Design of Masonry Structures
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Module – 03
Lecture – 13
Strength and Behaviour of Masonry Part – III

So, good morning, we continue examining the Behaviour of Masonry elements under compression, we looked at a derivation that would allow us to account for geometric second order effects right. And to begin with, we have the examined the effect of the slenderness of the wall and the effect of the eccentricity of the load, represented both as normalized values h/t for the slenderness and e/t for the eccentricity ratio. So, that is where; that is where we were in the last lecture, we will looked at a derivation that accounts for

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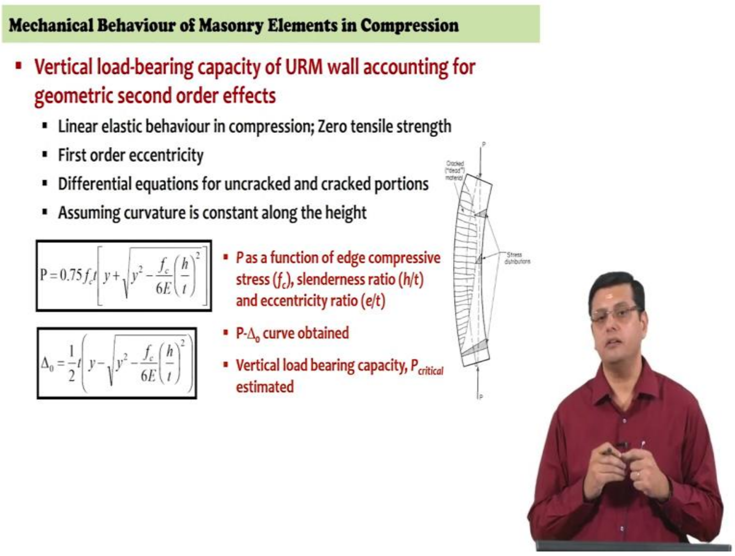

the estimate of the vertical load bearing capacity of a masonry wall; URM here is Unreinforced Masonry. Of course, you will agree that with reinforcement these values will of course, change and we will examine how in a reinforced masonry situation you can estimate capacities of the wall, but currently, we are examining the strength of masonry and now in the component or the element which is the masonry wall itself.

So, we make the derivation of the vertical load carrying capacity, with a certain set of assumptions which really become the basis of how you get these expressions- how you have these expressions or the curves that you can develop from these. International codes have different basis for developing such force-displacement or load-deflection curves for compression. They can vary from linear elastic basis to non-linear stress-strain definitions for the material itself, we have examined under linear elastic condition.

So, that was our first assumption- that the behaviour in compression of the material is linear elastic and that is why, we were able to simplify the distribution of stress to a triangular distribution in the in different sections along the height right. And that distribution, the triangular distribution, was simplifying the estimates of the cracked length and the un-cracked length in each section, which was essential to be able to arrive at an expression for the edge compressive stress f_c. So, this assumption is very important, the second assumption that goes into the calculation of the vertical load bearing capacity is that the material has no tensile strength.

There is some reserve tensile strength; however, we have seen in our previous lectures that this is not something that is uniformly available and dependable and hence assuming that it is a zero tensile strength material makes sense from an engineering estimate point of view. We also have examined the problem under first order eccentricity implying that the load eccentricity at the top and the bottom is the same which may not necessarily be the case if the contact areas at the base of the wall are different from the contact areas at the top of the wall.

So, load eccentricities at the top and bottom can be different, but here we have examined the problem under a first order eccentricity, where e_{top} is equal to e_{bottom}. We then develop the whole premise that the you need a differential equation to examine and solve the elastic buckling problem here, but these differential equations have to be considered differently, because part of the wall where maximum deflections are expected may be cracked where as part of the wall could be uncracked.

And so, we had different differential equations for the cracked and the uncracked portions and the classification between cracked and the uncracked portion was on the simple basis that, if the total eccentricity lies beyond the middle one third of the cross section or $e/t > 1/6$ the wall should have cracked otherwise the wall is uncracked.

Again, a further simplification was made of course, depending on how much of a section is cracked at different heights of the wall. The curvature will be different, but again that introduces the complexity in terms of arriving at a closed form solution in a simple manner and therefore, we assume that the curvature remains constant along the entire height of the of the wall itself. This again is a conservative estimate; with respect to the exact solution itself. So, what we finally had if you are examining the estimate of the vertical load at failure, we had P as a function of the edge compressive stress right.

So, as the wall is being loaded from 0 to maximum load at which failure is expected, the edge compressive stress is going to keep increasing. So, we have an estimate of the edge compressive stress and P the load is directionally proportional to this edge compressive stress. We have also built in two important aspects that affect the second order effects on the wall, geometric second order effects on the wall, which are the slenderness ratio h/t and the eccentricity ratio e/t.

The calculations have been made for unit length of wall, in a similar manner the displacement was also established in this case, we have assumed pinned-pinned condition for the top and bottom boundaries of the wall. And the force-displacement with the displacement at mid height being the maximum displacement is arrived at. So, with the expression, you should be able to arrive at a force-displacement curve, P-delta curve for the wall itself.

So, you start from zero load, keep increasing the load, you could estimate it in terms of f_c because you have the edge compressive stress which is directly proportional to P, you keep increase in the edge compressive stress, till the edge compressive stress reaches a maximum or you have instability in the system. So, a P-delta curve can be obtained for a certain geometry for the load till failure. So, from this single load-deflection curve that you estimate, you can identify what is the critical load or $P_{critical}$ as for as the wall is concerned.

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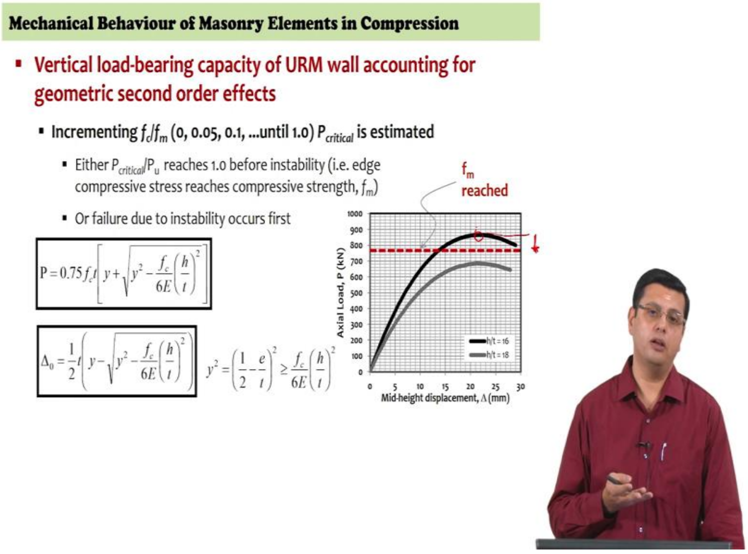

If you were to work on this problem, you can look at the edge compressive stress incrementally reaching the compressive strength of the material, f_c is the edge compressive stress. Let us say, you know the compressive strength of the material, it is this strength of masonry, the wall is not going to be able to fail in crushing because, second order effects will govern.

So, basically you can increment this ratio f_c by f_m from 0, keep increasing at till the value of 1 meaning that the edge compressive stress has now reach the crushing strength of masonry and you have material failure in the cross section right. So, as you do this, if before instability is reached, the material crushes then that is the load at which the wall is expected to fail. That becomes your critical value of P or the vertical load bearing capacity of the wall, but in some situations you might not have the crushing strength of the edge compression fibre reaching a value of f_m before the wall fails; before the wall fails due to instability.

So, you could either have $P_{critical}$ by P_u reaching a value of 1 before instability, that is the edge compressive stress reaching a compressive strength f_m or you could have a situation of instability being reached right. So, the P-delta curve that you see here, it has been worked out for two different values of h/t, two different slenderness ratio, and for specific values of compressive strength of the material f_m and modulus of elasticity, these are the values that are actually going. And these curves have been worked out for a single eccentricity ratio e/t, a single value of eccentricity ratio.

You can see that, you could have a situation with respect to h/t equal to 16, you see that the edge compressive stress reaches the value of the compressive strength of the material and for all practical purposes, you can assume that the wall has reached its failure load that becomes your $P_{critical}$, but you might have another situation, where the edge compressive stress is still less than the compressive strength of the material, but failure is happening due to instability itself.

So, this is what such set of equations can help you arrive at- the load-deflection excursion of the entire wall and the $P_{critical}$ and whether the $P_{critical}$ is occurring due to instability or $P_{critical}$ is occurring due to failure due to crushing of the material itself right. So, this is, what we were targeting. Now of course, this is mid-height displacement and as I told you, we have made these calculations considering a wall that is pinned at the top and the bottom. You could do the same calculations for a wall that is cantilevered- fixed at the bottom and free at the top, you could do the same calculation assuming that the top and the bottom are a fixed and not pinned right.

I mention that the assumption of the top and the bottom being pinned is a reasonable assumption because, very often you will have cracking at the top and the bottom of a wall occurring due to thermal movements not because of structural loads, dissimilar materials are present at the top and the bottom of the wall. Masonry is placed on reinforced concrete the reinforced concrete slab or if it is the ground floor you might have the damp proofing course.

So, typically due to thermal strains, you will have crack formed at the top and the bottom allowing rotations at the top and the bottom of the wall. So, it is acceptable to assume that the wall is pinned at the top and the bottom, but if you want to assume that it is closer to a fixed-fixed situation rather than a pinned-pinned situation, you have to go back to the boundary conditions that you are imposing in solving the differential equations.

Same is true for a wall, let us say a parapet wall and if you are estimating the load carrying capacity of a parapet wall. I would assume it is fixed at the base and free at the top and you might want to relook at the expressions and the boundary conditions used for estimating the expressions for arriving at the expressions of P and delta. Again, to recall what we have done, we have actually in this case considering the symmetry of deformations been working on one half of the height of a wall right. If you remember the integration was done from 0 to h/2, because of the symmetry. So, when you working on a cantilever you will have to do it from 0 to h. So, these are some aspects that you might want to keep in mind.

One aspect that I touched upon as I was completing the derivation in the previous lecture is that, this expression is going to be valid as long as $y^2 \geq \frac{f_c}{6E}\left(\frac{h}{t}\right)^2$. Physically, we are looking at instability and the effect of instability in the system.

So, you will be able to estimate a peak value and you will see that beyond the peak value this is going to be a reduction in the load carrying capacity of the wall, but values beyond the limit of $y^2 = \frac{f_c}{6E}\left(\frac{h}{t}\right)^2$ cannot be estimated. So, that is a limitation that you should be aware of and as I said we are interested in the peak value of load that the wall can carry. So, the peak value of load in this case is estimated as the maximum value that you can, you see that the wall is able to resist and typically the limitation that you are talking about would occur a few steps beyond this point right. So, you need to keep this in mind.

So, the curves that you see in the graph here, pertain to a single wall right or 2 walls, one wall which has a h/ t of 16 and other wall which has a h/t ratio of 18. So, if this is the basis to be able to arrive at the vertical load bearing capacity of a masonry wall of certain dimensions and strengths, compressive strengths of the material. How is that you can look at limits of slenderness ratio, limits of the eccentricity ratio of the load, eccentricity ratio on the wall and that is how typically codes would give you the response of a masonry wall under compression right.

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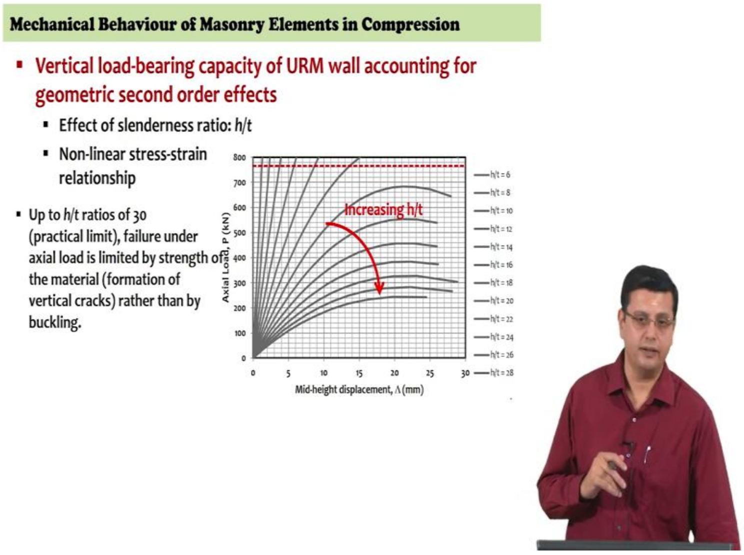

So, what I am trying to examine here is the effect of slenderness ratio and so, I look at several walls with changing slenderness ratios and changing eccentricity ratios and do we get a pattern and does that pattern give a some indication of how codes are going to be able to give you simple charts for arriving at the load that you must design the wall for and limit eccentricities and slenderness.

So, let us examine the effect of slenderness ratio to begin with. The graph that you see here has a number of slenderness ratios, you see that the ratio goes from something as low as 6 to something as high as h/t of 28 ok. And we are reaching practically limiting values and we are talking of slenderness ratio of 28. So, a slenderness ratio of h/t of 6 is a very thick wall right and is not going to be affected so much by second order effects, marginally by second order effects. Primarily if this wall is loaded till failure it should fail in crushing; it should not fail in fail in buckling.

So, this is something that, you can see with increasing h/t ratios the effect of buckling comes into the comes into the picture for initial values that are there in this graph, you really not looking at; really not looking at any effect, any practical effect of the slenderness itself. The red dotted line that you see there is the value at which for the strength considered to prepare this graph, the edge compressive stress actually reaches the value of f_m, which means the curves that you see above beyond the red dotted line really do not make sense because, for all practical purposes the wall cross section has reached with compressive strength and the curves below that red dotted line is what really matter .

Now, one fundamental aspect that we must keep in mind is that, the these curves have been arrived at assuming we have a triangle distribution of compressive stresses in the compressed zone. We have the entire cross section, part of it is cracked, part of the section is carrying loads and this part of the equilibrium of the system. We are assuming that till failure, the stresses there are linear elastic, the triangular distribution continues.

But I am sure all of you are familiar with the way we deal with stress distributions at failure in a material like reinforced concrete. There is softening of the material as the material edge compressive stress reaches the compressive strength of the material because of which the stress distribution cannot be linear, the stress distribution has to take a different shape. Parabolic shape is typically, what we start seeing, the edge compressive stress reaches the value of compressive strength and the consecutive fibres start softening.

Now, if one were to consider the softening, there is further level of accuracy in the estimates. These calculations have been based on the linear elastic distribution of stresses in the cross section implying, we have conservatism in this estimate right. So, if you were to examine different codes in different countries some of them use a non-linear stress-strain relationship for the cross section, the compressive stress in the cross section. So, if you were to do that you will get values that are different for same h by t or same e by t ratios.

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Right your question is again to do with what do all these graphs, what do all these curves in this graph represent with respect to the red dotted line that we are looking at right. Yes, that is exactly what I said that the red dotted line is the point at which the edge compressive stress reaches the value of compressive strength of the material ok. So, for all practical purposes with the for us, we are assuming linear elastic response of the cross section. So, we have not built in non-linearity or inelasticity in our estimates. So, we need an artificial way of saying ok this is where I stop the response.

So, I artificially come and stop the response, when the edge compressive stress reaches f m. Which means, you are right and saying that the curves above this red dotted line, are not to be considered right, you can estimate them, but you not to be considering them simply because crushing would have occurred in the cross section and that is now dictating the behaviour of the wall and not the instability as the equations are going to be prescribing.

So, we will examine a close up of the of the lower portion and it will become clearer to you, but the point that I wanted to make is you can get a deviation from these curves. If you use different code and that comes specifically because of the fact that, this is based on linear elastic distribution of stresses, but in reality you are going to have a non-linear distributional stresses.

Again, if you were to examine this against real data, you will see that this overestimates. This underestimates the load carrying capacity of the wall, which is airing on the conservative side and good as for as a design approaches concern, but the reason for the divergence between the real values and these is simply because of the fact that we are assuming an unrealistic distributional stresses even at failure. That is the point that I want to make.

So, if I were to zoom in on the portion of the graph capped by the load at which edge compression stress reaches compressive strength of the material. This is the valid region of the; valid region of the load deflection P delta graph itself for the different walls. And you can immediately see, that all those curves depicting walls which have a slenderness ratio that is rather small, which are thick walls are not governed by instability, you do not see the the curve goes way beyond the the red dotted line.

Meaning that, they not governed by instability the peak values according to those expressions have not been reached yet, but the material crushing has actually occurred which means, those walls are not being governed by second order geometrical effects. Those are being governed by material failure in the cross section itself. Whereas, the graphs below the last 1, 2, 3, 4, 5, 6, 7 graphs, which are basically pertaining to h/t ratios of 18 to 28 are the ones which are being governed by second order effects right.

So, I would; I wanted to focus on this to tell you the role h/t plays; it changes the failure mechanism in the wall itself. Now, question would be ok what is the maximum h/t that you can consider and in experimental tests it is been seen that up to h/t ratio of 30 right, you can have failure that is limited by the strength depending on the strength again this set of graphs have been made for a certain compressive strength of the material and a certain modulus of elasticity of the material. If I were to change that you will have a dependency based on the strength of the material and the stiffness governed by modulus of elasticity of the material; you have a very stiff construction versus a material which is more deformable you going to have differences again.

So, your assignment will actually make that quite clear to you, but if you look at a practical limit up to which the axial load is limited by strength over buckling that value is about 30. So, generally speaking and this statement is general irrespective of strength and stiffness of the wall itself. Now, what it means is if you have a h/t ratio 30 and above it so close to instability merely because of its slenderness ratios; nearly because of its slenderness ratios you do not need to load the wall even under its gravity under itself weight you can have instability in the wall itself.

So, this is the other aspect that I did not mention, but was there in our initial assumptions, is for our calculation, we have not used the self weight of the wall right. And that is an acceptable assumption, because the self weight of the wall is going to be a very small fraction of the total load carrying capacity the P critical of the wall itself and particularly so, for smaller values of h/t. So, that is another assumption that comes into the picture, but will not change your estimates significantly ok.

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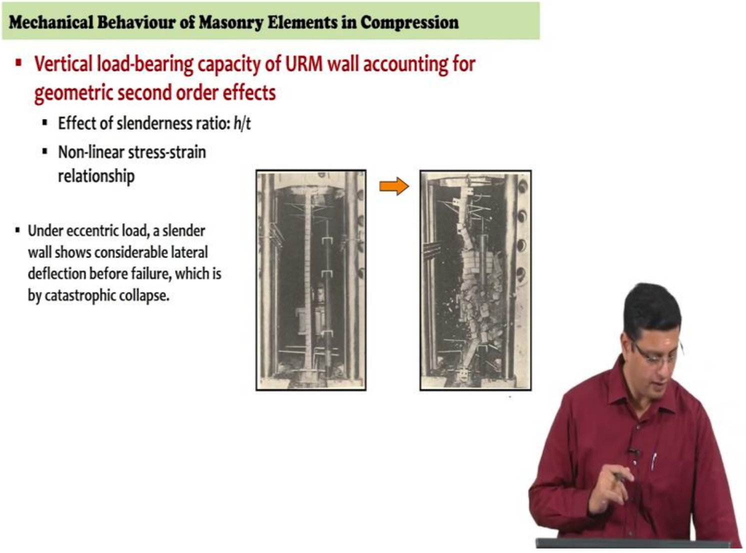

And it is observed that, when you have slender walls; when you have slender walls you have significant lateral deflection before it can actually fail ok. So, if you were look at a slender wall and look at the P delta for that wall you will see and in fact, in this figure you can visually see the lateral deflection that the wall is undergoing, but it is still stable right.

So, slender walls show significant lateral deflection before failure, but now the cross section is subjected to; quite part of the cross section is cracked, rest of the cross section is subject to quite a strong gradient in compression right. Strong gradient in compressive strains and failure will be catastrophic. You get sudden buckling of the cross section because, the edge compressive stress has given away your cross section is already limited because of cracking.

So, you can get catastrophic failure these are tests that have been conducted several years ago, but give us a very clear, very clear picture of what sort of a failure mechanism you can get. And that is fundamentally, the reason why slenderness ratios are limited by codes. You have cap on the slenderness ratios because, what you get by optimizing is a very brittle failure mechanism. And codes typically tend to avoid occurrence of brittle failure mechanisms.

You want to avoid the occurrence of brittle failure mechanisms. And you will start appreciating in a few lectures, that depending on the earthquake zone that we are working in, the slenderness ratios of walls is further limited. So, if you are in a high earthquake zone, slenderness ratios can come down to as low as 12 and that simply because, you have lateral forces along with the gravity forces. Under gravity forces to avoid buckling control behavior you limit the slenderness ratios, but in the presence of lateral forces you have if you have very slender walls you have out of plane failure which cannot be controlled.

And so, you have slenderness ratios, which are controlled to prevent premature out of plane failure. That is not primarily in compression, but because of the lateral forces, but we come back to the concept of slenderness ratios being controlled. In this case, it is being controlled for gravity, but in earthquake zones, its controlled to avoid significant lateral deflection and failure due to the inertial forces themselves. So, that is a point that we will come back to very soon ok.

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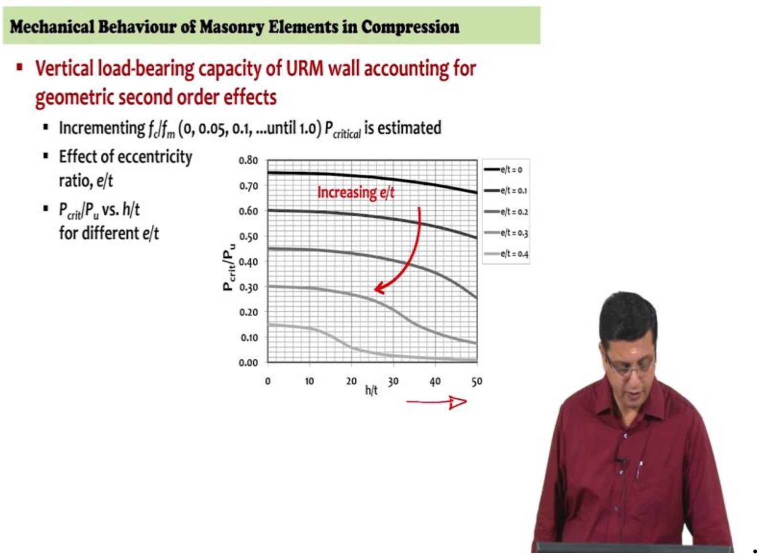

So, I wanted to show you, how with this sort of an estimate, a structural code would then give you the framework within which you design walls under second order effects. So, as I was mentioning earlier, I can take a single wall for a given strength of the masonry and modulus of elasticity. You remember that modulus of elasticity, we could have an empirical relationship between the modulus of elasticity and the compressive strength. So, E_m the modulus of elasticity of the material could be k times f_m. So, f_m known, I have the modulus of elasticity with me and so, for a single wall, I keep incrementing the edge compressive stress and I get the $P_{critical}$.

Now, I can look at different walls and the able to arrive at with respect to slenderness ratio and with respect to e/t, what sort of trends can I get? And that is what codes typically give you in terms of graphs. So, if you look at the effect of eccentricity ratio now, this graph here is making use of I would say several walls, where the eccentricity ratio is varying from no eccentricity of the axial load, which is concentric compression to a very high eccentricity ratio of e/t of 0.4 right.

And if you remember, we said, if you have an e/t of 1/6 right. Plus or minus 1/6 that is the middle third, you can start having cracking in the wall, but codes would typically allow you to go up to about one third even one half. One half is typically when you start having instability.

So, you can see that here; the here these graphs that have been made these curves that have been made of a five different values of eccentricity ratio e by t going from 0 all the way to e by t of 0.4. Now, I use walls of different h by t; so if you can see the X axis here; in the x axis, we have different h by t ratios going all the way from 0 to 50. Which means, I am basically looking at different geometrical configurations of height and the cross-sectional dimension t of the wall.

So, I have an entire range from 0 to 50 and in those walls I am changing the eccentricity ratio from 0 no eccentricity of load to an eccentricity of 0.4t. I will be able to estimate the $P_{critical}$ for each wall right. I have an entire matrix now. Different values of h/t, different values of e/t and for each of those I estimate what is the $P_{critical}$. Some of them will fail by instability being reached, some of them will fail by the crushing of the cross section, but I have now the entire set of values of $P_{critical}$ and if I normalize $P_{critical}$ by P_u. P_u being nothing but the compressive strength of the cross section right, the load at which compressive strength of the cross section is reached.

So, P_u is nothing but f_m multiplied by thickness of the wall into length of the wall right. If I do that, then this is the kind of effect that you would see with increasing eccentricity ratios, you see that there is a drastic drop in the critical load at which failure to compression is happening with respect to the material strength defined by P_u right. So, this is one graph, where you are looking at both the effect of the eccentricity ratio and the effect of and the effect of the slenderness ratio. And you will appreciate that the effect of the eccentricity ratios quite drastic; you see when you have no eccentricity, the black line at the top, with moderately increasing eccentricity ratios you get a drastic reduction in the load carrying capacity of the wall and compression. You are reaching values of 10 percent of the compressive strength of material. So, the second order effects have a very significant role and have to be accounted for in estimating the value of the vertical load carrying capacity of the wall itself.

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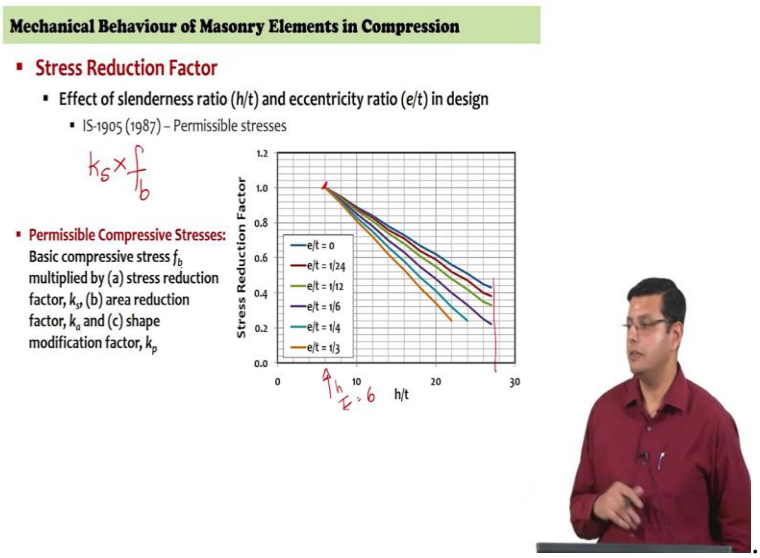

So, I have here a graph that looks at how our code, the IS code presents the reduction factors. So, what you saw in the previous slide was really the reduction factor that you must consider, if you have a certain eccentricity ratio of loading e/t or certain h/t ratio the slenderness of the wall itself. So, these are limiting curves, but mind you these curves are again from the equations that we have developed based on linear elastic distribution of stresses in the cross section.

So, with the example of IS 1905 (1987), which is code based on permissible stresses. The way the design occurs as far as 1905 is by limiting the stresses that will occur in the masonry for compression due to gravity forces, or compression due to gravity forces plus lateral forces. So, you have to arrive at, what is the permissible level of compressive stress. To be able to arrive at the permissible level of compressive, what the code requires is that the estimate of what is referred to as a basic compressive stress and we will examine this concept in a in future classes as well.

So, basic compressive stress, which is nothing, but the compressive strength of the material denoted as f_b is then multiplied by a factor that accounts for the slenderness ratio and the eccentricity ratio together right. So, what the code does is it requires that the slenderness h/t and the eccentricity e/t. Together gives you the factor that you can use as a stressed stress reduction factor, this stress reduction factor is in IS code referred to as K_s, you multiply this with the basic compressive stress f_b ok.

Now, this is then multiplied with other factors, which we will examine subsequently, but the point is the basic compressive stress is a factored strength of the material. For example, if I know that the strength of the masonry is 4 MPa at failure, this being a code that deals with working stresses will not take the value of 4 MPa. We will probably use a factor of safety of 4 and say that the basic compressive stress, which is your limiting compressive stress under working loads is 4 MPa by 4, 1 MPa.

So, 1 MPa would be the basic compressive stress, which is your limiting compressive stress. That limiting compressive stress is further reduced by what is referred to as a stress reduction factor and this value of the stress reduction factor is something that you can arrive at based on the eccentricity ratio of the load and the h by t ratio.

So, if you look at this graph here, this is the graphical representation of a table, table number 9 which is given in the IS code- IS 1905, which will tell you I know the total eccentricity ratio of the loads coming onto the wall, I know the slenderness that I am going to be adopting. What is the case factor, that I am further reduce the working stress, the basic compressive stress f_b by, to account for second order effects.

And you can again see here, that if I am looking at a wall of h/t of 6 that is where we start, you have a h/t of equal to 6, your correction factor is one. There is no correction required for a wall which has a slenderness ratio of h/t, 6. So, that is where the calculation starts from and you see that we reach values of 27 in the code we go up to value of 27 that is the value here and you see that the stress reduction factors are of the order of 0.2 and slightly more than 0.2. Which means the permissible stress is going to be reduced so much already of reduce the permissible stresses to account for the working stresses and then you further reduce it to account for the second order effects.

So, f_b in reality and we will come back and examine this later, f_b is multiplied by three factors, a stress reduction factor is what we talked about. There is an area reduction factor and shape reduction factor which come in later for the time being you can assume that both are one. Then the only factor that you multiply f_b with this K_s and the K_s is what you get from this graph itself right. Correct so, the you have to make an estimate of the eccentricity, have an estimate of the eccentricity of the load and that allows you to estimate the eccentricity ratio e/t.

So, you know e by t you have chosen a certain h/t for the wall construction and then you look at what the stress reduction factor is. Now, if the stress reduction factor into the basic compressive stress for the chosen material. Lets say, you have chosen a material which is of a compressive strength 8 MPa or 8 N/mm^2. The f_b would be about 1 quarter of that value, so I have a 2 MPa of basic compressive stress into a stress reduction factor.

Now, let say you arrive at the permissible stress by multiplying K_s into f_b, but its far lower than what you want as the load carrying capacity of the wall. Your only options are change the wall cross section or change the material by increasing the strength of the material. So, it gives you a framework within, which you can look at dimensioning of masonry walls or choosing the strength of the masonry wall itself right. That is as far as unreinforced masonry design is concerned and the use of these factors to account for stress reduction itself ok.

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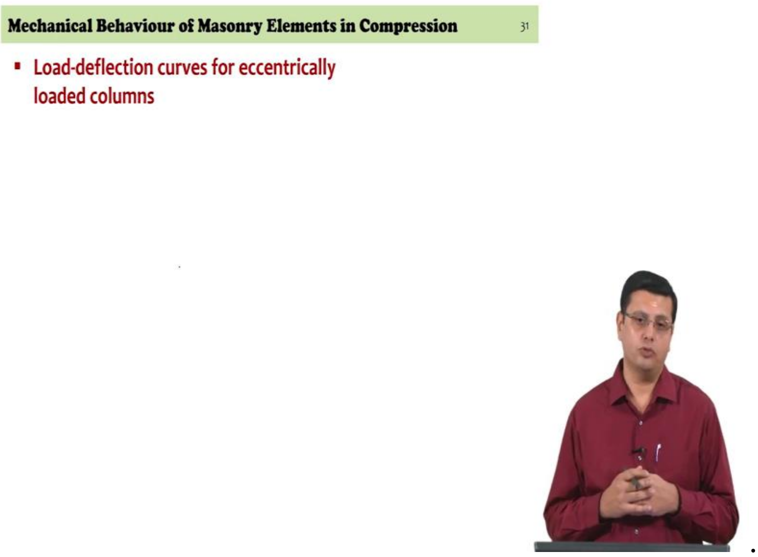

So, I think I will stop here and look at the rest of the international codes and how this is adopted. And also the effect of other geometrical effects like wall slab interaction and the rotation of the joint how it is going to affect your assumptions on the boundary conditions of the wall and whether that is going to affect the compressive strength or the load carrying capacity of the wall. See you in the next lecture.