Design of Masonry Structures
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Module – 03
Lecture – 20
Strength and Behavior of Masonry Part – X

So, good afternoon; we will continue examining the different mechanisms; in-plane mechanisms we looked at as an extension of the Mann Muller criterion which works at the level of stresses. The extension which looks at resultant forces and we have examined one of the criteria based on the flexural failure mechanism in the last class.

So, the flexural failure mechanism, when the formulation is made, the formulation is based on a flexural compression failure occurring at the compressed toe. However, we also saw that is a special case of that situation; if the axial compression level- the pre compression levels are low; then you could have rigid rocking of the wall in the in-plane direction itself. So, that is the special case, the overturning as a failure mechanism is a special case of the flexural in plane flexural mechanism itself; we will examine the other two mechanisms.

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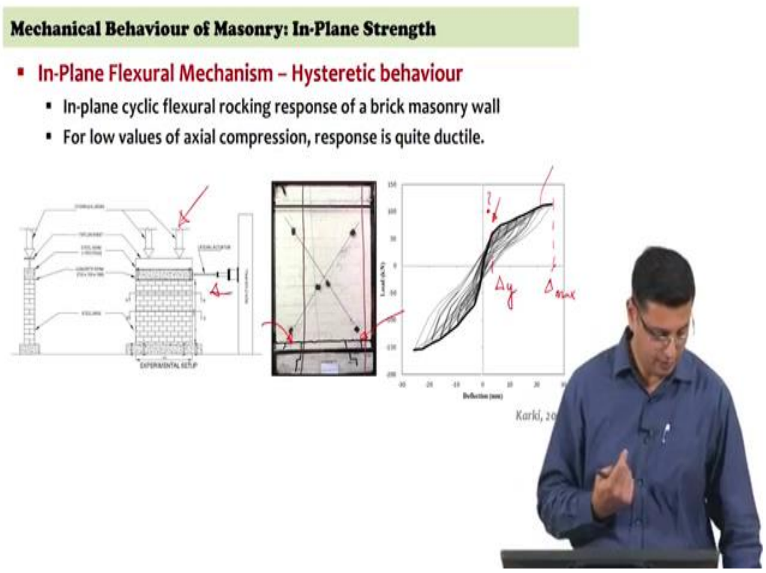

But before we move on to the shear dominated behavior; it is useful to look at what is the cyclic response of a wall that is governed by a flexural rocking or a flexural crushing mechanism itself. So, what I am going to be looking at is the hysteretic behavior right. So, since the elaboration of these capacities and force displacement behavior is being done primarily with respect to response of masonry; response of unreinforced masonry to earthquake actions, it is important to examine the hysteretic behavior which is cyclic behavior; earthquake being reverse cyclic behavior in nature. And therefore, if we were to examine and compare the in-plane response for the flexural mechanisms and the in-plane response for shear mechanisms, we get rather good understanding of how deformation capacities available in a masonry wall. What is the status of energy dissipation in the masonry wall and where does introducing steel reinforcement and making un-reinforced masonry to reinforced masonry really contribute.

So, it is in that context we will start examining hysteretic behavior and though some of you may not be familiar with cyclic response; static cyclic response is what we are trying to capture here. It is not from dynamic analysis or dynamic behavior, but from a static or a quasi-static behavior. But it will be useful to look at these graphs to get a better understanding of how masonry walls respond to reverse cyclic loading.

So, if we were to look at the flexural dominated mechanism, rocking or the rocking effect ultimately leading to crushing of the compressed toe in a brick masonry wall. I am looking at laboratory tests because you can examine these in a controlled environment; you can apply perfect in-plane loads in an earthquake, you have a combination of actions coming on to a masonry wall because of the randomness of the ground motion itself.

So, we are looking at a laboratory test where the wall panel that you are looking at is a shear wall; it is subjected to gravity loads. So, the actuators that you see at the top, the jacks that you see at the top are actually subjecting the wall to a desired level of pre compression and this is to idealize the gravity forces superimposed on the wall itself.

So, if we were to look at this wall as a ground storey wall in a multi storey building, then I can regulate the precompression level based on what is the expected precompression level in the ground storey wall. If it were a single storied wall, you reduce the pre compression levels such that probably only the weight of the parapet and the slab is the superimposed load on the wall itself.

So, it is a convenient way of introducing the level of superimposed load and then subjecting the wall to lateral forces. In this case, you have the you have the actuator in the lateral direction which is applying the in-plane shear forces, trying to simulate the kind of damage that is expected in an earthquake, but under a quasi static condition of loading itself.

So, the wall has boundary conditions, you can fix it at the base the top is free to rotate and so you again are able to mimic the boundary conditions expected in a real building. If you want to prevent the rotations at the top, then you can create a setup which does not allow rotations at the top and takes you towards the shear deformation profile of the wall itself. But those are important choices that have to be made before such a test is carried out.

So, what you are seeing are tests conducted in our laboratory; you see a masonry wall, this was constructed using fly ash bricks. And as you see, it is rather slender meaning the height is more than the the length of the wall itself. And typically, we refer to walls as slender walls when the height to length ratio is greater and about 1.5.

So, this is a slender wall that we are examining. And as you can see the black lines that you see at the bottom are actually the cracks that are formed as the wall is subjected to the lateral force under the presence of gravity forces. And this is reversed cyclic, so you go from the 0 position to a positive maximum, come back to 0 then you unload and then to go to the negative maximum and come back.

So, it is cyclic, but reversed cyclic positive and negative cycles and that is why you see that the crack at the base; the black line that you see at the base on both the ends, the tension, the heel cracking is happening at both the ends with the reversed cyclic loading. And if the level of pre compression is low; considering that this is a slender wall, you can even get the entire cross section that can get cracked. And depending on the pre compression level, again you might get a shear sliding failure or you might get rocking of the wall itself.

So, this is a case that underwent rocking and the hysteretic behavior which is captured from the experiment; you get the cyclic response of the structure of the wall. And then we look at an envelope curve over all the cycles that the wall is taken through. So, the thick black line that you see here; the thick black line that you see in the positive quadrant and the negative quadrant is the envelope curve. That is the envelop of the overall response and represents the force displacement behavior in the positive cyclic and in the negative cycle. And typically, we look at either both or an average of the two or the positive curve and the negative curve whichever is the absolute maximum of the response itself.

So, what we are actually observing here is a wall that has undergone rocking. There are a few aspects that you need to understand from such a hysteretic curve; what you are seeing is force versus displacement. So, from the envelope curve at the maximum displacement versus a sort of a yield displacement that could be defined.

And defining what is the yield displacement of the wall is something that is subjective empirical; it could vary depending on the method that you use to establish what the yield displacement is. Because you do not see very clearly like in reinforced concrete systems or in steel systems, the phenomenon of yielding is not seen so clearly in a material like masonry. And therefore, it becomes subjective depending on the method that you choose; the estimate of the yield displacement itself.

If you were to make an estimate of the yield displacement and then look at the maximum displacement that the wall is capable of resisting. And if I have now ratio of the maximum displacement to the yield displacement, the ratio gives me the displacement ductility of the wall itself.

So, from the envelope curve it is possible to get an estimate of the deformation capacity of the wall, quantified in terms of ductility available in the wall, 2. If you look at each loop which is the force-displacement excursion in the positive and the negative. And if you look at one excursion; the area under the curve is representing the energy dissipated in one cycle.

Of course, in the elastic cycles you do not have energy dissipation and as you start seeing damage in the wall that is when cracking is occurring and there is energy dissipation. So, particularly in the elastic cycles; in each force displacement loop in a cycle the area under the force displacement loop is the energy dissipated.

When you look at flexural rocking mechanisms in comparison to shear dominated mechanisms, there are two things that are typically observed. One is that significant deformation capacity is available in the wall. That is since the wall is rocking and if the pre-compression levels are low, it can continue to undergo rigid rocking. The precompression levels are heavy are high, then you can get flexural compression failure at the compressed toe.

So, in this particular mechanism, because of the rocking behavior deformation capacities or the ductility that is available in the system is significant. For an unreinforced system; this is the maximum displacement ductility that you might get. And if you actually qualitatively look at the graph, you can see that delta max is about 3 to 4 times the yield displacement, ok.

So, your question was in the case of significant pre-compression levels; failure in a flexural mechanism is expected by the crushing of the compressed toe. Yes, so what typically would happen in a reverse cyclic scenario is that one of the toes; one of the toes would reach crushing failure at the beginning of a cycle that is taking the wall to the maximum displacement.

So, with that we have to stop the test because we have seen failure in the wall and we do not bring it back because then it causes instability due to one end already crushed and symmetry lost in the wall. So, typically that is observed in the figure that we have been seeing in the other class. You see that the crushing failure is always at one end; so we stop at that point as far as a test is concerned.

So, what I was mentioning earlier is that if you look at this graph; qualitatively you will say that the maximum displacement is 3 to 4 or even more than the yield displacement. So, depending on the typology of masonry; this ratio can vary and we are not immediately examining this number. But I would like you to keep in mind the fact that the displacement ductility is significantly higher, when you have flexural rocking mechanisms in comparison to shear mechanisms.

The second thing that you see is of course, we have not compared this hysteretic behavior with any other mechanism; any other hysteretic curve particularly dominated by shear mechanism. But what you observe are these loops of hysteresis are rather narrow loops ok; are rather narrow loops implying that the area under the loop is going to be small and the energy dissipated. If you were to estimate what is the area under the loop for the force displacement curve, you will see that it is small in comparison to other mechanisms or if you were to reinforce a wall.

So, the point is and the reason why it is low; energy dissipation is low is that you get a tensile heel crack formed in the wall and probably that might extend to about 80, 85 percent of the length of the wall. And if the wall were not going to crush at the compressed toe; then the wall is simply going to go rocking back and forth.

There is no possibility of energy dissipation in the wall; it is already a pre-formed crack plane that you have and it is just going, rocking along that crack plane, there is no further inelasticity occurring in the system to dissipate energy. So, a hysteretic behavior in in-plane flexural mechanism is typically characterized by good deformation capacity, good ductility, but low energy dissipation capacity.

And that is a little bit of a paradox because when you have earthquake response when you want good earthquake response; you want good deformation capacity, but you also want good energy dissipation. And that is a difficult to achieve if your wall is unreinforced; once you reinforce your wall, it is then possible for you to achieve the deformation capacity, but also have good energy dissipation.

So, unreinforced masonry in earthquake applications does not make too much of sense because you will not be able to achieve these twin requirements for good earthquake response. Now, energy dissipation is the input energy into a system during an earthquake has to be dissipated by way of formation of damage, right.

A system is able to absorb the input energy and convert that input energy into something. Because that energy, if a large energy is input into the structure; it has to find a way of losing that energy or converting that energy into something. If you have a wall that is capable of deforming inelastically, but not failing, not reaching any failure mechanism that is a successful system because you are able to dissipate energy and yet remain stable for.

Student: Ductility right, sir that is ductility.

That is ductility. So, you have deformation capacity, but deformation capacity alone is not sufficient. Because there is input energy into a system and you need to be able to dissipate energy which is the earthquake energy that is input into the system from the ground.

Now, rocking is good but if you look at the hysteretic curve for rocking; it is typically a thin curve. You are not able to dissipate energy by rocking alone. You are able to get the displacement capacity the displacement good displacement behavior, but you are not able to dissipate energy, you need to achieve both yeah. So let us keep this in mind and comeback when we examine the shear deformation behavior itself.

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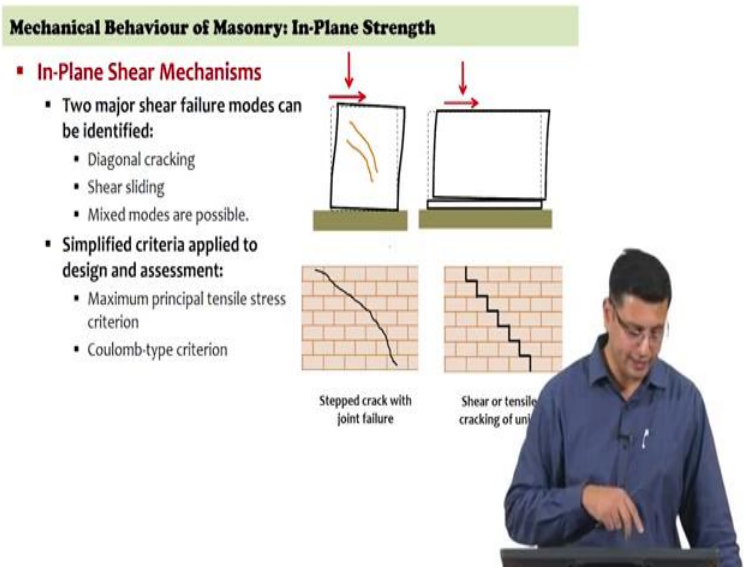

Let us move on to the in-plane shear mechanisms and the in plane shear mechanisms of interest to us in plane direction. There are two major shear failure modes that can be identified as independent shear failure modes. However, life is not so straight forward, we can have a combination of failure modes and that is a problem. Because it can start making it difficult to have clean closed form solution from each mechanism; when you have combined modes it becomes a problem to work with these expressions; however, this is a good starting point.

So, the two shear failure modes are the diagonal cracking mode; in the Mann Muller criterion we have examined this which is the formation of the tension cracks, the diagonal tension cracks in the center of the brick. And we used a formulation based on when the principal tension reaches the tensile strength of the unit. It is based on a similar approach diagonal cracking is occurring when the principal tension reaches the tensile strength of masonry. So, that is your first mechanism of shear failure.

The second mechanism of shear failure is what we have seen earlier, the shear sliding failure which way which is expected to occur at a bed joint. There will be a critical bed joint and typically we are looking at the bed joint; which has the maximum pre - compression level. Typically, where the maximum demand is also coming and therefore, it could be the first bed joint layer level or the second bed joint level or so on.

So, shear sliding is the second criterion; now this criterion is what you would, this failure mode is what you would link with in the Mann Muller criterion where we had the failure of the mortar joint. We had a formulation based on the Mohr Coulomb criterion where the failure of the mortar joint was one of the failure modes. In fact, the first failure mode that we looked at that was a Mohr Coulomb criterion; tau is equal to cohesion plus mu into sigma. The shear sliding criterion can be defined with respect to that sort of a basis.

So, these two failure modes of course, as I said mixed modes are possible. And you will see a body of literature that tries to understand how to capture the capacity and the failure mechanism; if you have a combination of modes. You will typically have a combination of modes, it does not mean that right from the beginning of the lateral force displacement; force displacement behavior of the wall; that is always going to be shear sliding dominated or diagonal cracking dominated or flexural dominated.

It might begin in one; you might get tensile heal cracking, area of cross section reduces because of the cracking of the critical section. And then depending on the axial stress levels, you can also see a change in the behavior of the wall. So, this is quite possible that you get mixed modes.

However, with respect to the diagonal tension failure, again you can have two mechanisms; if the crack is actually splitting the stone, splitting the brick; you can have the line crack. And then if you have the crack not splitting the stone, but actually following the joint; you will have the stepped failure. And here the criterion that matters is the the bed joint strength versus the unit strength in tension itself. So, this is a further aspect that needs to considered as far as the diagonal cracking mechanism is concerned.

So, extending the Mann Muller theory and then using it with respect to these force resultants; the criteria to define and have a closed form expression for the two shear failure modes are the maximum principal stress criterion; where we are saying that the maximum principal tension, when it approaches the tensile strength of masonry you get diagonal cracks. And the other failure criterion is based on the Mohr Coulomb criterion for explaining the joint sliding that is occurring in the masonry wall.

So, we will examine the formulations based on these two criteria.

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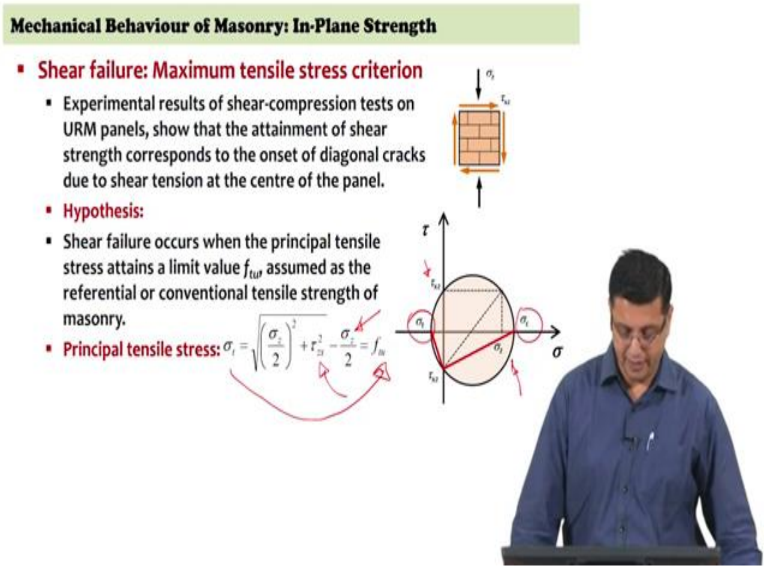

The first one being the maximum tensile stress criterion; here if you look at masonry panels that are subjected to both shear and compression. It has been demonstrated that the shear strength of the panel is reached when diagonal cracks are formed. And diagonal cracks are formed typically from the center and again I make reference to the Mann Muller criterion, where they were looking at the tension cracks in the brick unit forming at the center. And we have to take into account the dimension of the brick to be able to change the shear stress defined at the joint, to the shear stress at the center of the brick unit.

So, similarly here typically this sort of a failure mechanism sees the onset of diagonal cracks at the center of the panel. And these are diagonal tension cracks, they are due to shear tension which means we can actually we use a principal tension criterion to estimate the lateral force required to cross this sort of a failure mechanism.

So, we are talking of a combination of shear stress developed because of the lateral force in the presence of a normal stress an average normal stress in the wall σ_z and τ_{xz} assuming we are talking of the xz plane itself. The hypothesis in this particular criterion is that shear failure is occurring, when the principle tensile stress reaches a value of the tensile strength of masonry.

Now, again with respect to the Mann Muller criterion; for the Mann Muller criterion we were talking of the tensile strength of the brick unit. But here we are interested in the tensile strength of the masonry, right. Now, what is the tensile strength of the masonry that is relevant in this context, we are talking of a value that can be established with the test like the diagonal compression test.

The diagonal compression test was used to estimate the shear strength of masonry, but in that context, I was mentioning that the strength f_{tu} that you estimate from the diagonal compression test is also referred to as the conventional or referential tensile strength of masonry right. So, this is the sort of value, an estimate of the tensile strength of masonry that we are talking of because the failure is in the principal tension itself.

So, with this; in this context if you examine the Mohr circle; the state of stress in the wall τ_{xz} versus σ_z; τ_{xz} in the shear stress and sigma z the normal stress gives us the principal stresses σ_c and σ_t principal compression principal tension. This estimate of sigma t is what we require we equate to the tensile strength of masonry and we have a criterion.

So, the principal tensile stress itself here σ_t is the principal tension form the Mohr circle, you can estimate what is the actual value of σ_t itself; knowing the value of the shear stress and knowing the value of the average normal stress acting on the wall. This is then equated to f_{tu} that is the tensile strength of masonry; the assembly masonry itself. So, this is the criterion that we use.

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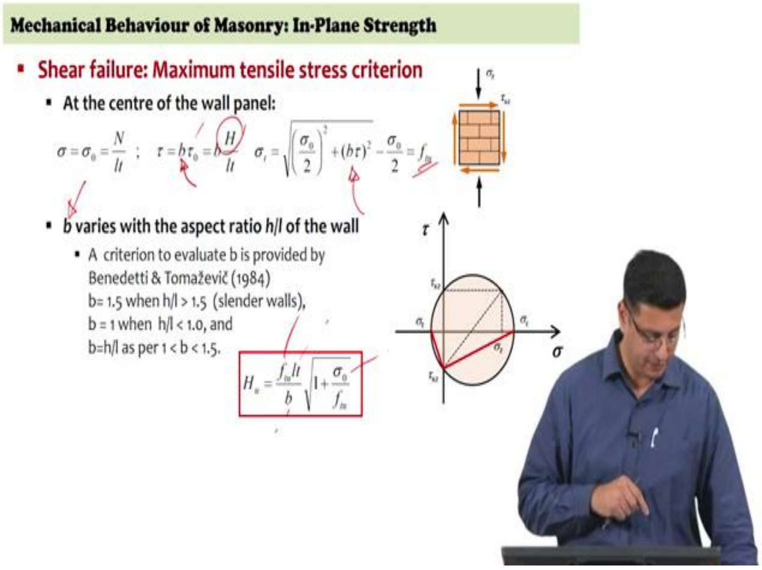

Now, of course, the previous expression was defined in terms of stresses. But we are talking of being able to do this in terms of force resultants. And therefore, we need to now transfer from the state of stress to the force resultants especially to define what is the ultimate lateral force capacity of the wall; if it were to fail in a diagonal tension criterion. So, since this crack occurs at the center of the panel; we are interested in looking at the stresses defined at the center of the panel. The normal stress at the panel; N being the superimposed load divided by the area of cross section l into t and the shear stress.

Now, the shear stress is going to be affected; you know that the shear stress distribution in any cross section along the height is not uniform. It reaches a maximum, if you take a rectangular cross section in a beam that is subjected to vertical shear itself; you get a parabolic distribution with the peak shear stress being at the neutral axis.

So, the shear stress distribution is not going to be uniform. So, working with the average stress is tricky because the average shear stress might not be able to give you a value that is close to the value for which cracking is going to occur. So, this is observed to be affected significantly by the aspect ratio of the wall.

So, we introduce an estimate of the shear stress corrected by a factor that actually takes into account what the aspect ratio of the wall is; what is the role of the aspect ratio of the wall is. So, while the average normal stress is adopted as it is for the shear stress, we make a correction based on the aspect ratio of the wall. And you know the aspect ratio of a wall has a role to play in whether it is going to be dominated by flexural mechanisms or whether it is going to be dominated by shear mechanisms.

So, here τ_0 is the average shear stress and you see that the average shear stress is nothing, but H/lt. So, that is going to give you the average shear stress, but there is a deviation from this average shear stress to be able to estimate what is the maximum shear stress we use this aspect ratio. We; we bring in the aspect ratio in the form of this constant term b and I will explain what is this constant term in a moment.

So, go back to the previous expression; now $(\tau_{xz})^2$ that was there in the second part of the under root is replaced with bτ. Now this factor is directly dependent on the aspect ratio of the wall; H being the height of the wall and l being the length of the wall. And it is observed that it is not possible to get an analytical form of what these should be. And researchers worked on this subject have proposed empirical expressions that take care rather well the effect of the geometry on the state of shear stress causing failure itself.

So, we are looking at this criterion developed in 1984; where the value of b ranges between 1 and 1.5. When you are looking at slender walls with an aspect ratio greater than 1.5, you take the value as 1.5; so, b will have a maximum value of 1.5. When you are looking at Squat walls, where h by l is less than 1; the wall is longer than it is taller in such a wall; it is a Squat wall, you limit value of b as 1; so b varies from 1 to 1.5. And in the range 1 to 1.5; I mean in the range of h by l 1 to 1.5; the value of h by l will be taken as b itself. So, in the range b 1 to 1.5, it will depend directly on the value of h by l.

So, with this expression, with this value that you can adopt based on the aspect ratio of the wall for b; you will have a number times τ in the under root term. And you can use the average stress; average normal stress value and the average shear stress value, in this expression and write down the expression in terms of H which is what we require.

In the first expression for the failure governing the failure governed by the toe crushing; we had the expression in terms of the ultimate moment. But knowing the height of the wall you can estimate what is the lateral force corresponding to the ultimate moment. In this expression again we bring in H into the expression and therefore, you can write down the final expression in terms of H; simplifying the expression in the under root, entire expression.

$$H_u = \frac{f_{tu} l t}{b} \sqrt{1 + \frac{\sigma_0}{f_{tu}}}$$

So, you know the average axial stress, you know the area of cross section of the wall, you know the tensile strength of masonry and you have an estimate based on the aspect ratio of the wall h by l. And you can estimate at what value of lateral force is the mechanism of diagonal tension cracking expected, ok.

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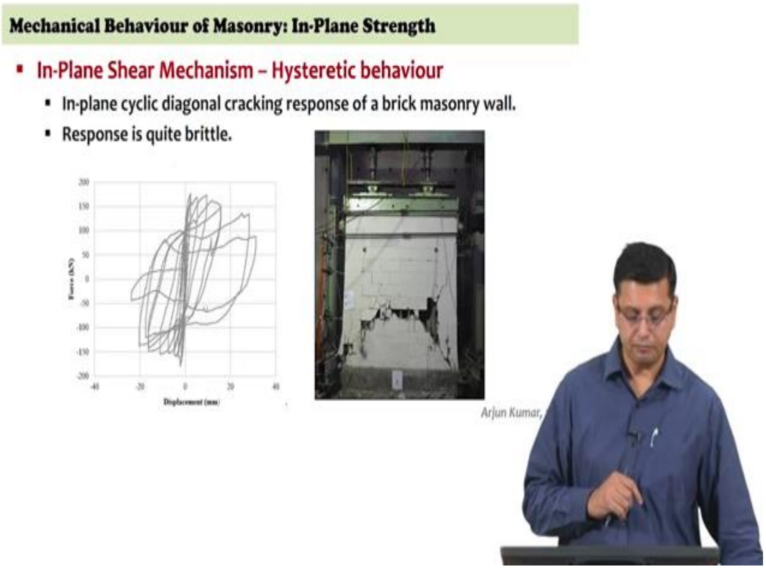

So, that is the second mechanism. So, in this case the hysteretic behavior in the discussion that we had for the flexural mechanism; let us extend it and examine what happens in the shear mechanism. We typically see two important things; this is again a test that was conducted by one of our masters students; the wall was made out of blocks, it was concrete blocks. So, you can a shear failure mechanism in the wall, but the two things that you will observe are first the deformation capacity is significantly lower than in the wall which is governed by flexural rocking.

So, the deformation capacity; the fact that which shear you going to get brittle response is true to an extent; you will get almost one half of the displacement capacity or the ductility that you were getting with a wall that is governed by flexural rocking mechanism; that is the first point. The second aspect is you do get energy dissipation which is more than what you would get for flexural rocking.

Now, if it an unreinforced wall; you are going to have one signal crack that propagates and becomes the surface on which the sliding is going to occur and can dissipate a limited amount of energy. With reinforcement you can have multiple cracks, multiple shear cracks that can actually help dissipate more energy.

So, the point that you need to note is with respect to the in-plane flexural rocking mechanisms, displacement capacity is lower for shear dominated mechanisms one. Almost one half is what you would get and the energy dissipation is comparatively higher in this mechanism compared to the rigid rocking mechanism of in-plane behavior. The behavior is therefore, brittle in this particular case, ok.

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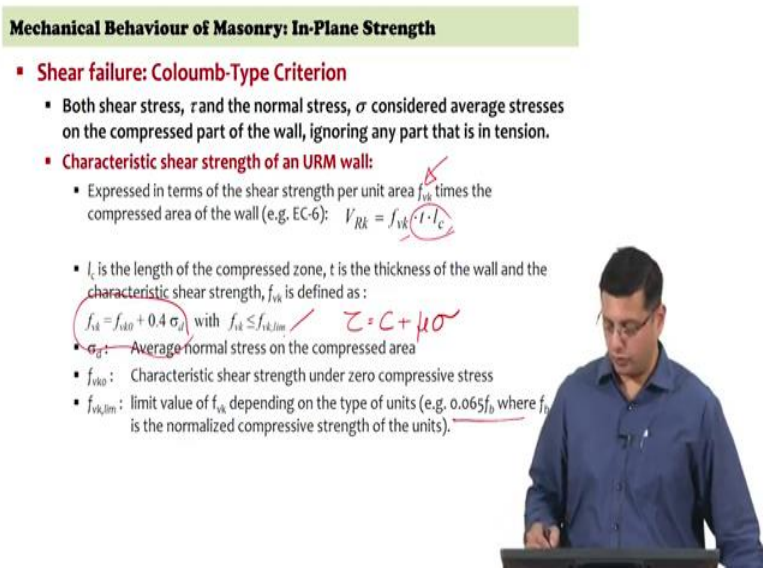

Let us examine the third criterion, the third criterion as I said is an extension of the joint failure, mortar joint failure criterion that we examined in under the Mann Muller set of expressions. So, we are examining again the normal stress and the shear stress as average stresses. But what is important is as far as this criterion is concerned; we have to examine the shear strength coming from only the compressed zone of the wall which means part of the wall has already cracked and that may be tensile heel cracking, right.

We were talking of mixed modes. So, it is not like right from the beginning you are going to have shear sliding occurring in the wall. Given geometry and given the level of stress axial stress; it is not likely that you have one governing mechanism. So, let us say you have a wall; which is subjected to lateral forces and axial forces. And as I mentioned yesterday tensile heel cracking is a serviceability criterion right; it is not an ultimate condition.

So, tensile heel cracking might happen depending on how much axial stress is acting on the wall. Once that has occurred, the compressed length of the wall that is available for equilibrating the lateral force and the gravity force is reduced. It is on that, that you will have to calculate the shear capacity; if you are considering a coulomb type criterion, right.

And in the earlier situation where we were looking at the diagonal tension failure; the calculations were being carried out at the mid height of the panel, in the middle of the panel. We do not have heel cracking at the middle of the panel, we have heel cracking at the bottom of the panel.

So, this is the criterion where one has to be careful about what is the length of the compressed zone in your estimate. And to reflect the importance of it I will just show you that codes, when they look at estimating the shear capacity always work on neglecting the zone in tension and keep only the compressed zone to estimate the shear strength of a wall.

So, I am just examining couple of them; the eurocode format which is of course, based on a different approach, in terms of design with respect to the is code. In the eurocode the shear strength of an unreinforced force masonry wall is estimated as the shear strength per unit area of masonry f_{vk} is the notation for the shear strength of the masonry that is used, multiplied by the estimate of the compressed length of the wall.

So, to check whether a wall has started cracking; you will need an estimate of whether the combination of the lateral force and the gravity force can cause cracking. And what is the eccentricity and then how would how much do you reduce the compressed length of the wall itself.

I am just flagging this off because it is an important aspect as you use these equations to estimate the strength according to the coulomb type failure criterion. So, code in this particular case gives the shear strength, the characteristic shear strength of masonry as being the shear strength per unit area into the length of the compressed zone into thickness the area of the compressed zone itself.

The shear strength itself; just shear strength of masonry itself actually follows nothing, but the Mohr Coulomb criterion that we have been talking of; even earlier.

$$f_{vk} = f_{vk0} + 0.4\sigma_d$$

So, this is really of the form shear strength $\tau = c + \mu\sigma$.

So, this is really of the form that we have seen earlier tau is equal to C plus mu into sigma which means f_{vk0} naught is really the shear strength of the joint; when there is no axial compression which is nothing, but cohesion itself the bond. So, f_{vk0} is bound; f_{vk0} is nothing but c and then the contribution from the friction depends on the level of pre compression. So, σ_d is the average normal stress on the compressed area.

Now, if the compressed area is less than the total length then the stress can increase, the normal stress can increase. So, the σ_d is defined on the compressed area and 0.4 is the value that codes typically used for brick masonry friction coefficient. It can be higher in reality, but 0.4 is a value that is used in the definition of a characteristic shear strength of masonry itself.

Of course, codes also put a limit on this value when you might get the higher value of the shear strength per unit area of the material. But it is the standard practice to limit this particular value and there are different ways in which this can limited. Some codes go empirical, some codes link it to the tensile strength the compressive strength of the unit. In this particular case, you see that it is about 0.065 times the compressive strength of the brick unit; it is just some information that we will complete the picture here.

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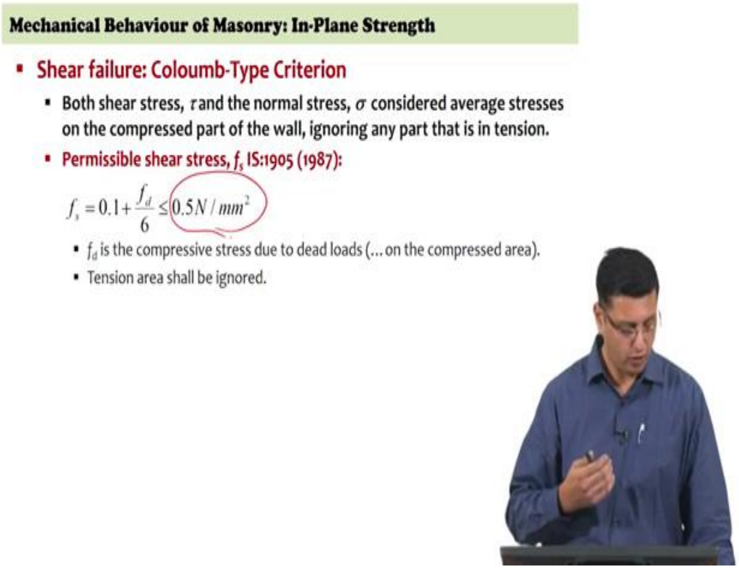

The IS code; what does the IS code do? Mind you, we working on permissible stresses as far is 1905 is concerned. So, IS code defines the permissible stress, the permissible shear stress which you will then compare with the level of shear stress due to the combination of loads in the structure itself.

And in this particular case the code uses a value of 0.1 + f_d/6; in this case. And again you have a limiting value for the permissible shear stress, but here f_d; the value of f_d is the compressive stress due to the dead loads. And this is to be calculated only on compressed area which means we are neglecting the tension zone. And if you actually look at the form of the expression again coming from a Mohr Coulomb criterion; f_s is τ, 0.1 is the cohesion.

So, the code is actually giving a number there and saying look we can; we can expect a bond strength of up to of about 0.1 MPa as cohesion available within a permissible stresses approach and f_d by 6 is the μσ component which means the friction coefficient assumed here is about one sixth or 0.167 as the friction coefficient available in the estimate of the permissible shear stress.

So, this is the standard, the Mohr Coulomb type criterion is the standard approach that is used for shear strength estimate in different codes across the world.

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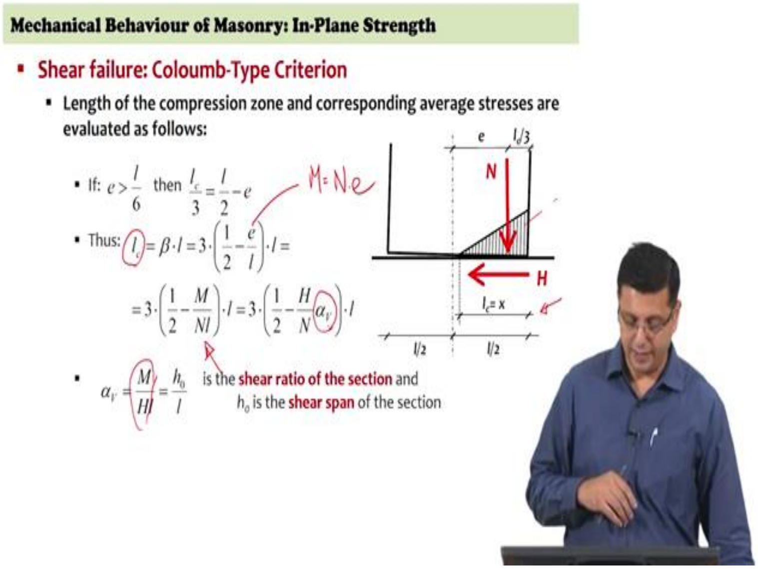

So, a little bit about the formulation itself; we need to be able to estimate the compressed length which is critical. And then it follows the same method that we had used earlier for the Mann Muller criterion of a reduced cohesion and a reduced friction coefficient at the joint.

So, we are looking at the wall where the compressed length, l_c is less than l. And based on a linear distribution of stresses; we are not assuming that the distribution of stresses starts becoming non-linear. Because the failure is happening due to sliding shear and we are not at a situation where there is plastification at the compressed toe because of flexural compression.

And so the assumption of the distribution of compressive stresses as a triangular distribution is a valid assumption here. So, based on the eccentricity of the axial force e and the estimate of the compressed length, we use the expressions based on the classical condition for which cracking is expected to begin.

If e/6 is greater than one sixth of the length of the wall itself; then based on the triangular distribution, based on the geometry itself it is possible to estimate e or the compressed length itself. We are; mind you using the estimate of moment as axial force in to the eccentricity N into e that we have seen earlier. So, the moment is N into e is something that you need to bring in.

So, the compressed length is some β times the total length itself. And we have an expression for the length of the compressed zone; the ratio that we need to multiply the length itself with which is this part that we have written down l/2 – e/2. And then you see that this can be expressed eccentricity e itself as I said; the moment is written as N into e.

So, bringing in an expression for e as M by N into this equation; we are able to get an expression which now depends on the aspect ratio of the wall. So, the shear span of the wall is brought in here; here α_V is nothing, but the shear span of the wall itself. So, M by H is the moment to shear; shear force ratio divided by l. We write that down as H naught by l; this M by H the ratio of the bending moment to the shear force expressed as H naught; H naught by l. And therefore, this expression for l_c brings in the shear span ratio; shear span of the section of the wall itself, the shear ratio of the wall section itself.

$$l_c = 3\left(\frac{l}{2} - \frac{H}{N}\alpha_v\right)\cdot l$$

So, with this estimate of the compressed length; it is on the compressed length that the joint shear failure criterion is going to be used.

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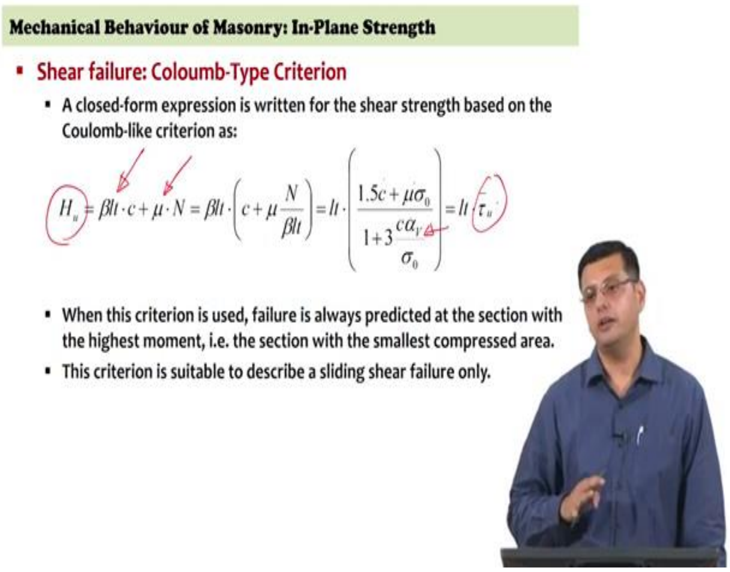

And therefore, if we were to use this within the Coulomb like criterion we ae basically saying that the ultimate the shear capacity is because of the cohesion available in the remaining compressed length of the wall. In the rest of the compressed length of the wall cohesion is lost, there is no bond anymore. And so βlt is the area over which cohesion is still active and of course, you have friction coefficient that is fully available to you and μ is multiplied into N. So, we see that we are working on forces here unlike the earlier expression which was in terms of stresses; so elaborating this expression and bringing in the form of the shear ratio that we had earlier. And you have the expression which brings in both cohesion and the friction coefficient and the shear ratio of the wall in estimating the ultimate capacity of the wall.

$$H_u = lt \cdot \left(\frac{1.5c + \mu\sigma_0}{1 + 3\frac{c\alpha_v}{\sigma_0}} \right)$$

So, this is sort of a reduced shear strength like we had in the previous case; where the friction coefficient and the estimate of the cohesion. Because of the geometry from the center of the block to the top or bottom of the block; required a reduction factor based on the geometry of the block. Similarly, the expression that we have here is looking at a reduced strength because of the compressed length being lesser than the overall length of the wall itself.

So, this criterion typically will always tell you that it is the section that is most compressed because of the maximum moment occurring at the bottom of the wall is where the failure is expected to occur. So, this will always be at the bottommost bed joint either and physically also; when we do a test we see that it is either the interface between the damp proofing course and the masonry wall or it is one of the first masonry joints themselves. And this particular criterion can be used only for the shear sliding criterion.

You cannot use this to predict shear strength of a masonry wall if the failure that is occurred in the wall is of any other type of failure. That is if the wall were to fail by flexural compression or wall where to fail by the formation of diagonal cracks; this estimate of H_u using the coulomb type criterion is not appropriate.

So, and the reason why I am raising this word of caution is many codes give only this expression to estimate the shear capacity of a wall. But the shear capacity of a wall depends on the mechanism of failure and using it across mechanisms; using this expression across mechanisms will give you erroneous results.

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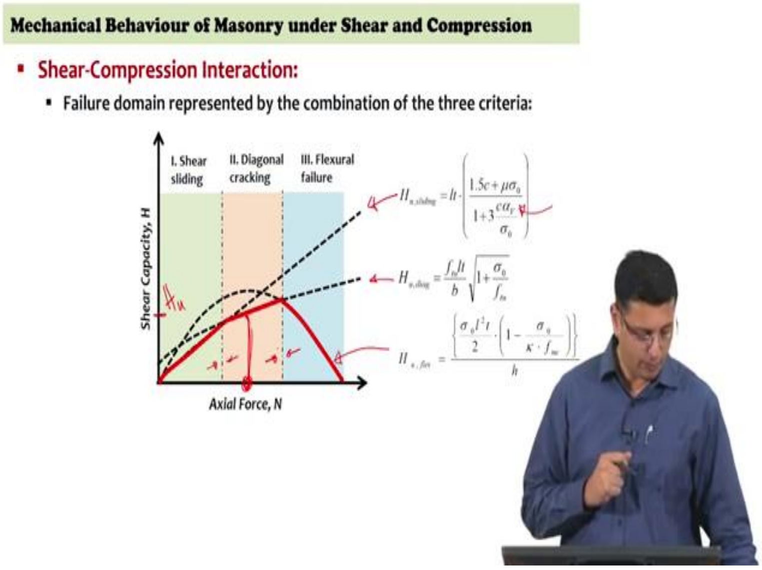

So, I just flag that off and then close by therefore, looking at having examine three different failure mechanisms; what is the shear force, axial force interaction for a given masonry wall for different levels of axial force acting on the wall.

So, like we had the Mann Muller criterion, then find; then used to define the failure domain as far as shear stress normal stress was concerned. We do the same thing for lateral force H and axial force N and you actually have three different zones; the sliding shear zone, the diagonal cracking zone and the flexural failure zone.

And you do see that the sliding shear is typically expected when the level of pre compression is low. Diagonal cracking is typically expected when you have intermediate levels of axial compression and flexural failure is expected, when you have significant pre compression in the wall. So, the three expression that we had developed the H_u for sliding; H_u for diagonal; so that is the H_u for sliding, H_u for diagonal compression and that is the big curve that you see there is H_u for flexure and they dominate in different zones.

So, if you were given a single panel of a known aspect ratio h by l known boundary conditions because now when we are talking of α_v; α_v directly brings in the effect of whether the wall is going to be bending in single bending; cantilevered profile or double bending shear profile. Because the shear ratio the shear span is going to be change; h_0

will change; the bending moment to lateral force ratio will change depending on whether your shear deformation profile or a cantilevered profile.

So, if I know the overall dimensions and then estimate h by l; I know the thickness of the wall, I know the boundary conditions; so I know whether it is going to be in double bending or single bending. And then if I have material properties like compressive strength of the masonry, tensile strength of the masonry, cohesion and friction coefficient and I know the level of axial stress; average axial stress, I have a closed form way of estimating in what mechanism should the wall fail and at what value.

So, I come to this interaction where axial force is increasing on the x axis. If I know the axial force corresponding to the average level of axial stress; I will be able to estimate ok, if I am somewhere there I can go and estimate that the axial the shear capacity of the wall is so much kilonewtons and the failure is expected to be in diagonal cracking.

You could also be somewhere in between the two, you might lie somewhere there or you might lie somewhere there, but which means that your estimates can actually overlap you are not very sure whether it is going to be one mechanism or the other. You can have those boundary issues, but you can also have mixed modes, but this gives you like we would use a PM interaction for a problem of bending compression in RC design, here you have something that is shear compression that you can play around with in design and for assessment.

So, with that we come to the end of shear capacity for in plane actions and we will be looking at PM interactions that can be developed for the unreinforced masonry; which will be of use for us later in design of reinforced masonry walls. So, I will stop here.