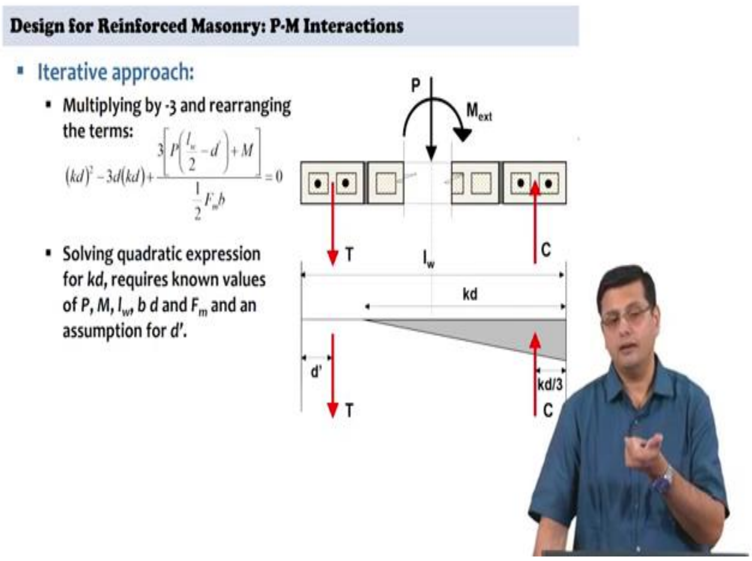

So, if you have to solve this expression, if you look at the values that you need to plug in there, you need the dimensions of the wall, you finalize the dimensions of the wall. You would need to have made an estimate of the steel reinforcement that you are going to give right. You need to have a priori decided what is the steel reinforcement as a first stage in your design. And therefore, some assumption on what the bar diameter and the location of the bar is becomes essential as your d' will get affected by that choice.

So, you will need to know what the axial force is that is acting on the wall, the external moment, the length, breadth and the thickness. And the d of the wall, what is the permissible compressive stress in masonry and what is the assumption that you want to make for d'. And therefore, your steel reinforcement configuration matters. With those values known you can solve for kd and establish what the value of kd itself is.

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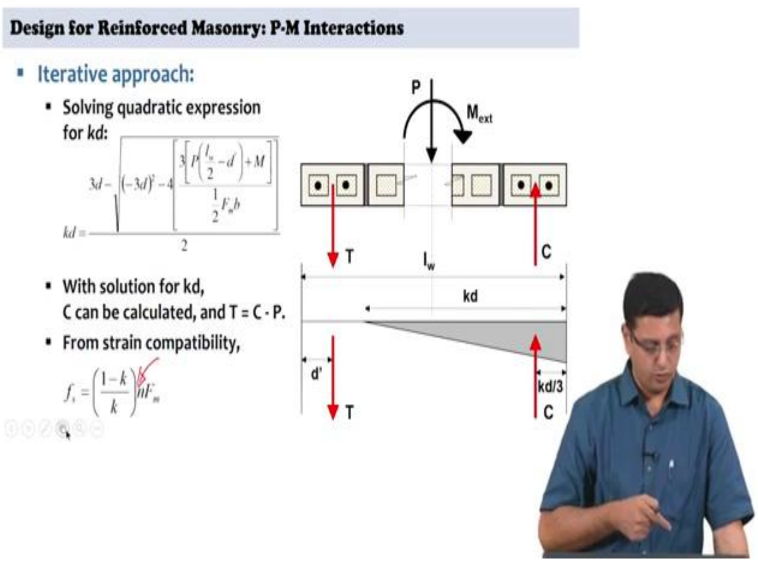

Now, for this kd, once you establish this kd, you know that the compression resultant C is written in terms of kd, as a function of kd; you can go and solve for kd. And then with the compression resultant known and P known, you can establish what T is. And you now have a way of checking if your first assumption that compression controls the interaction whether that is valid or not.

So, since you looked at compression-controlled situation, based on the compressive stresses you will establish from strain compatibility what the steel stresses are. So, the steel stresses are established, we are in the working stress approach, n here is the modular ratio of the steel to concrete moduli. And you establish f_s after you have established what the state of stress in the masonry is going to be.

F_m is considered as the stress in the permissible compressive stress in masonry. So you now have an estimate of f_s. However, if this estimate of f_s is larger than the permissible tensile stress in steel then your assumption is wrong that compression controls.

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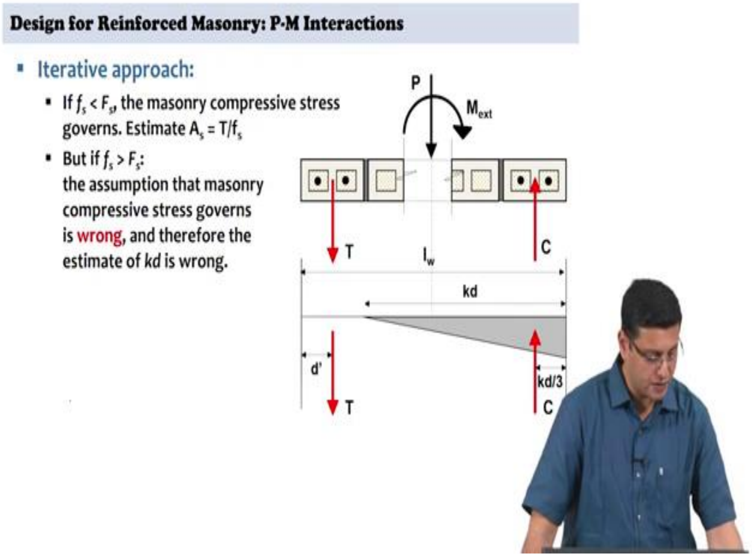

So, at this stage with f_s established you go and check, if this f_s is less than the permissible steel stress F_s. If permissible steel stress F_s is larger, then you have been working with the right assumption; in reality the load combination is such that in the interaction surface masonry compressive stress still governs.

And therefore, you can now make an estimate of what is the area of steel that is required to satisfy the design requirement. So, estimate area of steel from tension force that you have estimated divided by the stress in steel; the stress in steel is the actual stress in steel lesser than the permissible tensile stress in steel.

So, if this is true the cycle ends and you have establish how much steel is required to satisfy the interaction accounting for the interaction itself. However, if the stress in steel f_s is greater than the permissible steel stress, then your basic assumption that the masonry compressive stress governs is wrong. And you have to go back and make amends because the kd that you have established is the wrong kd now.

So, in this case you will have to go back and start using a tension-controlled scenario and for the tension control scenario check again if your f_s value is less than the permissible tensile stress. And then if it is right, establish what the compressive stress is and close the calculations, establish the amount of steel reinforcement required for that condition.

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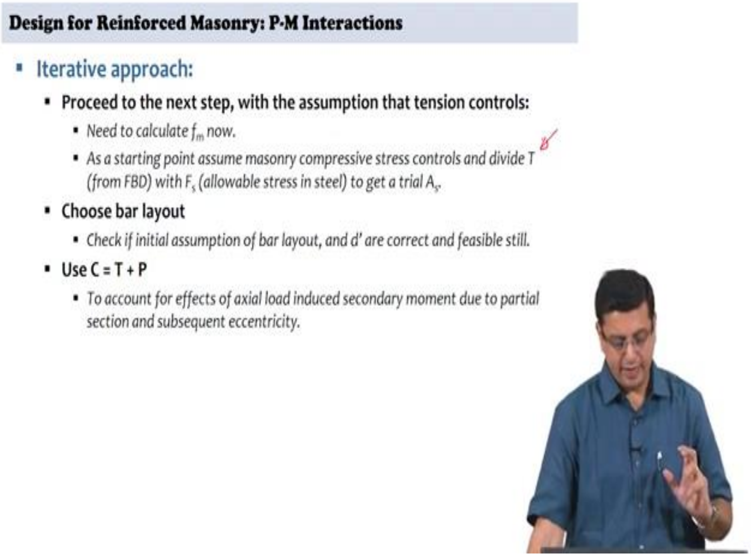

So, to proceed, we now have to look at a tension-controlled scenario and from the tension control scenario F_s being the permissible tensile stress, you will then calculate how much is the value of f_m. So, but you need an estimate of the amount of steel to actually begin; you also need an estimate of the masonry compressive stress.

So, we can actually start this part of the calculation assuming that the masonry compressive stress still controls. And divide the tension that you are getting from equilibrium with the allowable steel tensile stress. You have from the equilibrium established what T is and divide that T with F_s and get a trial area of steel because from the previous calculations whatever kd you established is wrong and area of steel is wrong.

And therefore, you now need to rework and start somewhere and to therefore, to get a trial area of steel you are taking the tension force that you arrived at dividing it by the allowable stress in steel. Because now allowable stress in steel is governing, tension controls get a trial area of steel and then start your calculations.

You might want to change bar layout or bar dimensions; your d' will change, check if it is still feasible to use the d' that you are using earlier. And then the total compression force as the tension resultant plus the axial force that is acting on the wall. This compression and tension is because of the bending itself.

So, you actually have to account for the secondary effects; there is going to be cracking in the wall shift of the centroid shift of the there is an eccentricity cost because of the cracking and due to the axial force there will be secondary moments. So, you account for the secondary moments, and the external moment acting on the wall. And then establish what the compression force for which you are going to be making the calculations themselves.

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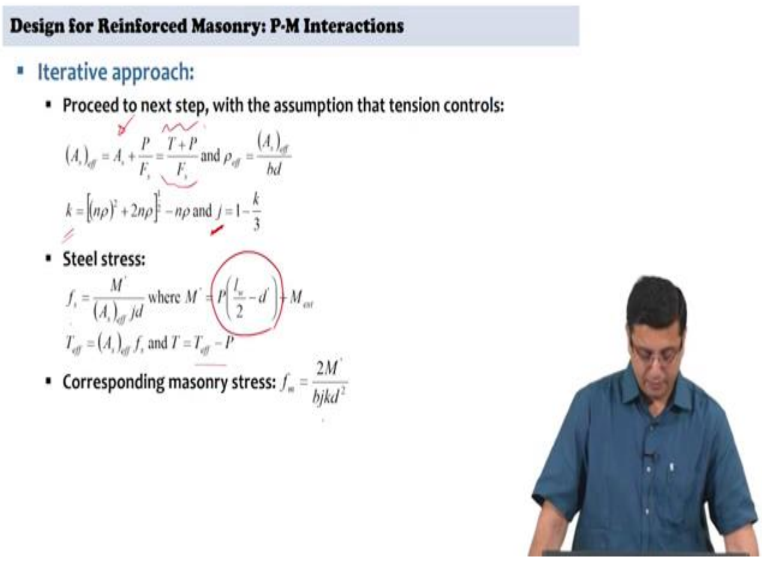

So, the area of steel that you are going to be using in your calculations is the area of steel that you estimated in the previous step plus the axial force divided by the permissible stress F_s. So, to account for the secondary effect tension force plus the axial force that is due to gravity T plus P divided by F_s. So, this becomes an estimate of the steel that you are going to be using in your calculations. For this area of steel rho effective is the percentage of steel in the wall itself; $(A_S)_{\text{effective}}$ divided by bd, where b is the width of is the thickness of the wall, and d is the effective depth. With that expression you will now estimate the value of k. And the corresponding j in the triangular distribution of compression, j = 1-k/3 in the triangular distribution of compression. Again you are solving a quadratic and getting the value of k; now this k thus established, if it satisfies the requirement of tensile stress and steel lesser than F_s you can actually conclude.

So, you estimate what the tensile stress in steel here is; this tensile stress in steel should not be more than F_s. So, f_s is calculated from the moment, the total moment you have a moment which is the secondary moment because of the shifting; because of the eccentricity caused by cracking plus the exterior the externally applied moment.

From kd and j calculated, you can then make an estimate of what the stress in steel is. And the stress in steel as the total moment divided by the effective area of steel divided by j into d. And it should satisfy the original set of assumptions that you have made in terms of what the effective tension is and what the tension from equilibrium itself is.

So, once that is established, with f_s known you can estimate what the corresponding masonry stress is. And with the triangular distribution of stresses you can estimate that it is given by,

$$f_s = \frac{2M'}{bj(kd)^2}$$

So, you are making an estimate of the stress in the masonry compressive stress and this value again has to be lesser than the value of f_m which is the permissible compressive stress in masonry.

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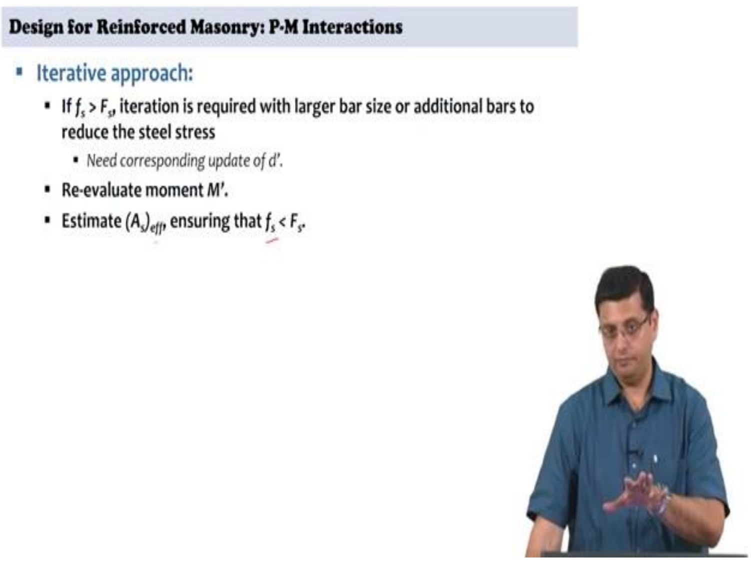

So, here again the f_s that you have estimated if it is lesser than the permissible tensile stress of steel you can stop your iteration. But if f_s is greater you basically have to look at changing the sizes of the bars, which would mean again some changes in your configuration or the values of d'. So, you might require additional bars or larger bar to account for f_s being less than or equal to F_s.

You need to reduce the steel stress, update your d' and then if d' is getting updated, your estimate of kd will change. And then your k and j will change and then you have to reevaluate your value of M'. And then for that value of M' again f_s has to be calculated; effective area of steel has to be calculated and corresponding masonry stress has to be evaluated.

So, that is the iterative approach which is basically taking you through a set of assumptions. There is an implicit reference to which zone of the interaction surface are you falling in, right. But there is no direct use of the interaction surface itself; the second approach which we will discuss subsequently is an approach that actually identifies explicitly in which zone you are.

And then uses a non dimensional parameter M/ Pd to estimate what expression should you use to check how much of steel is required for the interaction; the P-M load combination itself. So, this is the first iterative approach you are free to choose any iterative approach; even another third iterative approach is ok.

But this is necessitated because of the unknown condition particularly when you are looking at beyond the balance point of the reinforced masonry cross section itself. So, I will stop here with the iterative approach, the first iterative approach. And discuss the second iterative approach and then get into some design examples which will give you a feel for and a hold on how you make these iterations. And why you make these iterations depending on the P and M demands on the wall itself.