Design of Masonry Structures
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Module - 03
Lecture - 21
Strength and Behaviour of Masonry Part – XI

So, good morning. We will examine the last segment of the module that deals with Strength and Behaviour of Masonry Components.

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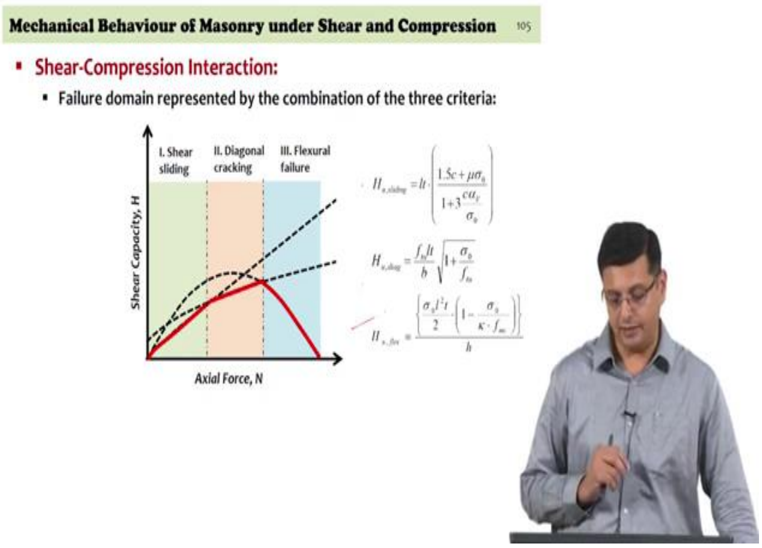

And in the last lecture we were examining the interaction surface between shear force and axial compression, in-plane shear force and axial compression. And we looked at this interaction in terms of the 3 criteria that we used to develop the failure domain, we developed expressions for each of these. And if you remember the expression dealing with the flexural compression mechanism is estimated based on the ultimate moment capacity and then knowing the geometry of the wall you can express it in terms of the lateral force, ultimate lateral force H required for developing the flexural mechanism.

The other two mechanisms are shear mechanisms which is based on the diagonal tension failure and then the sliding shear failure. So, once this is estimated for all values of axial force, you get the interaction surface which is the lowest shear force for a given mechanism, the lowest shear force of the 3 mechanisms which then gives you the failure domain itself.

Now, this is for in-plane capacities, in-plane behaviour and we have expressed it in terms of shear capacity versus the axial force. This can also be represented in terms of the moment versus the axial force, so the same can be represented in terms of moment and axial force which then becomes an interaction surface in moment and axial force. So, since the in-plane behaviour is governed by shear, the masonry load bearing wall acts as a shear wall we have had to resort to the shear mechanisms to get a full understanding of the behaviour of masonry under different levels of axial force.

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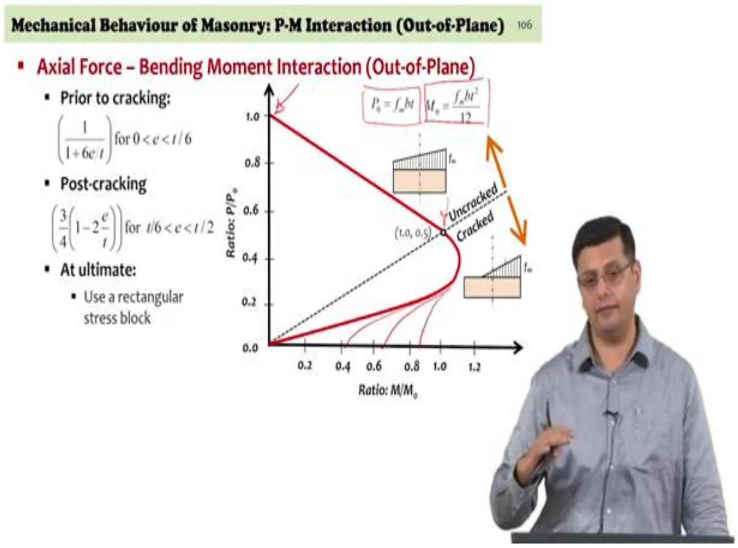

In a similar manner, we can also look at the axial force-moment interaction. As I said in the previous case though it is shear force versus axial force, you can represent it in terms of bending moment versus axial force. For in-plane mechanisms that is not so common. We tend to represent it more in terms of shear force versus axial force.

However, for out-of-plane mechanisms shear is not a dominating mechanism, shear is not the mechanism which for which you expect failure to occur. The out-of-plane mechanism is typically governed by flexural compression and so, P-M interaction curves can be developed for out-of-plane bending. And this is based on what we have looked at earlier. We looked at the effect of out-of-plane bending the response of a wall in the out-of-plane direction due to lateral forces. But here we are looking at the axial force to the bending interaction, and an interaction surface can be created assuming linear elastic distribution of stresses or assuming a non-linear distribution stresses at the at the ultimate atleast.

So, if you look at the interaction surface here, we are looking at a normalized interaction surface for a wall bending out-of-plane and the load acting is the axial force with a certain eccentricity. So, it is different from the lateral force based failure mechanisms that we have been looking on looking at in terms of the out-of-plane capacities. This is different representation because the action on the wall is different.

So, we are looking at normalized axes, P/P_0 on the y axis and M/M_0 the moment capacity on the x axis. And here P_0 is nothing, but the compressive strength of the masonry into the area of cross section with the length of the wall into the thickness. So,

$$P_0 = f_m bt$$
$$M_0 = \frac{f_m bt^2}{6}$$

So, we are making use of the section modulus to arrive at the value of moment capacity and it helps us represent the x and y axis in a normalized manner.

So, this can be developed based on 3 important zones in the interaction surface. The first one being where the entire section is in compression and we have developed these equations earlier prior to cracking which is the straight line that you see from a value of P/P_0 equal to 1, that is from this point all the way up to when the wall starts cracking.

So, we are looking at a situation where the eccentricity of the axial force is less than t/6. The classical expressions that we have seen earlier, so that is the that is the level of eccentricity in the wall between 0 eccentricity when you get the full axial force capacity of the wall which is P_0, all the way up to when cracking commences at eccentricity equal to t/6. At that point you can see that you get a moment capacity of 1 in the wall whereas, the axial force capacity drops to one-half and that is because of the triangular distribution that we have assumed in the distribution of compressive stresses in the wall.

So, beyond that point this critical point that you see here where the moment capacity is 1, the axial force capacity is 50 percent of the capacity P_0 the wall starts cracking. Once the wall starts cracking, we are looking at the post cracking phase, but in the post cracking phase we are still continuing with an elastic distribution linear distribution of stresses.

So, it is only at the ultimate that we would like to bring in, if you are using a non-linear approach the softening under compression with the use of a stress block parabolic, stress block replaced with an equivalent rectangular stress block. So, beyond that point we are looking at a cracked cross section and you do now work on the compressed thickness of the wall which is less than t after this point. So, in this case, you really looking at eccentricities that range from the critical eccentricity at which cracking commences t/6 to about 50 percent, eccentricity equal to about 50 percent of the cross section itself.

So, you can then estimate the moment capacities for the remaining section, but as you go close to a section which is very small in comparison to the thickness of the cross section, you can assume that the when the value of the compressive stress approaches the compressive strength of masonry you can start assuming for the ultimate condition that a rectangular stress block is present. And that is what is going to give you the moment capacity in the in the wall and the axial load capacity.

So, this is a basic framework with which you can work to develop the axial force bending moment interaction. You can fix the level of axial stress to be equal to f m and look at different eccentricities from 0 eccentricity to the cracking eccentricity. And then beyond that, start reducing the cross section then you come to the second phase of the cross section and its bending moment capacity and axial force capacity, and then you come to the final stage where you will want to replace the force resultant with that which corresponds to the rectangular stress block.

So, this is a simple framework that you can use to develop the axial force bending moment interaction surface which can then be used for design. Of course, here we have looked at an unreinforced masonry wall cross section. If you were to reinforce this wall cross section with one or two reinforcement bars then you will agree that it is only in the cracked phase, once the steel bar start experiencing tension you will have improved moment capacity which is in the lower half, in the lower one-third of the of the interaction surface itself.

So, with reinforcement, if reinforcement is to be added in the cross section we are actually going to be looking at the improvement of moment capacity in this zone depending on how much steel reinforcement you are bringing in you will have improved moment capacity in the post cracking phase of the interaction curve. You will be using for the reinforced masonry, interaction curves both for in-plane bending moment to axial force interaction and out-of-plane moment to axial force interactions. So, this is one last part in terms of the strength, behaviour and strength of masonry that I wanted to touch upon. There is one component that remains which again links us to design and that is really deformations.

Now, deformations and stiffnessess I would say, because you need an estimate of stiffness, you need to know what deformations you will get in a masonry wall to be able to estimate what is the demand that certain masonry component is going to experience. You need an estimate of the stiffnesses. So, let us look at a basic framework to understand how different elements in masonry interact with each other and how stiffness's can be estimated, what are the assumptions that you need to make at a component level to be able to estimate stiffnesses which is essential for distribution of forces.

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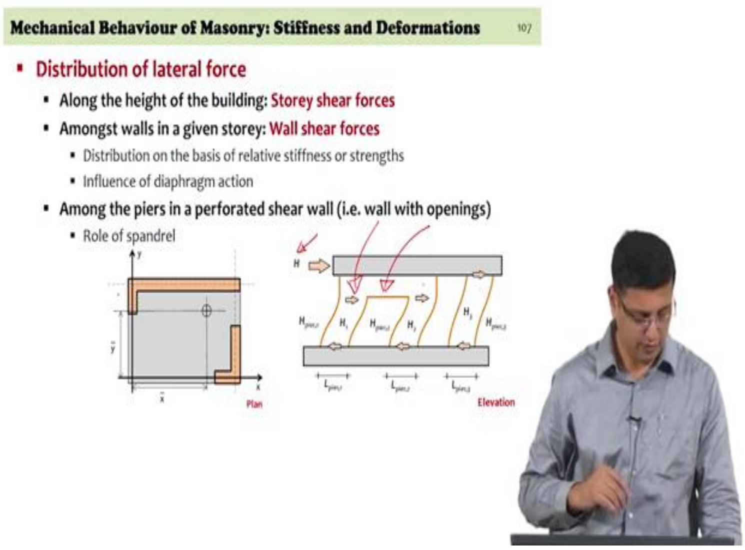

So, that is the last part of the of this particular module, where we are really examining distribution of lateral force, ok. The intricacies in terms of different methods that you will have at the scale of an entire wall we will examine as we start doing design problems. But, the overall framework for how lateral force distribution occurs in a masonry building all the way up to a masonry panel or masonry component is important to understand, because there are there are assumptions that you have to make on the behaviour of each of these elements and the system as a whole.

And my focus is primarily from the point of view of earthquake response of these of these structures of masonry structures. So, when I say distribution of lateral force I am making particular reference to earthquake forces, base shear that you can estimate under the action of an earthquake. And how this shear demand is going to be distributed right from the building the base shear demand that the building would experience all the way up to what shear force a single component is going to be experiencing. So, that that trajectory is what I am really looking at.

So, we will get into seismic design aspects, but the transition from the system level to the component level becomes important. We will be able to establish with the basic force is equal to mass into acceleration format of expressions that the code would have for estimating base shear. Once the base shear is estimated for a given building this base shear is then distributed along the height of a structure.

So, if you have a single storey structure all the base shear is equilibrated by the walls that are present in the single storey, in the ground storey structure itself. But if you have multiple floors then the storey shear, this shear force corresponding to each storey has to be estimated from the overall base shear, and that depends on a certain distribution that you will assume along the height. Codes give us simplified ways of doing this. You can assume a triangular distribution along the height you can use mode shapes to do that distribution. But that is one aspect that we will examine we will start examining when we do actual seismic design.

So, the first transition is once the base shear is established; you need to know what the storey shear is. Once the storey shear is established that storey shear has then to be distributed among the different walls in a given storey. So, you need to know the distribution of the walls. And this distribution is affected by several parameters.

You can have a situation where there is total symmetry in the plan layout of the structure. The center of mass of that storey and the center of stiffness of the distribution of the walls match. And, in such a situation if you have lateral force acting along the x direction or lateral force acting along the y direction you are going to be looking at direct shear components acting on the walls themselves. But in case if you have an eccentricity between the center of mass and the center of stiffness you have the possibility of torsion. And with the possibility of torsion, apart from the direct shear component coming from earthquake force acting in one direction or the other you have an additional torsional shear component that comes into the picture.

So, at the level of a storey it becomes important to first establish; what is the center of mass, what is the center of stiffness, and examine if there is an eccentricity between the center of mass and the center of stiffness. This eccentricity can be broken down into x eccentricity and y eccentricity based on the geometry and the Cartesian coordinates within which you are working.

So, this storey shear based on whether or not there is eccentricity in the plan between the center of mass and center of stiffness will then have to distributed between the walls. And this distribution of the storey shear to each of the walls. The plan that you can see here has long wall along the x direction at the top, a short wall along the x direction at the bottom and then it also has walls along the y direction of different lengths.

Now, in the x direction and the y direction, the shear force depending on the direction in which you are assuming the earthquake force to act and the direction in which you are doing the analysis you will have to distribute the shear force between these walls. So let me assume that, the earthquake force is acting along the x direction. And, if there is no eccentricity between the center of mass and center of stiffness that earthquake force now has to be distributed to the two walls: one wall at the bottom and one wall at the top in the x direction as direct shear components. If there is eccentricity, then apart from the direct shear components there is going to be a torsional shear component.

In the y direction similarly: I have two walls, so the shear force the storey shear and the y direction is then distributed between the two walls at the two extremities in the y direction of the building itself. This distribution from the storey shear to the wall shear force is affected by some assumptions and actual physical conditions of the structure itself. And what are they? This distribution is going to be on the basis of the relative stiffness or strength of the walls themselves. So, you need to know an estimate; you need to have an estimate of the stiffnesses of the walls in a given storey. It is based on this stiffness that the distribution is going to happen.

Strength is something you have already been able to estimate based on the you will be able to estimate based on the previous lectures which focused primarily on strength. But this shear force demand on to a storey is going to be distributed to the walls based on their relative stiffnesses. So, you need an estimate of the stiffness of each wall in a storey.;

However, this is affected completely going to be effected by whether or not you have diaphragm action in the structure. Diaphragm action meaning: the floor slab or the roof slab, if it is acting as an ideal diaphragm implying that you have the slab acting in a rigid manner because of which the displacements at any point on the slab are equal due to the lateral force. That is within the slab there is going to be no relative deformations.

Therefore, if two points on the slab cannot deform relative to each other; deform relative to each other they will displace, but they cannot deform relative to each other then the slab can be assumed to be ideally rigid. And you have diaphragm action. If you have diaphragm action then the distribution of forces is going to be based on the relative stiffnesss. If you do not have diaphragm action the distribution cannot be based on the stiffnesss. So, the diaphragm action has a very crucial role in how the demand is going to be distributed to the individual walls in a given storey in a given direction.

Now, once let us say you have completed this stage of distributing the base shear along the height of a structure and established your storey shears; the storey shears have now been distributed among the walls in a given storey in a given direction. You established a wall shears the wall itself need not necessarily be one single component, you can have a blank wall. And that is one single masonry panel. However, if you have openings then it is not one single panel. You have different elements there and what are those elements.

So, the next stage is after I have established what the wall shear is, and here the wall shear is let us say H acting on the wall and if this is the wall which has openings. Then I have two openings here: there is one large opening at the end between this pier and this pier here, and another door opening between the first two piers, right. So, I have pier 1, pier 2, and pier 3. We call them piers because they are the vertical elements; they are the gravity load carrying elements, and they are the once who are going to be resisting the lateral force.

So, the piers are defined as lateral load resisting vertical elements, and they form the most important parts of a load bearing masonry structure under the lateral action. So, I have 3 piers here, but you can see that the deformation of the pier under lateral forces is going to be affected by the size of the openings. So, the wall here is the entire wall along one direction, along one of the axis the resisting elements are the piers. And here we are talking of a perforated wall.

Now, one it is going to be affected by what is a size of the openings, and two, what is the coupling element between these piers. And when I talk of a coupling element what is there I am referring to? In a regular frame the coupling element is a beam, right. In a moment resisting frame, in a reinforced concrete structure you have columns as the lateral load resisting vertical elements, but in a masonry wall when we are talking of these coupling elements we are referring to the spandrels and these are horizontally aligned the elements of different aspect ratio and their primary role is transfer of forces between the piers when there is lateral action.

Under gravity they are basically distributing the load the loads or the forces coming from the floor to the piers and have to be provided with lintels, so that if it is unreinforced they do not crack. So, the second element that we are talking about is the spandrel itself, right. Now, depending on the strength of the spandrel in relation to the strength of the pier itself the distribution of shear forces in the wall can be effected.

Now, at the boundary between the piers and the spandrels you have something that is comparable to a beam column joint. In a moment resisting frame you would have between the columns and the beams beam column joint. In this case, in a similar manner between the spandrel which is the horizontally aligned element and the vertically aligned element which is the pier you have the beam column joint or what is referred to as a zone of low deformation and often that is idealized as a rigid node like a beam column joint in a moment resisting frame.

So, there are 3 elements that really interact with each other. One is the vertical lateral load resisting element the pier, the horizontal lateral load resisting element which does not carry gravity strictly speaking that is the spandrel and the joint between the spandrel and the pier. Depending on the strength and stiffness of the spandrel the boundary condition of the pier is established. So, a pier can actually have a cantilevered deformation profile under the lateral forces or can have a shear deformation profile under the action of lateral forces and this depends on the role played by the spandrel itself, ok.

So, once you have established wall shear, you have to now start looking at distributing the wall shear to the vertical lateral load carrying elements. So, the wall shear has to be distributed to the 3 piers in the figure here H has to be distributed among H_1, H_2 and H_3 assuming that we have rigid diaphragm action. And here in this particular case 3 vertical lateral load carrying elements are present you need a way of distributing the shear force to these 3 and that is done based on the relative stiffnesss.

So, you need an estimate of the relative stiffness of the pier in a wall and you need the overall stiffness of the wall to be able to establish what is the proportion of the storey shear that the wall will attract, right. So, that is where you really need to be able to make an estimate of deformations and stiffnesss in masonry panels. And this is affected by spandrels. If you have an opening with a coupling element or if you have a blank wall with no openings there is a difference in the way the wall will deform under lateral forces, but it also as I said depends on how strong and how stiff the spandrel itself is.

And the reason why it is isolated as a different element is a pier is a vertical lateral load carrying element. So, you construct a pier, there is a continuity between, there is a vertical continuity in a pier and there is pre-compression because of the superimposed loads and its dead weight and its own dead weight. The spandrel does not have the benefit of that action. The spandrel does not have pre-compression because it is an element that is sitting above an opening.

So, you do not have the kind of consolidation and the pre compression that the pier enjoys in a wall. This is one important difference that changes the way the pier and the spandrel behave. We will come to specific calculations one once we get into the design.

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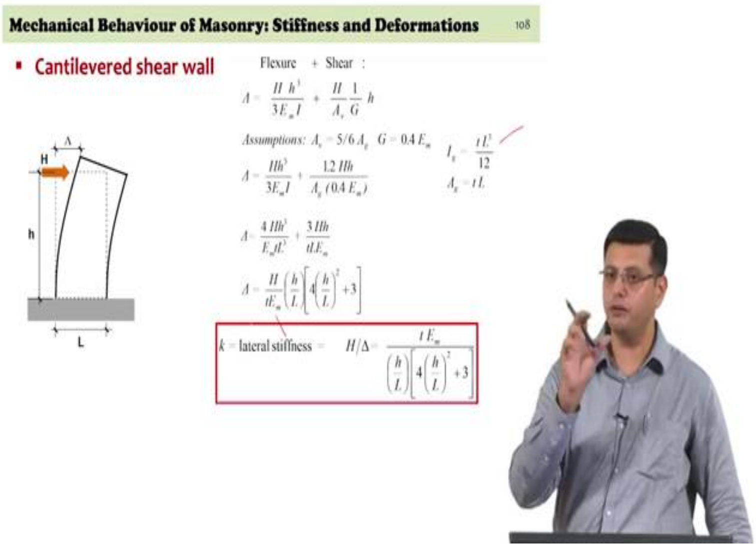

But it is instructive at this stage to look at how can you make an estimate of the stiffness of a pier because that is where you start, you need an estimate of the stiffness of the pier and then you need to put them together to be able to estimate the stiffness of the wall itself.

So, the two conditions under which the stiffness of the pier can be estimated is that the pier is acting as a cantilevered shear wall. So, if you are looking at a blank wall, if you looking at a wall without openings the deformed profile of the wall is closer to a cantilevered wall. You get a cantilevered deformation profile. The moment you have an opening and you have a spandrel which is the coupling element between the opening the spandrel can prevent the rotations at the top of the piers and it changes the boundary condition.

So, in the first case, if you were to consider a cantilevered shear wall of height h, lateral force H acting on it and length of the wall being L thickness of the wall being t, we are interested in being able to get an estimate of the lateral deformation delta and with that we can actually estimate the lateral stiffness of the wall itself. So, simplification, but this simplification and idealization of the boundary condition is essential for us to be able to go and estimate the stiffness of an entire wall composed of piers and spandrels.

So, if you look at the lateral deformation when we look at flexural elements when we look at reinforced concrete frames and we calculate deflections in reinforced concrete members given the slenderness ratios, we typically calculate the flexural deflection, but as the aspect ratio changes as the h by L ratios change the shear deformations cannot be neglected. And so, the total lateral deflection of a masonry wall in-plane lateral deflection has two components the flexural component and the shear component.

When we assume that the deformation profile is a cantilevered deformation profile, the flexural component is $Hh^3/3EI$, where E here is the masonry overall masonry, modulus of elasticity. The shear component is the shear force divided by the shear area which is different from the total area of cross section of the wall divided by the shear modulus into the height of the wall panel itself.

So, here there are to plug in numbers that are reasonable numbers in terms of the sectional properties and be able to make an estimate of what the lateral deformation and lateral stiffness can be an assumption that the shear area is about 5-6th of the total area of the cross section, gross area of the cross section is an acceptable assumption, A_g here is thickness into the total length of the wall.

And then the second moment of area of the wall in-plane direction bending along the major axis $tL^3/12$ and the shear modulus again is assuming that the material is an isotropic material; we are linking the shear modulus to the Young's modulus of the material E_m and assumption that the shear modulus is about 40 percent of the Young's modulus is a fair estimate for masonry.

So, we have numbers from A_v in terms of the gross area, the shear modulus and using those simplifications and expanding on the total deflection which is the flexural deflection plus the shear deflection, and simplifying such that we are able to write an expression purely in terms of the geometry. Simplifications are done here to be able to express it in terms of the modulus of elasticity of the material the h by L ratio which is the aspect ratio and the thickness of the wall.

So, lateral force is the quantity that you need to know. You need to know the geometry of the wall h, L and t and the modulus of elasticity of the masonry you get an expression for the deflection in a cantilevered wall subjected to in-plane lateral forces. If you want to write that in terms of the stiffnesss, H/Δ and you get an expression for the lateral stiffness, if you assume a cantilevered profile of the shear wall. Now, in a similar manner if it is a wall, if it is a pier between openings which then has the interaction of the spandrel coming into the picture.

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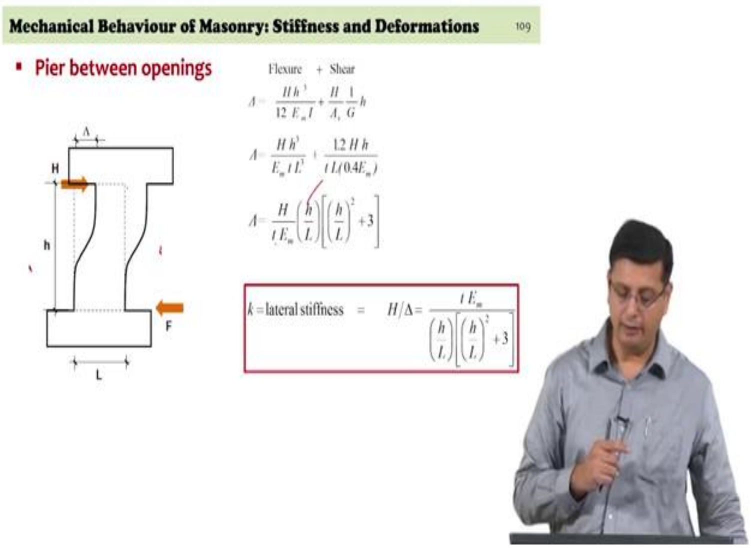

So, I am looking at the pier that is sitting between openings, I have an opening on this side I have another opening on the other side, which means there are spandrels interacting assume it is a window, if you have a window and then you have a spandrel above and a spandrel below and those are actually going to work towards preventing the rotations at the top and the bottom of the masonry pier itself. Earlier, you had a cantilevered deformation profile top was free to rotate, but when you have a spandrel which is significant in its geometry and strong then what it can do to the top of the pier is that it can prevent the rotations.

So, prevention of rotations would mean the pier is subjected to double bending now and therefore, the estimate of lateral stiffness we will change. So, here again you have a flexural component and the shear component, but given the fact that the wall is undergoing double bending I have $Hh^3/12EI$ would be the estimate of the deflection due to flexural mechanism. And we use the same expressions that we used earlier for the shear area and for the modulus of the shear modulus and again simplifying and writing it in terms of the aspect ratio h by L, geometry of the wall cross section t and modulus of elasticity. We have an expression of either in terms of lateral deflection or lateral stiffness of the wall.

So, it is important for you to examine a wall and understand whether a the behaviour of the pier is closer to a cantilevered profile or a shear profile. And then you now need to assemble the different piers and the spandrels to be able to estimate, to be able to establish the lateral stiffness of the entire wall. Then you have several walls there are relative stiffnesss between these walls is going to help you establish what is the proportion of shear force that the wall is going to get from the storey shear itself.

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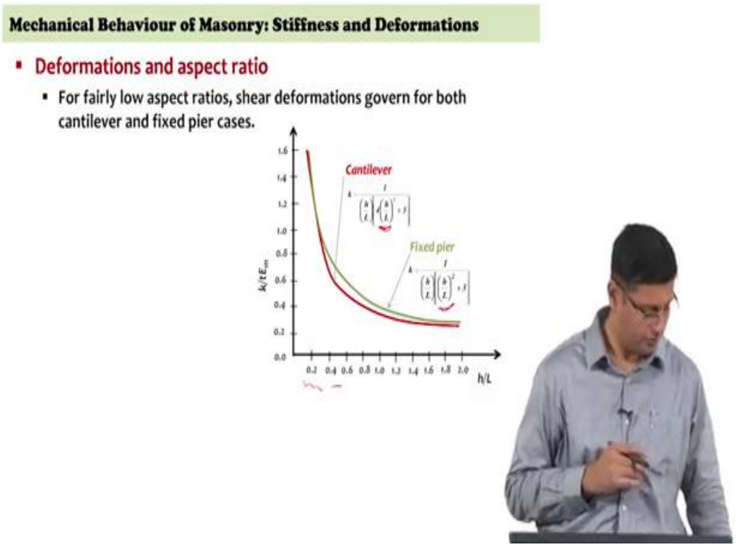

It is useful to examine how cantilevered piers and piers with shear deformation profile would look with respect to the aspect ratio itself. So, if away to look at this stiffness normalized stiffness on the y axis, I have stiffness k divided by thickness of the wall t and the modulus of elasticity of masonry normalized. And if I have the aspect ratio h by L on the x axis you see that for small aspect ratios implying that we looking at squat walls, the shear deformation prevails.

But as the aspect ratio increases as we go to longer aspect ratios you see that the shear deformation profile, the shear deformation quantity actually becomes negligible. And in a slender wall which is slender its aspect ratio such that h by L is greater than 1, the contribution of the shear deformation keeps reducing. So, for fairly low aspect ratios the shear deformation is what will govern for both the cantilevered profile and the fixed pier cases, and that is really because of the of the way the expressions pan out in terms of what is on the denominator, ok.

So, I just wanted this aspect which is a link to our design, where a point where we start aspects of design to be the concluding portion of this section that looks at strength and behaviour. So, I think with this we have an overall hold on the different actions and expected behaviours and therefore, what needs to be taken care of from the design perspective itself. And we will begin design in the next lecture, ok.

Thank you.