**Design of Masonry Structures**
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**Module - 03**
**Lecture - 18**
**Strength and Behaviour of Masonry Part-VIII**

So, very good morning we will get on to the next component of material strengths of masonry, Behavior of Masonry before we get to design.

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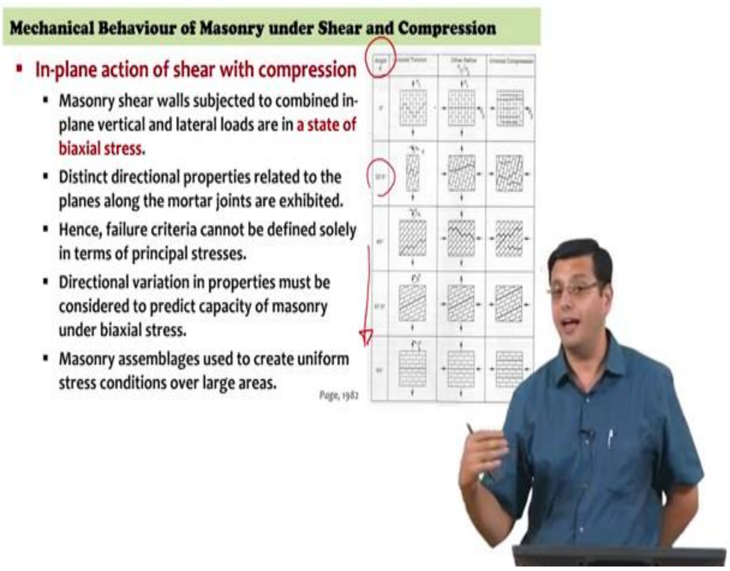


So, that is behavior primarily under shear, but we are not going to be examining shear in isolation because that is really not a practical state. Pure shear is an interesting state in mechanics, but very rarely would you get a pure shear state. So, it is always going to be in the presence of compression- axial compression when we are looking at load bearing constructions.

So, it is about axial compression coming from gravity and shear coming typically from and the lateral forces. Our focus primarily is going to be on lateral forces coming from earthquake effects. And therefore, it is in-plane shear because we are looking at load bearing shear walls in masonry in the presence of gravity. Now, the axial compression levels can vary and therefore, it is useful to examine the kind of interaction that you get between axial compression and shear.

So, the next three lectures would be dedicated to getting an understanding of this interaction leading to a biaxial state of stresses at the material level- masonry. And then using that basis to understand in terms of resultant forces; will we be able to define these interactions which then become the basis for design, you have shear governed behavior, a flexure governed behavior both in plane.

So, this becomes really the basis of what we are going to be looking at in design for shear walls. So, we are looking at the combination of shear and compression, but we are looking at in-plane action now. So, in the previous set of lectures that we were examining, bending that was out of plane bending ok.

So, now we going to be looking at in-plane action, looking at shear wall behavior in masonry and it is not that is shear is not going to act in the out of plane direction in the masonry wall. It is just that the failure mechanism because of shear in the out of plane direction is not something that is expected to occur; though you might have some shear stress. But failure is normally expected in bending, flexural compression, rocking failure. So, it is flexure dominated rather than shear dominated in the out of plane direction and in the in-plane direction, we need to consider this interaction between shear and compression.

So, we need to be examining this state of biaxial stress that develops and it is this which becomes the basis of the principal stresses which we need to examine and arrive at some failure criteria. So, we are going to be examining combination of the vertical gravity forces, superposed and self weight with the lateral forces acting on the wall leading to biaxial state of stress.

So, what is observed mechanically and experimentally? That there is very clear directional property that comes into the picture because of the joints; the orientation of the bed joints. So, depending on how the bed joint is oriented with respect to the stresses, to the principle stresses or if you want to take it. Further the direction of the applied forces to the orientation of the bed joints; the plane of orientation of the bed joints becomes critical ok.

And that is where there is a departure from materials that can easily be considered as homogeneous materials. So, in reality there is a heterogeneity; there is heterogeneity because we are looking at the composite, a very well defined composite, units and mortar and interface. And a second issue which is coming from directional properties; that the strengths are not going to be the same in one direction versus the other. So, this starts coming into the picture particularly when you have combination of shear and compression.

So, if we were to work on the concept of homogenization and that assumption of an isotropic homogeneous material if it were completely acceptable for a material like mortar masonry, then you could simply work with principal stresses. And when of the principal stresses reach some of the material strengths you have a failure criterion. But in this case unless you consider the orientation of the bed joint, you will not be able to explain the failure mechanisms completely.

So, that is what we are going to be examining and that leads us to how we can consider the orientation of the bed joint in the failure mechanism. So, this is a set of experiments that was conducted primarily to show that the failure mechanism is not dependent only on the principal stresses or the orientation of the principal stresses, but the directional property of the masonry governed by the orientation of the bed joint starts becoming extremely significant.

So, Page in 1982; this is in fact, the PhD thesis of Page; carried out a set of experiments ok. The table that you see there is examining simple wallets in masonry, small walls in masonry to a state of biaxial stresses. Now, you need to create a state of biaxial stresses; so if you have shear stresses and axial stresses acting on a planar element; you know that you can always arrive at an equivalent set of principal stresses $\sigma_1$ and $\sigma_2$.

So, here what has been done is that $\sigma_1$ and $\sigma_2$ is then used as the state of stress that is going to be what the panel is subjected to. You keep $\sigma_2$ zero, then you have only $\sigma_1$ acting and then $\sigma_1$ 1 itself is oriented differently with respect to the bed joint; so that was the first set of test. So, if you look at the first; if you look at the first column we have angles right this first column is looking at angles and that is the angle at which the load is being applied with respect to the bed joint orientation right. The second column is referring to uniaxial tension tests. Now, the panel if you have your principal stresses $\sigma_1$1 and $\sigma_2$; in this first case assumption is $\sigma_2$ is 0, you only have $\sigma_1$ tension.

But now the $\sigma_1$ tension is oriented with respect to the bed joint, goes all the way from 0 degrees, which means the tension uniaxial tension is being applied parallel to the bed joint itself and then keeps changing goes to 22.5 degrees, 45 degrees and so on up to 90 degree. So, in the last of the panels in the first column, you can see that the uniaxial tension is perpendicular to the bed joint.

In the second case, the principal stress $\sigma_2$ is not 0; it is a non-zero value. So, you have actually have a ratio of $\sigma_1$ to $\sigma_2$ right; you have $\sigma_1$ to $\sigma_2$ and in one direction there is tension. The other direction there is compression and there is a change in this ratio that is carried out with changing orientation of the bed joint with respect to $\sigma_1$ itself.

In the third set of tests, it is assumed that there is only compression; no tension in the other direction and test is repeated going all the way from an orientation of 0, which is compression as perpendicular to the bed joint to compression being parallel to the bed joint. So, this was the overall basis with which these experimental investigations were carried out and of course, there was one more which we will examine towards the end which is biaxial compression.

So, in both the directions there was compression and that led to another failure mechanism. So, it is very interesting for you to observe that in each of the panels; the thick line that you see is the failure plane; is the failure line, it differs based not only on the ratio of $\sigma_1$ to $\sigma_2$ or whether you have $\sigma_1$ or $\sigma_2$ alone. But it also varies depending on the angle at which the load; angle at which these principal stresses are oriented with respect to the bed joint.

So, you can see in the first case, you have some stepped failure, the stepped failure continues. But then by the time it reaches; by the time it reaches 90 degrees for uniaxial tension, you have a flat line; you have the bed joint failing and even in 67.5; you can see that it is the bed joint that is failing. As you look at the other cases, you see this changing depending on the orientation of the plane with respect to the principal stresses.

In compression again you can see, in the first case where the compression is perpendicular to the bed joint that is angle is 0; you have failure due to the formation of tensile cracks which are parallel to the direction of the compression itself. To the last case where compression is parallel to the bed joints and failure is by splitting the bed joints themselves.

So, this framework has been used by several researchers to be able to arrive at failure surfaces for masonry under biaxial state of stress. Page himself gives a set of curves which we will see in a few minutes. And appreciate the fact that it is not something that can be mathematically represented in a simple manner at all ok.

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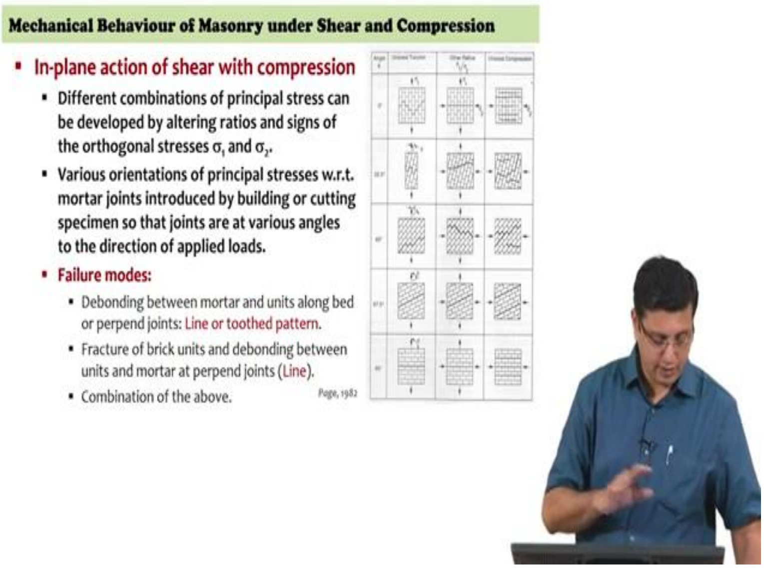


So, different combinations of principal stresses can be developed, that is basically what is been done with altering ratios of $\sigma_1$ and $\sigma_2$ in the orthogonal direction with different signs as well. So, this the you could ask me how were these tests carried out? It is just that big panels were constructed and cut at an orientation. So, you get a panel with a joint which is not perpendicular or parallel to the edges and that is how this is actually been carried out.

It was observed largely; if you were to if you were to club the different failure mechanisms observed; two distinct failure mechanisms and one which is a combination of all these could be observed. You had one failure mechanism where you had debonding between the mortar and the units. So, the interface was giving away you are debonding between the mortar and the units either along the bed joints or along the perpend joints.

So, we have been examining these two cases earlier as well; as a line failure or the toothed failure. This line failure that I am referring to here is not splitting the unit; in this case the line failure is just following the entire, the total bed joint and you get a line. Whereas, the toothed pattern is actually following the bed joint-head joint-bed joint-head joint sequence. The other possibility is that the units are fractured in tension and that is the line failure that we saw earlier.

So, the other possibility is that it is not the debonding between the unit and the mortar, but also it requires the fracture of the unit. So, the line failure that we have been seeing earlier is the other type of failure that can occur and the fracture of the brick unit is required in this case; along with the debonding between the mortar joint and the unit itself. And it is not that only type 1 or type 2 is occurring in isolation; you have situations in these examples on the right where you have a combination of the two as well; so it is as complex as it can get.

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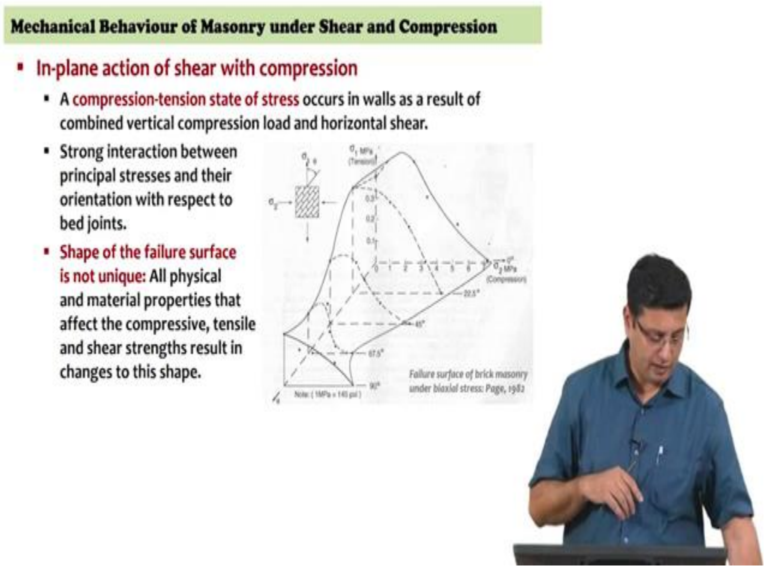


So, because of the combined state of loading that we are looking at; you have a compression-tension state that occurs in the wall. Basically, you can characterize the wall based on this compression-tension state of stress or it is observed that finally it is the interaction between the principal stresses and their orientation. What is the orientation of the principal stresses with respect to the bed joint; this needs to be captured. If that is not captured, the specific failure mechanism that is observed cannot be explained. And that is the failure surface, this is based on the test results of Page.

So, what you can actually see in this three dimensional figure is $\sigma_1$ and $\sigma_2$ as the two axis; the y axis and the x axis, $\sigma_1$ in tension, $\sigma_2$ in compression. And on the third axis- the z axis if you want, is the rotation of the load with respect to the bed joint. And you can actually see that each curve is made basically by the failure stresses $\sigma_1$, $\sigma_2$; failure stresses from the panels that were part of the test.

And you can actually see that for each orientation, the failure plane is different; for the same material. It is exactly the same material in all the tests - the unit and the mortar is kept common across all the specimens. So, this is the kind of failure surface that are getting. So, the shape of the failure surface is not unique and then you will definitely have material properties that will affect the shape of the failure surface. You can have changes to this based on the compressive strength of the unit, the bond strength, the tensile strength of the unit and all other specific material properties will contribute to changing this shape as well; so, the shape is definitely not unique.

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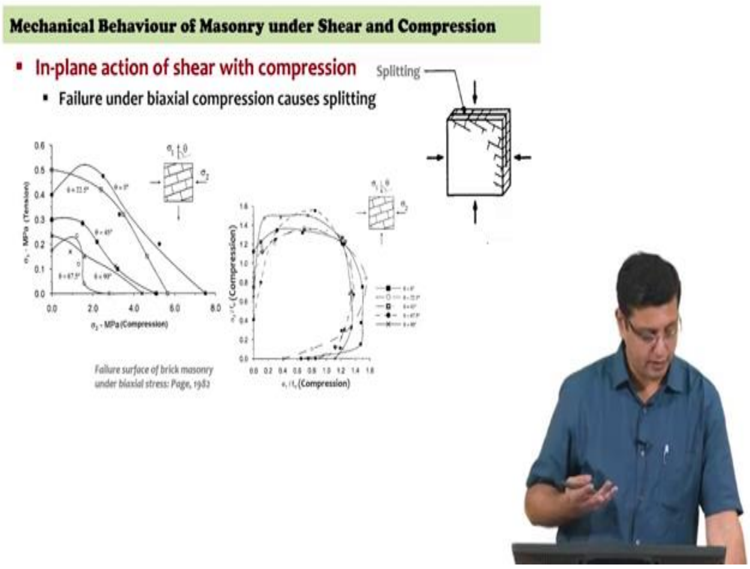


The set of experiments that we examined earlier were uniaxial tension; uniaxial compression and tension-compression as $\sigma_1$ and $\sigma_2$. However, when compression-compression was examined; behavior under compression-compression was examined, because of the confinement from two sides, the tendency of the masonry wall was to split along its thickness. You know, it is very difficult to imagine that masonry can fail in this manner. And it is interesting because you will think in what conditions would you expect this sort of a compression-compression state occurring in a masonry wall panel.

And we have seen this sort of a failure in a regular load bearing masonry wall, we were working with a reinforced masonry wall. And when it was subjected to lateral forces and gravity forces in our laboratory; the end block, the toe block where maximum compression is occurring as the wall is deforming laterally. Because of the confinement provided by the reinforcement and the hollow block at the edge, there was actually compression in two directions. There was compression due to the gravity and compression in the other direction and there was vertical splitting as you see in the end blocks and this is a possibility in given the multiple materials, multiple strengths and stiffnesses that we use in masonry.

So, the tension-compression $\sigma_1$, $\sigma_2$ three dimensional graph that you saw in the previous slide; if you were to collapse it, you can see that none of those failure curves actually overlap or have the same shape. So, that is a difficulty that we will observe and this is a failure surface, a failure plane for the case of compression-compression.

So, of course in compression you have more energy being absorbed before failure, more strength in compression is available in the masonry. So, the one on the right versus the one on the left that is because brittle failure in tension is observed versus; under compression-compresson, there is more plastification in the material and energy that can be dissipated.

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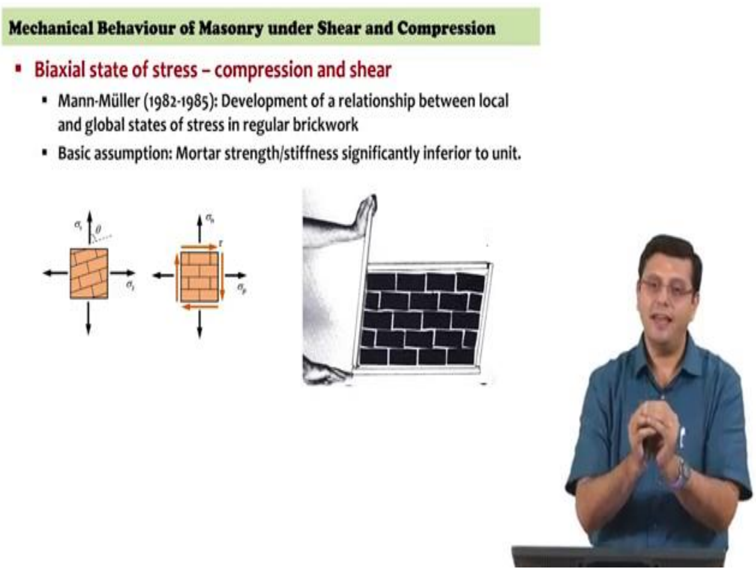


So, this is where we stand in terms of the complexity of the biaxial state of stresses in masonry. So, how do we then bring in an analytical form that can help us as far as analysis and design is concerned? Because, it is fine the complexity is there, but you need a mathematical basis that can help us define failure states under different combinations, under different conditions.

So, what we are going to be examining initially, this is basically at the state of stresses in a cross section defining the failure states in a cross section itself is theory that was developed that Mann Muller in 1982-1985. And here an understanding is developed at a micro level; that is at the mortar joint and the unit what is actually happening. And then that understanding is used to link it up to the global states of stresses or an average state of stress in the wall to define the failure surfaces or the failure criteria itself.

So, you have axial stresses acting on the wall and you have shear stresses that are occurring due to the lateral forces acting on the wall. But if it is possible for us to examine the principal stresses, orientation of those principal stresses and then work at the level of the unit and the mortar and then see if that local state of stress can then in some way be linked to the global state of stress meaning average states of stress, we have already a framework that we can adopt.

So, the basis of this experiment is very interesting to see how the physical behavior under compression and shear; is then being nicely translated to an analytical form idealized into an analytical form. The expressions that you will see now are developed with the understanding that both the strength of the mortar and the stiffness of the mortar is far lesser than the unit itself.

The difference is significant and hence it is ok to assume that the unit is almost a rigid block with respect to the to the joint material itself. So, that has been an important assumption in the work itself. So, the mortar joint if it were to be considered to be analogous to something like rubber; rubber sheet, thin rubber pads. And the brick itself is rigid; no deformation is observed in the brick. When this experiment was carried out, you are subjecting the panel made out of very deformable mortar and rigid blocks.

When subjected to shear, you can see that because of the deformability of the mortar, the unit has a tendency to rotate about its centers; individually rotate about its centers. So, it is an important test, it is an important demonstration because it then gives you a basis to say globally if the wall panel is subjected to lateral forces which is shear; in-plane shear, at the level of the unit, at the micro level of the wall, what is happening to unit is that because of the unit being stronger and rigid in comparison to the mortar; each unit is individually rotating. So, the rotation of the unit then becomes the basis for formulating the stress strain relationships at the unit level. And then you need to have to link it up to average state of stresses in the wall itself and that is why you are linking in up to the global.

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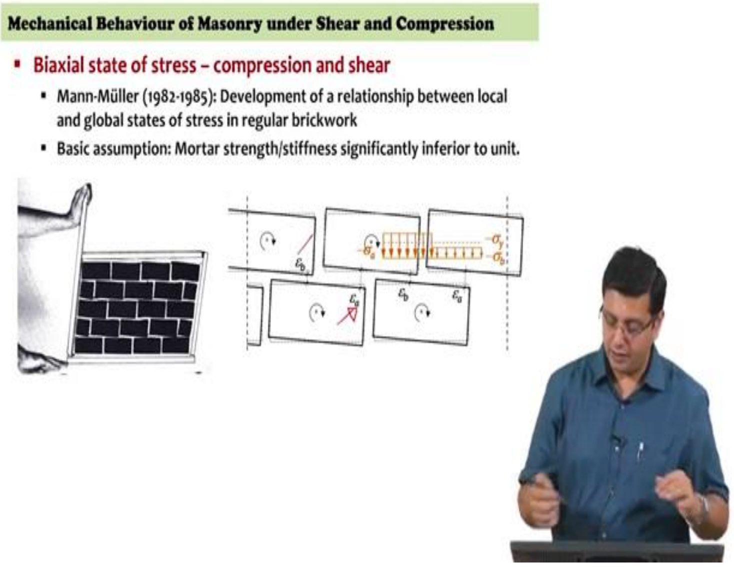


So, this fundamental observation that each unit under the action of the some gravity; there is some axial compression due to gravity forces and superimposed loads. But then in the presence of in-plane shear; each individual unit is rotating right. And because of that rotation and because of the fact that the mortar is the more deformable of the two materials and the weaker of the two materials; in the mortar joint right, we are examining some full bricks in some part bricks.

But then if you look at; if you look at the mortar joint that I am shading now right, if you look at this mortar joint. So, there is a unit at the top and two units at the bottom and if you look at that mortar joint; you will appreciate immediately that almost half of that mortar joint will have increased, the deformation increases there, the joint is subjected to some amount of tension because of the rotation of the unit.

The other half gets compressed right; almost one half of the joint is getting compressed further, whereas, the other half of the joint is getting relieved by the pre; there is a pre compression that exists because of the axial stresses. But the rotation of the unit is causing some of that to be lost; so that becomes the basis for the formulation. So, if you were to examine the joint; there is strain in that joint $\varepsilon_b$; when the mortar joint is getting compressed.

$\varepsilon_b$ is when the mortar joint is getting compressed whereas, in the other case where the mortar joint is opening up; it is more than what it was originally. $\varepsilon_a$ is that quantity of strain in the joint, where you are getting the reduction in the level of compression because of the rotation of the unit itself.

So, that is examining if in terms of the strains the deformation of the strains in a mortar joint. So, this is going to keep happening wherever there are units. If you examine this in terms of the stresses right; if you were to examine this in terms of the stresses; you have pre compression stresses in the wall because of the self-weight. And because of these superimposed loads that average level of pre compression in the wall is $\sigma_y$ (minus for compression); $\sigma_y$ is a uniform level.

But due to the action of lateral forces and the rotation of the unit, half the brick joint sees an increase in the compression and that is $-\sigma_a$. And the other half sees a reduction which is a which is going to be a level lesser than $\sigma_y$. So, this gives us a certain geometry to work with and the local effect of a global mechanism itself right.

So, $-\sigma_a$ is where the joint is getting compressed one half of the brick unit; the other half of the brick unit it is $-\sigma_b$ because the pre compression is being relieved to a certain extent. So, with this understanding of what $-\sigma_y$- σa and $-\sigma_y$ $\sigma_b$ are; we will then try to work on one unit of a certain dimension and see if that can be used to formulate the local stresses and the global stresses ok.

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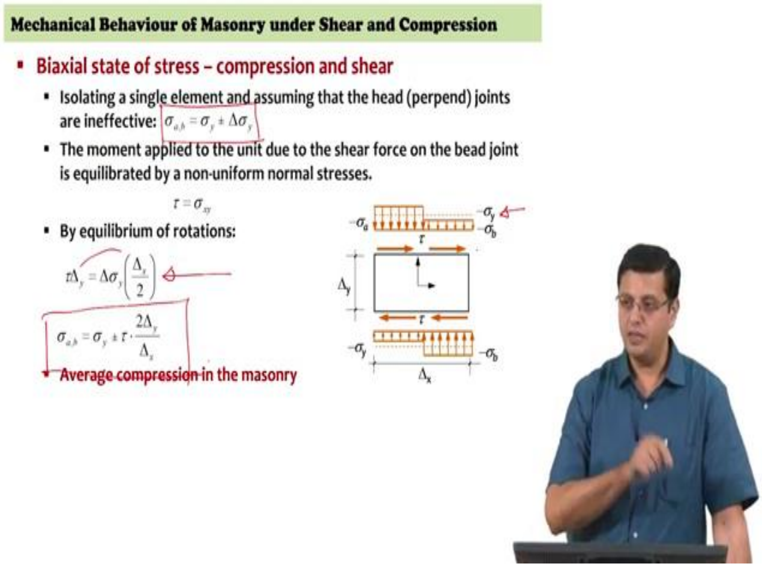


So, now let us reexamine what we talked about in the previous slide. One single unit is being considered the size of the unit is, it is a unit of height $\Delta_y$ and length $\Delta_x$ right. $\Delta_y$ by $\Delta_x$ is the unit itself; y axis along the vertical and x axis along the horizontal and we looked at the different state of compressive stress because of the action of lateral force itself.

So, $-\sigma_y$ is the uniform; average uniform pre compression in the wall, but due to the rotation of the unit, you have stress increasing in one part minus sigma a at the bottom, the other side is what is going to be compressed further. So, $-\sigma_a$; at the top and minus increased compression at the bottom and $-\sigma_b$ is where the level of compression is reduced ok.

So, we have isolated one single element and if you examine the stresses and the element, we have actually eliminated anything that is happening on the sides of the unit right. So, that is an important assumption because we are assuming that the perpend joints are inactive, we are assuming that the perpend joints are inactive.

We have talked about this earlier that the perpend joints are typically not as well consolidated as the bed joints and so expecting that to be a participating uniformly participating resisting mechanism is questionable. And so it is an acceptable assumption to assume that the head joints or the perpend joints are ineffective and that is why we are not attributing any of the resistance to those two sides. So, that is the other assumption that goes into the problem.

So, now $\sigma_a$ and $\sigma_b$; the maximum compressive stress that you will reach and the minimum compressive stress that you will reach is over and above the average compressive stress due to the pre compression; $\sigma_y \pm \Delta\sigma_y$ which is going to be the difference in the compression itself. So, within this block, if we were to take moment equilibrium and we have examined the stresses now; the normal stresses that are acting on the unit because of the deformation in the wall.

But when this wall deforms, the bed joints are now going to be subjected to a state of shear right. So, $\tau$ at the top bed joint and $\tau$ at the bottom bed joint is what is going to be equilibrating the changed state of normal stresses. So, the moment equilibrium; if we apply the moment equilibrium to this unit of size $\Delta_y$ by $\Delta_x$ and shear stresses generated on the top bed joint and bottom bed joint. Because of this changed state of normal stresses; sigma a and sigma b being equal to $\sigma_y \pm \Delta\sigma_y$; we then have a basis to bring in the shear stress in the wall.

So, we want to get a relationship between the shear stress and the normal stress and the principal and the resultant average stresses itself. So, our $\sigma_{xy}$ is the; for this block that we are assuming; $\sigma_{xy}$ is the shear stress $\tau$ and considering equilibrium of rotations, it is possible to relate the geometry to the shear stress and the normal stresses.

So $\tau$ into $\Delta_y$ by the equilibrium of rotations is equal to the difference in the normal stresses, $\Delta\sigma_y$ multiplied by one half of the block over which $\Delta\sigma_y$ is actually acting; so that is the equilibrium of rotations. Once you write down that, you can use you can write down an expression for $\tau$. If I take delta y to the other side, then I have $\tau =$ is equal to $\Delta\sigma_y$ into $\Delta_x$ by $\Delta_y$ by 2 there. We are assuming that the difference is $\Delta\sigma_y$ which is; which is equal, what is increases; there is some conservation within that that is assumed.

Now, the $\Delta\sigma_y$ is then equilibrated by the shear and that is how we are establishing or relationship between the two. So, taking this expression to our $\Delta\sigma_y$ in the first equation here where we have what is $\sigma_a$ and $\sigma_b$ as $\sigma_y \pm \Delta\sigma_y$, we plug in delta sigma y into this expression from here.

And you get the final expression, the average compressive stress under the action of both gravity forces and lateral forces as being the original peak pre compression in the wall $\sigma_y \pm \tau \cdot 2\left(\frac{\Delta_y}{\Delta_x}\right)$. So, this is the average compression in the masonry that we can define. Now, this basic form is then going to be used to understand how the level of average compressive stress becomes a determinant in the failure mechanism itself ok.

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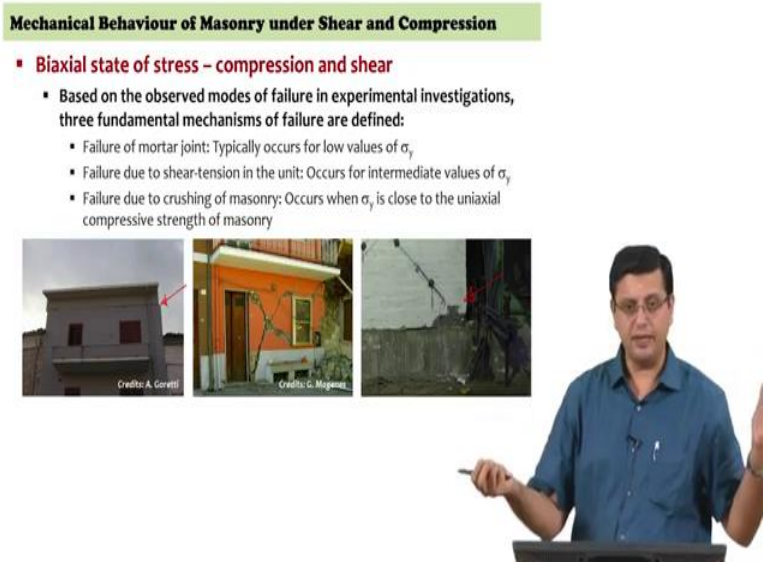


So, you have seen the set of experimental investigations based on those investigations; we can broadly classify three different mechanisms of failure; three different modes of failure. The first mode of failure is particularly when the pre compression levels are very low right.

When pre compression levels are very low the resistance is going to depend primarily on the shear strength of the joint itself right. Pre compression levels are very low, let us say it is a single storeyed structure or let us say it is a wall, a shear wall on the topmost floor of a masonry building. Then your values of pre compression sigma y are typically low; in this case the failure is controlled by the failure of the mortar joint ok. And we will actually look at this more closely, when we start examining the global failure, we are now trying to define it at a local state of stress, but we will come back to this.

But what you see in this particular picture here is failure of the joint; can you see how there is a failure plane that is initiated at the top of the wall? Now, at the top of the wall you will agree that the pre compression levels are going to be minimum. The pre compression levels are minimum, this wall panel has been subjected to sliding failure and the failure is about the bed joint itself.

So, in this particular case; classical case of low pre compression, shear failure is occurring when the joint strength is reached the bed joint strength is reached. So, when you have failure of the mortar joint; that is one case where you can examine and get a sort of a closed form expression to understand the value at which failure is expected.

The second case is when the pre compression levels are not too small; the pre compression levels are moderate levels of pre compression. So, you take a regular masonry structure 2 storeyed, 3 storeyed masonry structure pre compression levels are not small. Here the failure is governed by shear-tension right. So, here it is a case where the shear stresses in comparison to the pre compression; the normal stresses are significant enough.

And so you will have to start looking at the principal stresses and when the principal tension approaches the tensile strength of masonry; you will get a failure plane, you will get a failure or you will get of crack in the wall itself. So, this is a picture which we had seen earlier; we were talking about the classical x crack. And here what is actually happening is the pre compression levels are significant; it is a two storeyed structure at least and you can see that the failure plane is on the ground floor.

So, moderate levels of pre compression is expected, but under the action of lateral forces and this pre compression; you have the formation of these diagonal cracks. And this is happening when the principal tension actually approaches the tensile strength of the masonry. We are examining the behavior now at the level of the unit and mortar and so the second failure is when the shear tension in the unit is acting occurring; the shear tension in the unit is occurring right.

The third case is when the pre compression levels are not small, you have got significant pre compression levels. When you have heavy pre compression level due to this deformation of the unit, you have increase in the you have increase in the compression level. We will be looking at $\Delta\sigma_y$, so you will have an one edge where $\Delta\sigma_y$ can approach the maximum compression can approach the crushing strength of masonry.

If the compression level were to reproach the crushing strength of masonry under the action of lateral forces and gravity, you will have flexural compression failure right. So, three very distinct failure mechanisms; the first one is due to failure of the joint sliding shear. The second is due to shear tension cracking in the unit and the third is due to compression failure. As you can see, this is a test from our laboratory, you can see how the edge that is getting compressed due to the lateral force acting that compressed tau is crushing.

And if that were to crush, then you have reached the maximum lateral load carrying capacity of the wall. So, you have three distinct failure planes; three distinct failure mechanisms in the wall itself, we examine one by one to then arrive at an overall failure plane.

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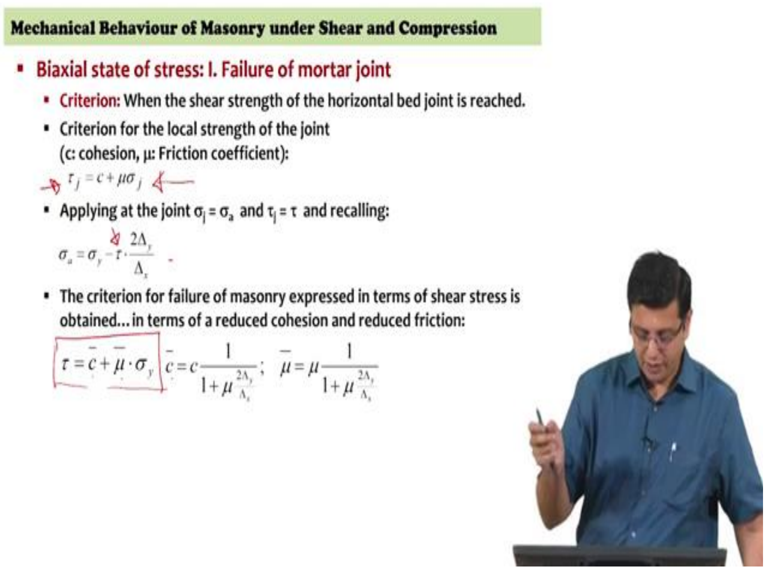


So, let us look at the first one; failure of the mortar joint. So, what we are saying is when the shear strength of the horizontal bed joint is reached, failure occurs in this units mortar assembly that we are examining at this state and defining the local states of stresses.

So, it is the shear strength of the bed joint that becomes essential here. If you remember when we were talking about the shear strength; there are tests to characterize the shear strength of the joint right. It is not the diagonal compression test, the diagonal compression test is going to give you the shear strength of masonry and is also referred to as a tensile strength test. Because the failure is occurring under principal tension; this is actually what we are talking about in this case is the joint failure the joint shear failure.

So, remember the concepts of cohesion and friction coefficient and this, if you are able to characterize what is the strength of the joint then that value is reached you get failure. So, earlier we were looking at $\sigma_a$, $\sigma_b$ as being delta as being $\sigma_y \pm \Delta\sigma_y$. So, we are here looking at the lower of the two stresses because the pre compression levels are lower right. So, to be able to use this failure criterion where we define the strength of the masonry using a criterion that we have examined earlier; the Mohr Coulomb criterion, where the joint shear strength is equal to cohesion plus the friction coefficient into the pre compression level in the joint; the normal stress in the joint $\sigma_j$.

So we use this form as the; as the limiting value for arriving at the failure surface. We apply this at the joint, our assumption, we have been looking at the states of stresses in the joint only. So, we assume that the $\sigma_a$ that we are talking about is really the joint compression levels and $\tau_j$ is the shear stress in the joint itself.

We make that assumption and we now assume that now use the previous expression that we had $\sigma_a$ is equal to $\sigma_y$, original level of average pre compression, minus; the other term that we had brought in for the difference in the difference in the compression level as $\tau \cdot 2\left(\frac{\Delta_y}{\Delta_x}\right)$ So, this expression for $\tau_j$ that we have earlier, the Mohr Coulomb criterion, we bring that into this expression, the second expression here and have the criterion for failure when the joint fails. So, basically we are looking at a sort of a reduced friction coefficient and a reduced cohesion that occurs in that local state itself, where we we are just introducing this part into the expression. We are bringing in our $c + \mu\sigma_j$ into this expression for $\tau$ here and rewrite it in terms of a reduced cohesion and a reduced friction coefficient where $\overline{c}$ and $\overline{\mu}$ are coming from the second expression on the page itself.

So, this is how in the first criterion we have the average state of stress; low pre compression level, failure coming from failure of the joint we bring the Mohr Coulomb criterion and then have an expression for $\sigma_a$. And in this particular case, the second expression is helping us arrive at a relationship between the geometry of the unit; the normal stresses, average normal stresses and the shear stress.

And we use that to get the failure criterion in terms of the shear stress at which failure is expected represented as a sort of a reduced cohesion and a reduced friction coefficient. Because we are looking at the states of stress at the middle of a joint and then we are going to the edge of a joint. So, this is the first failure criterion, we will examine the next two failure criterions and then have an understanding of for the entire range of levels of pre compression; how does this failure surface look like.