Design of Masonry Structures
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Module - 03
Lecture - 14
Strength and Behaviour of Masonry Part - IV

Good afternoon, we continue with our lecture which was looking at Behaviour of Masonry under Compression. And we examined the effect of slenderness ratio and the effect of eccentricity ratio and how we account for these two important geometrical second order effects in reducing the compression capacity of a masonry wall. So, that is what we were looking at in the last class.

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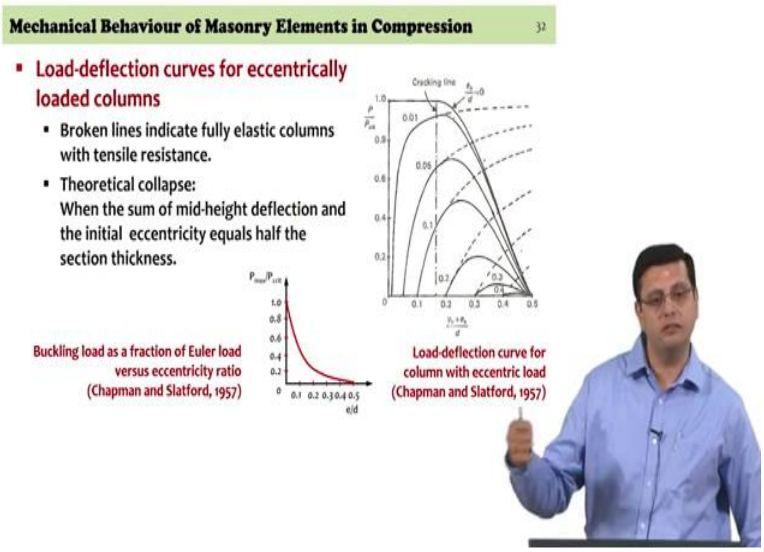

Of course, the vertical load carrying capacity of a masonry wall is also affected by other factors; we had briefly looked at some of these factors at the beginning of this module and we will examine this little closely today before we move on to behaviour in bending itself.

So, we were looking at how codes represent and that is where the concluded in our last lecture, how a code would give you design curves that you can use for estimating the dimensions for a given wall or the design loads for a given combination of geometry and eccentricity of loading. So, we did see how the IS code 1905 looks at stress reduction factor as a function of what the eccentricity ratio e by t, with t as the wall thickness and h by t was and we saw those curves. And I did mention to you in the last lecture that we make our calculations for the adopted approach based on linear elastic analysis- linear elastic behaviour of the material.

Now, that is going to be conservative and across the world different codes adopt different approaches: one of the aspects that can see a difference in the way these curves are presented in different codes is if a non-linear stress strain relationship has been used for the masonry compression behaviour, then you will see that you get a more accurate estimate of the second order effects.

So, I am just looking at some other forms in which you will see design curves made available for masonry design. So, in this particular example that we are seeing, you can see the normalized load on the y axis, $P/P_{critical}$ and on the x axis you have the total deflection; it includes the eccentricity of the load itself. So, you have y_c which is the mid height displacement that you would get in a pinned-pinned wall plus the eccentricity of the load itself normalized to the thickness of the wall given as d here.

So, in this set of curves that you see, if we were to consider the column to behave in an elastic manner even at ultimate and provide the material with tensile resistance, then the load deflection curves beyond the point where the continuous lines start reducing. If it were assuming fully elastic condition you will have you would actually follow those dotted lines there. So, this is another way of representing $P/P_{critical}$ to the eccentricity ratio and this is from work quite a few years ago. Now one aspect that you need to keep in mind is, it has been observed and most of the analytical forms, most of the design graphs would account for this aspect, for this phenomenon, the maximum mid height deflection that you get in the wall because of the second order effect and the initial eccentricity that is there because of the eccentricity of the load.

So, in this case we are looking at y_c as the mid height deflection and e_p is the eccentricity of the load so, when this reaches one half of the section thickness. So, when $y_c + \frac{e_p}{d}$ in this case is equal to 0.5 then you typically get loss of stability, collapse is observed at this limit and most codes would put a cap beyond this particular value.

So, you can see again that at e/d that is total eccentricity plus the mid height deflection reaches 0.5 you have reduction in the capacity coming right down to 0. Now if the dotted lines here imply that there is no inelasticity in the cross section and there is tensile resistance available in the cross section. So, in that case resistance is still there, but in reality you are looking at very low tensile resistance and plastification. So, once you assume plastification of the section and you assume that there is no tensile resistance, crack formation and subsequent plastification of the cross section is occurring which is what is crossing the reduction in the load carrying capacity.

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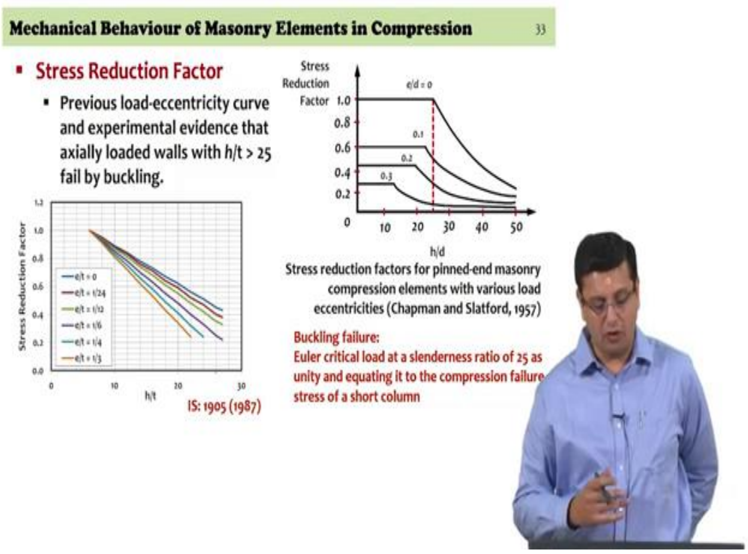

The stress reduction factor itself, we have seen how with h/t greater than 25 these are governed by buckling, but less than 25 you start seeing that you can have crushing in the cross section that can occur because before the instability due to buckling occurs itself.

So, typically the stress reduction factors that we saw for IS 1905 we will compare that in a moment, but this is the other way in which you would see codes giving you stress reduction factors that you should be using to account for second order effects again as a function of h/d and you would use different values for different eccentricity ratios as you can see. So, at about 25; at h/t of 25 which is the red dotted line with no eccentricity of the load itself you do not use a stress reduction factor. So, you can see that at h/t of 25 the stress reduction factor with no eccentricity of the load is one.

So, there is no reduction that is required and this comes from observations that beyond h/t of 25 is when failure due to buckling is expected to occur. So, this is what we had seen and in the IS code we have the values that go up to h/t of about 27 and we do see some differences, but more or less this is what is how codes would represent the effect of second order geometrical aspects such as h/t and e/t, ok.

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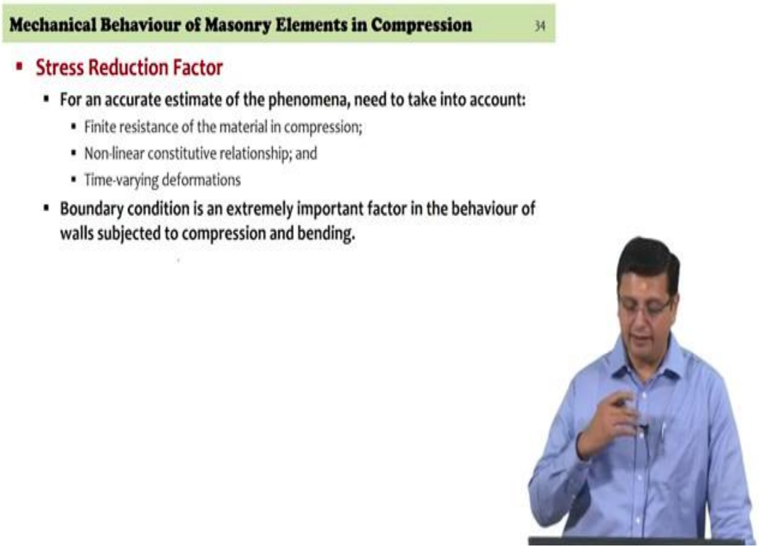

Now, as I had mentioned earlier there are other factors that will come into the picture and these phenomena particularly the interaction of the wall with the floor slabs and the rotation at that junction has an important role to play; it actually changes the boundary conditions and would alter the vertical load carrying capacity of the wall itself. So, that is an aspect that I would like to look at.

Now, if you were to make an estimate; an accurate estimate of the effect of second order P-delta in the load carrying wall, you need to consider the fact that beyond the compressive strength of the material you are going to have plastification of the section and that needs to be considered; a finite resistance of the material in compression needs to be considered. You can consider a non-linear constitutive relationship to get a more accurate estimate of the P-delta effects and the time varying deformations can also be considered. And your load configuration is going to change because of the deformation itself and these could also be considered to get a more accurate estimate itself. But boundary conditions getting affected is something that you need to keep in mind and can be accounted for in arriving at the vertical load carrying capacity of the wall itself.

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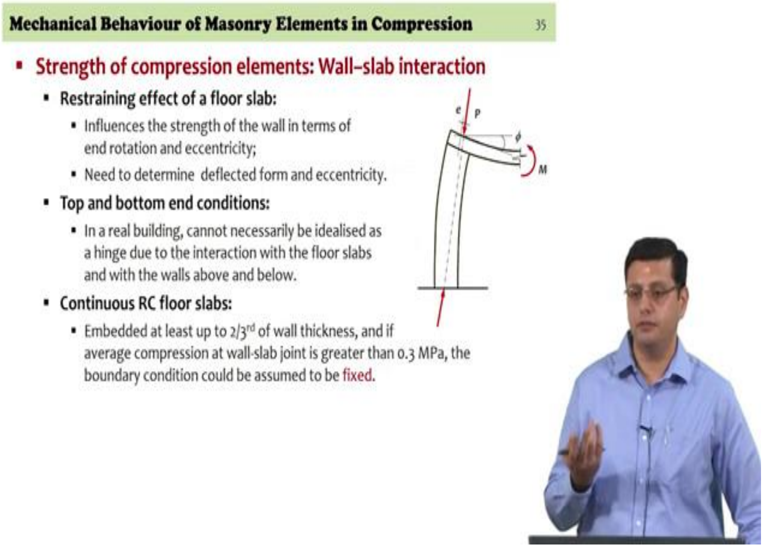

So, let us examine how the interaction between the wall and the slab is and if one were to be accounting for wall slab interactions in the vertical load carrying capacity of the wall, what sort of a framework could you use for this sort of this calculation itself.

So, the floor slab induces the restraining effect on the wall, when the wall is subjected to bending due to the gravity forces acting eccentrically or even when you have lateral forces acting on the wall the role of the floor slab is, it basically restrains the deformations in the wall. So, this slab has a restraining effect and it is essential therefore, to be able to understand what is this restraint offered by the slab and can this restraint actually be considered in the vertical load carrying capacity of the wall itself. It influences the strength of the wall, both particularly in terms of the end rotations and what the actual eccentricity of the load is on the wall itself.

So, in order to be able to arrive at an analytical form let us look at the deflected shape and then the eccentricity of the loading to then use then within an analytical expression itself. Now, as we were discussing earlier, the problem that we looked at and arrived at in analytical close form solution had first order eccentricity- we assumed that the top and the bottom eccentricities where equal, but you need not have that sort of a situation, you might have first different eccentricity to the top and the bottom and this can come from changed boundary conditions at the top and the bottom. So, the top and bottom end conditions need not necessarily be same, we idealize it as the same.

So, as I said the the first assumption that we have a hinge at the top and the bottom itself is based on a certain set of conditions of crack formation at the at the interface with the adjacent material. Now, this could be a situation that has not occurred so, your top and bottom boundary conditions could be different itself.

So, let us examine a few cases where you might want to make assumptions that are as appropriate as possible with respect to physical reality. If you were to consider a continuous reinforced concrete slab as the floor slab or the roof slab and let us assume that this reinforced concrete slab is embedded into the wall thickness. If wall thickness is t, then the we are assuming that the slab itself is embedded at least up to two thirds of the wall thickness and now if the average compression at the interface between the slab and the masonry wall, the bearing stresses are of the order of 0.3 MPa, 0.3 N/mm^2 that can be assumed to give fixity.

So, if you can make a quick estimate of what is the bearing stress at that interface, if you have embedment plus a bearing stress of the order of 0.3 MPa or higher, assuming that the joint is fixed is a reasonable estimate. You might have situations different from this ,if you have a flexible diaphragm, if you have a flexible floor then the situation definitely changes.

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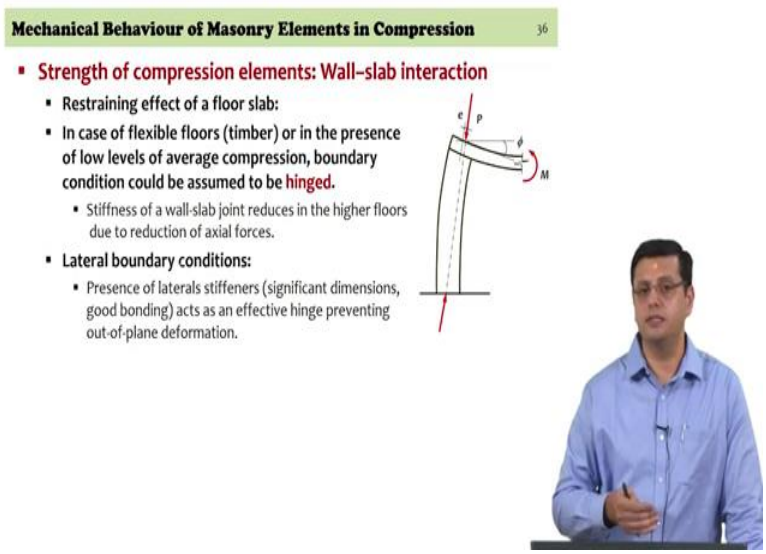

Now let us look at if you have a timber floor, which could be a which is a flexible floor or it could be the reinforced concrete slab that we examined earlier. But now let us say you do not have a bearing stress level of the order of 0.3 MPa; it is significantly low. In such a situation the flexible floor, the timber floor or reinforced concrete slab which has very little bearing stress, significantly low bearing stress, is not going to restrain the joint and assuming that the wall is hinged is a reasonable estimate. Plus, the other aspect that you need to know is, this is not just structural typology specific. Within the same building as you go to higher floors, the stress levels in the axial force levels are going to be different and therefore, the stress levels can also be different, in the higher floors compared to the lower floors.

And so, you might not have the bearing stresses that we are talking of, the thumb rule of 0.3 MPa in the higher reaches of the building compared to the lower floors so, this is something to be considered. If there is good bonding then the out of plane deformations, the return walls, if there is good bonding, then the return walls are going to be acting as stiffeners and you would be able to get further contribution as far as lateral restraint is concerned.

So, you are examining the wall and it is under axial forces, but now if the return walls are well bonded to the wall under examination there is an additional resistance that is available to this deformation by the return walls and it depends on how well the wall is bonded to the return walls. It depends on if you want to consider this effect and whether the effect are actually having a significant role to play in the vertical load carrying capacity of the wall, ok.

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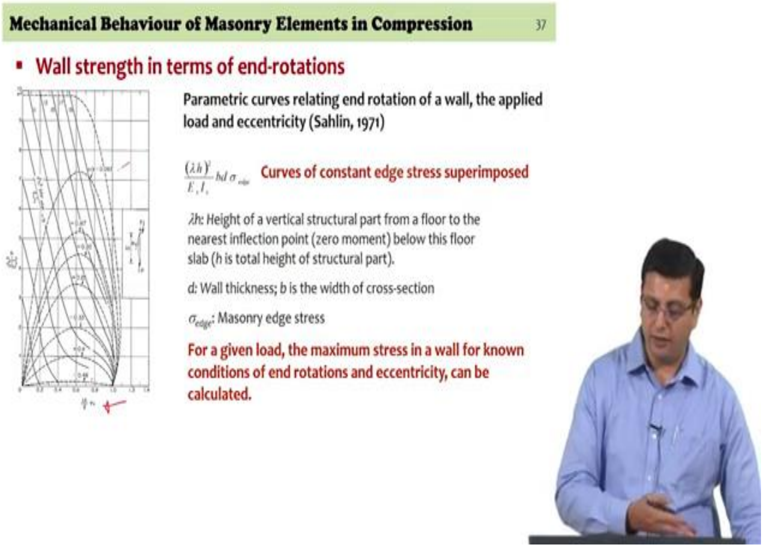

So, if you are able to make an estimate of the end rotations, can that be brought in analytically? Yes and this has been done, you need a certain amount of idealization, a set of parametric curves as you can see in this in this graph here have been created based on analytical formulations which link a certain set of conditions. The end rotations of the wall, you should be able to make an estimate of the end rotations of the wall the load that is being applied to the wall and the eccentricity of the load itself.

So, these set of parametric curves from this work that has been quoted here Sahlin (1971), looks at being able to estimate the stress in the wall, the stress level in the wall because of a combination of these affects- the end rotation of the wall, the total applied load and the eccentricity of the load itself. Here, let me make specific reference to the small figure that is there. λ_h is the height of the element that you are considering, it is the height of the load bearing wall of the bearing element from a floor to the nearest inflection point so, the point of zero moment.

So, it depends on what sort of an idealization of the boundary condition have you made and h is the total height of the of the wall and then depending on the deflected shape of the wall between the points of zero moment you have λh, d is the total wall thickness and b is the width of the cross section that is being considered and σ_{edge} is nothing, but the edge compression we have been using f_c in our earlier lectures and that edge compressive stress is what is given in this graph as sigma edge. So, these curves are actually curves of constant edge stress; you have constant edge stress curves superimposed on these on the other curves here.

So, depending on the level of rotation that you have, the end rotations are given on the y axis. You can see that the rotations are here, the end rotations are here, in terms of $\frac{\lambda h}{d}\varphi_v$ which is an estimate of how much of rotation do you have in the wall itself. And then on the x axis you have $\frac{(\lambda h)^2}{EI}$; E of the masonry and I again the second moment of area of the cross section.

And the second set of curves that you see here are curves of constant edge stress and that is $\frac{(\lambda h)^2}{EI} bd\sigma_{edge}$. So, the second set of curves that you see are curves of constant edge stress and they are superposed on this and the curves are calculated for different levels of eccentricity ratio. So, each curve is for e/d of 0.083 and keeps increasing to about 0.5 and as we said you will see that this e/d of 0.5 or e/ t of 0.5 is a limiting value in most codes.

So, this is if for a given load the maximum stress in a wall can be known if you can make an estimate of the end rotations and you go you know the extensity of the load. So, if you want to limit the maximum compressive stress in the wall let us say you are working within a permissible stresses approach, then you know what is the maximum permissible stress that is allowed for the combination of materials that you are looking at. Through this sort of a graph you will be able to check if for a given load and selected end rotations and eccentricity would you be able to satisfy the requirements of the allowable stress itself. So, this is another useful form in which the effect of eccentricity, effect of the end rotations have been captured in parametric design curves ok.

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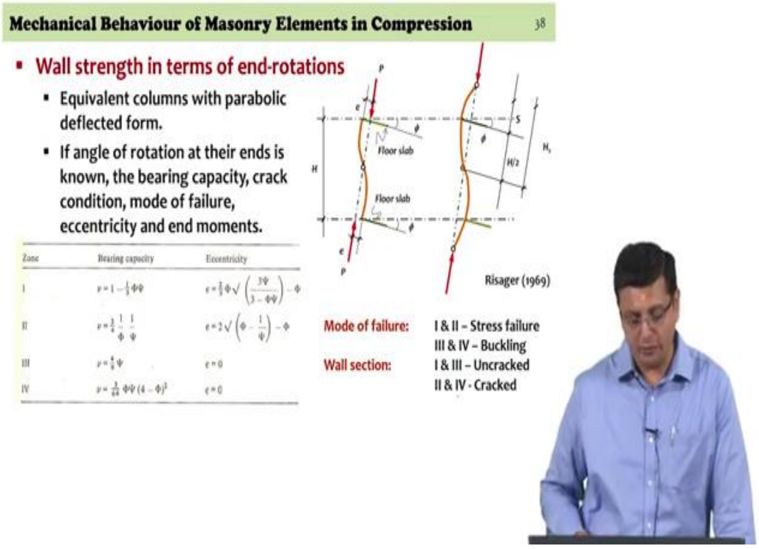

This is a work that has actually seen lot of research; this is an area that has seen a lot of research. So, there are different ways in which this has been accounted for. So, if you are looking at a wall which is carrying the vertical load and sitting between floor slabs and if the rotation the end rotations are designated as phi and e being the eccentricity of the axial load P, for a wall of height H. There are some other forms that have been developed in this particular work you can see that the deflected shape.

Now, the deflected shape of the column has been idealized as being close to a parabolic deflected shape and with that assumption you get an analytical form that you can use and in this particular situation, in this particular example, if the angle of rotation at the ends is estimated right. If you can make an estimate of the angle of rotation at the ends, the bearing capacity (the failure mode), that is in what way is the wall slab interaction expected to fail and the eccentricity and the end moments can be arrived at.

So, in terms of mode of failure you can either have failure in the section due to the compressive strength of the material being reached or you could have failure due to buckling and that depends on the h/t ratios that you are looking at and the relative strength that is being examined. You can have the entire wall section in the uncracked situation or the wall partly cracked or fully cracked. So, depending on the cracked condition in the wall and depending on the mode of failure you have a set of expressions which have been derived with the assumption of a parabolic deflected shape, for the column you get the bearing capacity and the eccentricity corresponding to the capacity estimate itself. So, this is another format in which the effect of end rotations has been considered.

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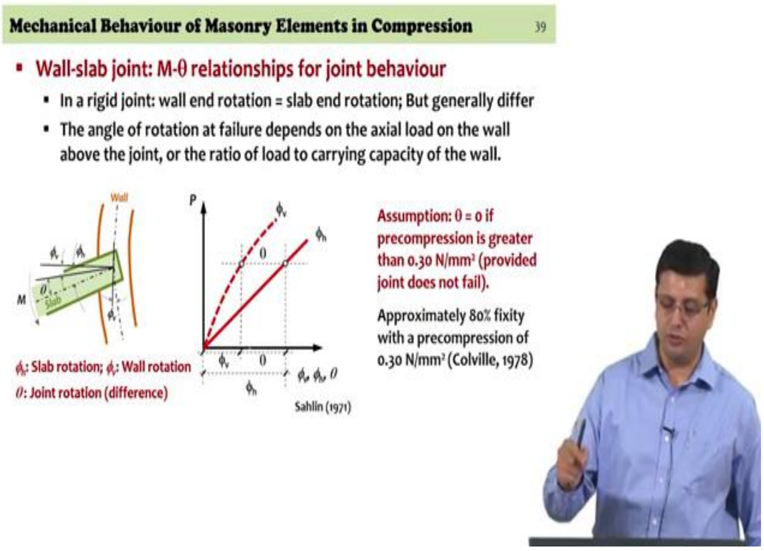

If you actually look at the kind of rotation that you have of the slab with respect to the rotations in the wall at the interface between the wall and the joint, it is instructive to see that the total rotation is effected by how much rotation the slab is going to undergo and how much rotation the wall is going to undergo. So, this is an interesting work that is been done to arrive at moment rotation relationships for the joint behaviour at the wall slab interface.

So, if you are assuming that the joint is rigid, if you are assuming that this wall slab interface is rigid, you are going to basically have a situation where you are assuming that the wall's end rotation and the slab's end rotation are similar, but this is not going to be the case, you are looking at materials which are definitely of different stiffnesses and these two values will not be the same.

So, is it possible to make an estimate of these two and if so, can a moment rotation relationship for the joint behaviour be developed and can that be used for your end rotation estimates and then use it within those example parametric curves if we were looking at to estimate, what the vertical load carrying capacity of the wall is going to be. So, this angle of rotation at failure depends on a few factors, it is how much of axial load is actually coming on to the joint that is, what is the axial stress level in the joint and also what is this load in relation to the load carrying capacity of the wall itself. So, this angle of rotation is going to be different at different stages of the loading itself.

So, if you look at the joint, this is basically examining a wall with a slab and the interface between the wall and the slab, here the rotation of the wall is φ_v. That is the rotation of the wall the vertical component the vertical element has a rotation φ_v, the slab itself undergoes rotation φ_h. Now, the difference between φ_h and φ_v is really the rotation of the rotation of the joint itself. So, there is an estimate of the joint rotation that is now dependent on how much is the slab rotating, how much is the wall rotating and the difference is going to give you the joint rotation.

So, some work that has been carried out in the past has looked at an elastic-plastic behaviour where the rotation of the wall is assumed to have a non-linear behaviour - with increasing load the rotation of the wall is not linear. Whereas, the rotation of the slab is considered to be linear and this is an acceptable assumption if you are looking at brick masonry and reinforced concrete as the 2 materials. I am assuming that the reinforced concrete material is continuing to behave in a linear elastic manner whereas, the wall has inelasticity coming in and so, the rotations are non-linear and that is a fairly good assumption.

So, in this graph you can see with increasing P on the y axis you have the different rotations the rotation of the wall the rotation of the slab and the difference θ. So, you can make an estimate of what theta is once the total rotation of the slab and the rotation of the wall is made. And as we were talking as we were discussing earlier in the case of the reinforced concrete slab, if the pre compression at that joint is about 0.3 MPa or greater you can assume that you do not get rotations there and it is a fixed joint and this is actually been seen quite clearly in experimental results that you get almost 80 percent fixity if you are looking at a compression stress level of about 0.3 MPa.

So, with this you will have an estimate of the rotation and then you can go use curves which are actually accounting for end rotations as well, to be able to estimate the vertical load carrying capacity of a wall.

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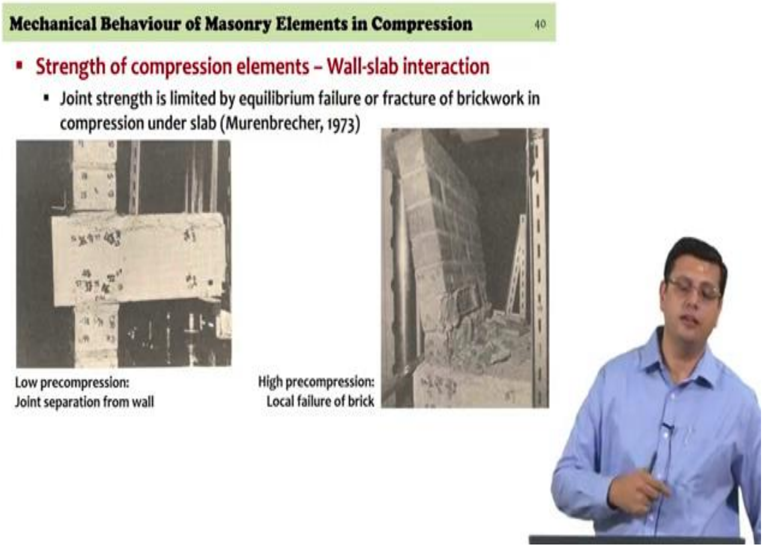

Just to show you the behaviour of such interfaces if you have very low level of pre compression in the wall. Let us say we are looking at a single storage structure and the roof slab of a single storage structure in reinforced concrete with low pre-compression levels you will have fracture crack formation as there is as the wall starts deforming. Whereas, if you have high pre-compression then the interface can see situations where there is crushing occurring in the brick masonry, these are of course, outcomes of tests and what you are seeing is that failure conditions.

So, the point is that interface will behave differently with respect to the level of pre compression. It can either crack or it can crack and rotate or crush and therefore, if it is crushing then you have the non-linear behaviour of the masonry wall the rotations in the masonry wall will be different and you cannot assume a linear response then.

So, that brings us to the end of examining masonry under compression we did it at 2 levels, we did it at the masonry cross section level, material compression and the element compressive behaviour itself. We continue examining different actions, the next that we will be examining is bending, but out of plane bending.

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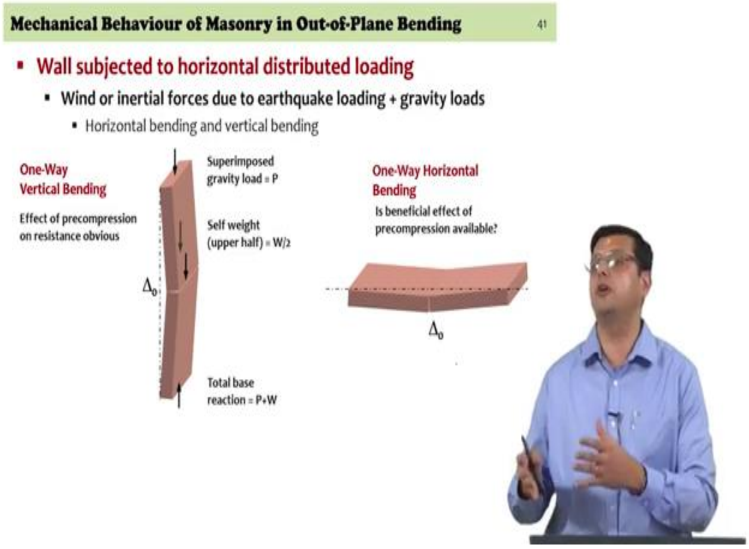

And when we examine out of plane bending, we did discuss this aspect at the beginning of our section, in the section on material behaviour and how the strength of the joint in flexural tension becomes important. If you remember we examined flexural tensile strength normal to the bed joint and flexural tensile strength parallel to the bed joint. Now, time has come when we start examining the role played by those parameters and the orthogonal strength ratio that I mentioned to you earlier in the behaviour of the masonry wall in bending ok.

Now, assuming we are looking at the wall subjected to horizontal distributed loading. So, now the load is actually acting laterally on the wall and we can examine typically distributed conditions, because we are interested in looking at the behaviour of the masonry wall when subjected to either inertial forces; inertial forces generated due to an earthquake and that is distributed load of course. We can idealize it as a concentrated force when we do seismic analysis, but basically the inertial forces are distributed forces; or wind forces (air pressure on a wall). So, we are examining a situation where the wall is subjected to horizontal distributed loading and this is in the presence of gravity load.

So, we are examining a combination of loads here, the gravity load is present the stresses due to gravity loads are already present in the wall, but in addition we have the lateral load acting on the wall. So, before we examine the analysis approaches to consider different situations it is useful to recollect that we talked of horizontal bending and vertical bending and you have situations where you have a combination of horizontal bending and vertical bending. We refer to that as diagonal bending and that is again a case that we will examine and horizontal bending and vertical bending is a differentiation that we make based on the predominant direction in which the bending is occurring and that is dictated by the boundary conditions.

So, both horizontal bending vertical bending are referred to as one - way bending, because you have the predominant bending only in about one axis whereas, in diagonal bending you will have two axis about which the bending is occurring and that is why we refer to it as diagonal bending. So, one way vertical bending, you have again the situation that I am looking at is with rotations at the top and the bottom being possible and then you see that there is a crack that is formed at the mid height roughly at the mid height.

So, these are some assumptions that the top and the bottom are free to rotate and you get a mid height crack also forming. We will examine under what conditions these are true, but most in most situations when you are looking at distributed load acting on the wall, lateral load acting on the wall this is the way in which the wall is expected to respond. So, this is one way vertical bending with a mid height crack at the crack somewhere at mid height, it can actually be at other locations away from the mid height due to specific conditions and we will examine that again. So, the maximum displacement could be delta acting because of the deformation of the wall itself.

Student: Sir here is (Refer Time: 34:53).

The question is the horizontal bending happening here? No there is no horizontal bending when we when we talk of one way vertical bending it is about the vertical axis that the bending is happening and you are getting cracking the resistances is offered by a joint and the flexural tensile strength of the material, normal to the bed joint becomes critical in one way vertical bending.

When I am talking of one way horizontal bending it is the wall which is actually opening inwards or outwards with a vertical crack that is formed whereas, in the one way horizontal bending you get a horizontal crack. That the orientation of the crack might mislead you to could get confused with the title, one way vertical bending is bending about the vertical axis, one way horizontal bending is one way is bending about the horizontal axis. The one way vertical bending ends up with the horizontal crack, the one way horizontal bending ends up with a vertical crack so, that is the fundamental difference.

Student: Sir.

Ok, so your question is what we have seen is the wall being loaded in the axial direction.

Student: Yes.

In the axial direction. We are talking of a wall which is carrying gravity forces, but is also subjected to lateral loading and the lateral loading that we examining in this case is either wind load or inertial load because of an earthquake. So, in this condition you have the horizontal load and deflections occurring in the wall because of the combination of the two.

So, you would have a similar situation in one way horizontal bending, now the point that we need to make is that in one way vertical bending you do have the effect of pre compression, the positive effect of pre compression on the resistance of the wall itself. So, if you are looking at a wall which has significant pre compression, then its out of plane capacity vertical bending capacity is going to be enhanced because of the presence of the pre compression right and that is obvious because the pre compression is going to act on the resistance side.

What about a horizontal bending situation? In one way horizontal bending can there be a beneficial effect of pre compression, if pre compression is there it will give you it will definitely have a beneficial effect, but would you get the beneficial effect of pre compression at all in horizontal bending. So, that is something that we will examine. Do you think you can encounter that sort of a situation? We know the beneficial effect of compression, pre compression it is available in one way vertical bending, but what about in one way horizontal bending.

Student: (Refer Time: 37:56).

Yes. So, it is available if you have non moving end supports and if there is no gap between the wall and those supports and if you have basically non-moving supports, the end constraints can induce pre compression as the wall starts deforming. As the wall starts deforming you will get clamping forces coming in and those clamping forces will augment the out of plane capacity of the wall. It is called an arching action and we will examine how one way horizontal bending estimates can be completely different if you consider arching action; conditions are available for arching action to occur.

So, and that is the reason why I put this here, because while in one case it is obvious, in the other case it is not obvious and it needs to be accounted for, if conditions are available for it to occur.