

The second approach was again bending analysis, conventional bending analysis, but the section now is non-linear. So, those were the two approaches that we looked at in the last

lecture and therefore, to sum up if you were to look at conventional bending analysis with linear elastic state of stresses till failure.

Then for the case that we considered where we had the base fixed and the top supported, lateral restraint coming from the supports, if we were to look at the progression of load resistance till failure, then you see a force displacement; applied lateral load to lateral deflection at mid height δ to be of this sort. So, let us see the critical points on this graph, which has been the critical points that we examined at different stages of loading and formation of inelasticity.

So, stage a, where we are considering only the presence of gravity forces, uniform compression at the base; the base is fixed and you have the lateral restraint at the top support. As the lateral forces act on the system, you have tensile stresses initially, these tensile stresses that develop because of bending are nullified by the compression. As the stresses due to the out of plane load (distributed load acting on the wall) cause bending in the wall, cause tension in the cross section, as long as the wind pressure or the lateral forces is low, they get nullified by the compressive stresses, but then you take it to the limiting case.

Limiting case where there is still no tensile stress, entire cross section is in compression, but that gives you limiting case as to what will be the wind pressure corresponding to entire cross section still being in compression. And as the lateral force continues to increase, we see the initiation of the first crack right and that is the first in elasticity that occurs in the system under this assumption and you should therefore, as the load resistance continues beyond that get a drop in stiffness, reduction in stiffness of the system.

So, stage c of our loading was when the first crack at the base is initiated and starts propagating as the lateral force acting on the wall, applied force acting on the wall increases. So, at this stage we have to start considering the fact that the system stops working as a propped cantilever and starts working more as a pinned-pinned boundary condition in the wall. And therefore, the maximum moment will now migrate to the mid height of the wall and as loading continues you get the crack initiation at the mid height and that is a stage that corresponds to maximum load carrying capacity of the wall itself. Because once the crack initiates at mid height it is basically only going to resist with

larger deformations, but additional capacity is not available at that stage, reserve capacity is not available at that stage.

Mind you in this assumption we are still continuing to assume that the linear distribution of compressive stresses at that cross section, the critical section is linear elastic is linear. So, the triangular distribution of compressive stresses continuous to be considered at the stage. And as the lateral force is increased, you will get the complete mechanism formation in the form of two rigid blocks that are rotating at it ends and about the mid height and you will have failure when the lateral displacement exceeds at least one half of the cross section thickness t . So, about $0.5 t$ you should get failure in the system due to instability.

So, in this way we have actually looked at considering both inelasticity, but under the assumption that the distributional stresses is linear elastic. So, what do you have is a non-linear stress strain a non-linear force displacement relationship ultimately with conventional bending analysis. So, this is something that we looked at and there are additional moments that start occurring in the in the wall and that is, because of the migration of the hinge towards the levered side, causing additional moments to come on to the wall. So, we have examined those aspects in the previous lecture.

However, if we were to assume that the wall is not bonded at the base that the fixity that we originally assumed at the base is not available for different conditions, if there is already a crack existing then it will not follow this route. You do not have have the formation of the crack at the base which we have looked at from a to c instead you will have a change in stiffness, once this section starts cracking at the mid height that is that is the first nonlinearity that we will be basically assuming. If we begin with an assumption that the wall is hinged-hinged as the boundary conditions in the initial condition itself.

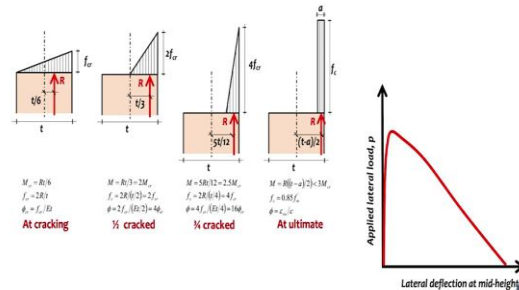
So, this is the possible difference that you should see the in the first case, you have some reserve capacity, because of the base fixity in the second. So, the overall energy that it can dissipate is definitely higher than compared to a situation, where you are assuming hinged-hinged at the top and the bottom. So, this is the possible difference that we can assume and examine in the conventional bending analysis with linear elastic distribution of stresses in compression.

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Mechanical Behaviour of Masonry in Out-of-Plane Bending

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Moments and curvatures at centre of face-loaded wall



However we then examined if we were to estimate the moments and curvature in the wall based on an assumption that the distribution of stresses can be considered as non-linear in the cross section particularly at ultimate, we examined how it is possible to write down the stresses in the cross section and up to the ultimate stage we follow the same process. But beyond that we assume a rectangular stress block that is replacing the parabolic distribution of stresses at ultimate due to softening of the material in compression. So, this is the other way in which the formulation can be done. In this particular example we had in this particular approach, we had looked at situation with the wall was pinned-pinned to begin with.

So, if you were to then examine the force displacement relationship, you will get the softening and failure in the system and the primary difference being close to the peak. When the peak capacity is being reached you have the softening behaviour occurring there which the multi-linear curve that you saw in the other case is the fundamental difference that you will get, because of the assumption of nonlinearity at ultimate that we are able to bring in, in the calculations.

So, this is what we have seen so far. What we now will start examining is one fundamental mechanism that allows for augmentation of the lateral load carrying capacity in the out of plane direction which is arching action.

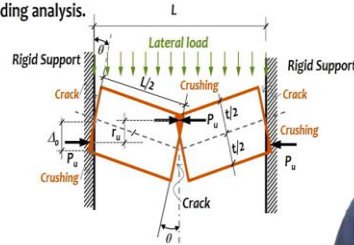
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Mechanical Behaviour of Masonry in Out-of-Plane Bending

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▪ Role of Arching Action in Out-of-Plane Flexure

- Large in-plane compressive forces are induced when a wall subjected to out-of-plane loads is butted up against rigid supports.
- Walls can resist much larger lateral loads than are predicted by conventional bending analysis.



So, arching action is something that you can consider, under a given set of conditions, but if those conditions do not exist, it will be erroneous to assume that arching action is going to occur, because arching action can significantly increase the load carrying capacity of a masonry wall in out of plane bending.

So, what is really happening, we will examine that figure in detail in a moment. Now, let us look at a situation where a wall is being loaded by wind forces or inertial forces and in this particular case we are examining horizontal bending. And therefore, I am interested in looking at the length of the wall; the total length of the wall l and in the original situation the wall is straight right. But as the wall starts getting face loaded perpendicular to the plane or by the out of plane forces, deflections are going to occur on the wall and as the wall starts deflecting, if the supports are rigid supports ok.

Now, having said that I think it is important to introduce here, what is this concept of a rigid support right. Now, if you consider a masonry wall itself- an unreinforced masonry wall with two return walls, you have the main wall that we are examining which is the out of plane wall and two return walls which are in plane walls, if the connections are poor and if the return walls are not strong enough to not be able to provide a rigid support, then you cannot assume that the out of plane wall is actually restrained by the in plane walls, the two return walls. That is one situation; the other situation is you can apply this even to infill panels.

If you have an infill panel that is sitting between reinforced concrete columns within a frame, then those could be assumed to be providing rigid support, because they are non-moving supports. So, an infill wall which is face loaded which is subjected to out of plane forces inertial forces or wind forces could also develop arching action, because the reinforced concrete columns could be considered as non-moving supports or rigid supports.

So, this is a framework that you can use, both for load bearing masonry constructions and for infill, which are partition walls in moment resisting frames. So, the concept of the rigid support is something you will have to here, it is ideally shown as fixity at the two ends, but you have to examine if the conditions prevail for that to be available to you.

Again to begin with we are examining what is called a rigid arching action and then we will move on to a concept called gapped arching action and there again if you have no gap between the out of plane wall and the return walls, then it is possible that you get arching action right at the beginning and that is referred to as rigid arching action. But in case there is a gap developing between the return walls and the out of plane wall, that gap has to be closed before the arching action can develop. And so, we will examine this arching action under rigid mechanism and the gap mechanism, but first we are examining the rigid mechanism.

So, what is really happening is as this wall, which is sitting between these non moving supports rigid supports starts deflecting, because of the action of lateral forces, the wall will deflect; the wall will deflect of course, depending on the boundary conditions that you are assuming you can assume fixed fixed boundary conditions, but then the point is if you assume fixed fixed boundary conditions you will get maximum negative moment at the fixed ends and crack initiation must occur at that point after which the midspan crack is going to happen. So, it is about how the propagation of inelasticity is occurring in the wall and the boundary conditions that you have assumed.

However, let us assume that the wall is rotating at both its ends and the midspan crack is also occurring, which means it is rotating about its ends and rotating about the mid span. So, when it starts rotating since, the supports are non-moving supports you start getting

clamping forces compression at the locations, where the wall is in contact, the out of plane wall is in contact with the rigid supports.

Originally, there was uniform contact and as the walls starts deflecting, you develop these clamping forces right. So, that is the whole the basis with which we are working. Now, these clamping forces depending on how much the wall is deflecting starts increasing right. They increase and reach a stage where you can even get crushing of the masonry at those at those hinged locations and you can get a certain finite length of the wall, which starts plastifying or crushing in the cross section itself. Therefore, your hinge itself can go from a point to at the centroid of a length of wall, which is undergoing compression and crushing, ok.

So, basically bottom line is there are large in plane forces that start developing, in plane compression forces that start developing when this wall starts deflecting and this is expected to happen when this out of plane wall is butted against rigid non-moving supports, that is the bottom line. Let us examine the figure for a moment. So, I have a wall of length L , subjected to out of plane lateral load, it starts rotating I have two non-moving supports.

The clamping force that develops as the wall deflects is designated as P_u , it is the compression and as that increases that section goes into plastification. So, we refer to that at the ultimate state. The black arrow marks that you see are the resultants sitting at the centroid of the triangular area that is been considered geometrically, being considered at the ends of the wall and at the center of the wall, where the contact points are.

So, this is the overall geometry with which we are going to be working. Now, assuming that the two blocks again a fundamental assumption that is going to make is midspan crack has formed, the ends are already cracked and free to rotate. So, you basically you have two rigid blocks, that are rotating. Now, based on that assumption your geometry is simplified and you can actually work on a simple analytical form that accounts for additional capacity coming from the arching action. So, if you were to use the conventional bending analysis in this case, we saw the bending analysis used for the vertical bending scenario.

If you were to use the same thing for the for the horizontal bending scenario and compare, you will see that there is a significant increase in the load carrying capacity and

that is one of our exercises. You will check what is the kind of augmentation that you get, if you were to consider rigid arching action as a contributory factor to lateral load carrying capacity of the out of plane wall. So, we will examine this in detail and develop a formulation.

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- **Rigid Arching Mechanism**
 - Flexural cracking occurs at the supports, location of maximum negative moments, followed by cracking at mid span, location of maximum positive moment.
 - As wall is pushed against unyielded supports, clamping forces (P_u) are formed.
 - A three-hinged arch forms, where the external moment is resisted by the internal couple $P_u r_u$ (r_u : height of the arch).

These clamping forces are what we are going to be trying to arrive at an expression for and these clamping forces as I said are actually, sitting at the centroids of the triangular area that is being considered at the ends of the wall that is a triangular area which is the one that is getting crushed and the contact area is now, Δ_0 there.

You see the length of contact and this length of contact is, because the masonry as the compression forces increase there is slowly getting plastified. So, as that happens, the contact changes from being a point to a finite length and that finite length is a delta naught that we are designating at this point in time, you will have to make some assumptions on how much that can be. Of course, that is going to be dependent on the material strength and the material strength that we are talking about here is the strength of masonry, but at a joint you might not have the unit and the mortar acting together it might simply be the mortar in a joint. So, it could even be the mortar compressive strength.

So, we will have to make an assumption on at what stage how much of length is going to be available as a contact for the wall to develop the maximum clamping force available.

Again, the thickness of the wall is considered as t and as you can see we have taken length L and thickness t in the wall. So, if you can actually examine what is happening in the wall, you have three hinges that are developed. Two hinges are at the supports, at the location of the supports with the non-moving ends which are the rigid supports, two hinges are there and the other hinge is actually at the midspan of the wall.

So, you are seeing formation of a three hinged arch, we are actually seeing the formation of a three hinged arch and now, it is a three hinged arch of low rise. Of course, the kind of deflection that you have seen in the wall is an exaggerated deflection and you are not going to be able to see so much of deflection it is going to be a very small amount of deflection.

We are talking about instability being reached when the wall when the deflection is of the order of one half of the cross section. So, at about 100 mm you can get failure in the system. So, this deflection that we are talking about 100 mm over the length of the wall is small and something that you will not be able to very easily visualize, but the point is the three hinged arch is an arch of a low rise right.

So, if you are actually looking at what is happening here, you have the formation of an arch and if you have the formation of a significant arch and that depends on how resistant the material is going to be, because the material should not crush immediately, if the material is able to withstand compression significantly, the rise of this arch will keep increasing. And so, if you can get a significant rise, you will be able to without failure and without movement of the supports, significant out of plane load can be carried by the wall itself ok.

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▪ Rigid Arching Mechanism

- Internal couple $P_u r_u$ (r_u : height of the arch) resists the external moment

$$M_u = P_u r_u$$

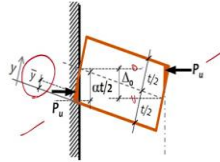
$$r_u = t - \Delta_y - 2\bar{y} = t \left(1 - \frac{\Delta_y}{t} - \frac{2\bar{y}}{t} \right)$$

$$\frac{\alpha t}{2} = 0.1t$$

- Assuming Δ_y (deflection due to axial shortening) is zero for length-thickness ratio less than 25:

$$r_u = t \left(1 - \frac{2\bar{y}}{t} \right) = t \left(1 - 2 \left(\frac{1}{3} \right) \left(\frac{\alpha t}{2} \right) \right) = t \left(1 - 2 \left(\frac{1}{3} \right) (0.1) \right)$$

$$t(1 - 0.067) = 0.933t$$



Now, let us just examine one half of this problem and create close firm solution to understand, what is happening in this system. So, if you actually go back to the concept of the three hinged arch, that is forming the compression resultant P_u into the rise of the arch r_u is really the moment resistance that you are getting in the wall. r_u is the final rise of the arch and P_u is the is the clamping force that you are getting and at the ultimate that we are examining. The $P_u \times r_u$ is what is the internal couple that is forming to resist the external moment that is acting on the wall itself.

So, the ultimate moment in this is going to be $P_u \times r_u$. So, let me just go back to the figure and examine the different notations that we are using. So, P_u is the clamping force, we talked about the clamping force at the two ends, in one half of the wall that we are examining. We have thickness of the wall which is t ; one half of the thickness is t by 2, the distance that the wall is actually in contact with the support is αt . This contact of the triangular area with the rigid support is our finite length and that depends on how quickly the material will start crushing and that distance is taken as αt .

The total deflection the midspan deflection of the wall itself is Δ_y . The line between this line here, at the centre line of the wall is the midspan deflection, the maximum midspan deflection that you are getting in the wall. So, if you are able to analytically arrive at an expression for the rise of the arch, then the moment capacity can be estimated. If you are again able to estimate how much the ultimate compression

resultant can be, which I said is based on how much crushing strength you have at that interface.

So, assuming that from the material strengths, I am able to arrive at an expression for P_u and from the geometry from the idealized geometry, if I am able to arrive at an expression for the rise of the arch r_u , then I have the moment capacity of the wall itself. So, r_u in this case is going to be the Δ_0 , which is the total midspan deflection that you are getting subtracted from the cross section of the wall itself t and again since, we are assuming that the cross section in contact is not a hinge, but is a finite length.

We need to know where the resultant is sitting at in assumption of a triangular area is being considered there and therefore, from the edge of the wall, the point at which the resultant is sitting is \bar{y} with the axis being considered with respect to the deflected shape of the wall itself.

So, this \bar{y} you have to consider twice, because you have got the crushing occurring in two sections; one at midspan and the other one at the non moving support.

$$r_u = t - \Delta_0 - 2\bar{y}$$

I take t out and simplify it further and we make an assumption. So, if it is a rigorous solution that you are looking at you can actually make an estimate of how much this can be, but you can also assume that this would be about 10 percent of the cross section.

So, this is taken here as about 10 percent of the total cross section; $\alpha t/2$ as $0.1 t$ is a fairly good assumption to work with you might not get something more than that that is that is a fairly good estimate to work with. Of course, what we are actually assuming in this case is that as the wall deflects, you know for geometry as the wall deflects the, there is axial shortening and that axial shortening is going to actually create a problem, because you have large deflections the axial shortening is more and then the contact is going to be lost.

Now, the deflections are small and so this axial shortening is not expected to occur which is a fairly good assumption if the length to thickness ratio, in this case the slenderness ratio is less than 25. So, this is again an important assumption that we make;

if you want to make a rigorous solution you will have to then account for any shortening, because of the deflected shape.

Student: Sir, I did not get it.

Δ_0 is equal to?

Student: Is it equal to $\alpha t/2$?

Yes, it would be fairly equal to it will be equal to almost equal to $\alpha t/2$, but; however, if you actually look at what $\alpha t/2$ is the contact area; $\alpha t/2$ is the contact area these are comparable values they are not equal to.

Student: $\alpha t/2$ is a contact area.

$\alpha t/2$ is the contact area.

Student: Δ_0

Δ_0 is the midspan deflection. Δ_0 in the figure delta naught is defined between the center lines of the deflected shape that is Δ_0 . However, αt is the projection of the contact area which is this height we are making an estimate of what that is that is being assumed to be 0.1 t in this case. It so happens that for the geometry that we are examining those two are comparable values ok.

So, with this assumption, if we were to get an estimate of r_u , if we assume that r_u if you assume that $\alpha t/2$ is 0.1 t you can go back to the previous expression and write down $\alpha t/2$ is about 0.1 t and then get an expression for what the r_u is going to be, the rise itself is going to be.

So, here you get a rise of about 0.933 t based on the geometry that we have assumed. So, analytically you need an expression for r_u and from the material strengths you need an expression for P_u to be able to estimate what the moment capacity of the wall itself is going to be ok.

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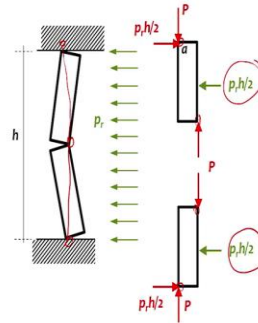
▪ Rigid Arching Mechanism

- For a symmetric arch:

$$\Sigma M \text{ about } a = 0$$

$$p_l \left(\frac{h}{2} \right) \left(\frac{h}{4} \right) = P_l$$

$$p_r = \frac{8P_l}{h^2}$$



We will come back to that and see how some codes do give you the possibility of accounting for the arching mechanism, most others just depend on the conventional bending analysis itself ok. So, if we were to assume this initial case, where the non-moving supports are present, but there is no gap that exists between the wall and the support.

So, in this particular case we are examining vertical bending and the top and the bottom are the supports that we are considering now. So, it could be a reinforced concrete slab at the top and reinforced concrete slab at the bottom and wall is subjected to an out of plane load. So, you have vertical bending in the wall and a similar mechanism that generates, you will have the contact points developing and because of the contact points developing here, here and here you have the arch that forms and resistance with rigid arching mechanism available for you.

So, in the case of rigid arching mechanism owing to the geometry, what you get is a symmetric arch; you would get a symmetric arch, also with the assumption that the crack is occurring at mid height right. So, if you were to examine the free body diagram of the rigid block above the mid height crack and rigid block below the mid height crack, you have the resultant of the lateral force acting at the mid height of the top block and mid height of the bottom block p_r here, we are assuming that rigid arching mechanism is happening. So, lateral load corresponding to rigid arching mechanism $p_r/(h/2)$ and

$p_r/(h/2)$ acting on the two blocks. And now, you have the symmetry, you have the hinge occurring at the windward edge at the mid height right and at the leeward edge at the top and the bottom.

So, that is giving you the symmetry in the system and if we take the moments about the contact point at the top, if you take the equilibrium of moments at point a which is at the top;

$$p_r \left(\frac{h}{2} \right) \left(\frac{h}{4} \right) = P t$$

$$\Rightarrow p_r = \frac{8Pt}{h^2}$$

Again, valid for both horizontal and vertical bending, one way bending. If we were now going to examine, what is happening with the presence of a gap, you do not have that symmetry that was previously available in the system right.

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Mechanical Behaviour of Masonry in Out-of-Plane Bending

- **Gapped Arching Mechanism**
 - Wall separated from one support by small gap: wall must rotate about its base enough to close gap and generate clamping thrust forces.

ΣM about a = 0 for bottom half

$$p_s \left(\frac{h}{4} \right) \left(\frac{h}{2} \right) + p_s \left(\frac{h}{2} \right) \left(\frac{h}{4} \right) = Pt$$

$$p_s = \frac{4Pt}{h^2}$$

- Results in an asymmetric arch.
- 50% reduction in capacity due to gap (resultant force at extreme fibre)

So, let us examine what would happen here, let us say there is a small gap at the top right, formed to do whatever reason, construction defect or shrinkage in the material and so on.

So, let us assume that there is a small gap which separates the wall and the support. However, this gap has to be limited, if you have too much of a gap it will never develop any arching action right. So, it will actually behave as a cantilever wall. However, in this

case we will examine what that limiting value of the gap must be, but let us assume, if we have this sort of situation, the wall must actually rotate and after rotating a certain finite rotation it will come into contact at the top point right. So, when that happens and if you were to examine the free body diagram, at the instant when contact is established and you have the hinges forming, you will have a hinge at the top windward side. A hinge that is formed at the mid span, at the windward side and one hinge that is formed on the leeward side, at the bottom right.

Now, you do not have the symmetry that was earlier available and you have an unsymmetric arch that develops in this case. If you were to look at the free body diagram again, the wind induced reactions here $p_g h/2$, the subscript g is here for the gapped arching action $p_g h/2$. You have two hinges on the windward side and therefore, the reactions $p_g h/4$ and $p_g h/4$ are on the hinge that is on the windward side, the bottom block will also have $p_g h/4$ on the windward side.

Whereas, you have only one contact point on the hinge on the levered side and therefore, by equilibrium;

$$p_g \left(\frac{h}{4} \right) \left(\frac{h}{2} \right) + p_g \left(\frac{h}{2} \right) \left(\frac{h}{4} \right) = P_t$$

$$\Rightarrow p_g = \frac{4P_t}{h^2}$$

And this is in comparison to the previous case about 50 percent lower. You, in the previous case we got for the rigid arching action $8P_t/h^2$ square. Here, with these assumptions that we have made in this particular case two fundamental assumptions that the crack is happening at mid height and the second assumption that we are making is simplified analysis, where we are assuming that the hinge itself is occurring that the area in compression is a point and that is a perfect hinge ah. So, this is another assumption, but plastification of the cross section is actually to be expected, but this simplified assumption tells us that if you have gapped arching and beyond the gap if you are able to mobilize clamping, then a 50 percent reduction is what you would see if there is a gap right ok.

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▪ Gapped Arching Mechanism

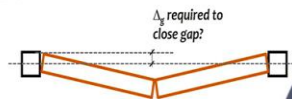
- Maximum gap to avoid rotation (loss of contact with support), for a wall of height h and thickness t :

$$g_{\max} = \sqrt{h^2 + t^2} - h$$



- If gap is $g/2$ at each end, mid span deflection to close the gap, approximately:

$$\Delta_s = \frac{gL}{4t}$$



So, the gapped arching mechanism again, because of the simplified geometry that we are looking at needs to have some limit on the gap that is that is available. So, if you can make an estimate of the gap or say that is the maximum gap that you can have so that arching action develops. Geometrically, you can arrive at what is that limiting gap that you can permit, in case of a vertical bending situation or horizontal bending situation. So, you want the wall to be able to rotate and come in to contact, but if the gap is too much it is just going to rotate and you will lose contact. So, what is the maximum gap, so that you avoid rotation of the wall with the simple geometry.

If the if the wall that we are looking at is of length L and we have two gaps the rigid nonmoving supports are the black boxes that you see at the two ends. If you have two gaps each assuming again that the gaps are equal $g/2$ and $g/2$, you can arrive at what this maximum g is going to be by simple geometry. Assumption here is that the block is moving as a rigid block once, the crack is formed itself ok. And this crack is actually going to be closed only on deflection; the crack is going to be closed only on deflection right.

So, how much should the wall deflect? Again, you can make an approximate estimate of how much should the midspan deflection be; which will ensure that contact is established and then the capacity can be estimated accounting for the gapped arching action.

So, for that let us assume that the gap itself is $g/2$ at each end, how much deflection at midspan is required so that you close the gap and again from geometry you can make an approximate estimate that this is about $gL/4t$ from the geometry of the deflection itself. So, simplified assumptions, but quick checks that you can do to ensure gapping can, the gap can be closed and arching action can be used for estimating the capacity enhancement of the capacity of the wall itself. So, you can have more rigorous, more detail calculations, but this is a simple framework that you could use.

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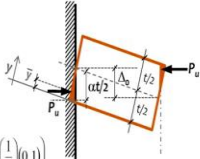
- **Gapped Arching Mechanism**
 - Internal couple $P_u r_a$ (r_a : height of the arch) resists the external moment.

$$M_u = P_u r_a$$

$$r_a = l \left(1 - \frac{gL}{4t} - \frac{2\bar{y}}{l} \right) = l \left(1 - \frac{gL}{4t^2} - 2 \left(\frac{1}{3} \right) \left(\frac{at}{2} \right) \right) = l \left(1 - \frac{gL}{4t^2} - 2 \left(\frac{1}{3} \right) (0.1) \right)$$

$$l \left(1 - .067 - \frac{gL}{4t^2} \right) = 0.933l - \frac{gL}{4t}$$

- A measure of reduction of arch height due to the presence of gap...
- Results in reduced moment



So, if you were to go back to the original set of expressions that we developed, if there is a gap and we have seen that the gap will lead to a reduction in the load carrying capacity of the wall right. Arching action is already enhancing the load carrying capacity of the wall in the out of plane direction, but if you have a gap with respect to the rigid arching case, you are going to get a drop in the load carrying capacity. Can an estimate of that we made? The answer is yes. So, we were earlier, making an estimate of an analytical estimate of what the rise of the arch was, we know that the gapping is going to lead to a reduction in the rise of the arch right.

So, in the same the expression that we were using earlier, we had the component that looked at the contact area, in that if we introduce the gap that needs to be closed or the gap that creates a loss in the rise, we can bring an $gL/4t$ into the expression here. And then expanding that have the total rise of the arch as a rise that is going to be

compromised by the deflection that is required to close the gap right. The wall must deflect to close the gap; the wall must deflect by $gL/4t$ that we saw earlier to be able to close the gap, but in doing that you are losing the original rise r_u and that rise is now compromised by $gL/4t$. So, your overall r_u in this case can be the original 0.933 t that we arrived at with the assumptions made, there further reduced by $gL/4t$.

So, here you get an estimate of r_u and then go back to the moment capacity M_u being P_u into r_u and see how much of reduction in the load carrying capacity you are getting, because of the, because of the presence of the gap. So, to interpret the gap as something that reduces the height of the arch that is available is the physical representation of the gapped arching mechanism itself and of course, M_u is going to be lower in this particular case.

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Mechanical Behaviour of Masonry in Out-of-Plane Bending

- **Rigid Arching Mechanism**
 - Thrust force per unit height or length (BS 5628):

$$P_u = (0.85 f_j) (0.1t)$$

$$\Sigma M \text{ about } a = 0$$

$$p \left(\frac{L}{2} \right) \left(\frac{L}{4} \right) = 0.1t (0.85 f_j) (0.9t)$$

$$p = 0.61 f_j \left(\frac{t}{L} \right)^2$$

f_j : Compressive strength of the mortar joint at contact surface (mortar cube strength, conservatively)

So, as I said it will be instructive to see how do codes account for this sort of a phenomenon that you can beneficially utilize in estimating the load carrying capacity of the wall in your design. So, the British code actually has a set of expressions that simplifies and helps in accounting for the arching action. It is BS 5628 that has this expression.

So, you are calculating the thrust force depending on vertical bending or horizontal bending as thrust force, the P_u is the thrust force per unit height or the length of the wall the thrust forces is the clamping force that we were looking at the P_u . So, the code is

estimating this in terms of the compressive strength of the mortar ok. As I was mentioning earlier, at the interface between the non-moving support and the brick wall its masonry; however, that interface actually the weaker material there is the mortar joint.

So, it is reasonable and probably better to make use of the strength of the weaker material, which is the mortar and hence, the expression in the British code actually makes use of the compressive strength of the mortar joint which is present at the contact itself and this is a conservative estimate. You can actually, if you have integral action of the composite masonry mortar and the masonry unit, you might get a higher compressive strength, but conservatively you are assuming the lower of the two and the code gives an expression $0.85f_j'(0.1t)$. So, let us just examine how that is arrived at.

So, now instead of assuming that the hinge is forming at the at a point, we are assuming a finite length and of finite length over which there is plastification of the material. So, the same expression that we saw earlier, but now that is not a perfect hinge, but over a finite length. And so, if you have examined the stress block in that location basically, bring in stress block parameters in the compression, in the mortar at ultimate. So, the stress block there is a finite length about $0.1 t$ is the contact area that we have also been assuming in the previous set of expressions.

So, 10 percent of the total cross section is the width of the stress block and softening of the material and therefore, we assume 85 percent of the compressive strength of mortar and with that stress block if you write down the moment equilibrium in this in the in the figure that we have assumed earlier, with the moment at the top hinge there a p into it could be h or L depending on vertical bending or horizontal bending.

$$p\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) = 0.1t(0.85f_j')(0.9t)$$

$$\Rightarrow p = 0.61f_j'\left(\frac{t}{L}\right)^2$$

So, this is the way of accounting for your in one way bending action vertical or horizontal at the clamping forces that can come, because of arching mechanism available in masonry. I will stop here and we will start examining two way bending in the next class as the last aspect to be considered in out of plane bending.