Design of Masonry Structures
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Module – 03
Strength and Behaviour of Masonry Part – II
Lecture – 12

Good morning so, we continue looking at the Strength and Behaviour of Masonry under different actions, we were examining the behaviour of masonry under compression and trying to look at the possibility of a closed form solution that gives you the compressive strength of masonry.

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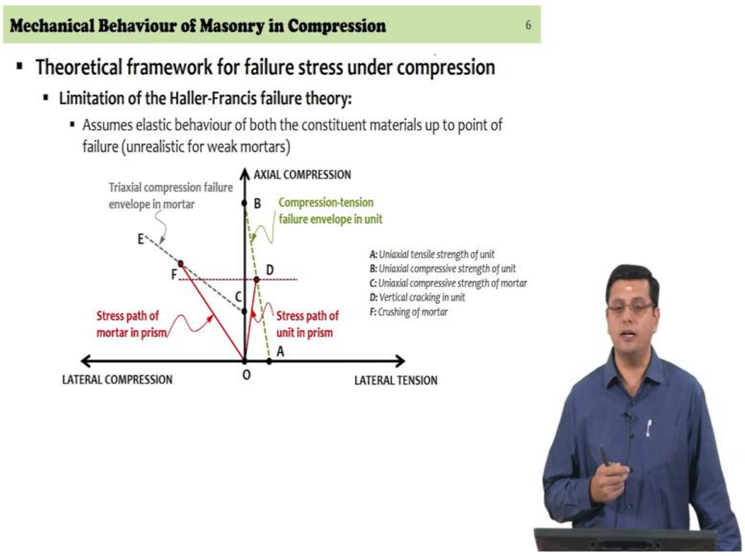

With the knowledge of its constituent properties- the unit and the mortar and the theoretical framework that we were looking for failure under compression on the basis of linear elastic; an assumption of linear elastic behaviour of the constituents was what we were looking at in the previous lecture. And we were examining, as we were concluding the lecture, how this important assumption that the constituents are behaving in a linear elastic manner actually does not explain completely the failure mechanism as physically observed, right.

So, the Haller-Francis theory which is what we actually defined, neglects this non-linear behaviour that you should expect of the constituent materials and the point is, if it is linear behaviour that you are going to assuming both in the unit and in the mortar it is going to be difficult to explain how the failure in the brick and the crushing of the mortar actually happen as simultaneous phenomena in reality, but the theoretical framework developed in this manner could not be able to give a justification for the physically observed phenomenon.

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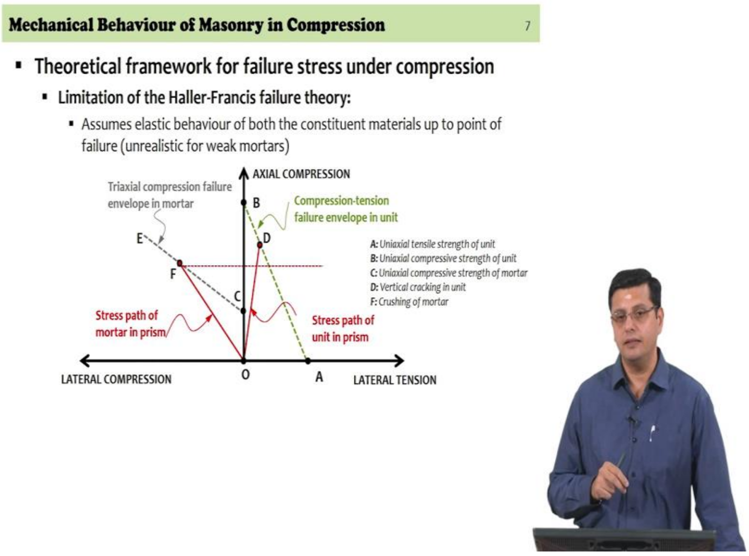

So, therefore, further developments in this direction have actually examined the possibility of looking at an inelastic behaviour of the mortar. This becomes an unrealistic, the elastic linear elastic assumption becomes an unrealistic assumption both for situations of the brick unit failing first or crushing happening in the mortar early on.

So, this slide that you can see is where the unit has reached level of biaxial tension that causes cracking following which is where the crushing of the mortar is expected to happen. However, if you are looking at very weak mortars then as per this theory you should have crushing in the mortar that that happens quite early on and the failure in the brick unit that happens after which again does not really explain the physical phenomenon of co-action between the two. So, you see the disparity in the values of F and D which do not give us clarity on how physically the phenomenon is occurring with the cracking in the brick unit leading to loss of confinement in the mortar causing crushing failure in the mortar itself.

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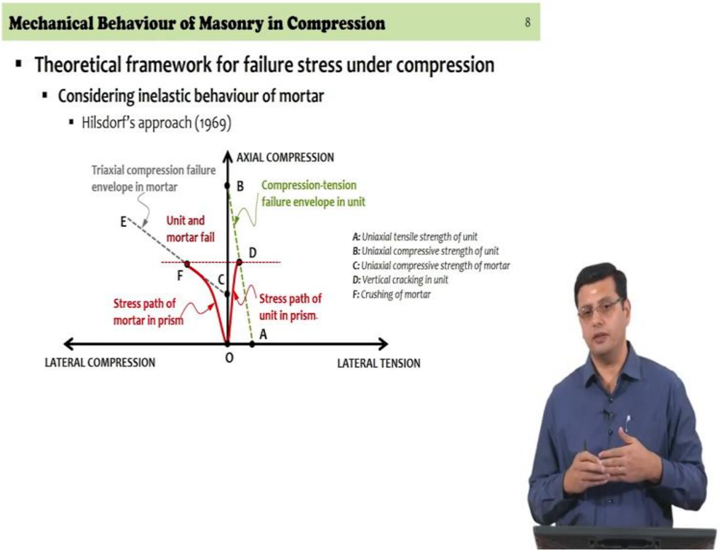

So, the extension that we will look at, which is useful to examine is one way we consider that the mortar behaves in an inelastic manner. Now between the mortar and the unit you will agree that the behaviour of the unit is being the brittle material, probably closer to a linear response and it is ok to consider a linear elastic behaviour there, but look at a non-linear behaviour and an inelastic behaviour of the mortar itself.

So, we looking at the basis of this approach is the Hilsdorf's approach of 1969 and if you remember the initial slide that I showed you when we will looking at co-action between the two, the mortar and the unit is where the stress path in the unit also has a inelastic behaviour of stress path of the mortar in the prism also has inelastic response and you can see that it is almost a simultaneous occurrence of failure in both the unit and the mortar.

So, if this is really the more appropriate basis to describe a theoretical framework for the failure, then what is important is that you have analytical form of the failure of the unit and an analytical form of the failure of the mortar and considering factors of compatibility and equilibrium be able to use these two and write the final expression. So, that is exactly what we will be looking at.

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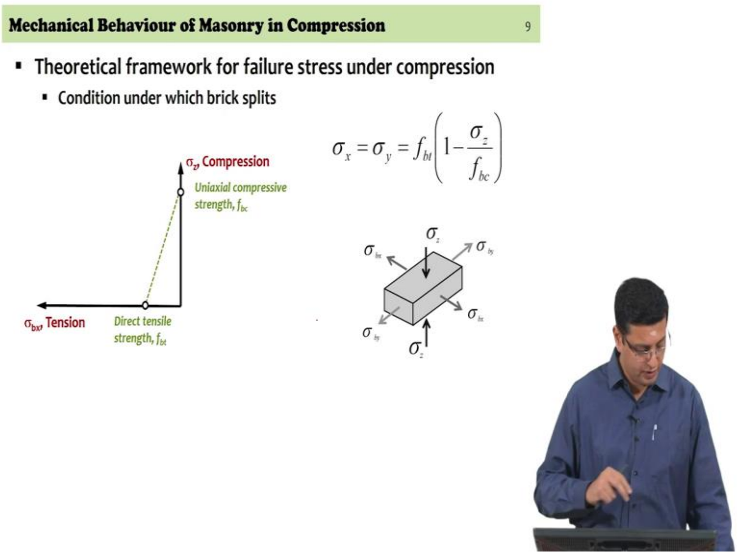

So, the condition under which the unit is going to split we looked at this earlier, the straight-line failure surface between uniaxial compression and uniaxial tension in the brick masonry unit itself. If you are looking at the axial compression which you can estimate the axial compressive strength of the brick unit uniaxial compressive strength of the brick unit from a flat-wise compression test and you get f_{bc} the uniaxial compressive strength of the brick. You do a direct tension test on the brick unit and establish the direct tensile strength of the brick unit and it is a straight-line failure plane that you get for the condition under which the brick is going to split. So, this is something we have used earlier in the Haller-Francis expression themselves.

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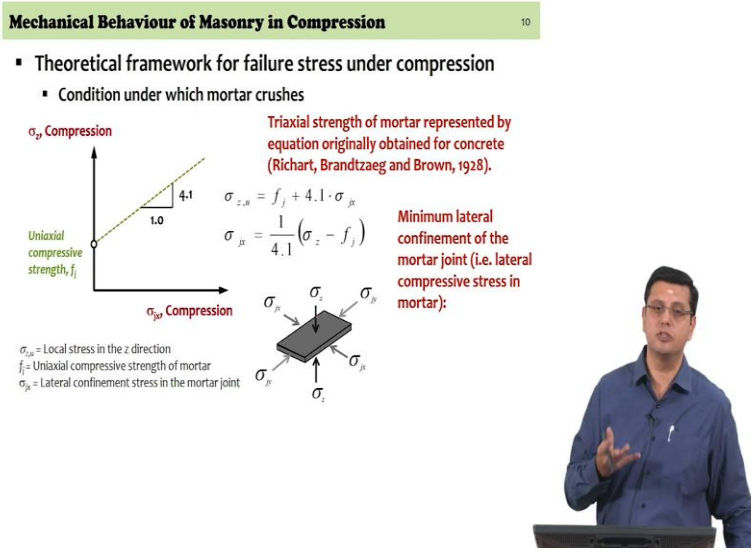

Now let us examine mortar and what happening to mortar, and we understand that when we are doing uniaxial compressive test on the mortar it is under uniaxial compression, but in the prism under compression the mortar goes into state of triaxial compression. So, you need to alter the uniaxial compressive strength of the mortar to be able to explain what strength you would get in mortar in a confined manner right.

So, the triaxial compressive stress, multiaxial state of stress with triaxial compression is something that we should be able to account for and so, if you are looking at the axial compression versus the bilateral compression that you have σ_{jy} and σ_{jx}. Knowing that the uniaxial compressive strength of the joint material from a uniaxial test, it is f_j, it has been observed that the behaviour that you would get for mortar if subjected to a triaxial condition in compression would follow a linear pattern. The important aspect to understand is what is the factor by which, this is going to increase so, the slope of that line is used and this is basically based on tests that have been conducted on concrete.

So, the model that we looking at here is coming directly from concrete, developed quite early in the 20th century and the is of the form that the failure strength in compression of the mortar is an additive form of the uniaxial compressive strength plus, the compressive stress in the lateral direction multiplied by a factor that comes empirically from experimental observations. So, this is the failure surface that we would use for the multi-axial state of stress in compression of the mortar.

We are interested in rewriting this in terms of the stress in the joint and therefore, it is just rewritten in terms of σ_{jx} with the knowledge of the uniaxial compressive strength of the mortar joint material itself.

$$\sigma_{jx} = \frac{1}{4.1}\left(\sigma_z - f_j\right)$$

So, this basically this last expression that you have seen is giving us literally at the point of failure is telling us how much minimum lateral confinement is available to the mortar joint just before the point of failure. So, σ_{jx} is the lateral compressive stress in the mortar. So, this expression now in terms of the uniaxial compressive strength and the stress in the z direction which is what we want to establish for the failure of the masonry assembly is written in terms of σ_{jx} ok.

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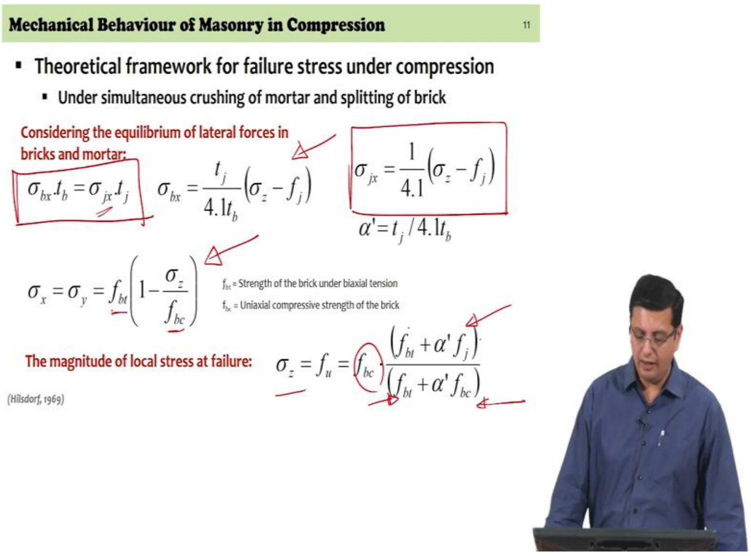

So, with the failure planes for the constituents defined if we assume that now these two phenomena are meant to occur together; that is the crushing of the mortar is happening as soon as or immediately after the splitting of the bricks in the bilateral tension. Then we use our original equilibrium equations and rewrite the expressions that we saw in the previous slide to be able to pull out an expression for σ_z now from the material constituent strengths themselves.

So, from equilibrium of forces we established that σ_{bx} which is the lateral stress in the brick into the thickness of the brick unit itself is equal to σ_{jx} which is the stress the lateral stress in the mortar into the joint thickness t_j established because there is bond at the interface considering compatibility and equilibrium you get their expression.

$$\sigma_{bx} = \frac{t_j}{4.1t_b}\left(\sigma_z - f_j\right)$$

And now we are able to write down the earlier expression that we had in σ_{jx} in the previous slide in terms of the lateral tensile strength; the lateral tensile stress in the unit itself.So, we write the previous expression, the previous expression if you remember in the previous slide was in terms of σ_{jx}. So, σ_{jx} was pulled out from the expression, the empirical expression of the failure criterion for mortar, we use that and with the consideration of equilibrium and compatibility, with this defined, we rewrite the expression in terms of σ_{bx}. That is what is done in this particular slide; give a factor this part $t_j/4.1t_b$ is then replaced with this α', we could simplify the expression a little.

So, now we have an expression that comes from the failure criterion for mortar and using equilibrium in compatibility and expression for the mortar and now an expression for the unit itself coming from the consideration of the linear failure surface in tension compression of the brick unit itself.

$$\sigma_x = \sigma_y = f_{bt}\left(1 - \frac{\sigma_z}{f_{bc}}\right)$$

So, this is again the lateral stresses σ_x or σ_y in the unit defined in terms of the failure stress in tension and the failure stress in compression under uniaxial conditions of the brick itself.

So, you have now these two expressions which are going to be linked now if you were to merge these two expressions and pull out σ_z from this, you get the at ultimate conditions, because you are considering the strength in the mortar uniaxial compressive strength of the mortar, the uniaxial compressive strength of the unit, the tensile strength of the unit, would then and the compressive strength of the uniaxial compressive strength of brick unit again you get the expression in sigma z.

$$\sigma_z = f_u = f_{bc} \cdot \frac{\left(f_{bt} + \alpha' f_j\right)}{\left(f_{bt} + \alpha' f_{bc}\right)}$$

This is the final expression that is used with respect to the Hilsdorf criterion.

However, in comparison to experimental results there is a slight deviation and that is really because of the fact that in this we are assuming that the stress is going to be uniform at all points in the cross section which is not the case, stress is typically defined at a point. But here there is implicit assumption that entire cross section is under the same state of stress. So, that non uniformity that is there in the distribution of stress owing to several factors needs to be accounted for.

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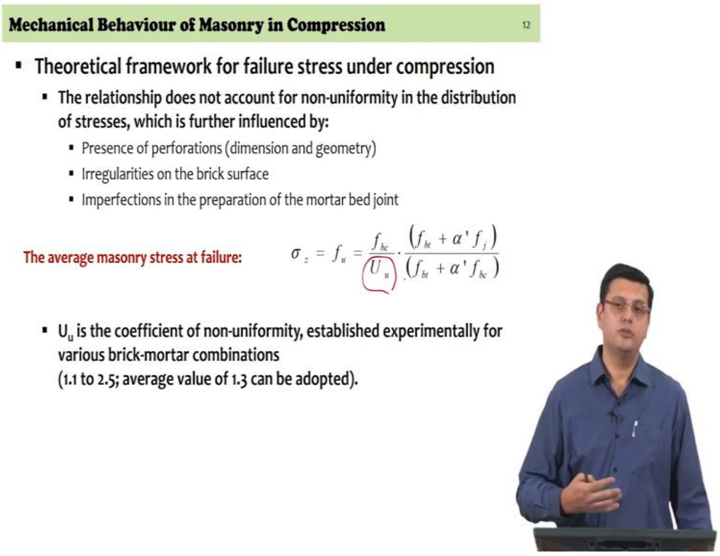

So, it is included in an empirical manner in this expression.

$$\sigma_z = f_u = \frac{f_{bc}}{U_u} \cdot \frac{\left(f_{bt} + \alpha' f_j\right)}{\left(f_{bt} + \alpha' f_{bc}\right)}$$
$$U_u = \text{coefficient of non-uniformity}$$

So, this non uniformity is actually coming from several sources, some of the sources are you can have perforations in the geometry, in the block itself and then the dimensions of these perforations can be different that can lead to a different state of stress in the block in the masonry prism itself. You can have irregularities on the brick surface and therefore, there will be concentration of stresses in some points. And smaller level of stresses in other points the mortar bed joint is again not something that is completely perfect.

And therefore, again can contribute to a non uniform state of compressive stress and therefore, the expression that we saw in the last slide is altered to factor in the effect of this non uniformity. And the non uniformity factor is actually brought in; it is to be able to match experimental results in a way. And of course, this is dependent on the type of masonry that you are looking at some more variable than other types of masonry.

So, this value actually comes from experimental investigations on different types of masonry you can see that if you can adopt an average value of 1.3. So, 30 percent correction is required; however, depending on the type of mortar and brick unit these values can actually vary quite significantly from about 1.1 to about 2.5. So, this theoretical framework, the Hilsdorf theoretical framework, actually gives you better physical interpretation of the failure mechanism and is appropriate primarily because it considers the inelasticity in the material, the weaker material the mortar in formulating the expression itself.

So, so have would looked at couple of formulations one based on linear elastic approach and one which bases base bases itself on the inelastic behaviour of the material. So, you could look at how well they are able to match experimental results. Both fairly well do their jobs in capturing the failure strength of masonry.

Student: Sir f_j and f_{bc} are both uniaxial compressive strength?

Yes. So, the yes that is true. f_j is uniaxial compressive strength, f_{bc} is the uniaxial compressive strength of the unit, j is of the joint material and f_{bt} is the tensile strength of the unit. So, the basic point is that it is an assembly of different constituents it is a composite masonry is a composite construction.

Now, if I know the strengths of the individual elements can I arrive at the strength of the assembly in compression it is we have seen that it is not additive, it lie somewhere in between the strength and deformability of the unit and the strength and deformability of the mortar. So, the whole attempt is to be able to look at an analytical framework for it and you have seen that it is possible to link it to the geometry. So, the joint thicknesses are what alpha prime is standing for; the ratio of the joint thicknesses. You also have the factor 4.1 that comes in here and then the uniaxial strengths of individual materials compression mortar and unit and tensile strength of the unit.

However, there is still variability and the non uniformity factor is something that is accounting for the variability that we are not able to capture analytically within the within a closed form solution itself ok.

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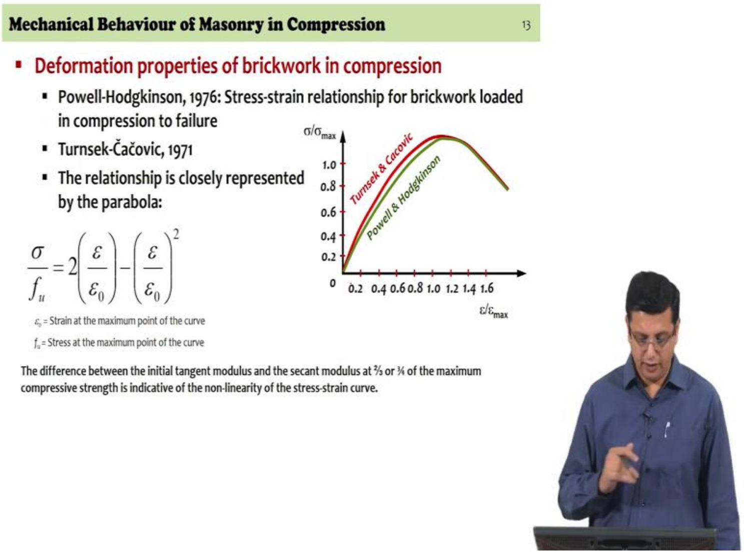

So, continuing it is useful to look at in terms of stress- strain curve what sort of what sort of a analytical formulation fits the stress-strain curve of masonry itself. Again ,we are examining compression and in a way it is not very different from concrete in its behaviour in compression and studies were carried out few decades ago and they still valid because they do a rather good job in capturing the stress strain behaviour of masonry in compression.

So, work carried out in the 70's- Powell and Hodgkinson and Turnsek and Cacovic are two typical initial works that tried to examine what sort of a stress strain relationship do you get in different types of brick work. They also examined do you get differences if the bond patterns change, do you get differences if you are looking at face-loaded, if you are looking at flat wise compressive strength that is a prism made out of bricks laid out in a flat wise manner versus brick on edge and so on.

So, several iterations were carried out in the configuration of the brick work and the type of masonry units and mortar as well. However, that is the typical stress strain curve and you can see a certain idealization the stress strain curve is possible because it tends to the close to a parabolic shape and here represented in terms of the ratio normalized stresses σ to σ_{max} and ε to ε_{max}. Then makes it possible to closely related to the shape of parabola and have if you need to use it for design purposes the equation of the parabola very well matches the stress strain curve of masonry itself.

The fact that a parabola is what is matching the stress strain curve of masonry, also tells you that if you were looked at in initial tangent as the description of the modulus of elasticity and secant modulus close to the peak. Let us say two-thirds or three quarters to the peak you will see a significant difference between the model I defined as a tangent modulus in initial elastic tangent modulus and the secant modulus. And this is principally telling you that the material starts showing non-linearity quite early on right ok.

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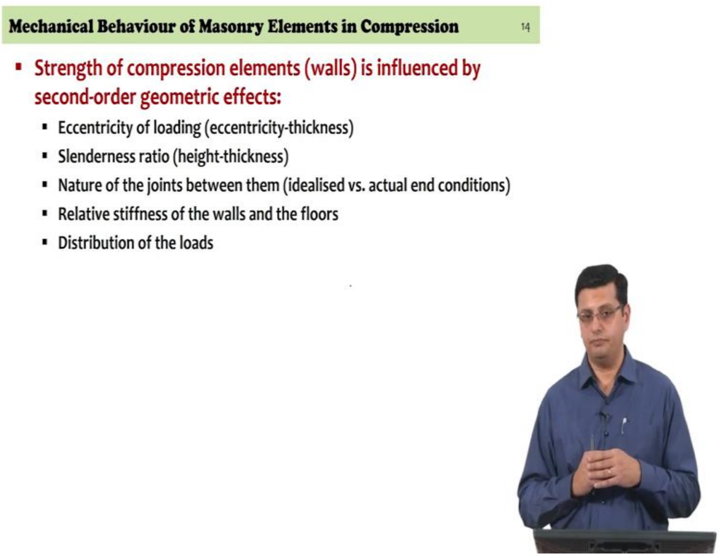

So, with that let me move on to elements. So, what we did examine in the previous lecture and today’s lecture in the beginning is the behaviour of masonry as a material in compression. And the expression is a representation of the strength of the material, but you do not get the same strength in a wall, for example, and that because you have other effects that come into the picture. So, we have now moved on from strength of the material of masonry to the strength of masonry element in compression and what effects the strength of walls in compression for example, and we will again examine if there is eccentricity of your axial loads, if there is eccentricity of your compression, what sort of effects occur and what sort of effects should you be able to analytically capture if you are estimating the strength of the masonry element itself.

So, the moment we look at structural elements like walls, the slenderness of the wall, the basic geometry of the wall can start inducing second order effects. We have just been looking at a first order effect as for as arriving at the compressive strength is concerned, but you will never get that compressive strength of masonry in your wall. And therefore, if you where to estimate the compressive strength of a masonry wall, you cannot use the compressive strength of the masonry and say I have arrived at the strength of the wall. You need to factor in other problems coming in from geometry and we grossly refer to that as second order effects.

But there are several aspects that contribute to the second order effects and some of them are listed here. It is almost exhaustive- you can have an eccentricity of the loading itself that the load is not acting concentric there is an eccentricity induced by the load. So, if the eccentricity is e you already have P into e acting on the wall which would mean there is a component that is just the gravity load P plus the moment acting on the wall.

So, this itself can cause stress gradient and a strain gradient in the cross section and the compressive strength of such a wall is not going to be the same as when you are considering pure concentric uniaxial compression. So, this is typically represented in terms of the effect of eccentricity to thickness as a ratio, we talk of the e/t ratio and we are interested in understanding what is the e/t ratio that you are examining, where does e/t sit with respect to the overall thickness of the cross section, are you talking of e/t within the middle third or, are you talking of e/t that is outside the middle third, how severe is e/t right and codes would like to classify the way you deal with the compressive strength such walls to account for second order effects into different categories of e/t.

So, low e/t, medium levels of e/t and high levels of e/t and a cap on what e/t is going to be as far as your design is concerned. And where does e/t, where does this effect come from. It is simply because your superimposed loads are not always design to sit concentrically with the wall you will have because of the geometry of the construction an eccentricity of the load transfer itself. So, code giving you a limit on the eccentricity ratio implies that you have to have construction detailing that ensures that the eccentricities induced by; the eccentricity of the load transfer by superimposed elements has to be curtailed.

So, this is one of the important contributing factors of second order effects, another aspect that definitely has role to play is how slender the wall is right. Now typically codes would prescribe limits on the slenderness ratio that is defined here as the height to thickness ratio. We are looking at the least lateral dimension. So, you take the height of the wall to the thickness of the wall and again there are limits on what should be the slenderness ratio, because slenderness and eccentricity loading together is going to lead to compromise in the compressive strength of the masonry ok.

So, we will examine eccentricity of loading first and then the effect of slenderness ratio and you will see as we start looking at design that most codes would give you the reduction in compressive strength because of a combination of e/t and h/t effects. So, the IS code for example, would give you a table which has h/t on one axis, e/t on the other axis and you look at what is the fact by which you may reduce the compressive strength because increase in slenderness ratios and increase in eccentricity ratios would mean lesser and lesser compressive strength of the element itself.

Now, the other aspect that does have a role to play which very often we do not give enough attention to is you have a wall, you have boundary conditions that are the real boundary conditions that occur because of the construction detailing and typology and then for our calculations we idealize the boundary conditions right. You would idealize it has a fixed-fixed boundary condition top and bottom, you would want to idealize it as a pin-pin condition, but reality does not always have to be exactly what the idealized conditions are going to give you.

Now, which means that there can be a deviation you can actually work out some estimates of what is the rotational restraint and the translation restraint partial restraints, you may be able to estimate those partial restraints and actually use those partial restraints to be able to arrive at effects in the wall due to eccentricity of loading or due to slenderness effects. So, this is an additional layer of complexity that would definitely come in and therefore, first being able to idealize rather correctly what the boundary condition should be and if there is significant deviation from idealized conditions being able to estimate what those partial rotational translation restraints could be is the other aspect that needs to be looked at.

There is another aspect the moment we consider the boundary conditions the point is when you are looking at walls and floors interacting the amount to joint rotation that you can get really depends on what is the relative stiffness of the floor and the wall itself. The joint rotation that is that the floor is permitting is something that can alter the boundary conditions and the deflections in the wall itself and then of course, the distribution of the loads.

Now, we are not talking about single load, you can have multiple loads, you can have loads that are varying over the length of a wall and that can introduce certain difference in the estimates of the strength of the masonry wall, and together these effects contribute to geometric second order effects also referred to P-delta effects. So, what we are going to be doing now is examining what is the role of second order geometric effect in reducing the strength of the masonry wall and if so can we have a simplified analytical framework based on a set of assumptions, of course, to estimate the force displacement relationship, the P-delta relationship of the masonry wall itself.

So, if I know the geometry and the material strengths can I for the geometry estimate what the strength in compression of the masonry wall is ok. So, that is what we are going to be looking at in this half of the lecture.

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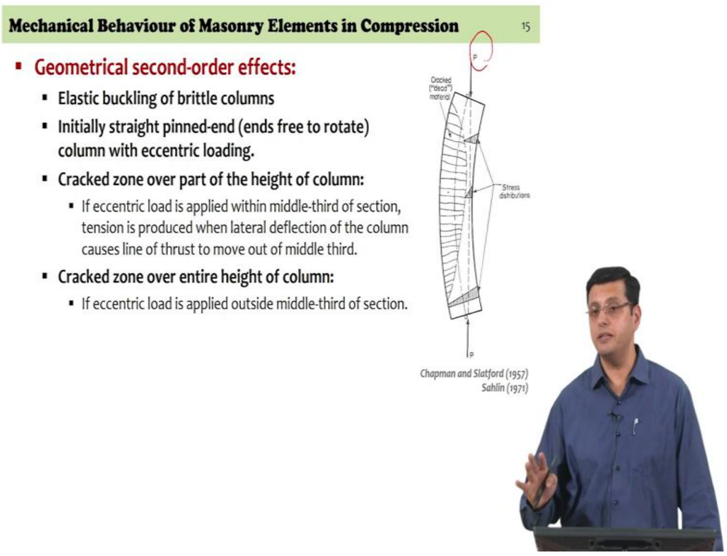

The assumption, the basic assumption that we make in this process here, we are looking at when you have eccentricity there is bending in the wall and that can basically lead to a situation of buckling. So, we looking at a brittle material masonry wall or masonry column and examining the phenomenon under elastic buckling itself.

For the case that we are going to examine we are going to be looking at pinned ends which means the ends are free to rotate, that is you might ask me when you construct a masonry wall how is that you get pin conditions at the ends of the wall, but typically what happens is you have a different material on which the wall is constructed. It is not brick work that is going to continue for the entire length of the building entire height of the building, you normally have a concrete slab that will come in at a certain point if you are looking at the ground storey and the plinth you might have a damp proofing course. So, you would normally have a different material at the boundaries of the wall and because you have different material thermal expansion coefficients will be different and that is sufficient to cause a cracking between these two surfaces.

So, you can work with the assumption that you have crack surfaces at the top and the bottom allowing for some rotation and therefore, the assumption of a pinned end, two ends being pinned is rather acceptable from the engineering calculations themselves. So, we looking at a wall that is initially straight subjected to eccentric loading and examine the deflections in the wall due to these eccentric loading and see if those deflections are actually going to compromise the force capacity, the strength capacity of the wall itself.

So, if this is the wall we are examining, we have an eccentricity of the load acting on it, P and we assume that the eccentricity is same at the top and the bottom; eccentricities can be different at the top and the bottom. Again, depends on support conditions, if you have full support and loads coming at a certain eccentricity different from what the reaction eccentricities are the base you will have top eccentricity and bottom eccentricities that are different.

Now, due to the deflection of the wall, due to the deflection of the wall because of the moment induced by the eccentricity of the load part of the wall will crack and part of the wall will remain un-cracked ok. So, there will be a part of the wall which goes into cracking along the height and you call that the cracked zone. This crack zone is really not going to be participating in the load carrying function, it is only the uncracked zone that is actually going to be actively participating in stress distribution.

So, we have seen earlier in our introductory lecture that if the resultant of the forces acting on the wall lie within the middle third of the section, then we know that the entire cross section is in compression, but the moment the resultant falls outside the middle third of the cross section you start getting tension and if the material is assume to have low or no tensile strength you can start getting cracking in a material which is brittle. So, under this assumption it is possible that for the deflection you have cracking in some part of the wall. If the crack has to if the entire height of the wall has to crack, it means a significant amount of the wall has thrust which is acting outside the middle one third of the cross section.

So, basically you need to account for the fact that part of your wall, based on your calculations should have cracked in part of the wall maybe in the un-cracked condition, which means you have now different cross sections to deal with as far as load equilibrium is concerned right.

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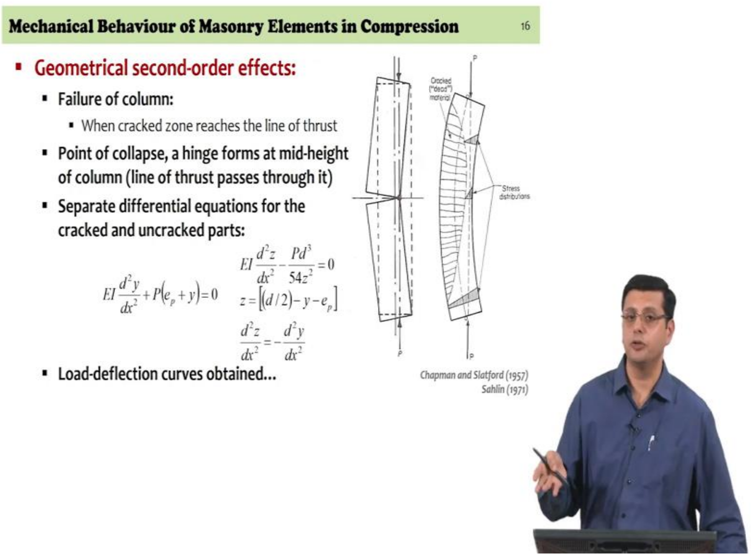

So, we will keep that in mind and use that as the fundamental difference between different resisting sections of the wall itself. The wall is expected to fail when the cracked zone reaches the line of thrust and that is at the moment of collapse itself you get a hinge that is forming at the mid height of the column the mid height of the column itself is the location where you have the maximum displacement and expect maximum cross section to have cracked.

So, let us say a significant amount of cross section has cracked at the mid height and now the resultant compression is basically passing through a point and when the stress level reaches the compressive strength of the material masonry, you get crushing and you can have failure. But now we have basically assuming that the mid height section is completely cracked, but the line of thrust is passing through a point which is now the hinge and we are considering that almost two blocks are capable of rotating about the hinge itself and still continue to equilibrium the axial load that is acting on the eccentric axial load that is acting on it.

Now, if you were to examine the wall along it is height which has some parts cracked and some parts uncracked, the deformations are going to be different and therefore, for the crack zone and the uncracked zone you should actually be using different differential equations to be able to estimate the deflections that occur because of the deflected form itself.

So, we will come to these aspect, you will have to consider a different differential equation for the cracked zone and the uncracked zone and then you could solve these equations for different boundary conditions and arrive at an expression to get a load deflection curves. The load deflection curves will actually be able to tell you how much the strength in compression of the element is going to be different from the compressive strength of material itself.

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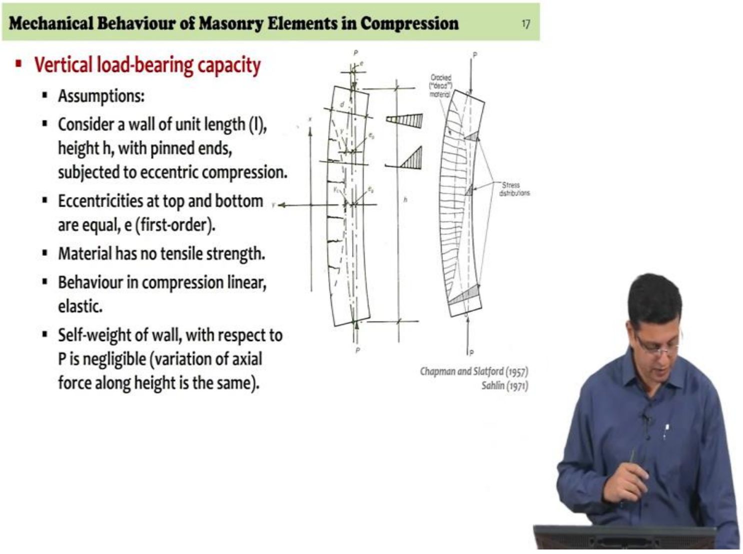

So, let us examine this we want to calculate the vertical load bearing capacity of the wall and these are the assumptions that we begin with. We are considering a wall of length l; height h, it has pinned ends and it subjected to eccentric compression. We have the same eccentricity at the top and the bottom; eccentricity represented with the notation e here top and bottom eccentricities are equal.

So, we are basically assuming a first order eccentricity; you can complicate this problem little more with and have e_{top} and e_{bottom} there are different you will have to then account for that in the differential equations. Your deflections are going to be different based on the relative values of e_{top} and e_{bottom}. We assume that the material has no tensile strength right, which means the moment you reach an eccentricity equal to the middle third of the cross section you get the conditions for cracking because zero tensile stress has been arrived at the extreme fiber.

And a fundamental assumption here, which can become a point that you take forward and improve with a non-linear stress strain relationship in compression. In this particular exercise we are going to be looking at the behaviour and compression as being linear elastic. To keep things a little simple the self weight is neglected is assumed to be very small in comparison to the superimposed load and this is also useful as a simplification, because then you can assume that the same load P is acting at all sections if you consider the self weight that is going to be incremently changing from the top to the bottom. So, this is again a simplification ok.

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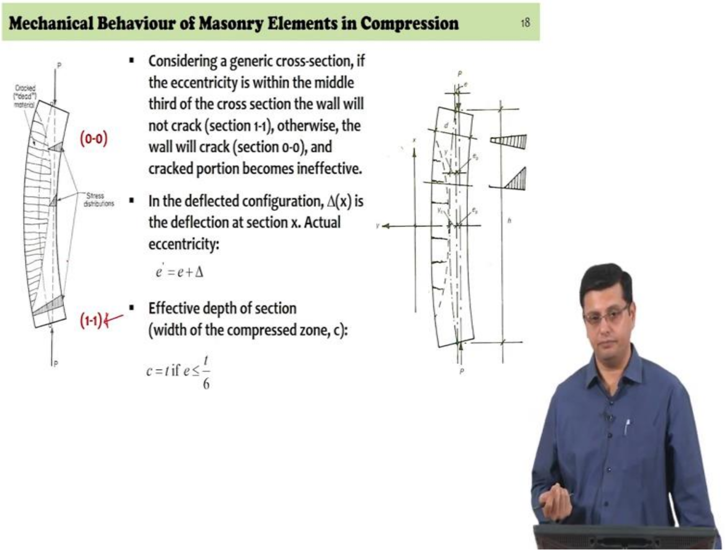

So, now we need to assume we need to start examining the cracked zone and the un-crack zoned and look at developing expressions differently for the cracked zone and the uncracked zone. So, let us examine one of the sections along the height of the wall itself. You look at a generic cross section. So, if the eccentricity is going to be within the middle third of the cross section, we are looking at section 1-1, if the eccentricity within the middle third the entire cross sections in compression. So, that is the uncracked region of the wall itself. However, in regions of the wall particularly the middle portions of the wall, the eccentricity is now going to be beyond the middle third of the cross section and so, you will have cracked conditions.

So, section 0-0 or other sections between section 1-1 and the 0-0 should possibly have the situation of cracking. So, you have got cracked sections defined and the uncracked sections defined and we are going to be assuming that the cracked zone is not going to participate in the load equilibrium itself ok. We are looking at sections along x, along the height x, delta x is the deflection at any specific section x that you are looking at. So, there is a total eccentricity- one is contributed by the eccentricity of the load, the other is contributed by the deflection of the wall itself, total eccentricity e'= e + Δ.

And now since we have assumed that part of the wall section is going to be cracked and part is uncracked, we are interested looking at what is the width of the compressed zone because you can use only the width of the compressed zone in the equilibrium equations. So, let us assume that the width of the compressed zone is c and it varies for the uncracked zone and the cracked zone. So, the total thickness is t the compress zone the width of the compress zone is equal to t if the eccentricity is within the middle one - third of the cross section. So, if the eccentricity is less than or equal to t/6 then c = t, right.

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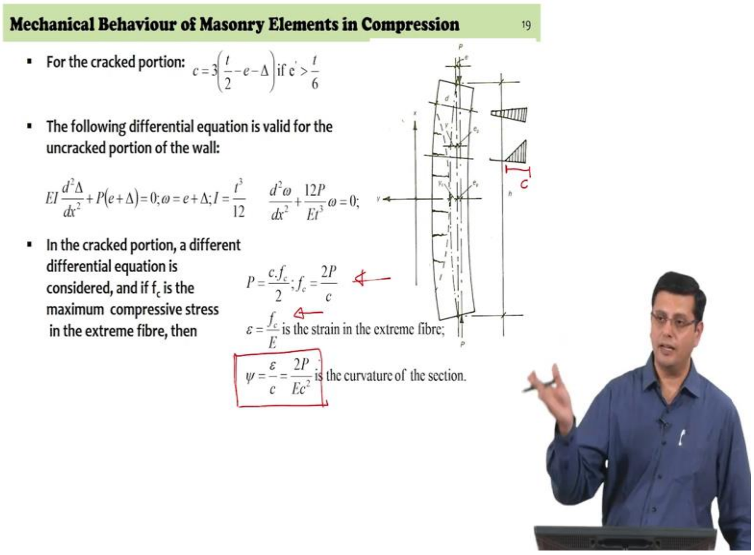

If eccentricity is greater than; if eccentricity is greater than t/6 then we have to then estimate what is going to be c which is less than t. Now for the cracked portion we look at a trianglur distribution the width of the compressed zone is c, the total length is t and we want to estimate what is this width of the cracked zone in case the eccentricity is greater than t/6.

So, we looking at a triangular distribution and the total width of the cracked zone is going to be based on a triangular distribution.

$$c = 3\left(\frac{t}{2} - e - \Delta\right) \text{ if } e > \frac{t}{6}$$

So, we have established the width of the compression zone for the two situations. Now as I said we need to established differential equations that are different for the two cracked and uncracked segments of the wall itself.

So, for the uncracked portion of the wall for the uncracked portion of the wall; the total eccentricity is the eccentricity of the load plus the deflection at that section. So, we write the differential equation,

$$EI\frac{d^2\Delta}{dx^2} + P(e+\Delta) = 0$$

$$\text{Let, } \omega = e+\Delta;\ I = \frac{t^3}{12}$$

$$\text{Then, } \frac{d^2\omega}{dx^2} + \frac{12P\omega}{Et^3} = 0$$

So, that the differential equation from the first expression taken forward and for the un-cracked portion in the cracked section you have a triangular distribution we are assuming that the material remains linear elastic it is a triangular distribution of stresses and the resultant is actually going to be acting at the centroid of that triangular distribution one-third. So, it is written in terms of the compressive stress now the compressive stress in the extreme fibre is considered as f_c.

$$P = \frac{c \cdot f_c}{2} \Rightarrow f_c = \frac{2P}{c}$$

Now, knowing the load knowing the compressive stress the strain in the cross section is defined merely in terms of the distress edge stress f_c by the modulus of elasticity of the material and having defined the stress the width the compressed zone and the strain the curvature in the section can be defined in terms of curvature of that section can be defined in terms of strain over the compress length.

$$\text{Strain}, \varepsilon = \frac{f_c}{E}$$

$$\text{Curvature}, \psi = \frac{\varepsilon}{c} = \frac{2P}{Ec^2}$$

So, now with the curvature established this is essential because we know that the curvature is going to be different in the cracked zone and the curvature is going to be different in the uncracked zone ok.

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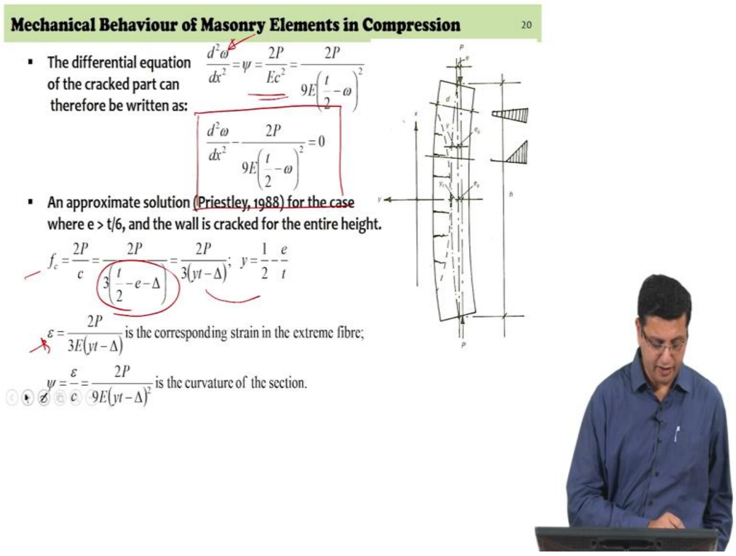

The curvature therefore, in the cracked zone is now written in terms of the curvature itself. So, we defined the total displacement eccentricity plus delta as ω. So, the second derivative of the displacement as the curvature with now the expression for curvature derived as we saw in the previous slide is going to help us write down the differential equation for the cracked zone of the wall right.

$$\frac{d^2\omega}{dx^2} = \psi = \frac{2P}{c^2} = \frac{2P}{9E\left(\frac{t}{2} - \omega\right)^2}$$

$$\frac{d^2\omega}{dx^2} - \frac{2P}{9E\left(\frac{t}{2} - \omega\right)^2} = 0$$

So, with the two differential equations available we can now proceed to work towards a force displacement relationship now ok. The fundamental problem that this is rigorous, because your curvature can actually be different at different sections can some simplification be useful. So, what is done here is we assume that the wall is cracked for the full height ok. This is an assumption of course, the wall is not cracked for the full height, but this would be a conservative estimate and this basically helps us to keep the curvature same along the entire height of the wall ok, that can help us work towards a single closed form solution for the situation itself.

$$f_c = \frac{2P}{c} = \frac{2P}{3\left(\frac{t}{2} - e - \Delta\right)} = \frac{2P}{3(yt - \Delta)};$$

$$\text{where } y = \frac{1}{2} - \frac{e}{t}$$

$$\varepsilon = \frac{2P}{3E(yt - \Delta)}; \Psi = \frac{2P}{9E(yt - \Delta)^2}$$

Double integration of that curvature can give us the displacements.

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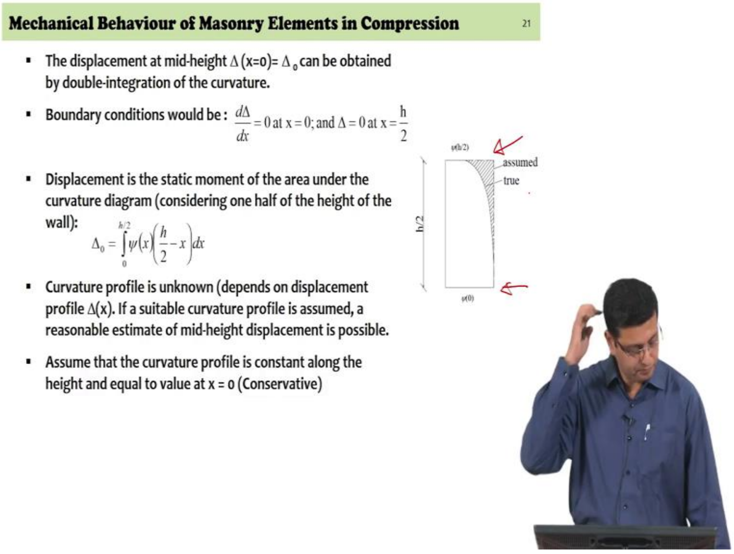

So, now in this particular case we know that the mid height displacement is maximum, delta naught maximum is at an x is equal to 0 you saw that the x that the origin is at the mid height of the wall and with the expression that we have developed for the curvature we can actually estimate what the displacements are at different height. The boundary conditions in this case if you were to have the curvature for all heights defined the slope of the displacement would be 0 at x is equal to 0 and looking at one half of the height of the wall itself and the displacement is going to be 0 at x is equal to h by 2.

So, you have boundary conditions and you can actually then be able to arrive at what is the displacement at mid height by double integration of the expression. However, this curvature profile is something that is not known to us because the cracking is different at different stages this complication can be overcome if we assume that the curvature profile is uniform throughout the cross section. But the fact is this is still conservative an estimate, conservative with respect to the exact solution of the problem itself.

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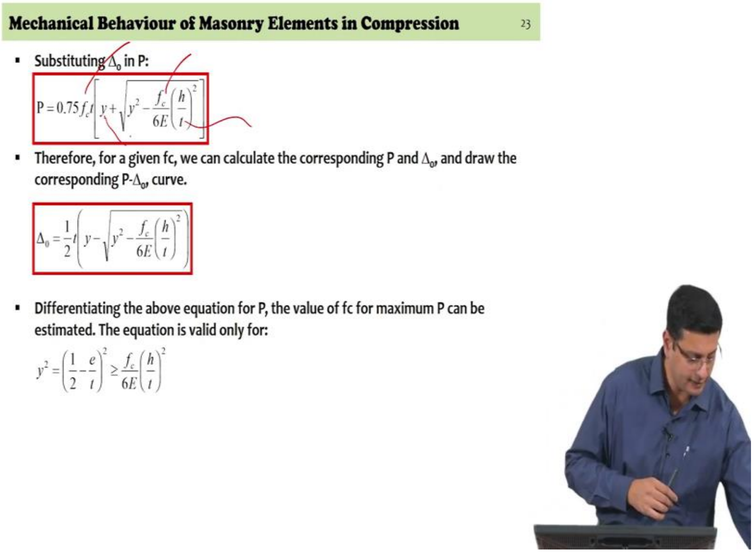

So, if we were to assume the curvature along the height being constant then the double integration becomes simpler you get from the expression for the curvature at any section plug in the values solve it and plug in the values and you get an expression in P. So, you get with this simplification you are able to pull out an expression in P you need a load displacement you need a load displacement finally, to be able to draw the effect of eccentricity of the load itself.

So, simplifying this expression by assumption of the curvature being constant along the height you get an expression in P and earlier we have actually written down that expression for f_c in P. So, we have 2 ways in which the axial load can be written, we have pulled out one from the integration of the curvature profile and the other one which was written down earlier with respect to the edge compressive stress. So, these 2 expressions you could, use these 2 expressions together to get an expression for delta, I need 2 independent expressions one for the axial load and the other for displacement at mid height.

So, you use the terms coming from the expression that you see there and the 2 ways in which the axial load has been derived use the 2 and get to get a quadratic expression and get the solution of the quadratic expression for mid height displacement Δ_0. So, now, you have an expression for P and an expression for Δ_0 and together you will be able to look at the force displacement relationship in the case of the wall with the dimensions as we started with.

So, if you substitute the expression for delta naught back into P you get the expression for the force P as you see here and for a what you can basically do you see that this is an expression in f_c in which is the edge compressive stress and the geometry of the wall and it also has the modulus of elasticity. So, for a given value of edge compressive stress, if you know that the edge compressive stress f_c is the stress that which the material is going to fail feel. So, if f_c is the compressive strength of the material for a given compressive strength of the material f_c you can estimate what is the load that wall can carry.

So, the final expression has P as a function of f_c, P as a function of f_c so, you know the compressive strength of the material, but the compressive strength of the material is not the strength of the wall because of the second order effects. So, knowing the geometry of the wall and therefore, you can see that there is h/t that comes into the picture directly that is the slenderness effect and y is the one that takes into account the eccentricity. So, the eccentricity effect eccentricity ratio and the slenderness effect together comes to reduce the the compression capacity, compression strength of the wall itself.

So, this expression and the earlier expression for delta can help you draw a force displacement relationship knowing the compressive strength of the masonry, height of the wall thickness of the wall modulus of elasticity and the eccentricity of the loading itself. However, if you look at the closed form solution and the equation is actually valid only for a certain range of y or the eccentricity ratios. So, this is basically an exercise that looks at arriving at the capacity in compression of a masonry wall considering two important effects the eccentricity ratio and the slenderness ratio of the wall itself.