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Module - 03 Lecture - 17 Strength and Behaviour of Masonry Part – VII

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Good afternoon. We will continue with the behavior under out-of-plane bending. We looked at formulations that were developed based on a linear elastic behavior of the cross section and a non-linear behavior of the cross section. And we also looked at how the arching mechanism can be beneficially used for the out of plane load bearing capacity of a masonry wall.

Now, we again within that, assumed that the rigid arching mechanism is possible, which is there is no gap between the wall and supports, so arching mechanism is available right from the beginning. But if that is not the case and there is a gap between the lateral supports or between the top support and the wall, then arching mechanism occurs with a certain delay and that leads to a loss in the out of plane flexural capacity. So, this is what we have examined so far.

So, to sum up horizontal bending and vertical bending one way and then moving on to two way bending, we will examine in the next set of course content. So, as far as one way horizontal flexure, we have been examining expressions under one way horizontal or vertical flexure, but the question is would you get purely horizontal flexure. It is an idealization, right. You will always have some effect of the bottom restraint at least if not if not both the bottom and the top restraint. So, assuming that you are going to get a condition of pure horizontal flexure is an idealization. And we saw that when we first looked at the wall as a propped cantilever and then looked at the wall as being hingedhinged. So, very often this situation is going to be restraint available at the bottom, right.

So, if you are using a purely horizontal flexural formulation it is an idealization and it could be conservatively so, just keep that in your mind. We did look at this figure when we examined the material behavior and the material strengths that we have to consider, particularly with horizontal bending or with one way vertical bending; the factors that material factors that start playing a role, so the head-joint flexure and the bed-joint torsion in a wall start becoming important as far as one way bending is concerned and typically to understand how failure will propagate it is the relative strengths that finally matter.

So, we will examine how a total progression of the failure happens in the systems. So, the tensile strength of the bond, the head-joint itself is typically the one which is the weakest link in a masonry wall, primarily because there is no consolidation of the head-joint, otherwise referred to as the perpend joints.

So, this is often neglected in our calculations or if you consider this will be lesser than most of the other tensile strengths or most of the other strengths in the masonry wall. So, the tensile bond strength as far as the head-joint is concerned is lower than the flexural tensile strength of the unit, right. So, if the course is subject to bending then the headjoint will should give up first even before the unit fails in tension.

So, this is something that we are agreeable upon. Plus, if you were to consider the two joints now the head-joint and the bed-joint, again the bed joint is far superior in terms its bond strength and that is of because of the consolidation under precompression under the dead weight of the wall itself and the head-joint will continue to be the weakest as far the strengths of all the constituents are concerned and the interfaces are concerned in the masonry wall itself, ok. And we have seen that we use an elastic theory to estimate the capacity. So, knowing the in this particular case what becomes important is the bed-joint torsion which means the bed-joint is subjected to shear stresses and since the bed-joint is subjected to shear stresses, the tension in the joint is acting parallel to the joint and therefore, the tensile strength of the joint with tension parallel to joint or  $f_{tp}$  becomes a parameter that is important for us. With an estimate of the flexural tensile strength parallel to the bed-joint available which is from a modified bond wrench test or a wall test we will be able to estimate what is the minimum capacity that is available in the wall before failure propagates, ok.

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So, if I were to examine what is the load deflection behavior in such a wall, when do you get critical points occurring and can you then make an estimate of the capacity at that point. So, initially you have the head-joints, you have the whole wall that is now bending. The head-joints and the units are subjected to bending, right. Cracks will start propagating at the head-joints. So, the head-joints give away first.

Now, since there is a vertical stagger of the joint, the head-joint should start opening in alternate courses, that is something we would expect unless a vertical stagger is not given, which is not the case because all bonds will ensure that you get a vertical stagger. So, the head-joints will start cracking under flexure in the alternate joints. Now, once the head-joints have given away, in the alternate courses it is the unit now which is taking

the flexure, right. And so moment will be resisted now by the units which will start acting under bending. But the point is you have now completely lost the role played by the head-joints, but the unit to transfer the moment from one layer to the other will now depend on the bed-joint itself, right; and that is when the torsion in the bed-joints starts becoming important or the strength  $f_{tp}$  that we were talking about starts becoming important.

So, as you start loading everything elastic, the whole system is responding but then you get cracking as the first phenomenon in the head-joints and this will occur in the alternate courses.

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As the applied lateral force increases, you will start seeing that after the head-joints have cracked the wall has to have a mechanism of resistance and that is where the moment is now been carried by the units, the units start bending, but layer to layer it is the torsion in the bed-joint which is going to be transferring and having load resistance.

We will examine what then is going to govern the capacity of the wall itself once this once this softening in the load deflection behavior occurs, at after the branch ab and when you are on the branch that goes b towards failure what governs is something interesting to look at. So, what sort of failure modes can occur? Let us say the headjoints have cracked and now the units and the bed-joint are participating in the load resistance. Can you have different failure mechanisms? If you have different failure mechanisms what governs that failure mechanism is important to examine, ok.

Now, if the torsional resistance, if the torsional resistance at the bed-joint is higher than the strength of the unit in tension, you now having a situation where the unit is going into tension and the bed-joint is going into torsion. If the bed-joint gives way then the failure will occur along the joint, but in the process of the unit gives way which is the tensile strength of the unit is weak in tension, then the torsional strength being higher in the joint resists, but the unit gives way, right.

So, the pattern will depend on whether the unit tensile strength is higher or the bed-joint torsional strength or the bed-joint strength which we have characterized as  $f_{tp}$  is higher. So, you will get what is called a toothed failure if the joint is weak, right. If bonding between the unit and the mortar is not good then the unit tensile strength will be higher, you will get the crack follow a pattern which is a toothed pattern between the alternate courses.

So, this is referred to as toothed failure. So, on this branch, after the head-joints are cracked and when we are on the branch b towards failure, the next phenomenon will be and that is the end of the capacity of the wall is when the joint gives way and the horizontal propagation of the crack occurs and comes to the next layer, right. So, toothed failure is expected to occur in this manner.

But when you have a situation where the bed-joint torsional strength is lower than the unit tensile strength, then if you have precompression, if the wall has heavy superimposed loads that will act beneficially in resisting the out of plane forces of the lateral forces, right. You understand. That, what is critical now is the horizontal bed-joint failing, but that can be prevented to an extent if you have heavy precompression.

So, this sort of a failure mechanism to the toothed failure mechanism is very sensitive to how much of precompression is available and therefore, you see that the branch b d now, the branch that we have we are we are talking about we are on the around the second branch, it could fail at a certain point or continue to resist depending on the precompression level that is available in the wall. So, if it is a top floor wall which has low superimposed loads versus a wall which is on the ground floor of a multi storied building, you will have additional resistance coming because of the precompression. So, this is how you would expect the formation of the final crack which is the horizontal crack and continuation of the head-joint failure which already occurred in the previous stage.

So, once this phenomenon that we are talking about is either occurring at d or occurring at c, where the bed-joint has cracked. So, once we are at point c or d beyond that there is going to be some residual capacity in the wall, that residual capacity in the wall is purely coming from the bed-joint friction.

If you remember when we were talking about the shear strength of a joint, when we talked about the shear strength of the joint you had two components, one component was cohesion and the other component was the joint friction, the frictional resistance that is available. Cohesion is nothing, but the bond. So, once the bond is overcome you have the formation of a crack. But still you have resistance because there is some precompression always, could be low or it could be high and therefore, the resistance after the bond has failed is coming from friction.

So, how is the post peak behavior beyond c to e or beyond d to f will really depend on how much precompression is there and therefore, it will affect; it will affect the frictional resistance. If you have good frictional strength good frictional friction coefficient and high precompression, you could get a rather stiff behavior in the post peak region.

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So, this is something that is going to be affected by the level of precompression due to two effects. One in the pre peak just before the peak depending on how the bed-joints failure can be delayed one or after failure how much of peak precompression is available and how much is the frictional resistance and the slope in the post peak is going to be affected by how much of frictional resistance is available in the set up.

So, if you where to now examine the other situation. We talked about the torsional strength of the bed-joint being lower than the unit tensile strength, right. That was the first case and we ended up with a toothed failure. However, if you have the reverse of the situation that the unit tensile strength is really poor and the bed-joint strength in torsion is fairly good, or  $f_{tp}$  is good then you will have what is called a line failure where the unit is going to be split, right.

So, in this particular case, if you are talking of weak units and as I have been reiterating, if you know the compressive strength of the unit all other material properties of the masonry can be linked to it. So, if the compressive strength of the unit is significantly low, is poor the unit tensile strength is also going to be low.

So, let us say you have weak units. If you have weak units it would typically follow a force displacement relationship, a force displacement trend a b and then sudden failure, because after b the unit is the one that is going to be resisting the flexure, but it fails even before the joint can give away. So, you would have the first possibility denoted by this

certain sudden drop, sudden vertical loss of strength that you see if you have very weak units.

However, if the units are not so poor and there is some finite strength in tension in the unit which the wall can actually depend on, then as you are on branch b c the unit is resisting moment the bed-joint torsion is active now. But before the bed-joint can fail in torsion the unit splits in tension and you will get and this is much more brittle failure where there is a sudden loss of capacity, almost complete loss of capacity of the system because you do not have any more plane that is counteracting a gravity forces.

In the previous case, because of the toothed failure the horizontal joints at the interface of failure were still active and carrying the gravity force, right. And so, you had some you had some residual resistance there. You had some residual resistance that is always going to be available till loss of contact happens. But in this case since the failure is a line failure, there is no capacity left in the system itself. So, beyond point c or beyond point b what you see is a vertical drop in the capacity and the line failure completes the mechanism itself.

And in this case whether the wall is provided with heavy superimposed loads or low superimposed loads, low precompression or high precompression it really does not matter, because you are not tapping into the bed-joint resistances the unit is so weak that in gives way and then the level of precompression in the bed-joint to augment friction is not available.

So, this is the fundamental difference between the two failure modes that one can capture as far as horizontal flexure is concerned. So, you have to be careful about two aspects the unit tensile strength and the  $f_{tp}$  or the bed-joint torsional strength that is available in the material in the composite itself, ok.

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And we have made capacity estimates, but we have not in the calculations assumed a difference at the section at the micro level, we have not really in our linear elastic assumption or the non-linear stress strain assumption made a distinction between unit and the joint.

We did not do that. If you remember, we used a homogenized cross section and we said that this stress strain curve of the cross section can be linear elastic in compression till failure or non-linear at failure at ultimate. So, we actually used a homogenized section a homogenized material model to make those estimates. But if you were to actually use some strengths to make this sort of an estimate between line failure, and a toothed failure, codes internationally give you some possibility of making an estimate of the capacity of walls.

So, this relationship that I have talked about in one of our previous classes is the tensile strength, the flexural tensile strength normal to the bed-joint to the flexural tensile strength parallel to the bed-joint,  $f_{tn}$  and  $f_{tp}$ , normal and parallel; and the bond wrench test, the modified bond wrench test as the as the test that you can use to estimate these. If this is known, if you have an estimate of these values use the lower of the two values to make the estimate of the capacities and mind you, variability is something that you cannot do away with.

So, if you have toothed failure and if the flexural tensile strength of the bed-joint, normal to the bed-joint is greater than about 0.15 MPa which is which is giving you a threshold and telling you look if it is a very low flexural tensile strength, normal to the bed-joint you cannot be using this expression.

So, if it is over and above about 0.15 MPa the flexural tensile strength normal to the bedjoint you can make an estimate of what is the flexural tensile strength parallel to the bedjoint with the knowledge of the flexural tensile strength normal to the bend joint, right.

This ratio between the flexural tensile strength parallel to the bed-joint and normal to the bed-joint is observed constantly and depending on the material properties, depending on the bond that can be established and the type of units you are using this ratio, can vary. But rather constant ratio is observed between  $f_{tn}$  and  $f_{tp}$ .

So,  $f_{tp}$  or the test to get  $f_{tp}$  being more complicated, the modified bond strength test because you have to use those half bricks or the wallete test we have to create significantly large specimen, the flexural tensile strength parallel to the bed-joint test may not be very easy to carry out in the laboratory. It may be easier for you to do a bond strength test and get  $f_{tn}$ , right.

So, if you know  $f_{tn}$  and if the  $f_{tn}$  works out to be greater than the number 0.15 MPa you can make an empirical estimate of how much the value of  $f_{tp}$  is, right; and of course, in that expression you also see  $f_a$ , which is going to the unit compressive strength the unit compressive strength is again made use of.

So, this is an empirical estimate. It is not an analytical, purely analytical form. It is an empirical estimate this expression fits the behavior. You could use that value of  $f_{tp}$  and estimate what is the one way flexural, one way horizontal bending capacity of the wall if toothed failure is happening because you need the bed-joint torsional strength. But if line failure were to happen you see that the line failure is just using capacities which is the unit tensile strength and the bed-joint tensile strength, right.

So, in case you have doubts on how strong the joint is going to be then you should expect a line failure to occur and you see that see in the estimate of the capacity with respect to line failure we use a rather, again an empirical relationship that is based on the unit tensile strength and the flexural tensile strength of the bed-joint, normal to the bed-joint. So, you can you can arrive at a value for  $f_{tp}$ , use that value of  $f_{tp}$  to make an estimate of the out-of-plane bending capacity of the wall itself. This again varies from code to code and this is calibrated on some experimental tests. So, we use them with. If we have a good knowledge of the type of masonry that we are working with, you should use them. You should not use them blindly the across different types of masonry, ok.

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That brings me to two-way flexure, right and that is something so far we have not examined and it is a complex mechanism that actually works in a two-way flexure, and strictly speaking this is what should be happening in most cases that there are lateral restraints, there are top and bottom restraints. So, most probably the wall is being restrained on 3 sides at least or all 4 sides, right.

So, it is an idealization that we said we are getting one way flexure in vertical bending or horizontal bending. So, most walls are going to be supported on 3 sides or 4 sides and we have seen that in this situation the wall under diagonal bending is going to be subjected to bed-joint torsion, head-joint torsion, bed-joint flexure and head-joint flexure. So, this is the kind of situation that we will be experiencing in a wall that is actually undergoing a proper diagonal bending itself. So, it is really a combination of horizontal and vertical bending.

So, the phenomenon is quite complex as I said and it is also be seen that if you were to take the horizontal bending capacity and the vertical bending capacity superpose them the diagonal bending capacity is not close to a mere addition of these two capacities, which means it is not something that is that easy to estimate; needs a lot of material parameters.

And hence, codes internationally have attempted to look at empirical methods of estimate, except for very few codes that actually give you rigorous analytical forms. Most codes actually give you an empirical approach where they say let us split the wall into two equivalent strips, a horizontal strip and a vertical strip based on the boundary conditions and estimate the capacity of the vertical strip in vertical bending and the horizontal strip in horizontal bending and add these two equivalent capacities and arrive at the overall bending capacity.

So, there are different ways in which this has been addressed in the past and the fundamental reason is there is indeterminacy in the in the problem and that closed form solutions are challenging to arrive at in this particular case.

We have talked about this earlier that the bed-joint tensile strength parallel to the bedjoint is typically higher than the bed-joint tensile strength normal to the bed-joint, right. So, tension causing bond failure when tension is acting parallel to the joint is something that is going to give you a higher strength than bed-joint tensile strength normal to the bed-joint and that is also because of that directions of strain gradient that you have in one versus the other.

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So, if you were to examine two-way flexure, this is a mechanism that is affected significantly by boundary conditions. But the type of boundary conditions that you have, the number of boundary conditions that you have and the aspect ratio of the wall, right. Depending on the aspect ratio of the wall the relative capacities in normal to the bed-joint, tensile strength normal to the bed-joint and parallel to the bed-joint start playing with the aspect ratio of the wall itself. So, let us examine first a wall supported on 3 sides, understand possible failure mechanism and then look at a wall supported on 4 sides.

So, wind pressure or inertial force acting on the wall and you have supports on the two lateral edges and the bottom of the wall. As the loading progresses the wall is acting like a plate now, elastically deforming and then when the crack, when the tensile stress due to the combination of the gravity load and the out of plane load reaches the tensile strength of masonry you should see the formation of the crack and typically, when you have a wall that is longer than it is tall, you would have a crack pattern that runs from somewhere at about 40 percent length of the wall at the top to the corner itself, right

So, the wall is trying to bend outwards, it is supported on 3 sides and it is trying to open out in a way forming these two. These cracks are not 45 degree cracks, these cracks would be about depending on the boundary condition would vary between 30 and 45 degrees, depending on the aspect ratio would vary and in this particular wall where the length is significantly larger than the height of the wall. This is how you would expect the failure pattern to look like.

Even if you where to look at if you were to look at a wall which has height to length ratio which is one or the height is more than the length and again supported on 3 sides the crack pattern would look like what you can see in this figure below, where again you have those almost diagonal cracks that begin. But then the mid span deflection is quite significant the top is free. Since the mid span deflection is significant you get a vertical crack, but then that vertical crack cannot propagate completely and split the wall to get the diagonal crack coming from the corners.

So, if you are actually look at it in the two figures the crack pattern above is actually a magnification of the crack pattern that you see below, because of the changing aspect ratio of the wall. So, what is really happening here is a wall is acting like plate now,

right; and the failure pattern very much matches with the kind of yield line failure that you get in a reinforced concrete slabs.

So, big difference being that in one you are talking of a reinforced system, here you are talking of a brittle system; you do not have the reinforcement. Therefore, one way of dealing with one of way of dealing with diagonal bending has been to use the yield line analysis which is actually being developed for reinforced concrete slabs to estimate a capacity for out of plane walls subjected to diagonal bending or subjected to two way bending. So, some codes actually adopt a yield line criterion to estimate the capacity of walls in two-way flexure.

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Again, it is instructive to examine, ok. Then if we were to examine the wall supported on 4 sides, it is again a mirror of what you saw for the wall with 3 sides. Now, the diagonal crack will propagate from all four corners and not from the bottom corners alone because the top is also supported.

So, let us examine a case, one case because as I said depending on the boundary condition if the boundaries are fixed if the boundaries are free to rotate if the aspect ratio is of a certain range such that the length of the wall is lesser than the height of the wall versus an aspect ratio where the length of the wall is significant, you can have differences. So, in the slides that you are seeing we are actually examining the behavior of a wall whose length is significant in comparison to the height probably almost twice.

So, as the wall starts deflecting elastically with increasing lateral force we are on the first branch of the load deflection curve, elastic behavior and all mechanism that we talked about earlier in terms of head-joint flexure, bed-joint flexure, head-joint torsion, bedjoint torsion are all being requested by the wall to actually resist the out of plane load itself, ok.

When the bed-joint capacity is reached, ok; now, we will mind you are examining of wall whose length is significant in comparison to the height, so it starts getting around the mid span around the mid span, not closer to the edges, around the mid span the bed-joint tensile strength normal to the bed-joint is reached, right. So, you have a situation for formation of a horizontal crack around mid span.

Now, once, so the once the flexural tensile stresses reach  $f_{tn}$  you should start getting the first crack which is a horizontal crack in the wall itself. So, that is your first, that is the first non-linearity that you should expect and then change in stiffness. So, you can see how we now come down either to branch b c or continue further depending on depending on the different capacities in different values of f tn and  $f_{tp}$  and precompression itself.

Now, beyond this point beyond b how is the propagation of failure going to be depends completely on the orthogonal strength ratio that we talked about, right. And as I said the orthogonal strength ratios  $f_{tp}$  over  $f_{tn}$ ; in some codes it is given the other way around, so you just have to be careful what is on the numerator what is on the denominator, but the point is this will be uniform for a given typology of masonry. So, depending on this relative capacity of  $f_{tn}$ ,  $f_{tp}$  to  $f_{tn}$ , we will see beyond this point b how does the force displacement curve go.

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So, let us examine that. In case  $f_{tn}$  is equal to  $f_{tp}$  which is rather rare, that you do not have any reserve capacity. Most often  $f_{tp}$  will be higher than  $f_{tn}$ . So, in this case, it simply means that the moment the bed-joint has cracked. It has to, the wall has to redistribute the demand that is coming to the other location. So, once the bed-joint has cracked it will expect that moment to be resisted more by the head-joints. But if the head-joints are going to crack at same level of flexural tension then you get a sudden propagation of the failure mechanism through the joints, right.

So, you we were initially along this and then we looked at coming down where do we come down to see and look at full failure mechanism getting propagated in the wall or depending on the orthogonal strength ratio is there is reserve capacity in the wall. So, let us say if there is situation of  $f_{tn}$  being equal to  $f_{tp}$ , meaning there is no reserve strength available in the wall per se, you get a sudden mechanism formation.

And what you observe now is that the horizontal crack has fully developed, it is now expecting the other mechanisms to start carrying the load, but you do not have reserve strength provided by the flexural tensile strength parallel to bed-joint that gives away and you get the diagonal crack that propagates to the four corners and again as you see this is very similar to the yield line mechanism that you would get in a simply supported reinforced concrete slab; very similar to it. However, here in this case you will have one crack or one low distributed cracking is to be expected in this particular case.

So, what you see here if you follow the branch a to b and then a sudden drop from b to c, and c to d and you have a residual, you can see that there is a certain residual capacity. Again this residual capacity where is it coming from? That simply coming because of the diagonal crack at the joints means that the hinging and the movement is happening on the bed-joint.

So, you have a horizontal surface which is actually carrying the load and this is depending this depends on the precompression level. So, you will always get a certain residual capacity because of friction. So, that lower branch is basically going to be available to you, but for all practical purposes the capacity is lost in the wall. The mechanism has propagated.

Now, if  $f_{tp}$  is greater than  $f_{tn}$  and the question is by how much because it depends on what this orthogonal strength ratio  $f_{tp}$  by  $f_{tn}$  is. Then, you will get a stabilization in the load carrying mechanism. The horizontal crack has propagated, but before the propagation of the diagonal cracks you will still have additional resistance in the wall you will go beyond b in terms of the load carrying capacity and therefore, wall stabilizes, it is able to resist till the formation of the mechanism when f tp is reached and at this stage you really will have to look at what the ratio  $f_{tp}$  to  $f_{tn}$  is.

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So, this propagates; basically you will have the sub panels that are formed because of the cracking that is happening and there is residual capacity till failure. So, let us just look at what then happens.

We did look at the overall failure mechanism, but if I were to increase the ratio that f tp is say twice the f tn or 2 and half, 3 times and so on you will basically go from a to b, from b stabilize to e and depending on the orthogonal strength ratio either keep going up this for further capacity and then once the bed-joints are, once the bed-joints are failed because of the tensile stress reaching  $f_{tp}$  you get the diagonal cracks initiating and you get failure. And so you would either go a b e f g or a b e f h and i that depends on the orthogonal strength ratio. So, this is, this is the expected propagation of failure.

Again, mind you the bed-joint is cracking,  $f_{tn}$  occurs on a joint, whereas, the diagonal crack can be a line crack or can be a tooth crack. So, again as far as the propagation of failure mechanism, the last branch capacity of the unit in tension versus the torsional strength of the joint matters, right.

So, this is the actual mechanism of failure propagation in the two-way flexure case and as I said few codes actually have a closed form analytical solution to make an estimate of this, but the demand is that you have an estimate of the tensile strength, the unit tensile strength, the joint strengths and is heavy on required mechanical parameters.

However, you will examine some of these methods in the assignment that is based on this segment of the module 3; with that we close out-of-plane bending.