Design of Masonry Structures
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Module - 03
Lecture – 19
Strength and Behavior of Masonry Part -IX

Good morning. We will continue with the lecture on the shear strength under the consideration of gravity forces in the masonry. We were looking at establishing a set of expressions that for different levels of axial compression give us a failure surface in terms of shear mechanisms.

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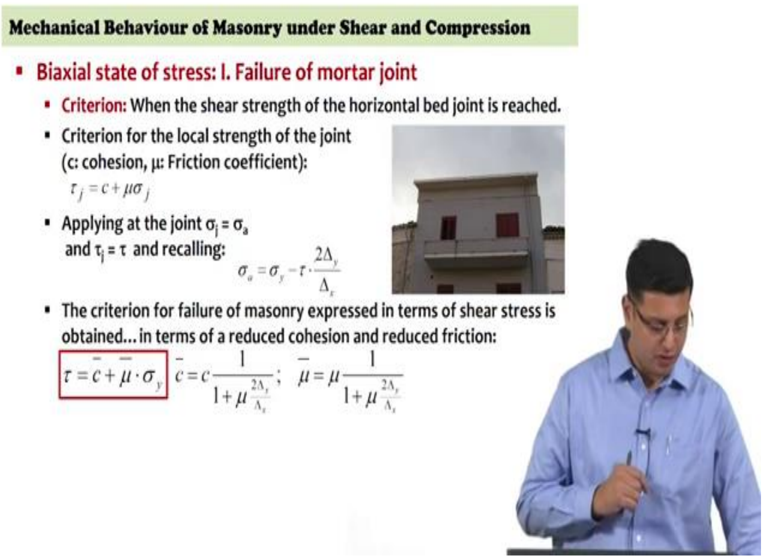

So, we had looked at the first of the cases based on the basic formulation of Mann and Muller. We are now trying to arrive at the state of stresses, the biaxial state of stresses a failure surface. So, the first failure criterion that we are examined to summarize was the failure of the mortar joint itself, where we were to talking of in the real scale of a structure subjected to lateral forces like an earthquake, you would have the formation of what is called sliding shear mechanism and that is what you see in the picture.

And this criterion is arrived when the shear strength of the horizontal bed joint is reached. And therefore, using the criterion the Mohr coulomb criterion, where the shear strength of the joint is represented as the combination of bond contribution which is cohesion in the joint and the friction coefficient which is affected by the pre-compression level itself.

So, we use this definition of failure of the joint itself to the set of equations that we have developed earlier for the Mann-Muller criterion in the case where the level of stress due to pre-compression is low. So, because of the rotation of the unit, one edge of the masonry unit is experiencing a reduced compression and that is σ_a, on the other side of the unit you have the increased pre-compression σ_b, the difference being delta $\Delta\sigma_y$ in the two cases.

So, we use this expression. And with the help of the failure criterion the Mohr coulomb failure criterion in terms of cohesion and the friction coefficient, describe the failure of the masonry joint the horizontal bed joint in terms of a reduced cohesion and a reduced friction coefficient. The reduction occurring because now you have the definition of the geometry of the joint with respect to the unit, where Δ_y and Δ_x are the unit dimensions Δ_y along the y axis and Δ_x the length of the unit itself.

So, we represent it in terms of a reduced cohesion and a reduced friction coefficient as being the criterion for failure of the horizontal bed joint. As I said this is expected to occur when the level of pre-compression is low. So, if it is a single-storied structure, and if the aspect ratio of the wall is such that the horizontal bed joint shear strength criterion is going to be reached or if it is a wall in the upper most storey where the pre-compression levels are low.

If the bed joint shear strength the bond strength is not good, good relative to the other material mechanical parameters of the masonry, you can have this failure criterion, so that is the first failure criterion under low level of axial compression.

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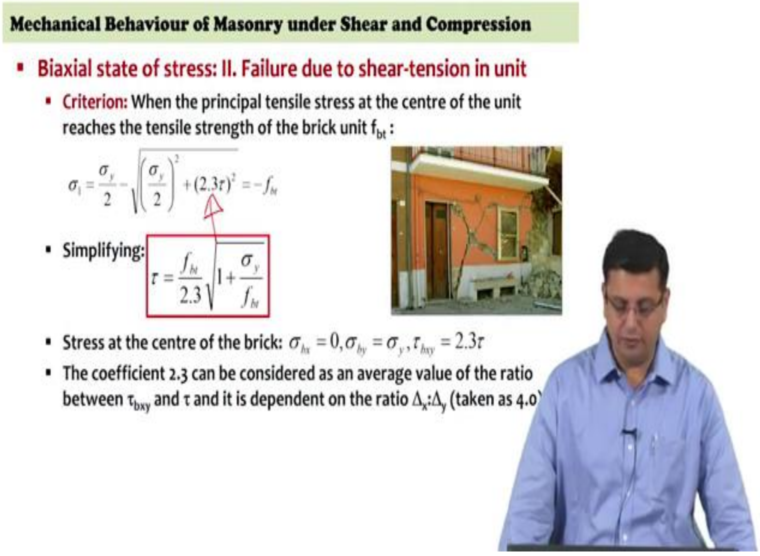

We will start we will look at the other two criteria one when the axial stress levels are intermediate. And you would expect that in a two storied, three storied masonry load bearing masonry construction in the ground storey and finally, a case where the pre-compression level is significantly high. So, the second criterion that we were examining was the shear tension failure in the unit itself.

Again, within this formulation, we are examining the local states of stress and trying to relate it to the global states of stress, the average pre-compression level in the wall itself. So, if you remember the second criterion that we were examining is the classical diagonal tension failure, the classical x-crack that is formed is the failure mechanism that we are actually referring to when we are examining failure due to shear tension in the unit.

So, in this particular case, failure is set to have been reached when the principal tension stress, when the principal tensile stress at the center of the unit reaches this strength of the brick unit in tension. So, the criterion says that if under a combination of axial compression and shear stress from the lateral force, the principal tension reaches the tensile strength of the unit, then you get the failure the crack occurring through the center of the unit.

So, in this particular criterion, we can then use the Mohr stress definition, the Mohr circle and define what the tensile stress, the principal tension σ_1 is with respect to the state of plane stress the biaxial state of stress τ and σ acting on the wall. σ_y is the normal stress acting on the wall, τ is the shear stress acting on the wall. We are considering condition at the center of the brick unit when the principal tension sigma one due to this reaches a value of f_{bt}, which is the tensile strength of the unit itself we get failure in the in the wall in the unit by the tension crack occurring in the unit.

But we have one little aspect to be taken care of which is we are defining the formation of the tension crack at the center of the unit. However, the earlier set of formulation were at the bed joint above or below the unit. So, you had a unit of dimension Δ_y by Δ_x and shear stress τ was defined at the top surface or the bottom surface, the bed joint, we were talking about the failure at the joint.

However, in this criterion, we are talking of tension crack occurring at the center of the brick unit and therefore, we must account for the translation of that stress now from the joint to the center of the brick. And this is affected by the geometric proportion of the unit itself. So, this is one aspect that needs to be considered. And there are simple numerical methods available to be able to arrive at this value.

So, it is from that this value 2.3 in the bracket that you see there 2.3τ that you see there, which is basically the magnitude of the shear stress from the joint to the center of the brick itself. If we now rewrite this expression in terms of τ, because the earlier expression was written in terms of τ as a function of σ_y, we again want for the second criterion the value of shear stress as a function of σ_y. So, expanding the under-root terms and rewriting the expression in terms of τ and making use of the failure strength here which is the tensile strength of the unit itself.

$$\tau = \frac{f_{bt}}{2.3}\sqrt{1+\frac{\sigma_y}{f_{bt}}}$$

The stress at the center of the brick, if you were to examine what is happening at the center of the brick, the axial compression level σ_{by} is σ_y, we are assuming that the average normal stress is the value which is at the which is the normal stress acting at the center of the brick. We are assuming that there is no stress, normal stress acting in the other orthogonal direction, and the shear stress is 2.3τ and I will be able to provide the reference available for arriving at this formulation, but it is a simplification of the state of stress at the joint to the center of the brick. So, this has been arrived at considering a proportion between the unit height and the unit width.

In this case, the unit width is the unit length is taken as 4 times the height of the unit itself. So, that is going to be geometry it is it is going to be dependent on the geometry. And therefore, if there is a deviation, if there is a significant deviation in terms of the unit dimensions, you should that that 2.3 is a number that comes because of the assumption of the ratio of Δ_x to Δ_y ok.

So, this is our second criterion, where we expect the level of average the level of pre-compression in the wall or the average normal stress in the wall to be of intermediate range right. It is neither too high nor too low and we expect the failure mechanism in that range to be because of the formation of tensile cracking due to the combination of at lateral force, and the axial force itself.

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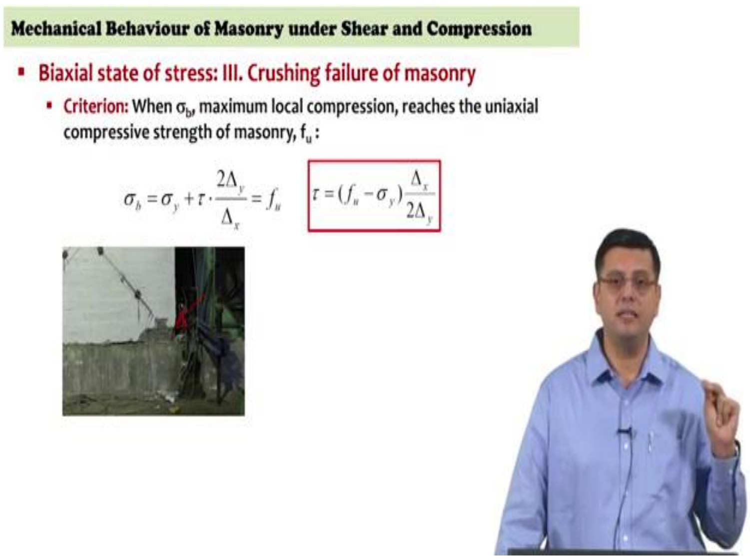

The third criterion that we look at is failure due to crushing of the masonry. And this was a third criterion that, third failure mechanism that I had mentioned. And here what we really talking of is under a combination of the lateral force and the high pre-compression that already exists in a wall, the compressed end of the wall- one end of the wall is experiencing uplift, whereas the other end of the wall under the action of the in-plane forces is experiencing increased compression.

Under their increased compression if the pre-compression level was originally high, the chances are that the compression the flexural compression at ultimate reaches a value closed to the crushing strength of masonry itself, so that becomes the third criterion for establishing failure in the masonry wall.

So, in this particular case, if you remember the photograph that I had shown you which is the compressed end of the wall starts experiencing crushing failure. This end is experiencing crushing failure; it is in flexural compression, it is not direct compression, but flexural compression and this is the basis that we use for the failure criterion. Therefore, you need the crushing failure strength of masonry, the crushing strength of masonry of masonry in this case in the previous criterion we were looking at the failure of the unit in tension, but here we are looking at the failure strength of masonry in compression the assembly itself.

So, when now if you remember again from the Mann-Muller criterion, we had σ_a where there is reduction in the compression level and σ_b where there is an increase in the compression level. We are going to be looking at σ_b to define what the failure criterion is when σ_b reaches the uniaxial compressive strength of masonry f_u you have failure established.

$$\sigma_b = \sigma_y + \tau \cdot \frac{2\Delta_y}{\Delta_x} = f_u$$

We rewrite this in terms of τ and we now have the relationship between the average shear stress related to the average normal stress σ_y in the wall itself.

$$\tau = (f_u - \sigma_y)\frac{\Delta_x}{2\Delta_y}$$

Here the failure criterion is f_u, in the previous tensile diagonal tension failure it is f_{bt}, the tensile strength of the unit, whereas, in the first case it is the shear strength of the joint. So, there are three material mechanical parameters that we bring in to define the failure of the system itself ok.

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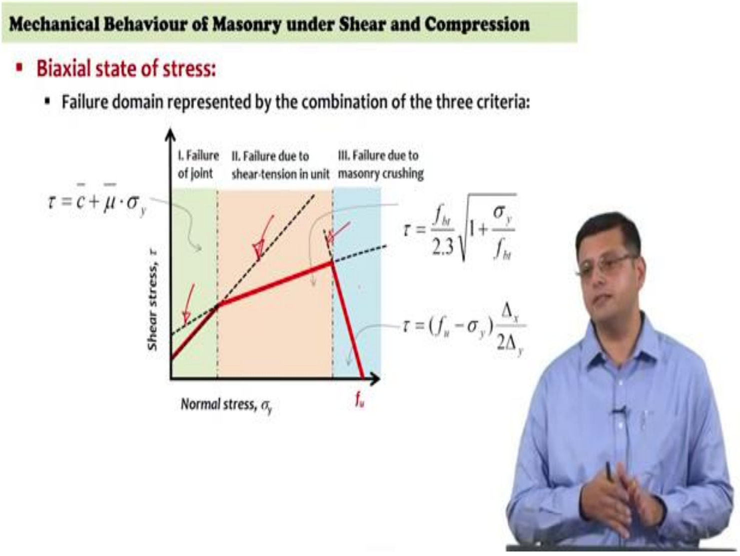

Therefore, now we have looked at three different; three different ranges, we had low pre-compression, intermediate pre-compression and high pre-compression levels. Now, you can expect a high pre-compression level let us say if the wall is not a very squat wall. If you have a very slender wall, if you have a relatively slender wall, then the area of cross section available for equilibrating the lateral forces and the gravity force is smaller in comparison to a squat wall.

So, in a slender wall with high pre-compression, you can get the flexural crushing mechanism that can occur in a masonry wall under the action of lateral forces and gravity forces. So, we are basically looking at all possible geometrical combinations in terms of the aspect ratio of the wall and in terms of the material strengths.

So, while we are defining this at the level of local stresses and relating it to the global stresses, this becomes the basis for us even later to establish in terms of stress resultants what is the lateral force-axial force interaction in a masonry wall right. You are familiar with axial force bending moment interaction P-M interactions that we use for design in concrete.

In masonry given that, in plane mechanisms are typically shear dominated mechanisms, shear capacity is what gets affected significantly by the lateral force and that is what we need to consider when we examine design of shear walls. And therefore, we will use a similar basis this is at the level of stresses, but we will be examining the same under the action of resultant forces. We will use the same basis to develop the interaction surface between shear forces and axial forces.

So, based on the three different failure mechanisms that we have examined so far which basically cover all possible failure mechanisms in masonry under in-plane actions. We first looked at failure of the joint. We then looked at failure due to shear tension, the second zone, the orange zone that you see there. And the third zone with high pre-compression level failure due to crushing of masonry itself.

So, we developed expressions for each zone based on the Mann-Muller criterion. We have τ is equal to reduced cohesion plus μ into σ_y, where μ again is reduced friction coefficient being the first criterion. If you use that criterion you would get the first set of dotted lines in the first zone. It can it will basically overlap at some point or become higher in value in terms of tau with respect to the second criterion right.

What I am talking about is, I am examining this line here I have the expression for τ, I have the expression. Now, beyond a certain range of σ_y, the second expression failure due to shear tension becomes the more critical one that is the lower of the three values would be for the intermediate ranges of axial compression, the criterion governed by shear tension failure.

So, you see that in the central zone, the equation represented by

$$\tau = \frac{f_{bt}}{2.3}\sqrt{1+\frac{\sigma_y}{f_{bt}}}$$

That becomes the lowest failure stress shear stress with respect to σ_y for the intermediate zone. And for the third zone we have the third line,

$$\tau = (f_u - \sigma_y)\frac{\Delta_x}{2\Delta_y}$$

And you see that value becomes the lowest value of shear stress τ as a function of σ_y.

So, you basically can use these three expressions for the material strengths available to you for a given case. Draw these three different lines governed by different equations the lowest of the three will form the failure surface. So, what you see in red overlapping the three black dotted lines is the failure plane. Of course, this failure plane is going to be affected by the relative strengths that we have considered here, the value of cohesion, the value of friction coefficient, the value of tensile strength of the unit, the value of failure crushing strength of masonry and it is also going to be dependent on the ratio Δ_x by Δ_y.

So, geometry and material properties are going to affect what the failure plane is for a given wall or a given failure zone that we are examining in a masonry wall itself. So, this is the failure domain that is representing the biaxial state of stress under lateral force and compression due to axial forces dead weight and pre-compression from superimposed loads. So, this is the extension of the Mann-Muller criterion which is now given as a basis to get a failure plane, but mind you we are still working at the level of stresses.

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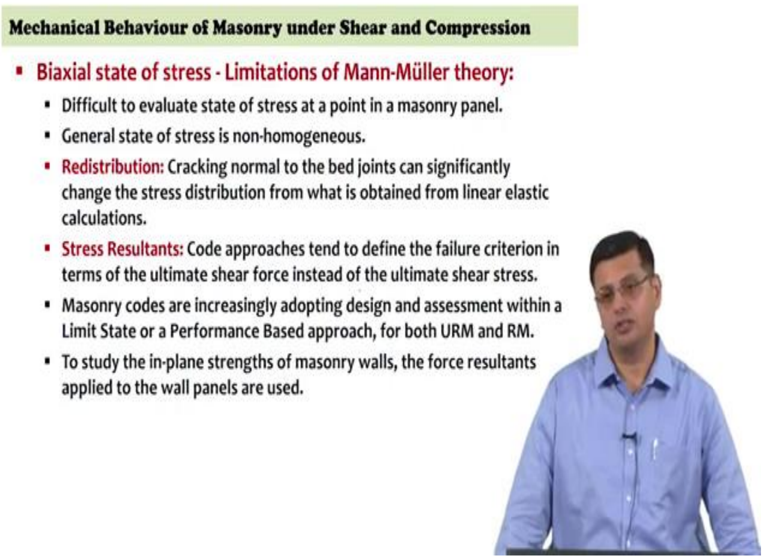

We need to do that because there are specific limitations of the Mann-Muller theory. Primarily, as I said if you were to use this sort of a criterion in design or you were to examine the states of stresses in a building, every masonry panel at every different point is going to have a different value of stress right. So, it is actually lot of simplification if we were to just estimate the failure stress at one point using this in a masonry panel using this, set of using the failure plane that we have just arrived at.

The other aspect is the state of stress is non-homogenous and therefore, defining failure planes at the level of stress is a problematic affair. Also considering the fact that we know that strengths in one direction versus the other direction is going to be different in masonry. We have also seen from the work, the experimental work that I presented of Page that there is going to be difference in the failure mechanism at different orientations of the principle stresses with the bed joint. So, you are not looking at a state of stress that is very easy to capture.

The other problem is Mann-Muller criterion is really not taking into account any redistribution after crack formation occurs in the masonry panel that is under examination. So, there is going to be stress redistribution the moment there is some inelasticity and therefore, the set of expressions that we have assumed are on the basis of a linear elastic set of calculations. And we are then using failure strength, then using material strengths to be able to establish criteria. So, this redistribution is something that is not going to be considered in your in the expressions or the failure plane itself.

And the other important aspect is if you look at the way codes define design procedures, it is not at the state of stresses that we work. We will it will be easier for us to work on stress resultants. And therefore, it is useful to have these expressions extended to stress resultants if possible. So, if you are looking at limit state approach or if you are looking at a performance based approach and if limit states can be designed both for unreinforced masonry and reinforced masonry if this then becomes an interesting criterion an interesting set of expression that you could use to define the interaction.

And I was mentioning the lateral force axial force lateral force H and the axial force N interaction in masonry walls. We could have that done for unreinforced masonry and extend the same thing to reinforce masonry as well. So, we will work here after on force resultants on the wall panels themselves ok.

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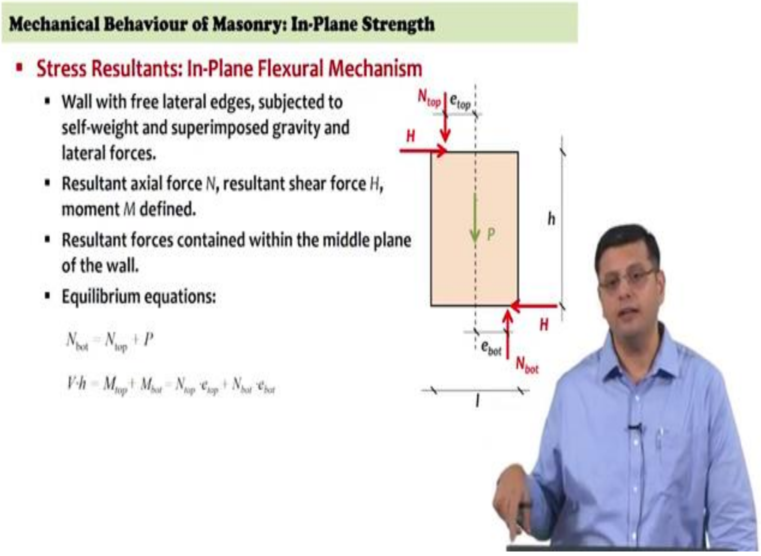

So, to start examining in-plane strength, we are now examining the in-plane strength of a masonry wall and again in a way similar to the different zones that we examined in the failure plane which was the end result of application of the Mann-Muller criterion, you had low pre-compression, moderate or intermediate levels of pre-compression and high pre-compression.

The same three situations can be considered where the first failure mechanism that we will try and examine is a flexural dominated mechanism, which means the wall when it is subjected to deformation, when it undergoes deformation due to the lateral force and in the presence of axial compression. So, here the geometry comes in, depending on the aspect ratio, if you are looking at a slender wall in all probability you can get a flexure dominated mechanism, meaning the you have one region where there is uplift in the wall and one region where there is increased compression in the wall. So, the increased compression if the pre-compression levels are high can lead us to flexural crushing mechanisms, whereas on the other end you are actually getting cracking which is tensile cracking of under flexure.

So, the first mechanism that we can examine and have overall expressions for is a flexure, flexure dominated mechanism or in-plane flexure dominate mechanism. And then we will examine the other two mechanisms which is again the shear sliding mechanism that we saw where the there is a failure in the joint, which is the shear dominated behavior, and then the formation of the diagonal cracks, the x-cracking which is again a shear mechanism rather than a flexure mechanism.

And in this clearly there is a role of the aspect ratio of the wall and that something that you must keep in mind; there is the role of the aspect ratio of the wall first and there is also the role played by the boundary conditions that exist in the wall, whether you have rotations free at the top, the wall is cantilevered with respect to its vertical boundary conditions, it is free to rotate at the top or is it restrained again rotations at the top.

So, two aspects that will come into play as far as the geometry and boundary condition are the aspect ratio and the whether the wall is free to rotate at the top or are there restraints to rotation at the top, we will examine a wall which is free to rotate cantilevered in the lateral deformed shape or shear deformation profile because of the top rotational restraint.

And then of course, we have already examined the role played by the material strengths. You can have the relative strengths between the bed joint shear strength, the compression strength of masonry and the tensile strength of masonry. Earlier we had looked at in the Mann-Muller criterion, the second criterion was when the tensile strength of the unit was reached, but now you are not going to examining it in terms of the unit or the mortar, you are going to be looking at tensile strength of masonry.

So, instead f_{bt}, we will start looking at the tensile strength of masonry and that is where the diagonal compression test that we looked at earlier as an estimate of the tensile strength of masonry starts becoming useful. So, let us examine the first criterion. The first mechanism, the in-plane flexural mechanism and here we really considering a wall that has boundary conditions at the top and the bottom.

It is free on the two the lateral edges of course, that is an ideal situation you might have a condition where there is a return wall, the wall is flanged you can have the spandrel of a wall can also be a lateral boundary condition. However, we are examining an idealized case, where the wall is free on the lateral edges and is subjected to its self-weight and there is superimposed gravity loads and there is lateral load acting on the wall.

So, we really examining self-weight of the wall given as P, there is superimposed load, we can assume that there is depending on the boundary conditions assume that there is some to eccentricity of the superimposed loads, and there is lateral force acting on the wall designated as H, length of the wall as l and height of the wall is h.

So, if you were to examine the resultant forces that we need to be working on you have resultant axial force N, you have resultant shear force H, and the moment which is acting on the wall because of the lateral force and the height of the wall h. So, we are going to be examining the forces in different segments and we are assuming that the resultant forces are all contained within the middle plane of the wall itself.

And we can write down the equilibrium equations for the two conditions the vertical equilibrium and the rotational equilibrium in the system. So, from the vertical equilibrium, the axial force at the bottom is the summation of the self-weight and the axial force superimposed at the top. Therefore, it is important for you to be careful where you are making the estimates of the stress.

If you are making the estimates of the stress distribution in the wall at the mid height, then be careful about the contribution of the self-weight that is being considered. If you are at the top there is no contribution of the self-weight, if you are at the bottom you have a full contribution of the self-weight of the wall, so this you will have to careful and in what plane are you making the calculations.

So, if you are typically looking at flexural compression failure right, I would be interested to look at the flexural compression failure at the bottom of the wall because that is where the flexural compression value is going to be the maximum- at the compressed edge. But, if you are interested to look at the diagonal tension failure, diagonal tension failure would typically begin from the mid height of the of the wall panel itself.

And therefore, for that remember we made the calculations at the center of the brick unit in a similar manner, so the entire wall panel typically shear cracks, diagonal shear cracks would start at the mid span because of the maximum shear stress distribution being at the mid height of the wall itself. And there your calculations are going to be at mid height of the wall.

And therefore, you should be considering the contribution of axial, the self-weight as being of one half of the wall itself. So, though we are working on stress resultants, you have to be very careful about where we are defining the failure in the wall itself. The lateral force into the height of the wall is equilibrated by the moment at the top and the moment at the bottom.

And here the moment at the top and the moment at the bottom are represented as the axial force resultant at the top into the eccentricity at the top and the axial stress resultant at the bottom into eccentricity at the bottom itself ok.

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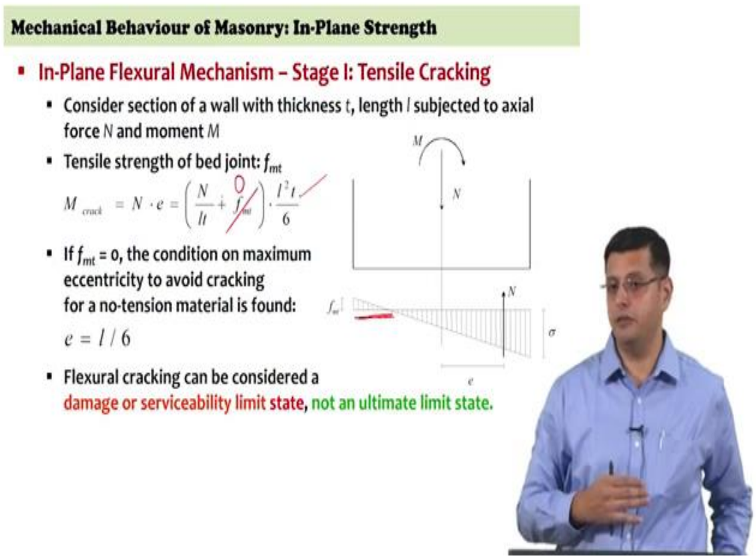

So, we start examining in-plane flexural mechanism. We look at three different stages. I would first refer to the tensile cracking stage. So, as the wall is being subjected to lateral forces and gravity, you can have cracking at the base of the wall right. And as cracking progresses, there is reduction in the area available for equilibrating the combination of lateral forces and gravity forces. And then finally, at ultimate you have reduced area and possibly crushing occurring in the wall that is the mechanism that we are examining.

So, stage I of tensile cracking, let us look at a wall which has a thickness of t, length l, it is subjected to axial force N and a moment M. So, what is actually happening is, at this stage, we are assuming that the distribution of stresses is linear elastic, triangular distribution is seen and part of the wall cross section can actually go in to tension depending on the relative values of M and N.

And therefore, if you are assuming that there is significant pre-compression in the wall, significant lateral forces required for cracking to occur. Let us assume a situation where tensile stresses have now occurred in the wall, you have a part of the wall cross section that is subjected to tensile stresses. The rest is in compression, edge compressive stress sigma, and the eccentricity of the axial force resultant N is e with respect to the centre line of the wall itself.

Now, here we have the wall is subjected to tension perpendicular to the bed joint and therefore, we are interested in the tensile strength of the bed joint with tension acting perpendicular to the bed joint f_{mt}. Now, you could assume that the wall has zero tensile strength or assume that f_{mt} is a finite value.

If f_{mt} is a finite value, cracking is going to occur only when the tensile stress reaches f_{mt}, if you assume that the wall has zero tensile strength, then the moment you have tension when we have the limiting eccentricity, you will start getting cracking in the wall itself.

So, assuming that the bed joint tensile strength here is f_{mt}, the cracking moment can be written with the knowledge of the tensile strength of the bed joint itself, again representing the moment as the axial stress resultant, axial force resultant N into the eccentricity e. And in this case, we are writing it as the axial stress plus the stress coming from the bending.

And with this additional resistance available which is the tensile strength of the bed joint f_{mt} non-zero value, we write down the, this is with respect to the this is the section modulus of the cross section of thickness t of thickness t and length l. Now, if you use the same expression and take f_{mt} to 0, then you get the classical condition where this cracking is occurring when eccentricity is equal to l/6.

So, this is the first stage, this is the first stage. So, I think a typical confusion we have been having with all students doing this particular exercise, this is the first stage of a three-stage loading. Therefore, this failure mechanism, this failure mechanism is not an ultimate failure mechanism. This is under serviceability condition. Cracking is occurring under serviceability conditions, it is not an ultimate limit state.

The ultimate limit state for me is the crushing failure of the wall. What is happened now is tensile flexure cracking, but that is not an ultimate limit state. It is only a; it is only one of the initial states or at least the serviceability limit state. So, as this progresses the ultimate limit state is going to be the flexural crushing in the wall itself.

So, what is important is when you are estimating the failure mechanism, the moment corresponding to the ultimate condition crushing you cannot compare that to M_{crack}. M_{crack} is at a serviceability level, M_{crack} is at a serviceability level, but M_u is ultimate limit state right. We will come back to this point because you cannot compare M crack. And M_u they are not at the same levels of demand they are at very different levels of demand, one is serviceability, the other is ultimate ok.

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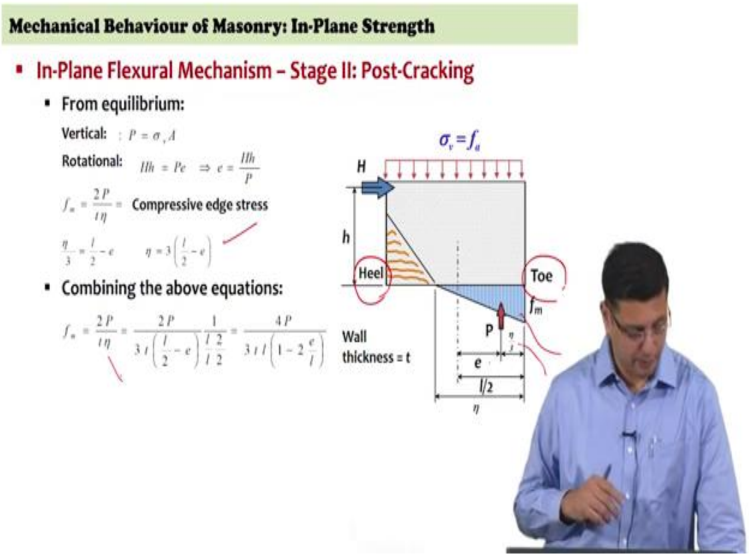

Stage II is post-cracking, post-cracking you have reduced cross section and now equilibrium is on the partial section of the cross section. So, the portion that has undergone uplift because of the direction of the lateral force, you refer to that as the heel of the wall, and the other end which is now experiencing higher pre-compression is the toe of the wall. So, you have toe going towards increased compression levels and can fail in crushing.

So, we refer to the mechanism as toe crushing, but what is happened that the serviceability state is heel cracking right. So, heel cracking is when tensile strength of the bed joint is reached serviceability limits state toe crushing will occur at the ultimate. So, let us examine this post-cracking phase in the wall, the axial stress levels in the wall at f_a, the vertical stress the average vertical stress in the wall equal to f_a. And we are considering now a partial section which is the hatched blue region that is what is equilibrating the combination of H and the axial load, the superimposed load and the self-weight of the wall.

However, what you see here is that the distribution of stresses is still considered to be linear elastic ok. So, we are in the post-cracking phase, but still linear elastic. The inelasticity that we have assumed is cracking and neglecting the area of cross section in tension itself. So, in this the set of notations that we are using the compressed length of the wall is η as a part of l, η is when the wall is not cracked eta is equal to l, but now the partial length is η given the triangular distribution of the stresses, the stress result the axial force resultant is sitting at one-third at the centroid of the triangular distribution.

And therefore, $\eta/3$ from the compressed edge is where the resultant is sitting and with respect to the center line of the wall the eccentricity is e. With this triangular distribution, we have done this earlier for the out of plane flexural mechanism. We can write down the equilibrium and then get an estimate for the compressed length eta which is the length over which the combination of lateral force and axial force is being achieved.

So, from the vertical equilibrium, the axial force P is σ_v into the area of cross section. And from the rotational equilibrium h lateral force into the height h of the wall is equal to the axial force P into e, and therefore, we get an estimate of the eccentricity which is H by P, lateral force by P, ratio of the lateral force to the axial force in to height h.

From this distribution, from the triangular distribution, we get the edge compressive stress f_m, currently the edge compressive stress is f_m, it is still within elastic range, but this will approach f_u or the crushing strength of masonry. And this is going to be equal to 2 times P by t into the length of the compressed zone which is the triangular area itself.

$$f_m = \frac{2P}{t\eta} = \frac{4P}{3tl\left(1-2\frac{e}{l}\right)}$$

So, when this edge compressive strength approaches the compressive strength, we have a we have the limiting value of lateral force for which the failure in crushing is expected.

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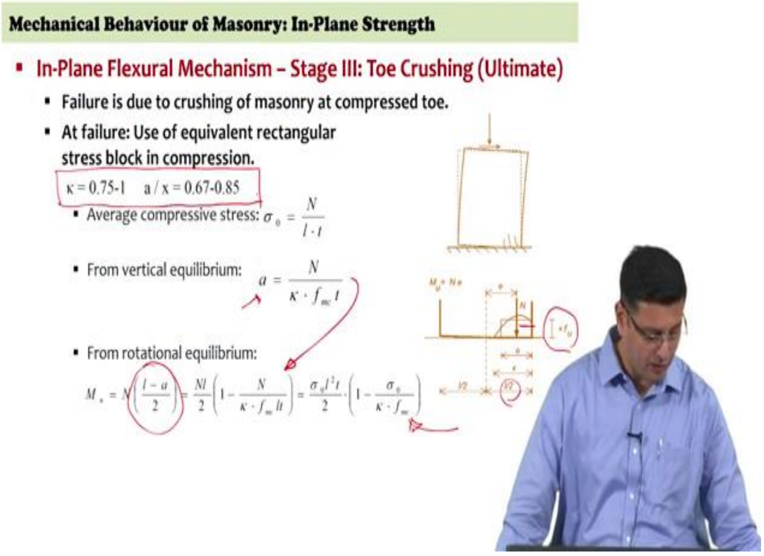

So, we have got this expression now of the edge compressive stress. We go to the third stage which is the ultimate stage of the demand and the capacity of the wall which is toe crushing itself. At the compressed toe, you will have crushing of the masonry. And so for the same condition, we now assume that the distribution of compression at the compressed segment of the wall is non-linear ok. So, it is only in the last stage that we assume a non-linear distribution of the stress in the compressed zone, the compressed zone is now further reduced.

In this case like we did for the case of out of plane bending, we can assume that an equivalent stress block can be used to represent the parabolic distribution of stresses. We use equivalent stress block parameters write own the equilibrium in the stress block and then use that with respect to the crushing strength of masonry to be able to arrive at the ultimate lateral force or represented in terms of the ultimate moment at which failure is occurring.

So, stress block parameter is here. We take the value of K here, Kappa here as varying between 0.75 and 1. And these are values that will actually depend on the type of material. So, in this case, value between 0.75 and 1 can be assumed depending on the type of masonry, you are looking at as the height of the stress block. And the dimensions of the stress block, the length of the stress block itself, ‘a’ being the side dimension of the rectangle divided by x which is the parabolic distribution, length of the parabolic distribution a by x varying between 0.67 and 0.85 is a good estimate.

So, the average compressive stress is the axial force divided by l into t and from vertical equilibrium we now have equilibrium provided by the stress block itself.

So, N into this is the eccentricity that we are looking at and therefore, you have the stress resultant sitting at the center of the rectangular block. So, N(l/2 – a/2), which is where the resultant is sitting, so that is your e. So, N into e is defined. We have expression for the value of a, we bring this into this expression and we have a final expression for the ultimate moment with the knowledge of the axial compressive stress level and the compressive strength of masonry f_{mc}, this.

Student: Sir, here x is the parabolic distribution, a is?

a is the rectangular stress block, x is the it is the values given here, just give you values that are used to equate the parabolic stress distribution to an equivalent stress block. And research suggests that the size of the stress block which depends on the type of masonry that you are looking at can have values ranging from 0.75 to 1 of f u as the height of the stress block and the width of the stress block as a ratio of a divided by x between 0.67 and 0.85.

So, you can assume values between these for different types of masonry. And therefore, now you have an expression for the ultimate moment M u given these given the knowledge of the axial compression level and the compressive strength of masonry, basically considering that failure is occurring under a flexure dominated mechanism of toe crushing itself.

$$M_u = \frac{Nl}{2}\left(1 - \frac{N}{\kappa \cdot f_{mc} lt}\right) = \frac{\sigma_0 l^2 t}{2}\left(1 - \frac{\sigma_0}{\kappa \cdot f_{mc}}\right)$$

So, this corresponds to the third failure criterion that we looked at in the Mann-Muller criterion right.

So, as I said earlier I repeat that M_{crack} and M_u are not at the same level. M_{crack} is at the serviceability condition. So, you cannot assume that M_{crack} is a failure criterion in the wall, it is only a cracking limit state; it is a serviceability limit state; M_u is the ultimate limit state in the wall itself.

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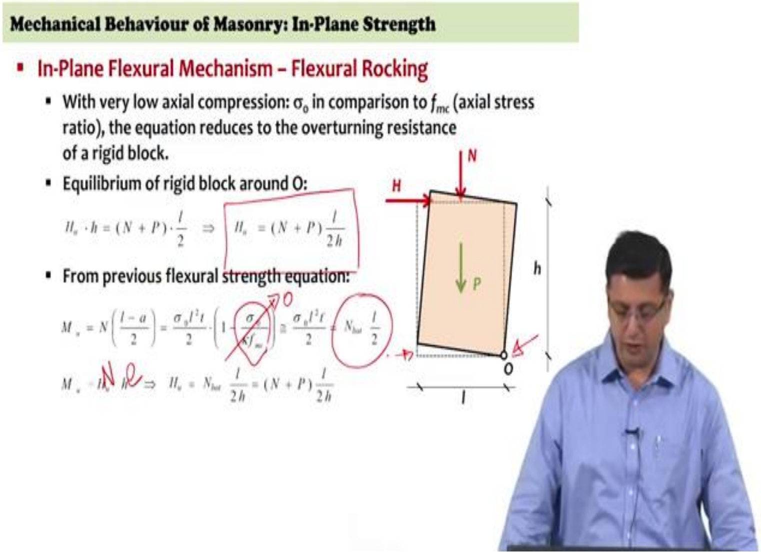

There is one there is a special case of this flexural mechanism, which is if the aspect ratio of the wall is such that the wall is still going to be dominated by flexural mechanisms. Let us assume the wall is of an aspect ratio such that the height of the wall is much larger than the length of the wall.

So, it is a slender wall not a squat wall. When it is a slender wall, most often it is going to be dominated by flexural mechanisms. But what we have looked at in the previous slides is when the pre-compression levels were significant, but if the pre-compression level is not significant can you still have a flexural mechanism that is the pre-compression level was, it was essential that the pre-compression level was high to have toe crushing in masonry, yes.

Now, if the level of pre-compression is not high, the problem is you might not reach crushing failure at the toe, which means the wall is going to continue to carry the gravity forces and not get crushed. This is particularly the case when the level of pre-compression is very low. So, you have a slender wall, but if you have this is top story wall or if it is a single storeyed structure and the wall its aspect ratio is such that it is slender, you can still have a flexure mechanism, but it was it will not fail in crushing, but it will rock.

So, in this particular case, we are really examining a situation of low axial compression. This axial compression expressed as a ratio with respect to the compressive strength of masonry. We refer to this as the axial stress ratio, which is pre-compression level sigma not divided by the crushing strength of masonry f_{mc}. If this value is really low and if it is a single storeyed structure slender wall, this is going to be a significantly low value.

We are going to be looking at point we are going to be looking about 2 percent of the compressive strength less than 5 percent of the compressive strength. When we are in that sort of a situation, it can still be dominated by a flexural mechanism. However, crushing will not occur, but rigid rocking will occur right. So, this is an extension of the flexural mechanism, crushing failure is not going to occur, but rocking is going to occur.

So, if the pre-compression level is low, then with even a small amount of lateral force if the bed joint tensile strength is not high, you can have cracking at the base of the wall. If the pre-compression level is low and the tensile strength of the joint is low with a little level of little magnitude of lateral force, you can start getting cracking heel cracking can occur.

But once heel cracking occurs and continues since the pre-compression levels are low, you can have significant part of the wall that undergoes cracking. With the condition that you only have almost a hinge at the other end which is equilibrating right and that is this, this point O, the level of pre-compression is so low and the bed joint tensile strength is also low in masonry.

So, with the little bit of lateral force, you can get the heel cracking that is going to be significant enough with respect to the cross section, leaving only a very small cross section in compression; still active in compression. Eta value is going to be significantly small in comparison to the length of the wall.

In the extreme case, it is a point you just have a point. But the level of pre-compression is so low that and the strength of masonry in compression is if it is good, then at that edge O you will never get crushing, the wall will continue to simply rock. And so it is overturning mechanism ultimate failure will be when the stability is reached and you will get a overturning, but you will not get a cross section failing in crushing in compression.

So, this is the special case. So, if we were to look at this rigid block now, the wall panel acts like a rigid block. It is got a failure plane that has formed at the bottom because of tensile cracking, but equilibrium is maintained by this hinge at O itself. So, if I take the equilibrium of the rigid block around O at ultimate the lateral force H u into height h equilibrated by summation of N and P into length by 2.

And therefore, you can get an expression,

$$H_u (N + P) \frac{l}{2h}$$

And you see that this is not governed by any material strength, this is not governed by any material strength, it is purely geometry. You can get rocking mechanism, you can get over turning mechanism. Under lying assumption is that the material strength in comparison is significantly high which is true for masonry.

And when you look at low axial compression levels, this value of sigma naught by f mc can be very low. So, the axial stress ratio being low you can get this special case of flexural mechanism which is flexural rocking. So, if you were to use the previous expression in the previous expression, this is going to 0.

This axial stress ratio is going to 0 for us and that goes to 0 your equilibrium comes merely from the axial force level and the geometry itself, you will get the overturning mechanism which is a rigid rocking that occurs in the wall. So, this is a special case that we can consider of flexural mechanism itself. So, if you have slender wall, flexural mechanism with low pre-compression can undergo overturning of flexural rocking, otherwise you can see the failure of the compressed toe itself. So, we have examined the first of the mechanisms and we have two more to examine which we will do in the next class.